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A COMMON FIXED POINT OF ISHIKAWA ITERATION WITH ERRORS FOR TWO QUASI-NONEXPANSIVE MULTI-VALUED MAPS IN BANACH SPACES

(COMMUNICATED BY TAKEAKI YAMAZAKI)

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Abstract. In this paper, we introduce a new two-step iterative scheme with errors for finding a common fixed points of two quasi-nonexpansive multi-valued maps in Banach spaces. We prove a strong convergence theorem of the purposed algorithm under some control conditions. The results obtained in this paper improve and extend the corresponding one announced by Shahzad and Zegeye [N. Shahzad, H. Zegeye, On Mann and Ishikawa iteration schemes for multi-valued maps in Banach spaces, Nonlinear Analysis 71 (2009) 838-844.].

1. Introduction

Let D be a nonempty convex subset of a Banach space E. The set D is called *proximinal* if for each $x \in E$, there exists an element $y \in D$ such that $\|x - y\| = d(x, D)$, where $d(x, D) = \inf\{\|x - z\| : z \in D\}$. Let $CB(D), K(D)$ and $P(D)$ denote the families of nonempty closed bounded subsets, nonempty compact subsets, and nonempty proximinal bounded subsets of D, respectively. The *Hausdorff metric* on $CB(D)$ is defined by

$$H(A, B) = \max\left\{\sup_{x\in A} d(x, B),\ \sup_{y\in B} d(y, A)\right\}$$

for $A, B \in CB(D)$. A single-valued map $T : D \to D$ is called *nonexpansive* if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in D$. A multi-valued map $T : D \to CB(D)$ is said to be *nonexpansive* if $H(Tx, Ty) \leq \|x - y\|$ for all $x, y \in D$. An element $p \in D$ is called a fixed point of $T : D \to D$ (respectively, $T : D \to CB(D)$) if $p = Tp$ (respectively, $p \in Tp$). The set of fixed points of T is denoted by $F(T)$.

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The mapping $T : D \to CB(D)$ is called
(i) *quasi-nonexpansive*[13] if $F(T) \neq \emptyset$ and $H(Tx, Tp) \leq \|x - p\|$ for all $x \in D$ and all $p \in F(T)$;
(ii) *L-Lipschitzian* if there exists a constant $L > 0$ such that $H(Tx, Ty) \leq L\|x - y\|$ for all $x, y \in D$;
(iii) *hemicompact* if, for any sequence $\{x_n\}$ in D such that $d(x_n, Tx_n) \to 0$ as $n \to \infty$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \to p \in D$. We note that if D is compact, then every multi-valued mapping $T : D \to CB(D)$ is *hemicompact*.

It is clear that every nonexpansive multi-valued map T with $F(T) \neq \emptyset$ is quasi-nonexpansive. But there exist quasi-nonexpansive mappings that are not nonexpansive, see [12]. It is known that if T is a quasi-nonexpansive multi-valued map, then $F(T)$ is closed.

A multi-valued map $T : D \to CB(D)$ is said to satisfy *Condition (I)* if there is a nondecreasing function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, $f(r) > 0$ for $r \in (0, \infty)$ such that $d(x, Tx) \geq f(d(x, F(T)))$ for all $x \in D$.

Two multi-valued maps $S, T : D \to CB(D)$ are said to satisfy *Condition (II)* if there is a nondecreasing function $f : [0, \infty) \to [0, \infty)$ with $f(0) = 0$, $f(r) > 0$ for $r \in (0, \infty)$ such that either $d(x, Sx) \geq f(d(x, F(S) \cap F(T)))$ or $d(x, Tx) \geq f(d(x, F(S) \cap F(T)))$ for all $x \in D$.

In 1953, Mann [6] introduced the following iterative procedure to approximate a fixed point of a nonexpansive mapping T in a Hilbert space H:

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)Tx_n, \quad \forall n \in \mathbb{N}, \tag{1.1}$$

where the initial point x_0 is taken in C arbitrarily and $\{\alpha_n\}$ is a sequence in [0,1].

However, we note that Mann's iteration process (1.1) has only weak convergence, in general; for instance, see [1, 3, 9].

In 2005, Sastry and Babu [10] proved that the Mann and Ishikawa iteration schemes for multi-valued map T with a fixed point p converge to a fixed point q of T under certain conditions. They also claimed that the fixed point q may be different from p. More precisely, they proved the following result for nonexpansive multi-valued map with compact domain.

In 2007, Panyanak [8] extended the above result of Sastry and Babu [10] to uniformly convex Banach spaces but the domain of T remains compact.

Later, Song and Wang [14] noted that there was a gap in the proofs of Theorem 3.1(see [8]) and Theorem 5 (see [12]). They further solved/revised the gap and also gave the affirmative answer to Panyanak [8] question using the following Ishikawa iteration scheme. In the main results, domain of T is still compact, which is a strong condition (see [14], Theorem 1) and T satisfies condition(I) (see [14], Theorem 1).

In 2009, Shahzad and Zegeye [10] extended and improved the results of Panyanak [8], Sastry and Babu [12] and Song and Wang [14] to quasi-nonexpansive multi-valued maps. They also relaxed compactness of the domain of T and constructed an iteration scheme which removes the restriction of T namely $Tp = \{p\}$ for any $p \in F(T)$. The results provided an affirmative answer to Panyanak [8] question in a more general setting. They introduced a new iteration as follows:
Let D be a nonempty convex subset of a Banach space E and $\alpha_n, \alpha'_n \in [0, 1]$. The

sequence of Ishikawa iterates is defined by $x_0 \in D$,

$$\begin{aligned} y_n &= \alpha_n' z_n' + (1-\alpha_n')x_n, \quad n \geq 0, \\ x_{n+1} &= \alpha_n z_n + (1-\alpha_n)x_n, \quad n \geq 0, \end{aligned}$$

where T is a quasi-nonexpansive multi-valued map, $z_n' \in Tx_n$ and $z_n \in Ty_n$.

Since 2003, the iterative schemes with errors for a single-valued map in Banach spaces have been studied by many authors, see [2, 4, 5, 7].

Question: How can we modify Mann and Ishikawa iterative schemes with errors to obtain convergence theorems for finding a common fixed point of two multi-valued nonexpansive maps ?

Motivated by Shahzad and Zegeye [12], we purpose a new two-step iterative scheme for two multi-valued quasi-nonexpansive maps in Banach spaces and prove strong convergence theorems of the purposed iteration.

2. Main results

We use the following iteration scheme:
Let D be a nonempty convex subset of a Banach space E, $\alpha_n, \beta_n, \alpha_n', \beta_n' \in [0,1]$ and $\{u_n\}, \{v_n\}$ are bounded sequences in D.
Let T_1, T_2 be two quasi-nonexpansive multi-valued maps from D into $CB(D)$. Let $\{x_n\}$ be the sequence defined by $x_0 \in D$,

$$\begin{aligned} y_n &= \alpha_n' z_n' + \beta_n' x_n + (1-\alpha_n'-\beta_n')u_n, \quad n \geq 0, \\ x_{n+1} &= \alpha_n z_n + \beta_n x_n + (1-\alpha_n-\beta_n)v_n, \quad n \geq 0, \end{aligned} \tag{2.1}$$

where $z_n' \in T_1 x_n$ and $z_n \in T_2 y_n$;

We shall make use of the following results.

Lemma 2.1. [15] *Let $\{s_n\}, \{t_n\}$ be two nonnegative sequences satisfying*

$$s_{n+1} \leq s_n + t_n, \quad \forall n \geq 1.$$

If $\sum_{n=1}^{\infty} t_n < \infty$ then $\lim_{n\to\infty} s_n$ exists.

Lemma 2.2. [11] *Suppose that E is a uniformly convex Banach space and $0 < p \leq t_n \leq q < 1$ for all positive integers n. Also suppose that $\{x_n\}$ and $\{y_n\}$ are two sequences of E such that $\limsup_{n\to\infty} \|x_n\| \leq r$, $\limsup_{n\to\infty} \|y_n\| \leq r$ and $\lim_{n\to\infty} \|t_n x_n + (1-t_n)y_n\| = r$ hold for some $r \geq 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.*

Theorem 2.3. *Let E be a uniformly convex Banach space, D a nonempty, closed and convex subset of E. Let T_1 be a quasi-nonexpansive multi-valued map and T_2 a quasi-nonexpansive and L-Lipschitzian multi-valued map from D into $CB(D)$ with $F(T_1) \cap F(T_2) \neq \emptyset$ and $T_1 p = \{p\} = T_2 p$ for all $p \in F(T_1) \cap F(T_2)$. Assume that*
(i) *$\{T_1, T_2\}$ satisfies condition (II);*
(ii) *$\sum_{n=1}^{\infty}(1-\alpha_n-\beta_n) < \infty$ and $\sum_{n=1}^{\infty}(1-\alpha_n'-\beta_n') < \infty$;*
(iii) *$0 < \ell \leq \alpha_n, \alpha_n' \leq k < 1$.*
Then the sequence $\{x_n\}$ generated by (2.1) converges strongly to an element of $F(T_1) \cap F(T_2)$.

Proof. We split the proof into three steps.

Step 1. Show that $\lim_{n\to\infty} \|x_n - p\|$ exists for all $p \in F(T_1) \cap F(T_2)$.

Let $p \in F(T_1) \cap F(T_2)$. Since u_n, v_n are bounded, therefore exists $M > 0$ such that $\max\{\sup_{n\in\mathbb{N}} \|u_n - p\|, \sup_{n\in\mathbb{N}} \|v_n - p\|\} \leq M$. Then

$$\begin{aligned} \|y_n - p\| &\leq \alpha'_n \|z'_n - p\| + \beta'_n \|x_n - p\| + (1 - \alpha'_n - \beta'_n)\|u_n - p\| \\ &\leq \alpha'_n d(z'_n, T_1 p) + \beta'_n \|x_n - p\| + (1 - \alpha'_n - \beta'_n)M \\ &\leq \alpha'_n H(T_1 x_n, T_1 p) + \beta'_n \|x_n - p\| + (1 - \alpha'_n - \beta'_n)M \\ &\leq (\alpha'_n + \beta'_n)\|x_n - p\| + (1 - \alpha'_n - \beta'_n)M \\ &\leq \|x_n - p\| + (1 - \alpha'_n - \beta'_n)M. \end{aligned} \tag{2.2}$$

It follows that

$$\begin{aligned} \|x_{n+1} - p\| &\leq \alpha_n \|z_n - p\| + \beta_n \|x_n - p\| + (1 - \alpha_n - \beta_n)\|v_n - p\| \\ &= \alpha_n d(z_n, T_2 p) + \beta_n \|x_n - p\| + (1 - \alpha_n - \beta_n)M \\ &\leq \alpha_n H(T_2 y_n, T_2 p) + \beta_n \|x_n - p\| + (1 - \alpha_n - \beta_n)M \\ &\leq \alpha_n \|y_n - p\| + \beta_n \|x_n - p\| + (1 - \alpha_n - \beta_n)M \\ &\leq \alpha_n \big(\|x_n - p\| + (1 - \alpha'_n - \beta'_n)M\big) + \beta_n \|x_n - p\| \\ &\quad + (1 - \alpha_n - \beta_n)M \\ &= (\alpha_n + \beta_n)\|x_n - p\| + \big(\alpha_n(1 - \alpha'_n - \beta'_n) + (1 - \alpha_n - \beta_n)\big)M \\ &\leq \|x_n - p\| + \big(\alpha_n(1 - \alpha'_n - \beta'_n) + (1 - \alpha_n - \beta_n)\big)M \\ &= \|x_n - p\| + \varepsilon_n, \end{aligned} \tag{2.3}$$

where $\varepsilon_n = \big(\alpha_n(1-\alpha'_n-\beta'_n)+(1-\alpha_n-\beta_n)\big)M$. By (ii), we have $\varepsilon_n \to 0$ as $n \to \infty$. Thus by Lemma 2.1, we have $\lim_{n\to\infty} \|x_n - p\|$ exists for all $p \in F(T_1) \cap F(T_2)$.

Step 2. Show that $\lim_{n\to\infty} \|z_n - x_n\| = 0 = \lim_{n\to\infty} \|z'_n - x_n\|$.

Let $p \in F(T_1) \cap F(T_2)$. By Step 1, there is a real number $c > 0$ such that $\lim_{n\to\infty} \|x_n - p\| = c$. Let $S = \max\{\sup_{n\in\mathbb{N}} \|v_n - y_n\|, \sup_{n\in\mathbb{N}} \|u_n - x_n\|\}$. From 2.2, we get

$$\limsup_{n\to\infty} \|y_n - p\| \leq c. \tag{2.4}$$

Next, we consider

$$\begin{aligned} \|z_n - p + (1 - \alpha_n - \beta_n)(v_n - x_n)\| &\leq \|z_n - p\| + (1 - \alpha_n - \beta_n)\|v_n - x_n\| \\ &\leq d(z_n, T_2 p) + (1 - \alpha_n - \beta_n)S \\ &\leq H(T_2 y_n, T_2 p) + (1 - \alpha_n - \beta_n)S \\ &\leq \|y_n - p\| + (1 - \alpha_n - \beta_n)S \end{aligned}$$

It follows that

$$\limsup_{n\to\infty} \|z_n - p + (1 - \alpha_n - \beta_n)(v_n - x_n)\| \leq c.$$

Also

$$\begin{aligned} \|x_n - p + (1 - \alpha_n - \beta_n)(v_n - x_n)\| &\leq \|x_n - p\| + (1 - \alpha_n - \beta_n)\|v_n - x_n\| \\ &\leq \|x_n - p\| + (1 - \alpha_n - \beta_n)S \end{aligned}$$

which implies that

$$\limsup_{n\to\infty} \|x_n - p + (1 - \alpha_n - \beta_n)(v_n - x_n)\| \leq c.$$

Since

$$\lim_{n\to\infty} \| \alpha_n\big(z_n - p + (1-\alpha_n-\beta_n)(v_n - x_n)\big) + (1-\alpha_n)\big(x_n - p + (1-\alpha_n-\beta_n)(v_n - x_n)\big)\| = \lim_{n\to\infty}\|x_{n+1}-p\| = c.$$

By Lemma 2.2, we obtain that

$$\lim_{n\to\infty}\|z_n - x_n\| = 0. \tag{2.5}$$

By the nonexpansiveness of T_2, we have

$$\begin{aligned} \|x_n - p\| &\leq \|x_n - z_n\| + \|z_n - p\| \\ &= \|x_n - z_n\| + d(z_n, T_2 p) \\ &\leq \|x_n - z_n\| + H(T_2 y_n, T_2 p) \\ &\leq \|x_n - z_n\| + \|y_n - p\| \end{aligned}$$

which implies

$$c \leq \liminf_{n\to\infty}\|y_n - p\| \leq \limsup_{n\to\infty}\|y_n - p\| \leq c.$$

Hence $\lim_{n\to\infty}\|y_n - p\| = c$. Since

$$\begin{aligned} y_n - p &= \alpha_n'\big(z_n' - p + (1-\alpha_n'-\beta_n')(u_n - x_n)\big) \\ &\quad + (1-\alpha_n')\big(x_n - p + (1-\alpha_n'-\beta_n')(u_n - x_n)\big), \end{aligned}$$

we have

$$\begin{aligned} \lim_{n\to\infty} \| &\alpha_n'\big(z_n' - p + (1-\alpha_n'-\beta_n')(u_n - x_n)\big) \\ + &(1-\alpha_n')\big(x_n - p + (1-\alpha_n'-\beta_n')(u_n - x_n)\big)\| = c. \end{aligned}$$

Moreover, we get

$$\begin{aligned} \|z_n' - p + (1-\alpha_n'-\beta_n')(u_n - x_n)\| &\leq \|z_n' - p\| + (1-\alpha_n'-\beta_n')\|u_n - x_n\| \\ &\leq d(z_n', T_1 p) + (1-\alpha_n'-\beta_n')S \\ &\leq H(T_1 x_n, T_1 p) + (1-\alpha_n'-\beta_n')S \\ &\leq \|x_n - p\| + (1-\alpha_n'-\beta_n')S. \end{aligned}$$

This yields that

$$\limsup_{n\to\infty}\|z_n' - p + (1-\alpha_n'-\beta_n')(u_n - x_n)\| \leq c.$$

Also

$$\begin{aligned} \|x_n - p + (1-\alpha_n'-\beta_n')(u_n - x_n)\| &\leq \|x_n - p\| + (1-\alpha_n'-\beta_n')\|u_n - x_n\| \\ &\leq \|x_n - p\| + (1-\alpha_n'-\beta_n')S. \end{aligned}$$

This implies that

$$\limsup_{n\to\infty}\|x_n - p + (1-\alpha_n'-\beta_n')(u_n - x_n)\| \leq c.$$

Again by Lemma 2.2, we have

$$\lim_{n\to\infty}\|z_n' - x_n\| = 0. \tag{2.6}$$

Step 3. Show that $\{x_n\}$ converges strongly to q for some $q \in F(T_1)\cap F(T_2)$

From Step 2, we know that $\lim_{n\to\infty}\|z_n - x_n\| = 0 = \lim_{n\to\infty}\|z'_n - x_n\|$. Also $d(x_n, T_1x_n) \leq \|z'_n - x_n\| \to 0$ as $n \to \infty$. Since $\{x_n\}, \{u_n\}$ are bounded, so is $\{u_n - z'_n\}$. Now, let $K = \sup_{n\in\mathbb{N}}\|u_n - z'_n\|$. By assumption and (2.6), we get

$$\begin{aligned}\|y_n - z'_n\| &\leq \|\alpha'_n z'_n + \beta'_n x_n + (1-\alpha'_n-\beta'_n)u_n - z'_n\| \\ &\leq \beta'_n\|x_n - z'_n\| + (1-\alpha'_n-\beta'_n)\|u_n - z'_n\| \\ &\leq \beta'_n\|x_n - z'_n\| + (1-\alpha'_n-\beta'_n)K \\ &\to 0 \end{aligned} \tag{2.7}$$

as $n \to \infty$. It follows from (2.6) and (2.7) that

$$\begin{aligned}\|y_n - x_n\| &\leq \|y_n - z'_n\| + \|z'_n - x_n\| \\ &\to 0\end{aligned} \tag{2.8}$$

as $n \to \infty$. It follows from (2.5) and (2.8) that

$$\begin{aligned}d(x_n, T_2x_n) &\leq d(x_n, T_2y_n) + H(T_2y_n, T_2x_n) \\ &\leq \|x_n - z_n\| + L\|y_n - x_n\| \\ &\to 0.\end{aligned}$$

Since that T_1, T_2 satisfy the condition (II), we have $d(x_n, F(T_1)\cap F(T_2)) \to 0$. Thus there is a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ and a sequence $\{p_k\} \subset F(T_1)\cap F(T_2)$ such that

$$\|x_{n_k} - p_k\| < \frac{1}{2^k} \tag{2.9}$$

for all k. From (2.3), we obtain

$$\begin{aligned}\|x_{n_{k+1}} - p\| &\leq \|x_{n_{k+1}-1} - p\| + \varepsilon_{n_{k+1}-1} \\ &\leq \|x_{n_{k+1}-2} - p\| + \varepsilon_{n_{k+1}-2} + \varepsilon_{n_{k+1}-1} \\ &\vdots \\ &\leq \|x_{n_k} - p\| + \sum_{i=0}^{n_{k+1}-n_k-1}\varepsilon_{n_k+i}\end{aligned}$$

for all $p \in F(T_1)\cap F(T_2)$. This implies that

$$\|x_{n_{k+1}} - p_k\| \leq \|x_{n_k} - p_k\| + \sum_{i=0}^{n_{k+1}-n_k-1}\varepsilon_{n_k+i} < \frac{1}{2^k} + \sum_{i=0}^{n_{k+1}-n_k-1}\varepsilon_{n_k+i}.$$

Next, we shall show that $\{p_k\}$ is Cauchy sequence in D. Notice that

$$\begin{aligned}\|p_{k+1} - p_k\| &\leq \|p_{k+1} - x_{n_{k+1}}\| + \|x_{n_{k+1}} - p_k\| \\ &< \frac{1}{2^{k+1}} + \frac{1}{2^k} + \sum_{i=0}^{n_{k+1}-n_k-1}\varepsilon_{n_k+i} \\ &< \frac{1}{2^{k-1}} + \sum_{i=0}^{n_{k+1}-n_k-1}\varepsilon_{n_k+i}.\end{aligned}$$

This implies that $\{p_k\}$ is Cauchy sequence in D and thus converges to $q \in D$. Since

$$d(p_k, T_iq) \leq H(T_iq, T_ip_k) \leq \|q - p_k\|$$

for all $i = 1, 2$ and $p_k \to q$ as $n \to \infty$, it follows that $d(q, T_i q) = 0$ for all $i = 1, 2$ and thus $q \in F(T_1) \cap F(T_2)$. It implies by (2.9) that $\{x_{n_k}\}$ converges strongly to q. Since $\lim_{n\to\infty} \|x_n - q\|$ exists, it follows that $\{x_n\}$ converges strongly to q. This completes the proof. □

For $T_1 = T_2 = T$ and $\alpha_n + \beta_n = 1 = \alpha'_n + \beta'_n$ in Theorem 2.3, we obtain the following result.

Theorem 2.4. (See [12], Theorem 2.3) *Let E be a uniformly convex Banach space, D a nonempty, closed and convex subset of E, and $T : D \to CB(D)$ a quasi-nonexpansive multi-valued map with $F(T) \neq \emptyset$ and $Tp = \{p\}$ for each $p \in F(T)$. Let $\{x_n\}$ be the Ishikawa iterates defined by (A). Assume that T satisfies condition (I) and $\alpha_n, \alpha'_n \in [a, b] \subset (0, 1)$. Then $\{x_n\}$ converges strongly to a fixed point of T.*

The main result of this paper holds true under the assumption that $Tp = \{p\}$ for all $p \in F(T)$. This condition was introduced by Shahzad and Zegeye [12]. The following examples give an example of a nonexpansive multi-valued map T which satisfies the property that $Tp = \{p\}$ for all $p \in F(T)$ and Tx is not a singleton for all $x \notin F(T)$.

Example 1. Consider $D = [0, 1] \times [0, 1]$ with the usual norm. Define $T : D \to CB(D)$ by

$$T(x, y) = \begin{cases} \{(x, 0)\}, & x \neq 0, y = 0 \\ \{(0, y)\}, & x = 0, y \neq 0 \\ \{(x, 0), (0, y)\}, & x, y \neq 0 \\ \{(0, 0)\}, & x, y = 0. \end{cases}$$

Example 2. Consider $D = [0, 1]$ with the usual norm. Define $T : D \to CB(D)$ by

$$Tx = [\frac{x+1}{2}, 1].$$

Example 3. Consider $D = [0, 1] \times [0, 1]$ with the usual norm. Define $T : D \to CB(D)$ by

$$T(x, y) = \{x\} \times [\frac{y+1}{2}, 1].$$

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