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COMMON FIXED POINT THEOREM FOR (ϕ, ψ)-WEAK CONTRACTION IN FUZZY METRIC SPACE

(COMMUNICATED BY DENNY H. LEUNG)

K. JHA, M. ABBAS, I. BEG, R.P. PANT AND M. IMDAD

Abstract. The aim of the present paper is to establish a common fixed point theorem for sequence of self mappings under (ϕ, ψ)-weak contractions in fuzzy metric space employing the control function which generalizes and improves various well-known comparable results.

1. Introduction and Preliminaries

The study of common fixed points of mappings in a fuzzy metric space satisfying certain contractive conditions has been at the center of vigorous research activity. The concept of fuzzy sets was initiated by Zadeh [43] in 1965. With the concept of fuzzy sets, the fuzzy metric space was introduced by Kramosil and Michalek [16]. Also, Grabiec [7] proved the contraction principle in the setting of fuzzy metric space which was further generalizations of results by Subrahmanyam [38] for a pair of commuting mappings. Also, George and Veeramani [5] modified the notion of fuzzy metric spaces with the help of continuous t-norm, by generalizing the concept of probabilistic metric space to fuzzy situation. In 1999, Vasuki [39] introduced the concept of R-commutativity of mappings in fuzzy metric space. Pant [21] introduced the notion of reciprocally continuity of mappings in metric space and proved some common fixed point theorems. Mishra et al. [20] introduced the notion of compatible maps under the name of asymptotically commuting maps in fuzzy metric spaces. Singh and Jain [36] studied the notion of weakly compatibility in fuzzy metric space that was introduced by Jungck and Rhoades [14] in metric space. Also, Balasubramaniam et.al. [1] proved a fixed point theorem, which generalizes a result of Pant [21] for self mappings in fuzzy metric space. Pant and Jha [24] proved a fixed point theorem that gives an analogue of the results by Balasubramaniam et. al. [1] by obtaining a connection between the continuity and reciprocal continuity for four mappings in fuzzy metric space.

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On the other hand, Khan et.al. [15] in 1984 introduced the notion of control function and established a fixed point theorem for single self map on a metric space by altering distances between points with the use of control functions. Also, Sastry and Babu [29] proved fixed point theorem for a pair of self maps. In 2000, Sastry et.al. [31] proved a unique common fixed point theorem for four mappings by using a control function in order to alter distances between the points with open problem. Pant et.al. [22, 23] obtained an answer to the open problem of Sastry et.al. [31] by establishing a connection between continuity and reciprocal continuity of maps in the setting of control function. Also, Jha et.al. [10] proved some common fixed point results under Meir-Keeler type contractive conditions for self mappings by altering distances.

Recently, Rhoades [28] proved interesting fixed point theorems for ψ-weak contraction in complete metric space. The significance of this kind of contraction can also be derived from the fact that they are strictly relative to famous Banach's fixed point theorem and to some other significant results. Also, motivated by the results of Rhoades [28] and on the lines of Khan et.al. [15] employing the idea of altering distances, Vetro et.al. [42] extended the notion of (ϕ, ψ)-weak contraction to fuzzy metric space and proved common fixed point theorem for weakly compatible maps in fuzzy metric space. Thus, an altering distance function is a control function which alter the metric distance between two points enabling one to deal with relatively new classes of fixed point problems. But, the presence of control function creates certain difficulties in proving the existence of fixed point under contractive conditions.

The purpose of this paper is to prove a common fixed point theorem for the sequence of self mappings in fuzzy metric space using weak contractive condition by altering distances between points. Our result generalizes and improves various other similar results of fixed points. We also give an example to illustrate our main theorem.

We have used the following notions:

Definition 1.1([43]). Let X be any set. A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 1.2([32]). A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous $t-$norm if, $([0, 1], *)$ is an abelian topological monoid with unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all a, b, c, d in $[0, 1]$.

Example: $a * b = ab$, $a * b = \min \{a, b\}$.

Definition 1.3([16]). The triplet $(X, M, *)$ is called a fuzzy metric space (shortly, a FM-space) if, X is an arbitrary set, $*$ is a continuous $t-$norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions: for all x, y, z in X, $s, t > 0$,
(i) $M(x, y, 0) = 0$, $M(x, y, t) > 0$;
(ii) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
(iii) $M(x, y, t) = M(y, x, t)$,

(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(v) $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left continuous and $s, t > 0$,

In this case, M is called a fuzzy metric on X and the function $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t. Also, we consider the following condition in the fuzzy metric space $(X, M, *)$:
(vi) $lim_{t\to\infty} M(x, y, t) = 1$, for all $x, y \in X$.

It is important to note that every metric space (X, d) induces a fuzzy metric space $(X, M, *)$ where $a * b =$ a b (or $a * b =$ min $\{a, b\}$) and for all $x, y \in X$, we have $M(x, y, t) = \frac{t}{t+d(x,y)}$, for all $t > 0$, and $M(x, y, 0) = 0$, so-called the fuzzy metric space induced by the metric d and it is often referred to as the standard fuzzy metric.

Definition 1.4([16]). A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is called G-Cauchy sequence (i.e., Cauchy sequence in the sense of Grabiec [7]) if,
$lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$ for every $t > 0$ and for each $p > 0$.

A fuzzy metric space $(X, M, *)$ is complete (respectively G-complete) if, every Cauchy sequence (respectively G-sequence) in X converges in X.
Vasuki and Veeramani suggested that the definition of G-Cauchy sequence is weaker than the definition of Cauchy sequence [40].

Definition 1.5([16]). A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be convergent to x in X if, $lim_{n\to\infty} M(x_n, x, t) = 1$ for each $t > 0$.

It is noted that since $*$ is continuous, it follows from the condition (iv) of Definition (1.3) that the limit of a sequence in a fuzzy metric space is unique.

Definition 1.6([20]). Two self mappings A and S of a fuzzy metric space $(X, M, *)$ are said to be compatible or asymptotically commuting if, for all $t > 0$,
$lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ whenever $\{x_n\}$ is a sequence such that $lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = p$ for some p in X.

It is noted that mappings A and S are noncompatible maps, if there exists a sequence $\{x_n\}$ in X such that $lim_{n\to\infty} Ax_n = p = lim_{n\to\infty} Sx_n$,
but either $lim_{n\to\infty} M(ASx_n, SAx_n, t) \neq 1$ or the limit does not exist for all p in X.

Definition 1.7([36]). Two self mappings A and S of a fuzzy metric space $(X, M, *)$ are said to be weakly compatible if, they commute at coincidence points. That is, $Ax = Sx$ implies that $ASx = SAx$ for all x in X.

It is important to note that a compatible mappings in a metric space are weakly compatible but weakly compatible mappings need not be compatible [37].

Definition 1.8([15]). A control function ψ is defined as $\psi : \Re_+ \to \Re_+$ which is continuous, monotonically increasing, $\psi(2t) \leq 2\psi(t)$ and $\psi(t) = 0$ iff $t = 0$.

Definition 1.9([28]). Let A and B be self mappings on a fuzzy metric space $(X, M, *)$. The map A is called a ψ-weak contraction with respect to B if there exists a function $\psi : [0, \infty) \to [0, \infty)$ with $\psi(r) > 0$ for $r > 0$ and $\psi(0) = 0$ such that $\frac{1}{M(Ax,Ay,t)} - 1 \leq (\frac{1}{M(Bx,By,t)} - 1) - \psi(\frac{1}{M(Bx,By,t)} - 1)$

holds for every $x, y \in X$ and each $t > 0$. If the map B is the identity map, then the map A is called a ψ-weak contraction.

Now, adopting the notion of altering distance function, Vitro *et.al.* [42] introduced the following notion of (ϕ, ψ)-weak contraction in fuzzy metric spaces.

Definition 1.10([42]) Let $(X, M, *)$ be a fuzzy metric space and A, B be self mappings on X. Then, the map A is called a (ϕ, ψ)-weak contraction with respect to B if there exists a function $\psi : [0, \infty) \to [0, \infty)$ with $\psi(r) > 0$ for $r > 0$, $\psi(0) = 0$ and an altering distance function ϕ such that

$$\phi(\frac{1}{M(Ax,Ay,t)} - 1) \leq \phi(\frac{1}{M(Bx,By,t)} - 1) - \psi(\frac{1}{M(Bx,By,t)} - 1)$$

holds for every $x, y \in X$ and each $t > 0$. If the map B is the identity map, then the map A is called a (ϕ, ψ)-weak contraction.

If $\{A_i\}, i = 1, 2, 3, ..., S$ and T are self mappings of fuzzy metric space $(X, M, *)$ in the sequel, we shall denote

$$M_{1i}(x, y, t) = min\{M(A_1x, Sx, t), M(A_iy, Ty, t), M(Sx, Ty, t),$$

$$M(A_1x, Ty, t), M(Sx, A_iy, t)\},$$

for all $x, y \in X$, and all $t > 0$.

Definition 1.11([42]). Let $(X, M, *)$ be a fuzzy metric space and $\{A_i\}, i = 1, 2, 3, ..., S$ and T be self mappings on X. Then, the pair $\{A_1, A_k\}$ is called a (ϕ, ψ)-weak contraction with respect to $\{S, T\}$ if there exists a function $\psi : [0, \infty) \to [0, \infty)$ with $\psi(r) > 0$ for $r > 0$, $\psi(0) = 0$ and an altering distance function ϕ such that for some $k > 1$,

$$\phi(\frac{1}{M(A_1x,A_ky,t)} - 1) \leq \phi(\frac{1}{M(A_1x,A_ky,t)} - 1) - \psi(\frac{1}{M(A_1x,A_ky,t)} - 1) \ (1)$$

holds for every $x, y \in X$ and each $t > 0$.

In particular, if $A = A_1 = A_k$ for some $k > 1$, then the map A is called a generalized (ϕ, ψ)-weak contraction with respect to $\{S, T\}$. Also, if $S = T$, then the pair $\{A_1, A_k\}$ is generalized (ϕ, ψ)-weak contraction with respect to $\{S\}$. If $A = A_1 = A_k$ for some $k > 1$ and $S = T = I$, an identity map, then the map A is called a (ϕ, ψ)-weak contraction

Lemma 12([2]) Let $(X, M, *)$ be a fuzzy metric space. If there exists $k \in (0, 1]$ such that $M(x, y, kt) \geq M(x, y, t)$, then we have $x = y$.

2. Main Result.

In this section, we prove the main result related to common fixed point theorem for (ϕ,ψ)-weak contraction in fuzzy metric space.

Theorem 2.1. Let $(X, M, *)$ be a fuzzy metric space. Let $\{A_i\}, i = 1, 2, 3, ..., S$ and T be mappings of a fuzzy metric space from X into itself such that
(i) $A_1X \subseteq TX$, $A_iX \subseteq SX$, for $i > 1$, and
(ii) for a function $\psi : [0,\infty) \to [0,\infty)$ with $\psi(r) > 0$ for $r > 0$, $\psi(0) = 0$ and an altering distance function ϕ such that for $i > 1$, the relation

$$\phi(\frac{1}{M(A_1x,A_iy,t)} - 1) \leq \phi(\frac{1}{M_{1i}(x,y,t)} - 1) - \psi(\frac{1}{M_{1i}(x,y,t)} - 1)$$

holds for every $x, y \in X$ and each $t > 0$.

If one of A_iX, SX and TX is a G-complete subspace of X; if the pair (A_1, S) and (A_i, T), for $i > 1$, are weakly compatible,then all the mappings A_i, S and T have a unique common fixed point in X.

Proof. Let x_0 be any point in X. Then, using condition (i), we define sequences $\{x_n\}$ and $\{y_n\}$ in X such that $y_{2n} = A_1x_{2n} = Tx_{2n+1}$;
and $y_{2n+1} = A_kx_{2n+1} = Sx_{2n+2}$, for some $k > 1$. We claim that $\{y_n\}$ is a Cauchy sequence.

If $y_{2n} = y_{2n+1}$ for some n. Then, using condition (ii) , we get $y_{2n+1} = y_{2n+2}$ and so $y_m = y_{2n}$ for each $m > 2n$. Thus, the sequence $\{y_n\}$ is a G-Cauchy sequence. Assume that $y_n \neq y_{n+1}$ for all n. Then, for some $k > 1$, setting $x = x_{2n}$ and $y = x_{2n-1}$ in condition (ii), we get

$$M_{1k}(x_{2n}, x_{2n-1}, t) = min\{M(A_1x_{2n}, Sx_{2n}, t), M(A_kx_{2n-1}, Tx_{2n-1}, t),$$

$$M(Sx_{2n}, Tx_{2n-1}, t), M(A_1x_{2n}, Tx_{2n-1}, t), M(Sx_{2n}, A_kx_{2n-1}, t)\},$$

and so,

$$\phi(\frac{1}{M(A_1x_{2n},A_kx_{2n-1},t)} - 1) \leq \phi(\frac{1}{M_{1k}(x_{2n},x_{2n-1},t)} - 1) - \psi(\frac{1}{M_{1k}(x_{2n},x_{2n-1},t)} - 1).$$

This implies

$$\phi(\frac{1}{M(A_1y_{2n},A_ky_{2n-1},t)} - 1) \leq \phi(\frac{1}{M_{1k}(y_{2n-1},y_{2n-2},t)} - 1) - \psi(\frac{1}{M_{1k}(y_{2n-1},y_{2n-2},t)} - 1),$$

that is, $\phi(\frac{1}{M(A_1y_{2n},A_ky_{2n-1},t)} - 1) < \phi(\frac{1}{M_{1k}(y_{2n-1},y_{2n-2},t)} - 1)$. (2)

Using the similar argument with $x = x_{2n-2}$ and $y = x_{2n-1}$ in condition (ii), we get the same inequality (2). Consequently, since the function ϕ is non-decreasing, we have that $M(y_n, y_{n-1}, t) > M(y_{n-1}, y_n, t)$ for all n and hence the sequence $\{M(y_{n-1}, y_n, t)\}$ is an increasing sequence of positive real numbers in $(0, 1]$.

Let $\gamma(t) = lim_{n\to\infty}M(y_n, y_{n+1}, t)$. Then, we show that $\gamma(t) = 1$ for all $t > 0$. If not, then there corresponds some $t > 0$ such that $\gamma(t) < 1$. So that, on making $n \to \infty$ in the above inequality (2), we get

$\phi(\frac{1}{\gamma(t)}-1) \leq \phi(\frac{1}{\gamma(t)}-1) - \psi(\frac{1}{\gamma(t)}-1)$,
which is a contraction. Therefore, we have $M(y_n, y_{n+1}, t) \to 1$ as $n \to \infty$.
So that, for each positive integer p, we have
$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, t/p) * M(y_{n+1}, y_{n+2}, t/p) * ... * M(y_{n+p-1}, y_{n+p}, t/p)$,

it follows that $lim_{n\to\infty} M(y_n, y_{n+p}, t) \geq 1 * 1 * ... * 1 = 1$.
Hence, $\{y_n\}$ is a G-Cauchy sequence.

Now, we assume that SX is a G-complete. Then, by definition, there exists $z \in SX$ such that $y_n \to z$ as $n \to \infty$. So that, we get $y_{2n} = A_1x_{2n} = Tx_{2n+1} \to z$ and $y_{2n+1} = A_kx_{2n+1} = Sx_{2n+2} \to z$, for some $k > 1$. Let $v \in X$ be such that $Sv = z$. We show that $A_1v = z$.
Suppose that $A_1v \neq Sv$. Then, for some $k > 1$ and for all $t > 0$, setting $x = v$ and $y = x_{2n+1}$ in condition (ii), we get

$M_{1k}(v, x_{2n+1}, t) = min\{M(A_1v, Sv, t), M(A_kx_{2n+1}, Tx_{2n+1}, t), M(Sv, Tx_{2n+1}, t),$

$M(A_1v, Tx_{2n+1}, t), M(Sv, A_kx_{2n+1}, t)\}$,
and so,
$\phi(\frac{1}{M(A_1v, A_kx_{2n+1}, t)}-1) \leq \phi(\frac{1}{M_{1k}(v, x_{2n+1}, t)}-1) - \psi(\frac{1}{M_{1k}(v, x_{2n+1}, t)}-1)$.

On taking $n \to \infty$, this implies
$\phi(\frac{1}{M(A_1v, z, t)}-1) < \phi(\frac{1}{M(A_1v, z, t)}-1)$, a contradiction. Therefore, we have $A_1v = z$.

Since the pair (A_1, S) is weakly compatible, so we have $Sz = SA_1v = A_1Sv = A_1z$. Now, we prove that $A_1z = z$.
If not, then for some $k > 1$ and for all $t > 0$, setting $x = z$ and $y = x_{2n+1}$ in condition (ii), we get

$M_{1k}(z, x_{2n+1}, t) = min\{M(A_1z, Sz, t), M(A_kx_{2n+1}, Tx_{2n+1}, t), M(Sz, Tx_{2n+1}, t),$

$M(A_1z, Tx_{2n+1}, t), M(Sz, A_kx_{2n+1}, t)\}$,
and so,
$\phi(\frac{1}{M(A_1z, A_kx_{2n+1}, t)}-1) \leq \phi(\frac{1}{M_{1k}(z, x_{2n+1}, t)}-1) - \psi(\frac{1}{M_{1k}(z, x_{2n+1}, t)}-1)$,

which on taking $n \to \infty$, reduces to
$\phi(\frac{1}{M(A_1z, z, t)}-1) < \phi(\frac{1}{M(A_1z, z, t)}-1)$, a contradiction.Therefore, we have $A_1z = z$.

Again, since $A_1X \subseteq TX$, so there exists some $u \in X$ such that $A_1z = Tu$.Therefore, we have $z = A_1v = Sv = Tu$. We claim that $A_ku = z$ for some $k > 1$.

If not, then for some $k > 1$ and for all $t > 0$, setting $x = z$ and $y = u$ in condition (ii), we get

$M_{1k}(z, u, t) = min\{M(A_1z, Sz, t), M(A_ku, Tu, t), M(Sz, Tu, t),$

$M(A_1z, Tu, t), M(Sz, A_ku, t)\}$,
and so,

$\phi(\frac{1}{M(A_1z,A_ku,t)}-1) \le \phi(\frac{1}{M_{1k}(z,u,t)}-1) - \psi(\frac{1}{M_{1k}(z,u,t)}-1)$,

which reduces to
$\phi(\frac{1}{M(z,A_ku,t)}-1) < \phi(\frac{1}{M(z,A_ku,t)}-1)$, a contradiction.This implies that $A_kz = z$.

Thus, we have $z = A_1v = Sv = Tu = A_ku$. By weak compatibility of the pair (A_k, T) for some $k > 1$, we get $A_kz = A_kTu = TA_ku = Tz$.
Similarly, using condition (ii) and for all $t > 0$, we can prove that $A_kz = z$. Thus, we have $z = A_1z = Sz = A_kz = Tz$ for some $k > 1$, and hence z is a common fixed point of all mappings A_i, S and T in X.

Uniqueness
The uniqueness of a common fixed point of the mappings A_i, S and T be easily verified by using condition (ii). In fact, if z' be another fixed point for mappings A_1, A_k, S and T, for some $k > 1$ and for all $t > 0$. Then, setting $x = z$ and $y = z'$ in condition (ii), we get

$M_{1k}(z, z', t) = min\{M(A_1z, Sz, t), M(A_kz', Tz', t), M(Sz, Tz', t),$

$M(A_1z, Tz', t), M(Sz, A_kz', t)\}$,
and so,
$\phi(\frac{1}{M(A_1z,A_kz',t)}-1) \le \phi(\frac{1}{M_{1k}(z,z',t)}-1) - \psi(\frac{1}{M_{1k}(z,z',t)}-1)$,

which reduces to
$\phi(\frac{1}{M(z,z',t)}-1) < \phi(\frac{1}{M(z,z',t)}-1)$, a contradiction.This implies that $z = z'$.

Similarly, instead of SX, if one of A_iX or TX is assumed to be a G-complete subspace of X, one can prove that all mappings A_i, S and T have a unique common fixed point in X.
This completely establishes the theorem.

We now give an example to illustrate the above theorems.

Example 2.1. Let $X = [2, 20]$ and M be the usual fuzzy metric space on $(X, M, *)$. Define A_i, S and $T : X \to X$ as follows:
$A_1x = 2$ for each x;

$Sx = x$ if, $x \le 8$, $Sx = 8$ if, $8 < x < 14$, $Sx = (x + 10)/3$ if, $14 \le x \le 17$ and $Sx = (x + 7)/3$ if $x > 17$;

$Tx = 2$ if, $x = 2$ or $x > 6$, $Tx = x+12$ if, $2 < x < 4$, $Tx = (x+9)/3$ if, $4 \le x < 5$ and $Tx = 8$ if, $5 \le x \le 6$;

$A_2x = 2$ if, $x < 4$ or $x > 6$, $A_2x = x+3$ if, $4 \le x < 5$, $A_2x = x+2$ if, $5 \le x \le 6$;

and for each $i > 2$,
$A_ix = 2$, if $x = 2$ or $x \ge 4$, $A_ix = (x + 30)/4$ if, $2 < x < 4$.

Also, we define $M(A_1x, A_ky, t) = \frac{t}{[t+d(x,y)]}$, for some $k > 1$, for all x, y in X and for all $t > 0$. Then, the pairs (A_1, S) and (A_k, T), for some $k > 1$, are weakly compatible mappings. Also,these mappings satisfy all the conditions of the above theorem with $\phi(t) = t$, an identity map and $\psi(t) = t/2$ and hence we have a unique common fixed point $x = 2$. However, it is important to note that the mappings A_1, A_2, S and T do not satisfy the contractive condition $M(A_1x, A_2y, kt) \geq \phi(M_{12}(x, y, t))$, where $k \in (0, 1)$ and $\phi : \Re^+ \to \Re^+$ is such that $\phi(r) > r$ for all $r > 0$.

Remarks: If, for some $k > 1$, $A_1 = T$, $A_k = S$, $T = f$ and $S = g$ in the above Theorem 1, then we get the main result of Vetro *et.al.*[42]. So, the results of Vetro *et.al.*[42] are the particular cases of our result. Also, Vetro *et.al.*[42] has defined control function without the subadditive property (please do refer Definition 2, page 574 of [42])and it is clear that this property certainly creates difficulties while establishing common fixed point results by altering distances between points. For details with examples, one can refer [10], [13], [23], [30]. In our result, we have employed this property and therefore, our result generalizes the results of Doric [3], Gregori and Sapena [6], Rhoades [28], Vetro and Vetro [41] and Vetro *et. al* [42]. Consequently, our theorem improves and unifies the results of Balasubramaniam *et. al.*[1], Chugh and Kumar [4], Imdad and Ali [8], Jha and Pant [9], Jha *et.al.*[10], Jha [11, 12], Kutukcu *et. al* [17], Mihet [18], Pant [25, 26], Sharma [33], Sharma *et. al* [34], Singh and Chauhan [35] and other similar results for fixed points.

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KANHAIYA JHA (Corresponding author), Department of Mathematical Sciences, School of Science, Kathmandu University, P.O. Box No. 6250, Kathmandu, Nepal.

E-mail address: jhaknh@yahoo.co.in; jhakn@ku.edu.np

ISMAT BEG, MUJAHID ABBAS, Department of Mathematics and Centre for Advanced Studies in Mathematics, Lahore University of Management Sciences', Lahore Pakistan - 54792.

E-mail address: ibeg@lums.edu.pk; mujahid@lums.edu.pk

RAJENDRA PRASAD PANT, Department of Mathematics, Kumaon University D.S.B. Campus, Nainital 263 002, India.

E-mail address: pant-rp@rediffmail.com

MUHAMAD IMDAD, Department of Mathematics, Aligarh Muslim University, Aligarh 202002, India.

E-mail address: mhimdad@yahoo.co.in