BULLETIN OF MATHEMATICAL ANALYSIS AND APPLICATIONS
ISSN: 1821-1291, URL: HTTP://WWW.BMATHAA.ORG
VOLUME 3 ISSUE 4(2011), PAGES 132-139.

MODIFIED NOOR ITERATIVE METHODS FOR A FAMILY OF STRONGLY PSEUDOCONTRACTIVE MAPS

(COMMUNICATED BY MOHAMMED ASLAM NOOR)

A.A. MOGBADEMU, J.O. OLALERU

ABSTRACT. We prove some convergence results using a newly introduced three-step iterative scheme with errors for three strongly pseudocontractive(accretive) mappings defined on Banach spaces. The results are generalizations of the work of several authors. In particular, they generalize the recent results of Fan and Xue (2008) which is in turn a correction of Rafiq (2006).

1. INTRODUCTION

We denote by J the normalized duality mapping from X into 2^{X^*} by

$$J(x) = \{f \in \mathrm{X}^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\}$$

where X^* denotes the dual space of X and $\langle ., .\rangle$ denotes the generalized duality pairing.

Definition 1.1 [15]. A mapping $T : X \to X$ with domain $D(T)$ and $R(T)$ in X is called strongly pseudocontractive if for all $x, y \in D(T)$, there exist $j(x-y) \in J(x-y)$ and a constant $k \in (0,1)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq k\|x-y\|^2.$$

Closely related to the class of strongly pseudocontractive operators is the important class of strongly accretive operators. It is well known that T is stronly pseudocontractive if and only if $(I-T)$ is strongly accretive, where I denotes the identity map. Therefore, an operator $T : X \to X$ is called strongly accretive if there exists a constant $k \in (0,1)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \geq k\|x-y\|^2$$

holds for all $x, y \in X$ and some $j(x-y) \in J(x-y)$. These operators have been studied and used by several authors (see, for example [2-3],[7,8,11-12,14,15]).

2000 *Mathematics Subject Classification.* 47H10, 46A03.
Key words and phrases. Three-step iteration; modified Noor iteration; strongly pseudocontractive mapping; strongly accretive operator;Banach spaces.

August 29, 2011. Published October 7, 2011.

The Mann iteration scheme [8], introduced in 1953, was used to prove the convergence of the sequence to the fixed points of mappings of which the Banach principle is not applicable. In 1974, Ishikawa [6] devised a new iteration scheme to establish the convergence of a Lipschitzian pseudocontractive map when Mann iteration process failed to converge. Noor et al.[13], gave the following three-step iteration process for solving non-linear operator equations in real Banach spaces.
Let K be a nonempty closed convex subset of X and $T : K \to K$ be a mapping. For an arbitrary $x_0 \in K$, the sequence $\{x_n\}_{n=0}^{\infty} \subset K$, defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n \\ y_n &= (1-\beta_n)x_n + \beta_n T z_n \\ z_n &= (1-\gamma_n)x_n + \gamma_n T x_n, \quad n \geq 0, \end{aligned} \tag{1.1}$$

where $\{\alpha_n\}_{n=0}^{\infty}$,$\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are three sequences satisfying $\alpha_n, \beta_n, \gamma_n \in [0,1]$ for each n, is called the three-step iteration (or the Noor iteration). When $\gamma_n = 0$, then the three-step iteration reduces to the Ishikawa iterative sequence $\{x_n\}_{n=0}^{\infty} \subset K$ defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n \\ y_n &= (1-\beta_n)x_n + \beta_n T x_n, \quad n \geq 0. \end{aligned} \tag{1.2}$$

If $\beta_n = \gamma_n = 0$, then (1.1) becomes the Mann iteration. It is the sequence $\{x_n\}_{n=0}^{\infty} \subset K$ defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, \quad n \geq 0. \tag{1.3}$$

Glowinski and Le Tallec [4] used a three-step iterative scheme to solve elastoviscoplasticity, liquid crystal and eigen-value problems. They have shown that the three-step approximation scheme performs better than the two-step and one-step iterative methods.
Haubruge et al.[5] studied the convergence analysis of three-step iterative schemes of Glowinski and Le Tallec [4] and applied these three-step iteration to obtain new splitting type algorithms for solving variational inequalities, separable convex programming and minimization of a sum of convex functions. They also proved that three-step iterations also lead to highly parallelized algorithms under certain conditions. Thus, it is clear that three-step schemes play an important part in solving various problems, which arise in pure and applied sciences.
Rafiq [15], recently introduced the following new type of iteration- the modified three-step iteration process, to approximate the unique common fixed points of a three strongly pseudocontractive mappings in Banach spaces.
Let $T_1, T_2, T_3 : K \to K$ be three mappings. For any given $x_0 \in K$, the modified three-step iteration $\{x_n\}_{n=0}^{\infty} \subset K$ is defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T_1 y_n \\ y_n &= (1-\beta_n)x_n + \beta_n T_2 z_n \\ z_n &= (1-\gamma_n)x_n + \gamma_n T_3 x_n, \quad n \geq 0 \end{aligned} \tag{1.4}$$

where $\{\alpha_n\}_{n=0}^{\infty}$,$\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are three real sequences satisfying some conditions. It is clear that the iteration schemes (1.1)-(1.3) are special cases of (1.4)
It is worth mentioning that, several authors, for example, Xue and Fan [17] recently used the iteration in equation (1.4) to approximate the common fixed points of three

pseudocontractive operators in Banach spaces. Infact,they stated and proved the corrected version of Rafiq's result [15] thus :

Theorem XF. Let X be a real Banach Space and K be a nonempty closed convex subset of X. Let T_1, T_2 and T_3 be strongly pseudocontractive self maps of K with $T_1(K)$ bounded and T_1, T_2 and T_3 uniformly continuous. Let $\{x_n\}_{n=0}^{\infty}$ be defined by (1.4),where $\{a_n\}_{n=0}^{\infty}, \{b_n\}_{n=0}^{\infty}$ and $\{c_n\}_{n=0}^{\infty}$ are three real sequences in [0,1] such that: (i) $a_n, b_n \to 0$ as $n \to \infty$ and (ii)$\sum_{n=0}^{\infty} a_n = \infty$. If $F(T_1) \cap F(T_2) \cap F(T_3) \neq \phi$, then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the common fixed point of T_1, T_2 and T_3.

This result itself is a generalization of many previous results (see[15] and the references there in).
For three mappings, it is desirable to devise a general iteration scheme which extend the Mann iteration, the Ishikawa iteration , the Noor iteration and the modified Noor iteration. We extend the modified three-step iteration (1.4) to what is called the modified three-step iteration process with errors for three strongly pseudocontractive operators defined as follows:

For $x_0, u_0, v_0, w_0 \in K$, the three step iteration sequence with errors $\{x_n\}$ is defined by

$$x_{n+1} = a_n x_n + b_n T_1 y_n + c_n u_n$$

where

$$\begin{array}{l} y_n = a'_n x_n + b'_n T_2 z_n + c'_n v_n \\ z_n = a''_n x_n + b''_n T_3 x_n + c'' w_n \quad n \geq 0, \end{array} \tag{1.5}$$

where $\{u_n\}$, $\{v_n\}$ and $\{w_n\}$ are arbitrary bounded sequences in K, $\{a_n\}$,$\{b_n\}$, $\{c_n\}$,$\{a'_n\}$,$\{b'_n\}$,$\{c'_n\}$,$\{a''_n\}$, $\{b''_n\}$ and $\{c''_n\}$ are real sequences in [0,1] satisfying certain conditions.
It may be noted that the iterative schemes (1.1)-(1.4) may be viewed as a special case of (1.5). In this paper, we use our newly introduced iteration process (1.5) and prove that it converges strongly to a unique common fixed point of a three strongly pseudocontractive mappings in Banach spaces. Thus,our results generalize the results given in Xue and Fan [17] which itself is a generalization of many of the previous results.

In order to obtain the main results, the following Lemmas are needed.
Lemma 1.1[15,16]. Let E be real Banach space and $J : E \to 2^{E^*}$ be the normalized duality mapping.Then,for any $x, y \in E$

$$\|x + y\|^2 \leq \|x\|^2 + 2 < y, j(x + y) >, \forall j(x + y) \in J(x + y)$$

Lemma 1.2[17]. Let (α_n) be a non- negative sequence which satisfies the following inequality

$$\alpha_{n+1} \leq (1 - \lambda_n)\alpha_n + \delta_n,$$

where $\lambda_n \in (0,1), \forall n \in N, \sum_{n=1}^{\infty} \lambda_n = \infty$ and $\delta_n = o(\lambda_n)$. Then $\lim_{n\to\infty} \alpha_n = 0$.

2. Results

Theorem 2.1. Let X be a real Banach space, K a nonempty closed and convex subset of X. Let T_1, T_2, T_3 be self maps of K with $T_1(K)$ bounded such that $F(T_1) \cap F(T_2) \cap F(T_3) \neq \emptyset$ and T_1, T_2 and T_3 uniformly continuous. Suppose T_1, T_2, T_3 are strongly pseudocontractive mappings. For $x_0, u_0, v_0, w_0 \in K$, the three step iteration sequence with errors $\{x_n\}$ defined by (1.5)
where $\{u_n\}$, $\{v_n\}$ and $\{w_n\}$ are arbitrary bounded sequences in K, $\{a_n\}$,$\{b_n\}$, $\{c_n\}$,$\{a'_n\}$,$\{b'_n\}$,$\{c'_n\}$,$\{a''_n\}$, $\{b''_n\}$ and $\{c''_n\}$ are real sequences in [0,1] satisfying the following conditions:

(i) $a_n + b_n + c_n = a'_n + b'_n + c'_n = a''_n + b''_n + c''_n = 1$
(ii)$b_n, b'_n, c_n, c'_n \to 0$ as $n \to \infty$
(iii) $\sum_{n=1}^{\infty} b_n = \infty$
(iv) $\lim_{n\to\infty} \frac{c_n}{b_n} = 0$,

converges strongly to the unique common fixed point of T_1, T_2 and T_3.

Proof: Let $\rho \in F(T_1) \cap F(T_2) \cap F(T_3)$. Since T_1 has bounded range, we denote
$D_1 = \|x_0 - \rho\| + \sup_{n\geq 0} \|T_1 y_n - \rho\| + \|u_n - \rho\|$. We prove by induction that $\|x_n - \rho\| \leq D_1$ for all n. It is clear that, $\|x_0 - \rho\| \leq D_1$. Assume that $\|x_n - \rho\| \leq D_1$ holds. We will prove that $\|x_{n+1} - \rho\| \leq D_1$. Indeed, from (1.5), we obtain

$$
\begin{aligned}
\|x_{n+1} - \rho\| &\leq \|(1-\delta_n)(x_n - \rho) + b_n(T_1 y_n - \rho) + c_n(u_n - \rho)\| \\
&\leq (1-\delta_n)\|x_n - \rho\| + b_n\|T_1 y_n - \rho\| + c_n\|u_n - \rho\| \\
&\leq (1-\delta_n)D_1 + b_n D_1 + c_n D_1 = D_1,
\end{aligned}
$$

where $\delta_n = (b_n + c_n)$. Hence the sequence $\{x_n\}$ is bounded.
Using the uniform continuity of T_3, we have $\{T_3 x_n\}$ is bounded.
Denote $D_2 = \max\{D_1, \sup\{\|T_3 x_n - \rho\|\}\}, \sup\{\|w_n - \rho\|\}\}$,
then

$$
\begin{aligned}
\|z_n - \rho\| &= \|a''_n(x_n - \rho) + b''_n(T_3 x_n - \rho) + c''_n(w_n - \rho)\| \\
&= \|(1-\delta''_n)(x_n - \rho) + b''_n(T_3 x_n - \rho) + c''_n(w_n - \rho)\| \\
&\leq (1-\delta''_n)\|x_n - \rho\| + b''_n\|T_3 x_n - \rho\| + c''_n\|w_n - \rho\| \\
&\leq (1-\delta''_n)D_1 + b''_n D_2 + c''_n D_2 \\
&\leq (1-\delta''_n)D_2 + b''_n D_2 + c''_n D_2 = D_2,
\end{aligned}
$$

where $\delta''_n = b''_n + c''_n$. By the virtue of the uniform continuity of T_2, we get that $\{T_2 z_n\}$ is bounded. Set $D = \sup_{n\geq 0} \|T_2 z_n - \rho\| + \sup_{n\geq 0} \|v_n - \rho\| + D_2$.
Applying Lemma 1.1 and (1.5), we have

$$
\begin{aligned}
\|x_{n+1}-\rho\|^2 &= \|(1-b_n-c_n)(x_n-\rho)+b_n(T_1y_n-\rho)+c_n(u_n-\rho)\|^2\\
&\leq (1-(b_n+c_n))^2\|x_n-\rho\|^2\\
&\quad +2\langle b_n(T_1y_n-\rho)+c_n(u_n-\rho), j(x_{n+1}-\rho)\rangle\\
&= (1-b_n)^2\|x_n-\rho\|^2+2b_n\langle T_1y_n-\rho, j(x_{n+1}-\rho)\rangle\\
&\quad +2c_n\langle u_n-\rho, j(x_{n+1}-\rho)\rangle\\
&\leq (1-b_n)^2\|x_n-\rho\|^2+2b_n\langle T_1y_n-T_1x_{n+1}, j(x_{n+1}-\rho)\rangle \qquad (2.1)\\
&\quad +2b_n\langle T_1x_{n+1}-\rho, j(x_{n+1}-\rho)\rangle+2D^2c_n\\
&\leq (1-b_n)^2\|x_n-\rho\|^2+2b_nk\|x_{n+1}-\rho\|^2\\
&\quad +2b_n\|T_1y_n-T_1x_{n+1}\|\|x_{n+1}-\rho\|+2D^2c_n\\
&\leq (1-b_n)^2\|x_n-\rho\|^2+2b_nk\|x_{n+1}-\rho\|^2\\
&\quad +2b_n\sigma_nD+2D^2c_n,
\end{aligned}
$$

where $\sigma_n=\|T_1y_n-T_1x_{n+1}\|$.

$$
\begin{aligned}
\|y_n-x_{n+1}\| &= \|(1-b'_n-c'_n)x_n+b'_nT_2z_n+c'_nv_n\\
&\quad -(1-b_n-c_n)x_n-b_nT_1y_n-c_nu_n\|\\
&= \|b'_n(T_2z_n-x_n)+c'_n(v_n-x_n)+b_n(x_n-T_1y_n)\\
&\quad +c_n(x_n-u_n)\|\\
&\leq b'\|T_2z_n-x_n\|+c'_n\|v_n-x_n\|+b_n\|x_n-T_1y_n\|\\
&\quad +c_n\|x_n-u_n\|\\
&\leq 2D(b'_n+c'_n+b_n+c_n).
\end{aligned}
$$

This implies that $\lim_{n\to\infty}\|x_{n+1}-y_n\|=0$, since $\lim_{n\to\infty}b_n=0$, $\lim_{n\to\infty}b'_n=0$, $\lim_{n\to\infty}c_n=0$,$\lim_{n\to\infty}c'_n=0$. Since T_1 is uniformly continuous, we have

$$\sigma_n=\|T_1x_{n+1}-T_1y_n\|\to 0, (\text{as} \quad n\to\infty).$$

And, since $b_n\to 0$ as $n\to\infty$, then there exist a positive integer N such that $b_n\leq\min\{\frac{1}{2k},\frac{1-k}{(1-k)^2+k^2}\}$ for all $n\geq N$. It follows from (2.1) that

$$
\begin{aligned}
\|x_{n+1}-\rho\|^2 &\leq \tfrac{(1-b_n)^2}{1-2b_nk}\|x_n-\rho\|^2+\tfrac{2b_nD\sigma_n}{1-2b_nk}+\tfrac{2D^2c_n}{1-2b_nk}\\
&\leq \tfrac{(1-b_n)^2}{1-2b_nk}\|x_n-\rho\|^2+\tfrac{2b_nD^2\sigma_n}{1-2b_nk}+\tfrac{2D^2c_n}{1-2b_nk}\\
&\leq (1-\tfrac{2-2k-b_n}{1-2b_nk})\|x_n-\rho\|^2+\tfrac{2b_nD^2}{1-2b_nk}(\sigma_n+\tfrac{c_n}{b_n})\\
&\leq (1-(1-k)b_n)\|x_n-\rho\|^2+\tfrac{2b_nD^2}{1-2b_nk}(\sigma_n+\tfrac{c_n}{b_n})
\end{aligned}
$$

Set $\alpha_n=\|x_n-\rho\|$, $\lambda_n=(1-k)b_n$ and $\delta_n=\frac{2b_nD^2}{1-2b_nk}(\sigma_n+\frac{c_n}{b_n})$. Applying Lemma 1.2, we obtain $\|x_n-\rho\|\to 0$ as $n\to\infty$. This complete the proof.

Remark 1. Theorem 2.1 extends Theorem 2.1 of Fan and Xue in the sense that, we replace modified three-step iterative scheme by a more general modified three-step iterative scheme with errors.

Theorem 2.2. Let $X, K, T_1, T_2, T_3, \{x_n\}, \{b_n\}$ and σ_n be as in Theorem 2.1. Suppose there exists a sequence $\{t_n\}$ with $\lim_{n\to\infty}t_n=0$ and $c_n=t_nb_n$ for any $n\geq 0$. Then $\{x_n\}$ converges strongly to the unique common fixed point of T_1, T_2, T_3 which is the unique fixed point of T_1.

Proof: Just as in the proof of Theorem 2.1, we have

$$\begin{aligned}\|x_{n+1}-\rho\|^2 &\leq (1-(1-k)b_n)\|x_n-\rho\|^2+\tfrac{2b_nD^2}{1-2b_nk}(\sigma_n+\tfrac{c_n}{b_n})\\ &= (1-(1-k)b_n)\|x_n-\rho\|^2+\tfrac{2b_nD^2}{1-2b_nk}(\sigma_n+\tfrac{t_nb_n}{b_n})\\ &= (1-(1-k)b_n)\|x_n-\rho\|^2+\tfrac{2b_nD^2}{1-2b_nk}(\sigma_n+t_n)\end{aligned}$$

Put $\alpha_n=\|x_n-\rho\|$, $\lambda_n=(1-k)b_n$ and $\delta_n=\frac{2b_nD^2}{1-2b_nk}(\sigma_n+t_n)$. Then, Lemma 1.2 ensures that $\|x_n-\rho\|\to 0$ as $n\to\infty$. This complete the proof.

Example Let K=$[0,\infty)$, X=$(-\infty,\infty)$ with the usual norm and let $R(T_1)=[0,\frac{1}{2})$, $R(T_2)=[0,1)$, $R(T_3)=[0,1]$.
The map $T_i: K\to K$ is given by

$$T_1x=\frac{x}{2(1+x)},\quad T_2x=\frac{x}{(1+x)},\quad T_3x=\frac{\sin^2 x}{4},\forall x\in K. \tag{2.2}$$

Then the following are true:
(i) T_1,T_2,T_3 are strongly pseudocontractive maps.
(ii) $F(T_1)\cap F(T_2)\cap F(T_3)=\{0\}$
Put

$$\begin{gathered}a_n=1-(n+1)^{\frac{-1}{4}},b_n=(n+1)^{\frac{-1}{4}},c_n=(n+1)^{\frac{-1}{2}}\quad n\geq 0\\ a'_n=1-(n+1)^{-1},b'_n=c'_n=(2(n+1))^{-1}\\ a''_n=1-2(n+1)^{-1},b''_n=(n+1)^{-1},c''_n=2(n+1)^{-1}\end{gathered}$$

Observe that the conditions of Theorem 2.1 are fulfilled. Thus, Theorem 2.1 is applicable.

Remark 2. Theorem 2.1 holds if the condition (iii) is replaced with $\sum_{n=1}^{\infty}c_n<\infty$.
Remarks 3. (i) We replace modified three-step iterative scheme by a more general modified three-step iterative scheme with errors.
(ii) Numerical example is given in our result.

Corollary 2.3. Let X be a real Banach space , K a nonempty closed and convex subset of X and $T: K\to K$ be a uniformly continuous strongly pseudocontractive map with bounded range such that$F(T)\neq\emptyset$. For $x_0,u_0,v_0,w_0\in K$, the Noor iteration sequence with errors $\{x_n\}$ defined by

$$x_{n+1}=a_nx_n+b_nTy_n+c_nu_n$$

where

$$\begin{aligned}y_n &= a'_nx_n+b'_nTz_n+c'_nv_n\\ z_n &= a''_nx_n+b''_nTx_n+c''w_n,\end{aligned} \tag{2.3}$$

and $\{u_n\}$, $\{v_n\}$, $\{w_n\}$ are arbitrary bounded sequences in K, $\{a_n\}$,$\{b_n\}$,$\{c_n\}$, $\{a'_n\}$,$\{b'_n\}$,$\{c'_n\}$,$\{a''_n\}$, $\{b''_n\}$ and $\{c''_n\}$ are real sequences in [0,1] satisfying the following conditions:
(i) $a_n+b_n+c_n=a'_n+b'_n+c'_n=a''_n+b''_n+c''_n=1$
(ii)$b_n,b'_n,c_n,c'_n\to 0$ as $n\to\infty$
(iii) $\sum_{n=1}^{\infty}b_n=\infty$
(iv) $\lim_{n\to\infty}\frac{c_n}{b_n}=0$

then, the sequence $\{x_n\}$ converges strongly to the unique fixed point of T.

Proof: If $T_1 = T_2 = T_3 = T$ in Theorem 2.1, then Corollary 2.2 follows immediately.

Theorem 2.4. Let X be a real Banach space , K a nonempty closed and convex subset of X. Let $T_1, T_2, T_3 : K \to K$ be strongly accretive and uniformly continuous. Define $S_i : K \to K$ by $S_i x = f + x - T_i x$. Let $F(T_1) \cap F(T_2) \cap F(T_3) \neq \emptyset$ with $R(I - T_1)$ bounded. Suppose $\{u_n\}$, $\{v_n\}$ and $\{w_n\}$ are arbitrary bounded sequences in K, $\{a_n\}$,$\{b_n\}$,$\{c_n\}$,$\{a'_n\}$,$\{b'_n\}$,$\{c'_n\}$,$\{a''_n\}$, $\{b''_n\}$ and $\{c''_n\}$ are real sequences in [0,1] satisfying the following (i)-(iii) in Theorem 2.1. For $x_0, u_0, v_0, w_0 \in K$, the modified three-step iteration sequence with errors $\{x_n\}$ defined by

$$x_{n+1} = a_n x_n + b_n S_1 y_n + c_n u_n$$

where

$$\begin{aligned} y_n &= a'_n x_n + b'_n S_2 z_n + c'_n v_n \\ z_n &= a''_n x_n + b''_n S_3 x_n + c'' w_n \quad n \geq 0, \end{aligned} \tag{2.4}$$

If the operator equations $T_i x = f \quad (i = 1, 2, 3)$ has a solution in K, then the sequence $\{x_n\}$ converges to a unique solution of operator equations $T_i x = f \quad (i = 1, 2, 3)$.

Proof: Obviously, if $x^* \in K$ is a solution of the equation $T_i x = f \quad (i = 1, 2, 3)$, then x^* is the common fixed point of $S_i \quad (i = 1, 2, 3)$. It is easy to prove that $S_i \quad (i = 1, 2, 3)$ is uniformly continuous and strongly pseudocontractive with the strongly pseudocontractive constant $(1 - k)$. Thus, Theorem 2.4 follows from Theorem 2.1.

Remark 4. Theorem 2.4 extends Theorem 2.2 of Xue and Fan [17] in the sense that, we replace modified three-step iterative scheme by a more general modified three-step iterative scheme with errors.

References

[1] F. E.Browder, *Nonlinear mapping of nonexpansive and accretive in Banach spaces*,Bull. Am. Math. Soc. **73**(1967)875-882.

[2] S. S. Chang, Y. J. Cho, B.S.Lee and S.H. Kang, *Iterative approximation of fixed points and solutions for strongly accretive and strongly pseudo-contractive mappings in Banach spaces*, J.Math.And.Appl. **224**(1998)194-165.

[3] L. J. Ciric, J. S. Ume, *Ishikawa iteration process for strongly pseudocontractive operator in arbitrary Banach space*,Math. Commun. 8 (**2003), 43-48.**

[4] R. Glowinski, P. LeTallee, *Augmented Lagrangian and operator-splitting methods in nonlinear mechanics*, SIAM publishing Co, Philadephia,(1989).

[5] S. Haubruge, V. H. Nguyen,J.J. Strodiot, *Convergence analysis and applications of the Glowinski-LeTallee splitting method for finding a zero of the sum of two maximal monotone operators*, J.Optim.Theory Appl.97(1998)645-673.

[6] S. Ishikawa, *Fixed points by a new iteration method*,Proc. Am. Math. Soc. 44(1974),147-150.

[7] L.S. Liu, *Fixed points of local strictly pseudo-contractive mappings using Mann and Ishikawa iteration with errors*, Indian J. Pure Appl.Math.26(1995),649-659.

[8] W. R. Mann, *Mean Value methods in iteration*, Proc. Am. Math. Soc.4(1953),506-510

[9] C.H. Morales, J.J. Jung, *Convergence of path for pseudoconntractive mappings in Banach spaces*, Proc. Amer .Math. Soc. 120(2000),3411-3419.

[10] **M. A. Noor,** ***New approximation schemes for general variational inequalities*****, J. Math. Anal. Appl. 251(2000),217-229.**

[11] **M. A. Noor,** ***Three-step iterative algorithms for multi-valued quasi variational inclusions*****,J.Math.Anal.Appl.225(2001),589-604.**

[12] **M. A. Noor,** ***Some developments in general variational inequalities*****, Appl. Math. Computation, 152(2004), 199-277.**

[13] **M. A. Noor, T. M. Rassias and Z.Y. Huang,** ***Three-step iterations for nonlinear accretive operator equations*****, J. Math. Anal. Appl. 274(2002)59-68.**

[14] **J.O. Olaleru and A.A. Mogbademu,** ***On the modified Noor iteration scheme for Non-linear maps*****, Acta Mathematica Universitatis Comenianae, vol., LXXX, 2(2011), 221-228.**

[15] **A. Rafiq,** ***Modified Noor iterations for Nonlinear equations in Banach spaces*****, Appl. Math. Comput.182(2006)589-595.**

[16] **X. Weng,** ***Fixed point iteration for local strictly pseudo-contractive mapping*****, Proc. Am. Math. Soc. 113(3)(1991) 727-731.**

[17] **Z. Xue and R. Fan,** ***Some comments on Noor's iterations in Banach spaces*****,Appl. Math. Comput. 206(2008),12-15**

[18] **Y.G. Xu,** ***Ishikawa and Mann iterative process with errors for nonlinear strongly accretive operator equations*****, J.Math.Anal.Appl.224(1998),91-101.**

[19] **K.S. Zazimierski,** ***Adaptive Mann iterative for nonlinear accretive and pseudocontractive operator equations*****, Math. Commun. 13(2008),33-44.**

[20] **H.Y. Zhou, Y. Jia,** ***Approximation of fixed points of strongly pseudocontractive maps without Lipschitz assumptions*****, Proc. Amer. Soc.125(1997), 1705-1709.**

Adesanmi Alao Mogbademu
Department of Mathematics, Faculty of Sciences,, University of Lagos, Akoka, Nigeria
E-mail address: prinsmo@yahoo.com

Johnson O. Olaleru
Department of Mathematics, Faculty of Sciences,, University of Lagos, Akoka, Nigeria
E-mail address: olaleru1@yahoo.com