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THRIAD GEODESIC COMPOSITIONS IN FOUR DIMENSIONAL SPACE WITH AN AFFINE CONNECTEDNESS WITHOUT A TORSION

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Abstract. Let A_4 be an affinely connected space without a torsion. Following [7] we introduce the affinors $a_\alpha^\beta, b_\alpha^\beta$ and $\tilde{c}_\alpha^\beta = ic_\alpha^\beta = -ia_\sigma^\beta b_\alpha^\sigma (i^2 = -1)$ which define the compositions $X_2 \times \bar{X}_2, Y_2 \times \bar{Y}_2$ and $Z_2 \times \bar{Z}_2$, respectively.The first two compositions are conjugate. The composition $U_2 \times \bar{U}_2$ generated by the affinor $d_\alpha^\beta = a_\alpha^\beta + b_\alpha^\beta + c_\alpha^\beta$ is considered too. We have found necessary and sufficient condition for any of the above compositions to be of the kind $(g-g)$. Characterisics of the spaces A_4 that contain such compositions are obtained. Conections between Richis tensor and fundamental density of $E_q A_4$ are establish when the space is equiaffine and the compositions $X_2 \times \bar{X}_2, Y_2 \times \bar{Y}_2, Z_2 \times \bar{Z}_2$, are simultaneously of the kind $(g-g)$.

1. Introduction

Let A_N be a space with a symmetric affine connectedness without a torsion, defined by $\Gamma_{\alpha\beta}^\gamma$. Let consider a composition $X_n \times X_m$ of two differentiable basic manifolds X_n and $X_m (n+m=N)$ in the space A_N. For every point of the space of compositions $A_N(X_n \times X_m)$ there are two positions of the basic manifolds, which we denotes by $P(X_n)$ and $P(X_m)$ [3]. The defining of a composition in the space A_N is equivalent to defining of a field of an affinor a_α^β, that satisfies the condition [2], [3]

$$a_\sigma^\beta a_\alpha^\sigma = \delta_\alpha^\beta \tag{1}$$

The affinor a_α^β is called an affinor of the composition [2]. According to [3] and [5] the condition for integrability of the structure is $a_\beta^\sigma \nabla_{[\alpha a_\sigma^\nu]} - a_\alpha^\sigma \nabla_{[\beta a_\sigma^\nu]} = 0$ The projective affinors ${}^n a_\alpha^\sigma$ and ${}^m a_\alpha^\sigma$ [3], [4], defined by the equalities ${}^n a_\alpha^\sigma = \frac{1}{2}(\delta_\alpha^\beta + a_\alpha^\beta)$, ${}^m a_\alpha^\sigma = \frac{1}{2}(\delta_\alpha^\beta - a_\alpha^\beta)$ satisfy the conditions ${}^n a_\alpha^\beta +^m a_\alpha^\beta = \delta_\alpha^\beta$, ${}^n a_\alpha^\beta +^m a_\alpha^\beta = a_\alpha^\beta$ For

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every vector $v^{\alpha} \in A_N(X_n \times X_m)$ we have $v^{\alpha} = \overset{n}{a}{}^{\beta}_{\alpha} v^{\beta} +^m a^{\beta}_{\alpha} v^{\beta} = \overset{n}{V}_{\alpha} + \overset{m}{V}_{\alpha}$, where $\overset{n}{V}{}^{\alpha} = \overset{n}{a}{}^{\beta}_{\alpha} v^{\beta} \in P(X_n), \overset{m}{V}{}^{\alpha} =^m a^{\beta}_{\alpha} v^{\beta} \in P(X_m)$[4].

The composition $X_n \times X_m \in A_N(n+m=N)$ for which the positions $P(X_n)$ and $P(X_m)$ are parallelly translated along any line of X_n and X_m, respectively is called a composition of the kind $(g-g)$[3] or geodesic composition [6]. According to [3] the geodesic composition is characterized with the equality

$$a^{\sigma}_{\alpha} \nabla_{\beta} a^{\nu}_{\sigma} + a^{\sigma}_{\beta} \nabla_{\sigma} a^{\nu}_{\alpha} = 0 \tag{2}$$

Let A_4 be an space with affine connectedness without a torsion, defined by $\Gamma^{\sigma}_{\alpha\beta}(\alpha, \beta, \sigma = 1;2;3;4)$. Let $\underset{1}{v}{}^{\alpha}$, $\underset{2}{v}{}^{\alpha}$, $\underset{3}{v}{}^{\alpha}$, $\underset{4}{v}{}^{\alpha}$ are independent vector fields in A_4. Following [7] we define the covectors $\overset{\sigma}{v}_{\alpha}$ by the equalities

$$\underset{\alpha}{v}{}^{\beta} \overset{\alpha}{v}_{\sigma} = \delta^{\beta}_{\sigma} \leftrightarrow \underset{\alpha}{v}{}^{\beta} \overset{\sigma}{v}_{\beta} = \delta^{\sigma}_{\alpha} \tag{3}$$

According to [6], [7] we can define the affinor

$$a^{\beta}_{\alpha} = \underset{1}{v}{}^{\beta} \overset{1}{v}_{\alpha} + \underset{2}{v}{}^{\beta} \overset{2}{v}_{\alpha} - \underset{3}{v}{}^{\beta} \overset{3}{v}_{\alpha} - \underset{4}{v}{}^{\beta} \overset{4}{v}_{\alpha} \tag{4}$$

that satisfy the equalities (1). The affinor (4) defines a composition $(X_n \times X_m)$ in A_4. The projective affinors of the composition $(X_n \times X_m)$ are [7]

$$\overset{1}{a}{}^{\beta}_{\alpha} = \underset{1}{v}{}^{\beta} \overset{1}{v}_{\alpha} + \underset{2}{v}{}^{\beta} \overset{2}{v}_{\alpha} \ , \ \overset{2}{a}{}^{\beta}_{\alpha} = \underset{3}{v}{}^{\beta} \overset{3}{v}_{\alpha} + \underset{4}{v}{}^{\beta} \overset{4}{v}_{\alpha} \tag{5}$$

Following [7] we choose the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v})$ for a coordinate one. Then we have

$$\begin{gathered} \underset{1}{v}{}^{\alpha}(1,0,0,0) \ , \ \underset{2}{v}{}^{\alpha}(0,1,0,0) \ , \ \underset{3}{v}{}^{\alpha}(0,0,1,0) \ , \ \underset{4}{v}{}^{\alpha}(0,0,0,1) \\ \overset{1}{v}_{\alpha} \ (1,0,0,0) \ , \ \overset{2}{v}_{\alpha} \ (0,1,0,0) \ , \ \overset{3}{v}_{\alpha} \ (0,0,1,0) \ , \ \overset{4}{v}_{\alpha} \ (0,0,0,1) \end{gathered} \tag{6}$$

Let consider the vectors [7]

$$\underset{1}{w}{}^{\alpha} = \underset{1}{v}{}^{\alpha} + \underset{2}{v}{}^{\alpha} \ , \ \underset{2}{w}{}^{\alpha} = \underset{2}{v}{}^{\alpha} + \underset{4}{v}{}^{\alpha} \ , \ \underset{3}{w}{}^{\alpha} = \underset{1}{v}{}^{\alpha} - \underset{2}{v}{}^{\alpha} \ , \ \underset{4}{w}{}^{\alpha} = \underset{2}{v}{}^{\alpha} - \underset{4}{v}{}^{\alpha} \tag{7}$$

We define the covectors $\overset{\alpha}{w}_{\sigma}$ by the equalities

$$\underset{\alpha}{w}{}^{\nu} \ \overset{\alpha}{w}_{\sigma} = \delta^{\nu}_{\sigma} \Leftrightarrow \underset{\alpha}{w}{}^{\sigma} \ \overset{\beta}{w}_{\sigma} = \delta^{\beta}_{\alpha} \tag{8}$$

From (3) and (8) follow

$$\overset{1}{w}_{\alpha} = \frac{1}{2}\left(\overset{1}{v}_{\alpha} + \overset{2}{v}_{\alpha}\right) \ , \ \overset{2}{w}_{\alpha} = \frac{1}{2}\left(\overset{2}{v}_{\alpha} + \overset{4}{v}_{\alpha}\right) \ , \ \overset{3}{w}_{\alpha} = \frac{1}{2}\left(\overset{1}{v}_{\alpha} - \overset{2}{v}_{\alpha}\right) \ , \ \overset{4}{w}_{\alpha} = \frac{1}{2}\left(\overset{2}{v}_{\alpha} - \overset{4}{v}_{\alpha}\right) \tag{9}$$

Let consider the affinor

$$b^\beta_\alpha = \underset{1}{w}^\beta \overset{1}{w}_\alpha + \underset{2}{w}^\beta \overset{2}{w}_\alpha - \underset{3}{w}^\beta \overset{3}{w}_\alpha - \underset{4}{w}^\beta \overset{4}{w}_\alpha \,, \tag{10}$$

which according to [7] satisfies the equality $b^\beta_\alpha b^\alpha_\sigma = \delta^\beta_\sigma$. Therefore the affinor (10) defines a composition $Y_2 \times \overline{Y}_2$ in A_4. According to [7] the compositions $X_2 \times \overline{X}_2$ and $Y_2 \times \overline{Y}_2$ are conjugate. By (3), (7), (8) and (10) we obtain

$$b^\beta_\alpha = \underset{1}{\upsilon}^\beta \overset{3}{\upsilon}_\alpha + \underset{3}{\upsilon}^\beta \overset{1}{\upsilon}_\alpha + \underset{2}{\upsilon}^\beta \overset{4}{\upsilon}_\alpha + \underset{4}{\upsilon}^\beta \overset{2}{\upsilon}_\alpha \;. \tag{11}$$

Following [7] let consider the affinor $c^\beta_\alpha = -a^\beta_\alpha b^\alpha_\sigma$, which satisfies the equality $c^\beta_\sigma c^\sigma_\alpha = -\delta^\beta_\alpha$. With the help of (3), (4), (11) we establish

$$c^\beta_\alpha = \underset{3}{\upsilon}^\beta \overset{1}{\upsilon}_\alpha - \underset{1}{\upsilon}^\beta \overset{3}{\upsilon}_\alpha + \underset{4}{\upsilon}^\beta \overset{2}{\upsilon}_\alpha - \underset{2}{\upsilon}^\beta \overset{4}{\upsilon}_\alpha \;. \tag{12}$$

The affinor $\tilde{c}^\beta_\alpha = i\, c^\beta_\alpha$, where $i^2 = -1$, defines a composition $Z_2 \times \overline{Z}_2$ in A_4.

§2. Geodesic compositions in spaces A_4

According to [8] we have the following derivative equations

$$\nabla_\sigma \underset{\alpha}{\upsilon}^\beta = \underset{\alpha}{\overset{\nu}{T}}_\sigma \underset{\nu}{\upsilon}^\beta \;,\; \nabla_\sigma \overset{\alpha}{\upsilon}_\beta = - \underset{\nu}{\overset{\alpha}{T}}_\sigma \overset{\nu}{\upsilon}_\beta \;. \tag{13}$$

Let consider the composition $X_2 \times \overline{X}_2$ and let accept:

$\alpha, \beta, \gamma, \sigma, \nu, \tau \in \{1,2,3,4\}$; $i,j,k,s \in \{1,2\}$; $\bar{i}, \bar{j}, \bar{k}, \bar{s} \in \{3,4\}$.

Theorem 1.1. *The composition $X_2 \times \overline{X}_2$ is of the kind $(g-g)$ if and only if the coefficients of the derivative equations (13) satisfy the conditions*

$$\underset{k}{\overset{\bar{i}}{T}}_\alpha \underset{s}{\upsilon}^\alpha = 0 \;,\; \underset{\bar{k}}{\overset{i}{T}}_\alpha \underset{\bar{s}}{\upsilon}^\alpha = 0. \tag{14}$$

Proof: According to (4) and (13) we have

$$\begin{aligned}\nabla_\beta a^\nu_\beta = {} & \underset{1}{\overset{\tau}{T}}_\beta \underset{\tau}{\upsilon}^\nu \overset{1}{\upsilon}_\sigma - \underset{\tau}{\overset{1}{T}}_\beta \underset{1}{\upsilon}^\nu \overset{\tau}{\upsilon}_\sigma + \underset{2}{\overset{\tau}{T}}_\beta \underset{\tau}{\upsilon}^\nu \overset{2}{\upsilon}_\sigma - \underset{\tau}{\overset{2}{T}}_\beta \underset{2}{\upsilon}^\nu \overset{\tau}{\upsilon}_\sigma \\ & - \underset{3}{\overset{\tau}{T}}_\beta \underset{\tau}{\upsilon}^\nu \overset{3}{\upsilon}_\sigma + \underset{\tau}{\overset{3}{T}}_\beta \underset{3}{\upsilon}^\nu \overset{\tau}{\upsilon}_\sigma - \underset{4}{\overset{\tau}{T}}_\beta \underset{\tau}{\upsilon}^\nu \overset{4}{\upsilon}_\sigma + \underset{\tau}{\overset{4}{T}}_\beta \underset{4}{\upsilon}^\nu \overset{\tau}{\upsilon}_\sigma \;.\end{aligned} \tag{15}$$

Taking into account the independence of the covectors $\overset{\sigma}{\upsilon}_\alpha$ and using (2), (3), (4), (15) we find the equalities

$$\begin{aligned} & (\delta^\sigma_\beta + a^\sigma_\beta)(\underset{1}{\overset{3}{T}}_\sigma \underset{3}{\upsilon}^\nu + \underset{1}{\overset{4}{T}}_\sigma \underset{4}{\upsilon}^\nu) = 0 \,,\; (\delta^\sigma_\beta + a^\sigma_\beta)(\underset{1}{\overset{3}{T}}_\sigma \underset{3}{\upsilon}^\nu + \underset{1}{\overset{4}{T}}_\sigma \underset{4}{\upsilon}^\nu) = 0 \,, \\ & (\delta^\sigma_\beta - a^\sigma_\beta)(\underset{3}{\overset{1}{T}}_\sigma \underset{3}{\upsilon}^\nu + \underset{3}{\overset{2}{T}}_\sigma \underset{4}{\upsilon}^\nu) = 0 \,,\; (\delta^\sigma_\beta - a^\sigma_\beta)(\underset{4}{\overset{1}{T}}_\sigma \underset{3}{\upsilon}^\nu + \underset{4}{\overset{2}{T}}_\sigma \underset{4}{\upsilon}^\nu) = 0 \,. \end{aligned} \tag{16}$$

Because of the independence of the vectors $\underset{\alpha}{\upsilon}^\nu$ it follows an equivalence of (16) to the following equalities

(17)
$$\overset{3}{\underset{1}{T}}{}_\beta + a^\sigma_\beta \overset{3}{\underset{1}{T}}{}_\sigma = 0,\ \overset{4}{\underset{1}{T}}{}_\beta + a^\sigma_\beta \overset{4}{\underset{1}{T}}{}_\sigma = 0,\ \overset{3}{\underset{2}{T}}{}_\beta + a^\sigma_\beta \overset{3}{\underset{2}{T}}{}_\sigma = 0,\ \overset{4}{\underset{2}{T}}{}_\beta + a^\sigma_\beta \overset{4}{\underset{2}{T}}{}_\sigma = 0,$$
$$\overset{1}{\underset{3}{T}}{}_\beta - a^\sigma_\beta \overset{1}{\underset{3}{T}}{}_\sigma = 0,\ \overset{2}{\underset{3}{T}}{}_\beta - a^\sigma_\beta \overset{2}{\underset{3}{T}}{}_\sigma = 0,\ \overset{1}{\underset{4}{T}}{}_\beta - a^\sigma_\beta \overset{1}{\underset{4}{T}}{}_\sigma = 0,\ \overset{2}{\underset{4}{T}}{}_\beta - a^\sigma_\beta \overset{2}{\underset{4}{T}}{}_\sigma = 0.$$

Now it is easy to see that the equalities (14) follow after contraction by $\underset{1}{v}^\beta$ and $\underset{2}{v}^\beta$ for the first four equalities of (17) and by $\underset{3}{v}^\beta$ and $\underset{4}{v}^\beta$ for the last four equalities of (17). Let?s note that the equalities (17) are proved in [6] by another approach.

Corollary 1.2. *If the net* $\left(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v}\right)$ *is chosen as a coordinate one then the composition* $X_2 \times \overline{X}_2$ *from the kind* $(g - g)$ *characterizes by the following equalities for: (1.1) the coefficients of the derivative equations*

$$\overset{\bar{i}}{\underset{k}{T}}{}_s = 0\,,\ \ \overset{i}{\underset{\bar{k}}{T}}{}_{\bar{s}} = 0 \tag{18}$$

1.2) the coefficients of the connectedness

$$\Gamma^{\bar{i}}_{sk}\,,\ \ \Gamma^{i}_{\bar{s}\bar{k}} = 0. \tag{19}$$

Proof: Let choose the net $\left(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v}\right)$ for a coordinate one. Then by (6) and (14) we find (18). According to [1] and (13) we can write $\partial_\sigma \underset{\alpha}{v}^\beta + \Gamma^\beta_{\sigma\nu} \underset{\alpha}{v}^\nu = \overset{\nu}{\underset{\alpha}{T}}{}_\sigma \underset{\nu}{v}^\beta$ from where using (6) we obtain

$$\Gamma^\beta_{\sigma\alpha} = \overset{\beta}{\underset{\alpha}{T}}{}_\sigma. \tag{20}$$

The equalities (19) follow from (18) and (20). Let's note that the equalities (19) are obtained in [3], when the coordinates are adaptive with the composition $X_2 \times \overline{X}_2$. This happens so, because the chosen coordinate net raises adaptive with the composition coordinates.

From (19) and $R_{\alpha\beta\sigma}{}^{\nu}_{\cdot} = 2\partial_{[\alpha}\Gamma_{\beta]\sigma} - 2\Gamma^\tau_{\sigma[\alpha}\Gamma_{\beta]\tau}$ [1] we establish the validity of the following statement:

Fact 1. *When the composition* $X_2 \times \overline{X}_2$ *is of the kind* $(g - g)$ *then in the parameters of the coordinate net* $\left(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v}\right)$ *the tensor of the curvature satisfy the conditions* $R_{ijk}{}^{\bar{s}}_{\cdot} = 0$, $R_{\overline{ijk}}{}^{s}_{\cdot} = 0$.

Theorem 1.3. *The composition* $Y_2 \times \overline{Y}_2$ *is of the kind* $(g - g)$ *if and only if the coefficients of the derivative equations satisfy the conditions*

$$\left(\overset{1}{\underset{1}{T}}{}_\sigma - \overset{3}{\underset{3}{T}}{}_\sigma\right)\underset{1}{v}^\sigma + \left(\overset{1}{\underset{3}{T}}{}_\sigma - \overset{3}{\underset{1}{T}}{}_\sigma\right)\underset{3}{v}^\sigma = 0,\ \left(\overset{1}{\underset{1}{T}}{}_\sigma - \overset{3}{\underset{3}{T}}{}_\sigma\right)\underset{3}{v}^\sigma + \left(\overset{1}{\underset{3}{T}}{}_\sigma - \overset{3}{\underset{1}{T}}{}_\sigma\right)\underset{1}{v}^\sigma = 0,$$

$$
(21)\quad
\begin{aligned}
&\left(\overset{1}{\underset{1}{T}}_{\sigma}-\overset{3}{\underset{3}{T}}_{\sigma}\right)\underset{2}{\upsilon}^{\sigma}+\left(\overset{1}{\underset{3}{T}}_{\sigma}-\overset{3}{\underset{1}{T}}_{\sigma}\right)\underset{4}{\upsilon}^{\sigma}=0,\ \left(\overset{1}{\underset{1}{T}}_{\sigma}-\overset{3}{\underset{3}{T}}_{\sigma}\right)\underset{1}{\upsilon}^{\sigma}+\left(\overset{1}{\underset{3}{T}}_{\sigma}-\overset{3}{\underset{1}{T}}_{\sigma}\right)\underset{2}{\upsilon}^{\sigma}=0,\\
&\left(\overset{2}{\underset{1}{T}}_{\sigma}-\overset{4}{\underset{3}{T}}_{\sigma}\right)\underset{1}{\upsilon}^{\sigma}+\left(\overset{2}{\underset{3}{T}}_{\sigma}-\overset{4}{\underset{1}{T}}_{\sigma}\right)\underset{3}{\upsilon}^{\sigma}=0,\ \left(\overset{2}{\underset{1}{T}}_{\sigma}-\overset{4}{\underset{3}{T}}_{\sigma}\right)\underset{3}{\upsilon}^{\sigma}+\left(\overset{2}{\underset{3}{T}}_{\sigma}-\overset{4}{\underset{1}{T}}_{\sigma}\right)\underset{1}{\upsilon}^{\sigma}=0,\\
&\left(\overset{2}{\underset{1}{T}}_{\sigma}-\overset{4}{\underset{3}{T}}_{\sigma}\right)\underset{2}{\upsilon}^{\sigma}+\left(\overset{2}{\underset{3}{T}}_{\sigma}-\overset{4}{\underset{1}{T}}_{\sigma}\right)\underset{4}{\upsilon}^{\sigma}=0,\ \left(\overset{2}{\underset{1}{T}}_{\sigma}-\overset{4}{\underset{3}{T}}_{\sigma}\right)\underset{4}{\upsilon}^{\sigma}+\left(\overset{2}{\underset{3}{T}}_{\sigma}-\overset{4}{\underset{1}{T}}_{\sigma}\right)\underset{2}{\upsilon}^{\sigma}=0,\\
&\left(\overset{1}{\underset{2}{T}}_{\sigma}-\overset{3}{\underset{4}{T}}_{\sigma}\right)\underset{1}{\upsilon}^{\sigma}+\left(\overset{1}{\underset{4}{T}}_{\sigma}-\overset{3}{\underset{2}{T}}_{\sigma}\right)\underset{3}{\upsilon}^{\sigma}=0,\ \left(\overset{1}{\underset{2}{T}}_{\sigma}-\overset{3}{\underset{4}{T}}_{\sigma}\right)\underset{3}{\upsilon}^{\sigma}+\left(\overset{1}{\underset{4}{T}}_{\sigma}-\overset{3}{\underset{2}{T}}_{\sigma}\right)\underset{1}{\upsilon}^{\sigma}=0,\\
&\left(\overset{1}{\underset{2}{T}}_{\sigma}-\overset{3}{\underset{4}{T}}_{\sigma}\right)\underset{2}{\upsilon}^{\sigma}+\left(\overset{1}{\underset{4}{T}}_{\sigma}-\overset{3}{\underset{2}{T}}_{\sigma}\right)\underset{4}{\upsilon}^{\sigma}=0,\ \left(\overset{1}{\underset{2}{T}}_{\sigma}-\overset{3}{\underset{4}{T}}_{\sigma}\right)\underset{1}{\upsilon}^{\sigma}+\left(\overset{1}{\underset{4}{T}}_{\sigma}-\overset{3}{\underset{2}{T}}_{\sigma}\right)\underset{2}{\upsilon}^{\sigma}=0,\\
&\left(\overset{2}{\underset{2}{T}}_{\sigma}-\overset{4}{\underset{4}{T}}_{\sigma}\right)\underset{1}{\upsilon}^{\sigma}+\left(\overset{2}{\underset{4}{T}}_{\sigma}-\overset{4}{\underset{2}{T}}_{\sigma}\right)\underset{3}{\upsilon}^{\sigma}=0,\ \left(\overset{2}{\underset{2}{T}}_{\sigma}-\overset{4}{\underset{4}{T}}_{\sigma}\right)\underset{3}{\upsilon}^{\sigma}+\left(\overset{2}{\underset{4}{T}}_{\sigma}-\overset{4}{\underset{2}{T}}_{\sigma}\right)\underset{1}{\upsilon}^{\sigma}=0,\\
&\left(\overset{2}{\underset{2}{T}}_{\sigma}-\overset{4}{\underset{4}{T}}_{\sigma}\right)\underset{2}{\upsilon}^{\sigma}+\left(\overset{2}{\underset{4}{T}}_{\sigma}-\overset{4}{\underset{2}{T}}_{\sigma}\right)\underset{4}{\upsilon}^{\sigma}=0,\ \left(\overset{2}{\underset{2}{T}}_{\sigma}-\overset{4}{\underset{4}{T}}_{\sigma}\right)\underset{4}{\upsilon}^{\sigma}+\left(\overset{2}{\underset{4}{T}}_{\sigma}-\overset{4}{\underset{2}{T}}_{\sigma}\right)\underset{2}{\upsilon}^{\sigma}=0.
\end{aligned}
$$

Proof: Because of the equalities (11) and (14) we have

$$
(22)\quad
\begin{aligned}
\nabla_{\sigma}b_{\alpha}^{\beta}=&\overset{\nu}{\underset{1}{T}}_{\sigma}\underset{\nu}{\upsilon}^{\beta}\overset{3}{\upsilon}_{\alpha}-\overset{3}{\underset{\nu}{T}}_{\sigma}\underset{1}{\upsilon}^{\beta}\overset{\nu}{\upsilon}_{\alpha}+\overset{\nu}{\underset{3}{T}}_{\sigma}\underset{\nu}{\upsilon}^{\beta}\overset{1}{\upsilon}_{\alpha}-\overset{1}{\underset{\nu}{T}}_{\sigma}\underset{3}{\upsilon}^{\beta}\overset{\nu}{\upsilon}_{\alpha}\\
&+\overset{\nu}{\underset{2}{T}}_{\sigma}\underset{\nu}{\upsilon}^{\beta}\overset{4}{\upsilon}_{\alpha}-\overset{4}{\underset{\nu}{T}}_{\sigma}\underset{2}{\upsilon}^{\beta}\overset{\nu}{\upsilon}_{\alpha}+\overset{\nu}{\underset{4}{T}}_{\sigma}\underset{\nu}{\upsilon}^{\beta}\overset{2}{\upsilon}_{\alpha}-\overset{2}{\underset{\nu}{T}}_{\sigma}\underset{4}{\upsilon}^{\beta}\overset{\nu}{\upsilon}_{\alpha}\,.
\end{aligned}
$$

Transforming the condition $b_{\alpha}^{\sigma}\nabla_{\beta}b_{\sigma}^{\nu}+b_{\beta}^{\sigma}\nabla_{\sigma}b_{\alpha}^{\nu}=0$ with the help of (3), (11), (22) and using the independence of the covectors $\overset{\sigma}{\upsilon}_{\alpha}$ we obtain the following equalities

$$
(23)\quad
\begin{aligned}
&\overset{1}{\underset{1}{T}}_{\beta}-\overset{3}{\underset{3}{T}}_{\beta}+b_{\beta}^{\sigma}\left(\overset{1}{\underset{3}{T}}_{\sigma}-\overset{3}{\underset{1}{T}}_{\sigma}\right)=0\,,\ \overset{2}{\underset{1}{T}}_{\beta}-\overset{4}{\underset{3}{T}}_{\beta}+b_{\beta}^{\sigma}\left(\overset{2}{\underset{3}{T}}_{\sigma}-\overset{4}{\underset{1}{T}}_{\sigma}\right)=0,\\
&\overset{1}{\underset{2}{T}}_{\beta}-\overset{3}{\underset{4}{T}}_{\beta}+b_{\beta}^{\sigma}\left(\overset{1}{\underset{4}{T}}_{\sigma}-\overset{3}{\underset{2}{T}}_{\sigma}\right)=0\,,\ \overset{2}{\underset{2}{T}}_{\beta}-\overset{4}{\underset{4}{T}}_{\beta}+b_{\beta}^{\sigma}\left(\overset{2}{\underset{4}{T}}_{\sigma}-\overset{4}{\underset{2}{T}}_{\sigma}\right)=0.
\end{aligned}
$$

Now after contraction by $\underset{\sigma}{\upsilon}^{\alpha}$ it is easy to see the equivalence of (23) to (21).

Corollary 1.4. *If the net* $\left(\underset{1}{\upsilon},\underset{2}{\upsilon},\underset{3}{\upsilon},\underset{4}{\upsilon}\right)$ *is chosen as a coordinate one then the composition* $Y_2\times\overline{Y}_2$ *from the kind* $(g-g)$ *characterizes by the following equalities for:*
2.1) the coefficients of the derivative equations

$$
(24)\quad
\begin{aligned}
&\overset{1}{\underset{1}{T}}_{\alpha}-\overset{3}{\underset{3}{T}}_{\alpha}=\overset{3}{\underset{1}{T}}_{\overline{\alpha}}-\overset{1}{\underset{3}{T}}_{\overline{\alpha}}\quad,\quad\overset{2}{\underset{1}{T}}_{\alpha}-\overset{4}{\underset{3}{T}}_{\alpha}=\overset{4}{\underset{1}{T}}_{\overline{\alpha}}-\overset{2}{\underset{3}{T}}_{\overline{\alpha}}\quad,\\
&\overset{1}{\underset{2}{T}}_{\alpha}-\overset{3}{\underset{4}{T}}_{\alpha}=\overset{3}{\underset{2}{T}}_{\overline{\alpha}}-\overset{1}{\underset{4}{T}}_{\overline{\alpha}}\quad,\quad\overset{2}{\underset{2}{T}}_{\alpha}-\overset{4}{\underset{4}{T}}_{\alpha}=\overset{4}{\underset{2}{T}}_{\overline{\alpha}}-\overset{2}{\underset{4}{T}}_{\overline{\alpha}}\quad,
\end{aligned}
$$

2.2) the coefficients of the connectedness

$$\Gamma^{\alpha}_{11}+\Gamma^{\alpha}_{33}=2\Gamma^{\overline{\alpha}}_{13}\ ,\ \Gamma^{\alpha}_{22}+\Gamma^{\alpha}_{44}=2\Gamma^{\overline{\alpha}}_{24}\ ,\ \Gamma^{\alpha}_{12}+\Gamma^{\alpha}_{34}=\Gamma^{\overline{\alpha}}_{14}+\Gamma^{\overline{\alpha}}_{23}\ , \tag{25}$$

as when α *accepts consecutively the values* $1,2,3,4$ *then* $\overline{\alpha}$ *accepts the values* $3,4,1,2$, *respectively.*

Proof: Let choose the net $\left(\underset{1}{\upsilon},\underset{2}{\upsilon},\underset{3}{\upsilon},\underset{4}{\upsilon}\right)$ for a coordinate net. With the help of (6) and (21) we find (24).
Then by (20) and (24) we obtain (25).

Theorem 1.5. *The composition* $Z_2\times\overline{Z}_2$ *is of the kind* $(g-g)$ *if and only if the coefficients of the derivative equations (13) satisfy the conditions*

$$\begin{gathered}
\left(\overset{1}{\underset{1}{T}}_\sigma-\overset{3}{\underset{3}{T}}_\sigma\right)\underset{1}{\upsilon}^\sigma=\left(\overset{1}{\underset{3}{T}}_\sigma+\overset{3}{\underset{1}{T}}_\sigma\right)\underset{3}{\upsilon}^\sigma,\ \left(\overset{1}{\underset{1}{T}}_\sigma-\overset{3}{\underset{3}{T}}_\sigma\right)\underset{2}{\upsilon}^\sigma=\left(\overset{1}{\underset{3}{T}}_\sigma+\overset{3}{\underset{1}{T}}_\sigma\right)\underset{4}{\upsilon}^\sigma,\\
\left(\overset{3}{\underset{3}{T}}_\sigma-\overset{1}{\underset{1}{T}}_\sigma\right)\underset{3}{\upsilon}^\sigma=\left(\overset{1}{\underset{3}{T}}_\sigma+\overset{3}{\underset{1}{T}}_\sigma\right)\underset{1}{\upsilon}^\sigma,\ \left(\overset{3}{\underset{3}{T}}_\sigma-\overset{1}{\underset{1}{T}}_\sigma\right)\underset{4}{\upsilon}^\sigma=\left(\overset{1}{\underset{3}{T}}_\sigma+\overset{3}{\underset{1}{T}}_\sigma\right)\underset{2}{\upsilon}^\sigma,\\
\left(\overset{2}{\underset{1}{T}}_\sigma-\overset{4}{\underset{3}{T}}_\sigma\right)\underset{1}{\upsilon}^\sigma=\left(\overset{2}{\underset{3}{T}}_\sigma+\overset{4}{\underset{1}{T}}_\sigma\right)\underset{3}{\upsilon}^\sigma,\ \left(\overset{2}{\underset{1}{T}}_\sigma-\overset{4}{\underset{3}{T}}_\sigma\right)\underset{2}{\upsilon}^\sigma=\left(\overset{2}{\underset{3}{T}}_\sigma+\overset{4}{\underset{1}{T}}_\sigma\right)\underset{4}{\upsilon}^\sigma,\\
\left(\overset{4}{\underset{3}{T}}_\sigma-\overset{2}{\underset{1}{T}}_\sigma\right)\underset{3}{\upsilon}^\sigma=\left(\overset{2}{\underset{3}{T}}_\sigma+\overset{4}{\underset{1}{T}}_\sigma\right)\underset{1}{\upsilon}^\sigma,\ \left(\overset{3}{\underset{4}{T}}_\sigma-\overset{2}{\underset{1}{T}}_\sigma\right)\underset{4}{\upsilon}^\sigma=\left(\overset{2}{\underset{3}{T}}_\sigma+\overset{4}{\underset{1}{T}}_\sigma\right)\underset{2}{\upsilon}^\sigma,\\
\left(\overset{1}{\underset{2}{T}}_\sigma-\overset{3}{\underset{4}{T}}_\sigma\right)\underset{1}{\upsilon}^\sigma=\left(\overset{1}{\underset{4}{T}}_\sigma-\overset{3}{\underset{2}{T}}_\sigma\right)\underset{3}{\upsilon}^\sigma,\ \left(\overset{1}{\underset{2}{T}}_\sigma-\overset{3}{\underset{4}{T}}_\sigma\right)\underset{2}{\upsilon}^\sigma=\left(\overset{1}{\underset{4}{T}}_\sigma+\overset{3}{\underset{2}{T}}_\sigma\right)\underset{4}{\upsilon}^\sigma,\\
\left(\overset{3}{\underset{4}{T}}_\sigma-\overset{1}{\underset{2}{T}}_\sigma\right)\underset{3}{\upsilon}^\sigma=\left(\overset{3}{\underset{2}{T}}_\sigma+\overset{1}{\underset{4}{T}}_\sigma\right)\underset{1}{\upsilon}^\sigma,\left(\overset{3}{\underset{4}{T}}_\sigma-\overset{1}{\underset{2}{T}}_\sigma\right)\underset{4}{\upsilon}^\sigma=\left(\overset{1}{\underset{4}{T}}_\sigma+\overset{3}{\underset{2}{T}}_\sigma\right)\underset{2}{\upsilon}^\sigma,\\
\left(\overset{2}{\underset{2}{T}}_\sigma-\overset{4}{\underset{4}{T}}_\sigma\right)\underset{1}{\upsilon}^\sigma=\left(\overset{2}{\underset{4}{T}}_\sigma+\overset{4}{\underset{2}{T}}_\sigma\right)\underset{3}{\upsilon}^\sigma,\left(\overset{2}{\underset{2}{T}}_\sigma-\overset{4}{\underset{4}{T}}_\sigma\right)\underset{2}{\upsilon}^\sigma=\left(\overset{2}{\underset{4}{T}}_\sigma+\overset{4}{\underset{2}{T}}_\sigma\right)\underset{4}{\upsilon}^\sigma,\\
\left(\overset{4}{\underset{4}{T}}_\sigma-\overset{2}{\underset{2}{T}}_\sigma\right)\underset{3}{\upsilon}^\sigma=\left(\overset{2}{\underset{4}{T}}_\sigma+\overset{4}{\underset{2}{T}}_\sigma\right)\underset{1}{\upsilon}^\sigma,\left(\overset{4}{\underset{4}{T}}_\sigma-\overset{2}{\underset{2}{T}}_\sigma\right)\underset{4}{\upsilon}^\sigma=\left(\overset{2}{\underset{4}{T}}_\sigma+\overset{4}{\underset{2}{T}}_\sigma\right)\underset{2}{\upsilon}^\sigma.
\end{gathered}\tag{26}$$

Proof: By the equalities (12) and (14) we obtain

$$\begin{aligned}
\nabla_\sigma c^\beta_\alpha=&\overset{\nu}{\underset{3}{T}}_\sigma\,\underset{\nu}{\upsilon}^\beta\,\overset{1}{\upsilon}_\alpha-\overset{1}{\underset{\nu}{T}}_\sigma\,\underset{3}{\upsilon}^\beta\,\overset{\nu}{\upsilon}_\alpha-\overset{\nu}{\underset{1}{T}}_\sigma\,\underset{\nu}{\upsilon}^\beta\,\overset{3}{\upsilon}_\alpha+\overset{3}{\underset{\nu}{T}}_\sigma\,\underset{1}{\upsilon}^\beta\,\overset{\nu}{\upsilon}_\alpha\\
&+\overset{\nu}{\underset{4}{T}}_\sigma\,\underset{\nu}{\upsilon}^\beta\,\overset{2}{\upsilon}_\alpha-\overset{2}{\underset{\nu}{T}}_\sigma\,\underset{4}{\upsilon}^\beta\,\overset{\nu}{\upsilon}_\alpha-\overset{\nu}{\underset{2}{T}}_\sigma\,\underset{\nu}{\upsilon}^\beta\,\overset{2}{\upsilon}_\alpha-\overset{4}{\underset{\nu}{T}}_\sigma\,\underset{2}{\upsilon}^\beta\,\overset{\nu}{\upsilon}_\alpha\ .
\end{aligned}\tag{27}$$

Transforming the condition $c^\sigma_\alpha\nabla_\beta b^\nu_\sigma+c^\sigma_\beta\nabla_\sigma b^\nu_\alpha=0$ with the help of (3), (12), (27) and using the independence of the covectors $\overset{\sigma}{\upsilon}_\alpha$ we obtain the following equalities

$$\overset{3}{\underset{3}{T}}_\beta-\overset{1}{\underset{1}{T}}_\beta+c^\sigma_\beta\left(\overset{1}{\underset{3}{T}}_\sigma+\overset{3}{\underset{1}{T}}_\sigma\right)=0\,,\ \overset{4}{\underset{3}{T}}_\beta-\overset{2}{\underset{1}{T}}_\beta+c^\sigma_\beta\left(\overset{2}{\underset{3}{T}}_\sigma+\overset{4}{\underset{1}{T}}_\sigma\right)=0, \tag{28}$$

$$\overset{3}{\underset{4}{T}}_\beta - \overset{1}{\underset{2}{T}}_\beta + c^\sigma_\beta\left(\overset{3}{\underset{2}{T}}_\sigma + \overset{1}{\underset{4}{T}}_\sigma\right) = 0\,,\ \overset{4}{\underset{4}{T}}_\beta - \overset{2}{\underset{2}{T}}_\beta + c^\sigma_\beta\left(\overset{2}{\underset{4}{T}}_\sigma + \overset{4}{\underset{2}{T}}_\sigma\right) = 0.$$

Now after contraction by $\underset{\sigma}{v}^\alpha$ it is easy to see the equivalence of (28) to (26).

Corollary 1.6. *If the net* $\left(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v},\right)$ *is chosen as a coordinate one then the composition* $Z_2 \times \overline{Z}_2$ *from the kind* $(g-g)$ *characterizes by the following equalities for: 3.1) the coefficients of the derivative equations*

(29)
$$\begin{array}{ll} \overset{1}{\underset{1}{T}}\alpha - \overset{3}{\underset{3}{T}}\alpha = \epsilon\left(\overset{3}{\underset{1}{T}}\overline{\alpha} + \overset{1}{\underset{3}{T}}\overline{\alpha}\right), & \overset{3}{\underset{1}{T}}\alpha - \overset{4}{\underset{3}{T}}\alpha = \epsilon\left(\overset{4}{\underset{1}{T}}\overline{\alpha} + \overset{2}{\underset{3}{T}}\overline{\alpha}\right) \\ \overset{1}{\underset{2}{T}}\alpha - \overset{3}{\underset{4}{T}}\alpha = \epsilon\left(\overset{3}{\underset{2}{T}}\overline{\alpha} + \overset{1}{\underset{4}{T}}\overline{\alpha}\right), & \overset{2}{\underset{2}{T}}\alpha - \overset{4}{\underset{4}{T}}\alpha = \epsilon\left(\overset{4}{\underset{2}{T}}\overline{\alpha} + \overset{2}{\underset{4}{T}}\overline{\alpha}\right) \end{array}$$

3.2) the coefficients of the connectedness

(30)
$$\Gamma^\alpha_{11} - \Gamma^\alpha_{33} = 2\epsilon\Gamma^{\overline{\alpha}}_{13},\ \Gamma^\alpha_{22} - \Gamma^\alpha_{44} = 2\epsilon\Gamma^{\overline{\alpha}}_{24},\ \Gamma^\alpha_{12} - \Gamma^\alpha_{34} = \Gamma^{\overline{\alpha}}_{14} + \Gamma^{\overline{\alpha}}_{23},$$

as when α *accepts consecutively the values 1; 2; 3; 4 then* $\overline{\alpha}$ *accepts the values 3; 4; 1; 2; respectively and*
$\epsilon = 1$ *for* $\alpha = 1, 2, \epsilon = -1$ *for* $\alpha = 3, 4$.

Proof. Let choose the net $\left(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v},\right)$ for a coordinate net. Then the equalities (29) follow by (6) and (26), and the equalities (30) follow by (20) and (29).

Using (19) (25) and (30) we establish the validity of the following statement:

□

Fact 2. *If two of the compositions* $X_2 \times \overline{X}_2, Y_2 \times \overline{Y}_2, Z_2 \times \overline{Z}_2$ *are from the kind (g - g) then and the third composition is of the kind (g - g).*

Since from (19), (25) and (30) it follows

(31)
$$\Gamma^\alpha_{ij} = \Gamma^{\alpha}_{\overline{ij}} = 0, \Gamma^\alpha_{13} = \Gamma^\alpha_{44} = 0, \Gamma^\alpha_{14} + \Gamma^\alpha_{23} = 0$$

then we can formulate

Fact 3. *When the compositions* $X_2 \times \overline{X}_2, Y_2 \times \overline{Y}_2, Z_2 \times \overline{Z}_2$ *are of the kind (g-g) then in the parameters of the coordinate net* $\left(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v},\right)$ *the tensor of the curvature satisfy the conditions* $R_{ijks}{}^{\overline{8}}_{\cdot} = R_{\overline{ijks}}{}^{8}_{\cdot} = 0, R_{133}{}^{\alpha}_{\cdot} = R_{244}{}^{\alpha}_{\cdot} = R_{311}{}^{\alpha}_{\cdot} = R_{422}{}^{\alpha}_{\cdot} = R_{143}{}^{\alpha}_{\cdot} = R_{234}{}^{\alpha}_{\cdot} = R_{321}{}^{\alpha}_{\cdot} = R_{412}{}^{\alpha}_{\cdot} = 0.$

Let consider the affinor

(32)

$$d_\alpha^\beta = a_\alpha^\beta + b_\alpha^\beta + c_\alpha^\beta$$

.

According to (3), (4), (10) and (12) we have

(33)

$$a_\alpha^\beta b_\alpha^\beta + b_\alpha^\beta a_\alpha^\beta = 0, b_\alpha^\beta c_\alpha^\beta + c_\alpha^\beta b_\alpha^\beta = 0, c_\alpha^\beta a_\alpha^\beta + a_\alpha^\beta c_\alpha^\beta = 0.$$

From (32) and (33) it follows $d_\alpha^\beta = a_\alpha^\beta\, a_\alpha^\beta + b_\alpha^\beta b_\alpha^\beta + c_\alpha^\beta\, c_\alpha^\beta = \delta_\alpha^\beta + \delta_\alpha^\beta - \delta_\alpha^\beta = \delta_\alpha^\beta$, which means that the affinor d_α^β defines a composition $U_2 \times \overline{U}_2$ with the positions P(U2) and $P(\overline{U}_2)$:

Theorem 1.7. *The composition $U_2 \times \overline{U}_2$ is of the kind (g-g) if and only if the coefficients of the derivative equations (13) satisfy the conditions*

(34)

$$\overset{s}{\underset{\bar{k}}{T}}{}_\beta - d_\beta^\sigma \overset{s}{\underset{\bar{k}}{T}}{}_\sigma = 0$$

(35)

$$\overset{\bar{s}}{\underset{k}{T}}{}_\beta + \overset{\bar{s}}{\underset{k+2}{T}}{}_\beta - \overset{\bar{s}-2}{\underset{k}{T}}{}_\beta - 2\overset{\bar{s}-2}{\underset{k+2}{T}}{}_\beta + d_\beta^\sigma \left(\overset{\bar{s}}{\underset{k}{T}}{}_\sigma + \overset{\bar{s}}{\underset{k+2}{T}}{}_\sigma - \overset{\bar{s}-2}{\underset{k}{T}}{}_\sigma \right) = 0$$

Proof According to (2) the composition $U_2 \times \overline{U}_2$ will be of the kind (g - g) if and only if

(36)

$$d_\alpha^\sigma \nabla_\beta\, d_\sigma^\nu + d_\beta^\sigma \nabla_\sigma\, d_\alpha^\nu = 0$$

With the help of (4), (10), (12) and (32) we find

(37)

$$d_\sigma^\nu = \alpha_\sigma^\nu + 2(\underset{3}{v}^\nu \overset{1}{v}_\nu + \underset{4}{v}^\nu \overset{2}{v}_\nu = \underset{i}{v}^\nu \overset{i}{v}_\nu - \underset{\underline{i}}{v}^\nu \overset{\bar{i}}{v}_\nu + 2\underset{2+i}{v}{}^\nu \overset{i}{v}_\sigma .$$

Then (36) can be written in the form

(38)

$$d_\alpha^\sigma(\overset{\delta}{\underset{i}{T}}{}_\beta \underset{\delta}{v}^\nu \overset{i}{v}_\sigma - \overset{i}{\underset{\delta}{T}}{}_\beta \underset{i}{v}^\nu \overset{\delta}{v}_\sigma - \overset{\delta}{\underset{\bar{i}}{T}}{}_\beta \underset{\delta}{v}^\nu \overset{\bar{i}}{v}_\sigma + \overset{\bar{i}}{\underset{\delta}{T}}{}_\beta \underset{\bar{i}}{v}^\nu \overset{\delta}{v}_\sigma + 2\overset{\delta}{\underset{2+i}{T}}{}_\beta \underset{\delta}{v}^\nu \overset{i}{v}_\sigma - 2\overset{i}{\underset{\delta}{T}}{}_\beta \underset{2+i}{v}{}^\nu \overset{\delta}{v}_\sigma)+$$

$$d_\beta^\sigma(\overset{\delta}{\underset{i}{T}}{}_\sigma \underset{\delta}{v}^\nu \overset{i}{v}_\alpha - \overset{i}{\underset{\delta}{T}}{}_\sigma \underset{i}{v}^\nu \overset{\delta}{v}_\alpha - \overset{\delta}{\underset{\bar{i}}{T}}{}_\sigma \underset{\delta}{v}^\nu \overset{\bar{i}}{v}_\alpha + \overset{\bar{i}}{\underset{\delta}{T}}{}_\sigma \underset{\bar{i}}{v}^\nu \overset{\delta}{v}_\alpha + 2\overset{\delta}{\underset{2+i}{T}}{}_\sigma \underset{\delta}{v}^\nu \overset{i}{v}_\alpha - 2\overset{i}{\underset{\delta}{T}}{}_\sigma \underset{2+i}{v}{}^\nu \overset{\delta}{v}_\alpha) = 0$$

The equalities received from (38) after contraction by $\underset{k}{v}^\alpha$ and $\underset{\bar{k}}{v}^\alpha$ are contracted once again by $\overset{s}{v}_\nu$ and $\overset{\bar{s}}{v}_\nu$. As a result of these operations we reach to (34) and (35).

Corollary 4 *If the net $\left(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v}\right)$ is chosen as a coordinate one then the composition $U_2 \times \overline{U}_2$ from the kind (g - g) characterizes by the following*

equalities for:
4.1) the coefficients of the derivative equations

$$\underset{\bar{k}}{\overset{s}{T}}\,{}_{\bar{i}} = 0$$

$$\underset{k}{\overset{\bar{s}}{T}}\,{}_{i} + \underset{k+2}{\overset{\bar{s}}{T}}\,{}_{i} - \underset{k}{\overset{\bar{s}-2}{T}}\,{}_{i} - \underset{k+2}{\overset{\bar{s}-2}{T}}\,{}_{i} + \underset{k}{\overset{\bar{s}}{T}}\,{}_{i+2} + \underset{k+2}{\overset{\bar{s}}{T}}\,{}_{i+2} - \underset{k}{\overset{\bar{s}-2}{T}}\,{}_{i+2} = 0$$

4.2) the coefficients of the connectedness
(40)

$$\Gamma^{s}_{\bar{k}\,\bar{i}} = 0,$$

$$\Gamma^{\bar{s}}_{i\,k} + \Gamma^{\bar{s}}_{i\,k+2} - \Gamma^{\bar{s}-2}_{i\;\;k} - \Gamma^{\bar{s}-2}_{i\;\;k+2} + \Gamma^{\bar{s}}_{i+2\,k} + \Gamma^{\bar{s}}_{i+2\,k+2} - \Gamma^{\bar{s}-2}_{i+2\,k} = 0$$

Proof: Let choose the net $\left(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}, \underset{4}{v}\right)$ as a coordinate one. Then taking into account (4), (6) and (37) we find the following presentation of the affinor d^{β}_{α}

(41)

$$(d^{\beta}_{\alpha}) = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

From (34), (35) and (41) we obtain the equalities (39), from where according to (20) follow (40) From [2] and the first equation of (39) it follows the validity of the statement:

Fact 4 *If the composition $U_2 \times \overline{U}_2$ is of the kind (g - g) , then the composition $X_2 \times \overline{X}_2$ is of the kind $(X_3 - g)$, i.e. the positions $P(\overline{X}_2)$ are parallelly translated along any line of $\overline{X}_2$.*

3. Geodesic compositions in spaces EqA_4

Let A_4 by an equiaffine space with a fundamental 4-vector $e_{\alpha\beta\gamma\delta}$. The quantity $e_{\alpha\beta\gamma\delta}\underset{1}{v}^{\alpha}\underset{2}{v}^{\beta}\underset{3}{v}^{\gamma}\underset{4}{v}^{\delta}$ is called an extent, defined by the vector fields $\underset{\alpha}{v}^{\sigma}$ It is known that the extent V preserves when the vectors $\underset{\alpha}{v}^{\sigma}$ are parallelly translated along any line in EqA_4 [1]. We denote by $e = e_{1234}$ the fundamental density of the equiaffine space EqA_4. The following characteristics for the equiaffine spaces are known: $\Gamma^{\sigma}_{\alpha\sigma} = \partial - \alpha \ln e, \nabla_{\nu} e_{\alpha\beta\gamma\delta} = 0, R_{\alpha\beta} = R_{\beta\alpha}$[1]

Proposition 1 *If in the equiaffine space EqA_4 with a fundamental 4-vector $e_{\alpha\beta\gamma\delta} = \overset{1}{v}_{[\alpha}\overset{2}{v}_{\beta}\overset{3}{v}_{\gamma}\overset{4}{v}_{\delta]}$, where $\overset{\sigma}{v}_{\alpha}$ are defined by (3), the compositions $X_2 \times \overline{X}_2, Y_2 \times \overline{Y}_2$ are the kind of $(g - g)$, then the space EqA_4 is affine*

Proof: From $\nabla e_{\alpha\beta\gamma\delta} = \nabla \overset{1}{v}_{[\alpha}\overset{2}{v}_{\beta}\overset{3}{v}_{\gamma}\overset{4}{v}_{\delta]} = 0$ and derivative equations (13) we obtain $\underset{\sigma}{\overset{\sigma}{T}}_{\nu} = 0$ which, according to (20), is equivalent to $\Gamma^{\delta}_{\nu\delta} = 0$. Since the compositions $X_2 \times \overline{X}_2, Y_2 \times \overline{Y}_2$ are the kind of (g-g), then on basis of Fact 2, (31) and $\Gamma^{\delta}_{\nu\delta} = \partial_{\nu} \ln e = 0$ we establish $\Gamma^{\delta}_{\nu\delta} = 0$, i.e. the space EqA_4 is affine.

Proposition 2 *If in the equiaffine EqA_4 with a fundamental density e the compositions $X_2 \times \overline{X}_2, Y_2 \times \overline{Y}_2$ are the kind of $(g-g)$ hen the components of the Richi?s tensor of the space EqA_4 have the following presentation:*
(42)

$$R_{13} = -\sigma_{13} \ln e, \quad R_{14} = -\sigma_{14} \ln e + \Gamma^{\alpha}_{14} \partial_{\alpha} \ln e + \partial_{\alpha} \Gamma^{\alpha}_{14},$$
$$R_{24} = -\sigma_{24} \ln e, \quad R_{23} = -\sigma_{23} \ln e - \Gamma^{\alpha}_{23} \partial_{\alpha} \ln e - \partial_{\alpha} \Gamma^{\alpha}_{23},$$

(43)

$$R_{ij} = -\delta_{ij} \ln e - \epsilon \Gamma^{\bar{i}}_{14} \Gamma^{\bar{j}}_{14}, \quad R_{\overline{i}\overline{j}} = -\delta_{\overline{i}\overline{j}} \ln e - \epsilon \Gamma^{i}_{14} \Gamma^{j}_{14}$$

where $\epsilon = 1$ for $i = j$ or $\bar{i} = \bar{j}, \epsilon = -1$ for $i \neq j$ or $\bar{i} \neq \bar{j}$ and for the indexes are fulfilled: $1 \leftrightarrow 4, 2 \leftrightarrow 3$.

Proof. According to [1] for the tensor of Richi in the space EqA_4 we have $R_{\beta\upsilon} = R_{\alpha\beta\upsilon}{}^{\alpha} \because = \delta_{\alpha} \Gamma^{\alpha}_{\beta\upsilon} - \delta_{\beta\upsilon} \ln e + \Gamma^{\rho}_{\beta\upsilon} \delta_{\rho} \ln e - \Gamma^{\alpha}_{\beta\rho} \Gamma^{\rho}_{\alpha\upsilon}$: Then applying (31) we find

(44)

$$R_{ij} = -\sigma_{ij} \ln e - \Gamma^{\bar{8}}_{i\bar{k}} \Gamma^{\bar{k}}_{\bar{8}j}, \qquad R_{\overline{i}\overline{j}} = -\sigma_{\overline{i}\overline{j}} \ln e - \Gamma^{8}_{\bar{i}k} \Gamma^{k}_{8\bar{j}}$$
$$R_{i\bar{j}} = \sigma_{\alpha} \Gamma^{\alpha}_{i\bar{j}} - \sigma_{i\bar{j}} \ln e + \Gamma^{\alpha}_{i\bar{j}} \sigma_{\alpha} \ln e$$

Now using (31) and (44) we obtain

$$R_{11} = -\delta_{11} \ln e - (\Gamma^{4}_{14})^2, \quad R_{33} = -\delta_{33} \ln e - (\Gamma^{2}_{14})^2,$$
$$R_{22} = -\delta_{22} \ln e - (\Gamma^{3}_{14})^2, \quad R_{44} = -\delta_{44} \ln e - (\Gamma^{1}_{14})^2,$$
$$R_{12} = -\delta_{12} \ln e + \Gamma^{4}_{14} \Gamma^{3}_{14}, \quad R_{34} = -\delta_{34} \ln e + \Gamma^{1}_{14} \Gamma^{2}_{14},$$

and (42). It is obviously that $R_{14} + R_{23} = -\delta_{14} \ln e - \delta_{23} \ln e$. □

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