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ON GENERALIZED RICCI-RECURRENT δ-LORENTZIAN TRANS-SASAKIAN MANIFOLDS

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Abstract. In this paper we study generalized Ricci-recurrent trans-Sasakian manifolds. It is proved that a generalized Ricci-recurrent δ-Lorentzian cosymplectic manifold is always recurrent. Generalized Ricci-recurrent δ-Lorentzian trans-Sasakian Manifolds of dimension ≥ 5 are locally classified. We have also proved that if M is one of the δ-Lorentzian Sasakian, δ-Lorentzian α- Sasakian, δ-Lorentzian Kenmotsu or δ-Lorentzian β-Kenmotsu manifoldswhich is generalized Ricci-recurrent with cyclic Ricci tensor and non-zero A(ξ) everywhere; then M is an Einstein manifold.

1. Introduction

Many authors recently have studied Lorentzian α - Sasakian manifolds [1] and Lorentzian β- Kenmotsu manifolds [9], [5]. In 2011, S.S.Pujar and V.J. Khairnar [12] have initiated the study of Lorentzian Trans-Sasakian manifolds and studied the basic results with some of its properties. Earlier to this , S. S. Pujar [14] has initiated the study of δ -Lorentzian α- Sasakian manifolds [5] and δ-Lorentzian β-Kenmotsu manifolds [12]

In 2010, S.S. Shukla and D.D.Singh [15] have studied ϵ- trans-Sasakian manifolds and its basic results and using these they deduced some of its interesting properties. Earlier to this in 1969 Takahashi [17] had introduced the notion of almost contact metric manifold equipped with pseudo Riemannian metric. In particular, he studied the Sasakian manifolds equipped with Riemannian metric g.These indefinite almost contact metric manifolds and indefinite Sasakian manifolds are also known as ϵ-almost contact metric manifolds and ϵ-Sasakian manifolds respectively.

Recently [16] and [10], we have observed that there does not exists a light like surface in the ϵ- Sasakian manifolds . On the other hand in almost para contact manifold defined by Motsumoto [7], the semi Riemannian manifold has the index 1 and the structure vector

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field ξ is always a time like.This motivated the Thripathi and others [17] to introduce ϵ-almost para contact structure where the vector field ξ is space like or time like according as $\epsilon = 1$ or $\epsilon = -1$.

A non-flat Riemannian manifold M is called a generalized Ricci- recurrent manifold [18] if its Ricci tensor S satisfies the condition

$$(\nabla_X S)(Y, Z) = A(X)S(Y, Z) + B(X)g(Y, Z), \tag{1.1}$$

where ∇ is Levi-Civita connection of the Riemannian metric g, and A, B are 1-forms on M. In particular, if the 1-form B vanishes identically, then M reduces to the well known Ricci-recurrent manifold [8].

In [16], S. Tanno classified connected almost contact metric manifolds whose automorphism groups possess the maximum dimension. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. He showed that they can be divided into three classes: (1) homogeneous normal contact Riemannian manifolds with $c > 0$, (2) global Riemannian products of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature if $c = 0$ and (3) a warped product space $R \times_f C^n$ if $c < 0$. It is known that the manifolds of class (1) are characterized by admitting a Sasakian structure. Kenmotsu [8] characterized the differential geometric properties of the manifolds of class (3); the structure so obtained is now known as Kenmotsu structure. In general, these structures are not Sasakian [8]. The paper is organized as follows:

In section 2,we introduce notion of $\delta-$Lorentzian trans-Sasakian manifold with an example and some basic results regarding such type of manifolds are also given. In section 3 for generalized Ricci-recurrent δ -Lorentzian trans-Sasakian manifold the relation between 1 forms A & B is establised. It is proved that a generalized Ricci-recurrent $\delta-$Lorentzian cosymplectic manifold is always Ricci-recurrent & generalized Ricci-recurrent $\delta-$Lorentzian trans-Sasakian manifolds of dimension ≥ 5 are also classified. In the last section, an expression for Ricci tensor of a generalized Ricci-recurrent $\delta-$Lorentzian trans-Sasakian manifold with cyclic Ricci tensor is obtained. It is also proved that if M is one of $\delta-$Lorentzian Sasakian, $\delta-$Lorentzian $\alpha-$Sasakian, $\delta-$Lorentzian Kenmotsu or $\delta-$Lorentzian $\beta-$Kenmotsu manifolds which is generalized Ricci-recurrent manifold with cyclic Ricci tensor and non-zero $A(\xi)$ everywhere, then M is an Einstein manifold.

2. Preliminaries

A $(2n + 1)$ dimensional manifold M, is said to be the δ- almost contact metric manifold if it admits a $(1, 1)$ tensor field ϕ, a structure tensor field ξ, a 1-form η and an indefinite metric g such that

$$\phi^2 X = X + \eta(X)\xi, \ \eta(\xi) = -1, \tag{2.1}$$

$$g(\xi, \xi) = -\delta, \ \eta(X) = \delta g(X, \xi), \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X, Y) + \delta\eta(X)\eta(Y) \tag{2.3}$$
$$g(X, \phi Y) = g(\phi X, Y)$$

for all vector fields X and Y on M , where δ is such that $\delta^2 = 1$ so The above structure $(\phi, \xi, \eta, G, \delta)$ on M is called the the $\delta-$ Lorentzian structure on M. If $\delta = 1$ and this is the usual Lorentzian structure [7] on M , the vector field ξ is the time like [1], that is M contains a time like vector field.

From the above equations , one can deduce that

$$\phi\xi = 0, \;\; \eta(\phi X) = 0 \tag{2.4}$$

Example 1. *Let us consider the* 3*-dimensional manifold* $M = \{(x, y, z) \in R^3\}$*,where* x, y, z *are the co-ordinates of a point in* R^3*.Let* $\{e_1, e_2, e_3\}$ *be the global frames on* M *given by*

$$e_1 = e^z(\frac{\partial}{\partial x} + y\frac{\partial}{\partial z}), e_2 = e^z\frac{\partial}{\partial y}, e_3 = e^z\frac{\partial}{\partial z}$$

Let g be the δ- Lorentzian metric on M defined by

$$g(e_1, e_2) = g(e_2, e_3) \;= g(e_1, e_3) = 0$$

and

$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = -\delta$$

where $\delta = \pm 1$. Then δ-Lorentzian indefinite metric g on M is in the following form:

$$g = \{e^{-2z} - \delta y^2\}(dx^2) + e^{-2z}(dy^2) - \delta e^{-2z}(dz^2) + 2\delta y e^{-2z}dxdy$$

Let $e_3 = \xi$. Let η be the 1-form defined by

$$\eta(U) = \delta g(U, e_3)$$

for any vector field U on M. Let ϕ be the $(1, 1)$ tensor field defined by

$$\phi(e_1) = e_2, \phi(e_2) = e_1, \phi(e_3) = 0$$

Then using linearity of ϕ and g and taking $e_3 = \xi$, one obtains

$$\phi(e_1) = e_2, \phi(e_2) = e_1, \phi(e_3) = 0$$

and

$$g(\phi U, \phi W) = g(U, W) + \delta\eta(U)\eta(W)$$

for any vector fields X and Y on M. Also putting $W = \xi$ in above equation,one can see that

$$\eta(U) = \delta g(U, \xi)$$

Putting $W = U = \xi$ in both the above equations respectively, we have

$$g(\xi, \xi) = -\delta, \eta(\xi) = -1$$

Clearly from $g(\phi U, \phi W) = g(U, W) + \delta\eta(U)\eta(W), \phi$ is symmetric. Thus $(\phi, \xi, \eta, g, \delta)$ defines δ- Lorentzian contact metric structure on M.

A δ- Lorentzian manifold with structure $(\phi, \xi, \eta, g, \delta)$ is said to be δ- Lorentzian trans-Sasakian manifold M of type (α, β) if it satisfies the condition

$$(\nabla_X \phi)(Y) = \alpha\{g(X,Y)\xi - \delta\eta(Y)X\} + \beta\{g(\phi X, Y)\xi - \delta\eta(Y)\phi X \tag{2.5}$$

for any vector fields X and Y on M.

If $\delta = 1$, then the δ- Lorentzian trans-Sasakian manifold is the usual Lorentzian trans-Sasakian manifold of type (α, β) [12]. δ-Lorentzian trans-Sasakian manifold of type $(0,0), (0,\beta), (\alpha, 0)$ are the Lorentzian cosympletic, Lorentzian β- Kenmotsu and Lorentzian α-Sasakian manifolds respectively. In particular if $\alpha = 1, \beta = 0$, and $\alpha = 0$, $\beta = 1$, then δ- Lorentzian trans-Sasakian manifold reduces to δ-Lorentzian Sasakian and δ- Lorentzian Kenmotsu manifolds respectively.

3. Generalized Ricci-recurrent $\delta-$Lorentzian Trans-Sasakian Manifolds

Let M be a $(2n+1)$ dimensional δ- Lorentzian trans-Sasakian manifold. From (2.5), we have

$$\nabla_X \xi = \delta\{-\alpha\phi X - \beta(X + \eta(X)\xi)\}, \tag{3.1}$$

$$(\nabla_X \eta)(Y) = \alpha g(\phi X, Y) + \beta\{g(X,Y) + \delta\eta(X)\eta(Y)\} \tag{3.2}$$

From equations (2.5) , (3.1), (3.2) we have following lemma.

Lemma 3.1. *In a $(2n+1)$ dimensional $\delta-$Lorentzian trans-Sasakian manifold, we have*

$$\begin{aligned} R(X,Y)\xi &= (\alpha^2+\beta^2)\{\eta(Y)X - \eta(X)Y\} + 2\alpha\beta\{\eta(Y)\phi X - \eta(X)\phi Y\} \\ &\quad +\delta\{-(X\alpha)\phi Y + (Y\alpha)\phi X - (X\beta)\phi^2 Y + (Y\beta)\phi^2 X\}, \end{aligned} \tag{3.3}$$

$$\begin{aligned} S(X,\xi) &= \{2n(\alpha^2+\beta^2) - \delta(\xi\beta)\}\eta(X) + (2n-1)\delta(X\beta) \\ &\quad +\{2\alpha\beta\eta(X) + \delta(X\alpha)\}f + \delta(\phi X)\alpha \end{aligned} \tag{3.4}$$

where R & S are curvature and Ricci curvature tensors. In particular, we have,

$$S(\xi,\xi) = -2n(\alpha^2 + \beta^2 - \delta(\xi\beta) \tag{3.5}$$

$$2\alpha\beta - \delta(\xi\alpha) = 0 \tag{3.6}$$

Now we prove the following

Theorem 3.2. *Let M be a $(2n+1)$ dimensional generalized Ricci-recurrent $\delta-$Lorentzian trans-Sasakian manifold. Then the $1-$forms A & B are related by*

$$\begin{aligned} \delta B(X) &= 2n\{X(\alpha^2+\beta^2-\delta(\xi\beta)) - (\alpha^2+\beta^2-\delta(\xi\beta))A(X)\} \\ &\quad -2(2n-1)\{\alpha\phi X + \beta\phi^2 X\}\beta - 2\{\alpha\phi^2 X + \beta\phi X\}\alpha \\ &\quad -2\{(\alpha\phi X + \beta\phi^2 X)\alpha\}f \end{aligned} \tag{3.7}$$

In particular, we get

$$\delta B(\xi) = 2n\{\xi(\alpha^2+\beta^2-\delta(\xi\beta)) - (\alpha^2+\beta^2-\delta(\xi\beta))A(\xi)\} \tag{3.8}$$

Proof. Using (1.1) in

$$(\nabla_X S)(Y,Z) = XS(Y,Z) - S(\nabla_X Y, Z) - S(Y, \nabla_X Z), \tag{3.9}$$

we get

$$A(X)S(Y,Z) + B(X)g(Y,Z) = XS(Y,Z) - S(\nabla_X Y,)Z - S(Y, \nabla_X Z). \tag{3.10}$$

Putting $Y = Z = \xi$,in the above equation we obtain

$$S(\xi,\xi)A(X) + B(X) = XS(\xi,\xi) - 2S(\nabla_X \xi, \xi), \tag{3.11}$$

which in view of (3.5), (2.3) & (3.1) yields (3.7). The equation (3.8) is obvious from (3.7). □

Let A^* & B^* be the associated vector fields of A & B, that is,

$$g(X, A^*) = A(X) \text{ and } g(X, B^*) = B(X).$$

Corollary 3.3. *In a (2n+1)-dimensional generalized Ricci-recurrent δ-Lorentzian $\alpha-$Sasakian (resp. δ-Lorentzian Sasakian) manifold, we have*

$$\delta B = -2n\alpha^2 A \ (resp. \delta B = -2nA) \tag{3.12}$$

Thus, the associated vector fields A^* & B^* are in opposite direction if $\delta = 1$ that is, structure vector field ξ is space like.

Proof. A δ-Lorentzian trans-Sasakian manifold of type $(\alpha, 0)$ is δ-Lorentzian α-Sasakian [12]. In this case α becomes a constant. If $\alpha = 1$, then δ-Lorentzian α-Sasakian manifold is δ-Lorentzian Sasakian. Thus, from the equation (3.7), the proof follows immediately. □

Corollary 3.4. *In a (2n+1)-dimensional generalized Ricci-recurrent normal almost δ-Lorentzian $f-$structure (or $f-$Kenmotsu) manifold we have*

$$\delta B(X) = 2n\{X(f^2 - \delta(\xi f)) - (f^2 - \delta(\xi f))A(X)\} - 2(2n-1)f(\phi^2 X)f. \tag{3.13}$$

Proof. A $\delta-$Lorentzian trans-Sasakian structure with $\alpha = 0$ and $\beta = f$ is a normal almost cosympectic $\delta-$Lorentzian f-structure [12] (or $\delta-$Lorentzian f-Kenmotsu structure [12]). Thus, putting $\alpha = 0$ and $\beta = f$ in the equation (3.7), we get (3.13). □

Corollary 3.5. *For a $(2n+1)$-dimensional generalized Ricci-recurrent $\delta-$Lorentzian β-Kenmotsu (resp. $\delta-$Lorentzian Kenmotsu) manifold, we have*

$$\delta B = -2n\beta^2 A \ (resp. \ \delta B = -2nA) \tag{3.14}$$

Thus, the associated vector fields A^* and B^* are in same direction if $\delta = -1$, that is structure vector field ξ is time like.

Proof. A $\delta-$ Lorentzian trans-Sasakian structure is $\delta-$ Lorentzian β-Kenmotsu [12] if $\alpha = 0$ and $\beta =$ constant. In particular, 1-Kenmotsu structure is a Kenmotsu structure. Putting $f = \beta =$ constant (resp. $f = 1$) in (3.13), we obtain (3.14). □

A $\delta-$Lorentzian trans-Sasakian structures of type $(0, 0)$ is cosymplectic [12]. Thus, putting $\alpha = 0 = \beta$ in (3.7), we get $B = 0$. Hence, we have the following theorem:

Theorem 3.6. *A generalized Ricci-recurrent $\delta-$Lorentzian cosymplectic manifold M is always Ricci-recurrent.*

Now for a generalized Ricci-recurrent $\delta-$Lorentzian trans-Sasakian manifold of dimension ≥ 5 locally, we give the following classification.

Theorem 3.7. *Let M be a generalized Ricci-recurrent $\delta-$Lorentzian trans-Sasakian manifold of dimension $(2n+1) \geq 5$. Then*

1. either M is Ricci-recurrent,
2. or $\delta B + 2n\alpha^2 A = 0$,
3. or $\delta B + 2n\beta^2 A = 0$,

where α & β are non-zero constants.

Proof. We know that locally a $\delta-$Lorentzian trans-Sasakian manifold of dimension $\geq$ 5 is either $\delta-$Lorentzian cosymplectic, or $\delta-$Lorentzian α-Sasakian or $\delta-$Lorentzian $\beta-$Kenmotsu manifold [12]. Hence, in view of Corollaries 1, 3 and Theorem 2, the proof is complete. □

4. Generalized Ricci-recurrent $\delta-$Lorentzian trans-Sasakian manifolds with cyclic Ricci tensor

A Riemannian manifold is said to admit cyclic Ricci tensor if

$$(\nabla_X S)(Y,Z) + (\nabla_Y S)(Z,X) + (\nabla_Z S)(X,Y) = 0 \tag{4.1}$$

Now we prove the following:

Theorem 4.1. *In a $(2n+1)$-dimensional generalized Ricci-recurrent $\delta-$Lorentzian trans-Sasakian manifold with cyclic Ricci tensor, the Ricci tensor satisfies*

$$\begin{aligned}
& \delta A(\xi)S(X,Y) \\
= \ & 2n\{(\alpha^2+\beta^2-\delta(\xi\beta))A(\xi) - \xi(\alpha^2+\beta^2-\delta(\xi\beta))\}g(X,Y) \\
& -(2n-1)\delta\{A(X)Y\beta + A(Y)X\beta\} \\
& -(2n-1)\delta(\xi\beta)\{\eta(Y)A(X)+\eta(X)A(Y)\} \\
& -\delta\{A(X)(\phi Y)\alpha + A(Y)(\phi X)\alpha\} \\
& -2\alpha\beta\{\eta(Y)A(X)+\eta(X)A(Y)\}f \\
& -\delta\{A(X)Y\alpha + A(Y)X\alpha\}f \\
& -2n\{\eta(X)Y(\alpha^2+\beta^2-\delta(\xi\beta)+\eta(Y)X(\alpha^2+\beta^2-\delta(\xi\beta)\} \\
& +2(2n-1)\{\eta(X)(\alpha\phi Y+\beta\phi^2 Y)\beta + \eta(Y)(\alpha\phi X+\beta\phi^2 X)\beta\} \\
& +2\{\eta(X)(\alpha\phi^2 Y+\beta\phi Y)\alpha + \eta(Y)(\alpha\phi^2 X+\beta\phi X)\alpha\} \\
& +2[\{(\alpha\phi X+\beta\phi^2 X)\alpha\}f]\eta(Y) + 2[\{(\alpha\phi Y+\beta\phi^2 Y)\alpha\}f]\eta(X)
\end{aligned} \tag{4.2}$$

Proof. Suppose that M is a generalized Ricci symmetric manifold admitting cyclic Ricci tensor. Then in view of (1.1) and (4.1), we get,

$$\begin{aligned} 0 &= A(X)S(Y,Z)+B(X)g(Y,Z)+A(Y)S(Z,X) \\ &\quad +B(Y)g(Z,X)+A(Z)S(X,Y)+B(Z)g(X,Y) \end{aligned} \tag{4.3}$$

Put $Z=\xi$ in the above equation, we get

$$\begin{aligned} A(\xi)S(X,Y) &= -B(\xi)g(X,Y)-A(X)S(Y,\xi)-A(Y)S(X,\xi) \\ &\quad -B(X)g(Y,\xi)+B(Y)S(X,\xi) \end{aligned} \tag{4.4}$$

Using (3.8) & (3.4) in (4.4) , we get (4.2). □

Corollary 4.2. *For a $(2n+1)$-dimensional generalized Ricci-recurrent manifold M with cyclic Ricci tensor, we have the following results:*

1. If M is an $\delta-$Lorentzian α-Sasakian manifold, then

$$\delta A(\xi)S(X,Y)=2n\alpha^2A(\xi)g(X,Y)$$

2. If M is an $\delta-$Lorentzian Sasakian manifold, then

$$\delta A(\xi)S(X,Y)=2nA(\xi)g(X,Y)$$

3. If M is a $\delta-$Lorentzian $f-$Kenmotsu manifold, then

$$\begin{aligned} \delta A(\xi)S(X,Y) &= 2n\{A(\xi)(f^2-\delta(\xi f))-\xi(f^2-\delta(\xi f))\}g(X,Y) \\ &\quad -(2n-1)\delta\{A(X)Yf+A(Y)Xf\} \\ &\quad -(2n-1)\delta(\xi f)\{\eta(Y)A(X)+\eta(X)A(Y)\} \\ &\quad -2n\{\eta(X)Y(f^2-\delta(\xi f))+\eta(Y)X(f^2-\delta(\xi f))\} \\ &\quad +2(2n-1)\{\eta(X)(f\phi^2Y)f+\eta(Y)(f\phi^2X)f\} \end{aligned}$$

4. If M is a $\delta-$Lorentzian $\beta-$Kenmotsu manifold, then

$$\delta A(\xi)S(X,Y)=2n\beta^2A(\xi)g(X,Y)$$

5.If M is a $\delta-$Lorentzian Kenmotsu manifold, then

$$\delta A(\xi)S(X,Y)=2nA(\xi)g(X,Y)$$

6.If M is a $\delta-$Lorentzian cosymplectic manifold, then

$$\delta A(\xi)S(X,Y)=0$$

Since $\delta\neq 0$,we have

$$A(\xi)S(X,Y)=0$$

As we know that a Riemannian manifold is Einstein if

$$S(X,Y)=\rho g(X,Y).$$

Therefore, in view of corollory (4), we have following theorem;

Theorem 4.3. *Let M be generalized Ricci-recurrent manifold with cyclic Ricci tensor. If M is one of $\delta-$Lorentzian $\alpha-$Sasakian, $\delta-$Lorentzian Sasakian, $\delta-$Lorentzian Kenmotsu and $\delta-$Lorentzian $\beta-$Kenmotsu manifolds with non-zero $A(\xi)$ everywhere, then M is Einstein.*

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