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Some Properties of θ-open Sets

Algunas Propiedades de los Conjuntos θ-abiertos

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Abstract

In the present paper, we introduce and study topological properties of θ-derived, θ-border, θ-frontier and θ-exterior of a set using the concept of θ-open sets and study also other properties of the well known notions of θ-closure and θ-interior.
Key words and phrases: θ-open, θ-closure, θ-interior, θ-border, θ-frontier, θ-exterior.

Resumen

En el presente ertículo se introducen y estudian las propiedades topológicas del θ-derivedo, θ-borde, θ-frontera y θ-exterior de un conjunto usando el concepto de conjunto θ-abierto y estudiando también otras propiedades de las nociones bien conocidas de θ-clausura y θ-interior.
Palabras y frases clave: θ-abierto, θ-clausura, θ-interior, θ-borde, θ-frontera, θ-exterior.

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1 Introduction

The notions of θ-open subsets, θ-closed subsets and θ-closure where introduced by Veličko [14] for the purpose of studying the important class of H-closed spaces in terms of arbitrary fiberbases. Dickman and Porter [2], [3], Joseph [9] and Long and Herrington [11] continued the work of Veličko . Recently Noiri and Jafari [12] and Jafari [6] have also obtained several new and interesting results related to these sets. For these sets, we introduce the notions of θ-derived, θ-border, θ-frontier and θ-exterior of a set and show that some of their properties are analogous to those for open sets. Also, we give some additional properties of θ-closure and θ-interior of a set due to Veličko [14]. In what follows (X, τ) (or X) denotes topological spaces. We denote the interior and the closure of a subset A of X by Int(A) and $Cl(A)$, respectively. A point $x \in X$ is called a θ-adherent point of A [14], if $A \cap Cl(V) \neq \emptyset$ for every open set V containing x. The set of all θ-adherent points of A is called the θ-closure of A and is denoted by $Cl_\theta(A)$. A subset A of X is called θ-closed if $A = Cl_\theta(A)$. Dontchev and Maki [[4], Lemma 3.9] have shown that if A and B are subsets of a space (X, τ), then $Cl_\theta(A \cup B) = Cl_\theta(A) \cup Cl_\theta(B)$ and $Cl_\theta(A \cap B) = Cl_\theta(A) \cap Cl_\theta(B)$. Note also that the θ-closure of a given set need not be a θ-closed set. But it is always closed. Dickman and Porter [2] proved that a compact subspace of a Hausdorff space is θ-closed. Moreover, they showed that a θ-closed subspace of a Hausdorff space is closed. Janković [7] proved that a space (X, τ) is Hausdorff if and only if every compact set is θ-closed. The complement of a θ-closed set is called a θ-open set. The family of all θ-open sets forms a topology on X and is denoted by τ_θ. This topology is coarser than τ and it is well-known that a space (X, τ) is regular if and only if $\tau = \tau_\theta$. It is also obvious that a set A is θ-closed in (X, τ) if and only if it is closed in (X, τ_θ).

Recall that a point $x \in X$ is called the δ-cluster point of $A \subseteq X$ if $A \cap Int(Cl(U)) \neq \emptyset$ for every open set U of X containing x. The set of all δ-cluster points of A is called the δ-closure of A, denoted by $Cl_\delta(A)$. A subset $A \subseteq X$ is called δ-closed if $A = Cl_\delta(A)$. The complement of a δ-closed set is called δ-open. It is worth to be noticed that the family of all δ-open subsets of (X, τ) is a topology on X which is denoted by τ_δ. The space (X, τ_δ) is called sometimes the semi-regularization of (X, τ). As a consequence of definitions, we have $\tau_\theta \subseteq \tau_\delta \subseteq \tau$, also $A \subseteq Cl(A) \subseteq Cl_\delta(A) \subseteq Cl_\theta(A) \subseteq \bar{A}^\theta$, where $\bar{A}^\theta$ denotes the closure of A with respect to (X, τ_θ) (see [1]).

A subset A of a space X is called preopen (resp. semi-open, α-open) if $A \subset Int(Cl(A))$ (resp. $A \subset Cl(Int(A))$, $A \subset Int(Cl(Int(A)))$). The complement of a semi-open (resp. α-open) set is said to be semi-closed (resp.

α-closed). The intersection of all semi-closed (resp. α-closed) sets containing A is called the semi-closure (resp. α-closure) of A and is denoted by $sCl(A)$ (resp. $\alpha Cl(A)$). Recall also that a space (X, τ) is called extremally disconnected if the closure of each open set is open. Ganster et al. [[5], Lemma 0.3] have shown that For $A \subset X$, we have $A \subseteq sCl(A) \subseteq Cl_\theta(A)$ and also if (X, τ) is extremally disconnected and A is a semi-open set in X, then $sCl(A) = Cl(A) = Cl_\theta(A)$. Moreover, it is well-known that if a set is preopen, then the concepts of α-closure, δ-closure, closure and θ-closure coincide. In [13], M. Steiner has obtained some results concerning some characterizations of some generalizations of T_1 spaces by utilizing θ-open and δ-open sets. Also, quite recently Cao et al. [1] obtained, among others, some substantial results concerning the θ-closure operator and the related notions. In general, we do not know much about θ-open sets and dealing with them are very difficult.

2 Properties of θ-open Sets

Definition 1. Let A be a subset of a space X. A point $x \in X$ is said to be θ-limit point of A if for each θ-open set U containing x , $U \cap (A \backslash \{x\}) \neq \emptyset$. The set of all θ-limit points of A is called the θ-derived set of A and is denoted by $D_\theta(A)$.

Theorem 2.1. *For subsets A, B of a space X, the following statements hold:*
(1) $D(A) \subset D_\theta(A)$ where $D(A)$ is the derived set of A.
(2) If $A \subset B$, then $D_\theta(A) \subset D_\theta(B)$.
(3) $D_\theta(A) \cup D_\theta(B) = D_\theta(A \cup B)$ and $D_\theta(A \cap B) \subset D_\theta(A) \cap D_\theta(B)$.
(4) $D_\theta(D_\theta(A)) \backslash A \subset D_\theta(A)$.
(5) $D_\theta(A \cup D_\theta(A)) \subset A \cup D_\theta(A)$.

Proof. (1) It suffices to observe that every θ-open set is open.
(3) $D_\theta(A \cup B) = D_\theta(A) \cup D_\theta(B)$ is a modification of the standard proof for D, where open sets are replaced by θ-open sets.
(4) If $x \in D_\theta(D_\theta(A)) \backslash A$ and U is a θ-open set containing x, then $U \cap (D_\theta(A) \backslash \{x\}) \neq \emptyset$. Let $y \in U \cap (D_\theta(A) \backslash \{x\})$. Then since $y \in D_\theta(A)$ and $y \in U$, $U \cap (A \backslash \{y\}) \neq \emptyset$. Let $z \in U \cap (A \backslash \{y\})$. Then $z \neq x$ for $z \in A$ and $x \notin A$. Hence $U \cap (A \backslash \{x\}) \neq \emptyset$. Therefore $x \in D_\theta(A)$.
(5) Let $x \in D_\theta(A \cup D_\theta(A))$. If $x \in A$, the result is obvious. So let $x \in D_\theta(A \cup D_\theta(A)) \backslash A$, then for θ-open set U containing x, $U \cap (A \cup D_\theta(A) \backslash \{x\}) \neq \emptyset$. Thus $U \cap (A \backslash \{x\}) \neq \emptyset$ or $U \cap (D_\theta(A) \backslash \{x\}) \neq \emptyset$. Now it follows similarly from (4) that $U \cap (A \backslash \{x\}) \neq \emptyset$. Hence $x \in D_\theta(A)$. Therefore, in any case

$D_\theta(A \cup D_\theta(A)) \subset A \cup D_\theta(A)$.

In general the equality of (1) and (3) does not hold.

Example 2.2. (i) Let $X = I \times 2$ has the product topology, where $I =< 0, 1 >$ has the Euclidean topology and $2 = \{0, 1\}$ has the Serpiński topology with the singleton $\{0\}$ open. Then $A \subset X$ is θ-closed (θ-open, respectively) if and only if $A = B \times 2$, where $B \subset I$ is closed (open, respectively).

Observe that if $A \subset X$ is θ-closed, then $Cl_\theta(A) = A$. Let $B = \pi_I(A) \subset I$. Obviously, $A \subset B \times 2$. Let $(x, y) \in B \times 2$. Then $x \in B$, so there is some $(x', y') \in A$, such that $\pi_I(x', y') = x$. Hence $x' = x$, so $(x, y') \in A$. Let H be a closed neighborhood of (x, y). Then H contains both of the points $(x, 0)$, $(x, 1)$ and so H contains (x, y') as well. It follows that $H \cap A \neq \emptyset$ and then, $(x, y) \in Cl_\theta(A) = A$. Hence, $A = B \times 2$. Let $z \in I \backslash B$. Then $(z, 0) \notin A$. Since A is θ-closed, there exist $\epsilon > 0$ such that $(< z - \epsilon, z + \epsilon > \times 2) \cap A = \emptyset$. Then $< z - \epsilon, z + \epsilon > \cap B = \emptyset$, which means that B is closed.
Let $A = I \times \{1\}$. Then $D_\theta(A) = X$ but $D(A) = A$. Hence $D_\theta(A) \not\subset D(A)$.

(ii) A counterexample illustrating that $D_\theta(A \cap B) \neq D_\theta(A) \cap D_\theta(B)$ in general can be easily found in regular spaces (e.g. in $\mathbf{R}$), for which open and θ-open sets (and hence D and D_θ) coincide.

Example 2.3. Let $(Z, \mathcal{K})$ be the digital n-space –the digital line or the so called Khalimsky line. This is the set of the integers, Z, equipped with the topology $\mathcal{K}$, generated by :
$\mathcal{G}_\mathcal{K} = \{\{2n - 1, 2n, 2n + 1\} : n \in Z\}$. Then [4]: If $A = \{x\}$
(i) $Cl_\theta(A) \neq Cl(A)$ if x is even.
(ii) $Cl_\theta(A) = Cl(A)$ if x is odd.

Theorem 2.4. $A \cup D_\theta(A) \subset Cl_\theta(A)$.

Proof. Since $D_\theta(A) \subset Cl_\theta(A)$, $A \cup D_\theta(A) \subset Cl_\theta(A)$.

Corollary 2.5. *If A is a θ-closed subset, then it contains the set of its θ-limit points.*

Definition 2. A point $x \in X$ is said to be a θ-interior point of A if there exists an open set U containing x such that $U \subset Cl(U) \subset A$. The set of all θ-interior points of A is said to be the θ-interior of A [9] and is denoted by $Int_\theta(A)$.

It is obvious that an open set U in X is θ-open if $Int_\theta(U) = U$ [[11], Definition 1].

Theorem 2.6. *For subsets A, B of a space X, the following statements are true:*
(1) $Int_\theta(A)$ is the union of all open sets of X whose closures are contained in A.
(2) A is θ-open if and only if $A = Int_\theta(A)$.
(3) $Int_\theta(Int_\theta(A)) \subset Int_\theta(A)$.
(4) $X \backslash Int_\theta(A) = Cl_\theta(X \backslash A)$.
(5) $X \backslash Cl_\theta(A) = Int_\theta(X \backslash A)$.
(6) $A \subset B$, then $Int_\theta(A) \subset Int_\theta(B)$.
(7) $Int_\theta(A) \cup Int_\theta(B) \subset Int_\theta(A \cup B)$.
(8) $Int_\theta(A) \cap Int_\theta(B) = Int_\theta(A \cap B)$.

Proof. (5) $X \backslash Int_\theta(A) = \cap\{F \in X \mid A \subset Int(F), (F closed)\} = Cl_\theta(X \backslash A)$.

Definition 3. $b_\theta(A) = A \backslash Int_\theta(A)$ is said to be the θ-border of A.

Theorem 2.7. *For a subset A of a space X, the following statements hold:*
(1) $b(A) \subset b_\theta(A)$ where $b(A)$ denotes the border of A.
(2) $A = Int_\theta(A) \cup b_\theta(A)$.
(3) $Int_\theta(A) \cap b_\theta(A) = \emptyset$.
(4) A is a θ-open set if and only if $b_\theta(A) = \emptyset$.
(5) $Int_\theta(b_\theta(A)) = \emptyset$.
(7) $b_\theta(b_\theta(A)) = b_\theta(A)$
(8) $b_\theta(A) = A \cap Cl_\theta(X \backslash A)$.

Proof. (5) If $x \in Int_\theta(b_p(A))$, then $x \in b_\theta(A)$. On the other hand, since $b_\theta(A) \subset A$, $x \in Int_\theta(b_p(A)) \subset Int_\theta(A)$. Hence $x \in Int_\theta(A) \cap b_\theta(A)$ which contradicts (3). Thus $Int_\theta(b_p(A)) = \emptyset$.
(8) $b_\theta(A) = A \backslash Int_\theta(A) = A \backslash (X \backslash Cl_\theta(X \backslash A) = A \cap Cl_\theta(X \backslash A)$.

Example 2.8. Let $X = \{a, b, c\}$ with $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$. Then it can be easily verified that for $A = \{b\}$,
we obtain $b_\theta(A) \not\subset b(A)$, i.e., in general equality of Theorem 2.7(1) does not hold.

Definition 4. $Fr_\theta(A) = Cl_\theta(A) \backslash Int_\theta(A)$ is said to be the θ-frontier [6] of A.

Theorem 2.9. *For a subset A of a space X, the following statements hold:*
(1) $Fr(A) \subset Fr_\theta(A)$ where $Fr(A)$ denotes the frontier of A.
(2) $Cl_\theta(A) = Int_\theta(A) \cup Fr_\theta(A)$.
(3) $Int_\theta(A) \cap Fr_\theta(A) = \emptyset$.
(4) $b_\theta(A) \subset Fr_\theta(A)$.
(5) $Fr_\theta(A) = Cl_\theta(A) \cap Cl_\theta(X \backslash A)$.

(6) $Fr_\theta(A) = Fr_\theta(X\backslash A)$.
(7) $Fr_\theta(A)$ *is closed.*
(8) $Int_\theta(A) = A\backslash Fr_\theta(A)$.

Proof. (2) $Int_\theta(A) \cup Fr_\theta(A) = Int_\theta(A) \cup (Cl_\theta(A)\backslash Int_\theta(A)) = Cl_\theta(A)$.
(3) $Int_\theta(A) \cap Fr_\theta(A) = Int_\theta(A) \cap (Cl_\theta(A)\backslash Int_\theta(A)) = \emptyset$.
(5) $Fr_\theta(A) = Cl_\theta(A)\backslash Int_\theta(A) = Cl_\theta(A) \cap Cl_\theta(X\backslash A)$.
(8) $A\backslash Fr_\theta(A) = A\backslash(Cl_\theta(A)\backslash Int_\theta(A)) = Int_\theta(A)$.

In general, the equalities in (1) and (4) of the Theorem 2.9 do not hold as it is shown by the following example.

Example 2.10. Consider the topological space (X, τ) given in Example 2.8. If $A = \{b\}$. Then $Fr_\theta(A) = \{b, c\} \not\subset \{c\} = Fr(A)$ and also $Fr_\theta(A) = \{b, c\} \not\subset \{b\} = b_\theta(A)$.

Remark 2.11. Let A and if B subsets of X. Then $A \subset B$ does not imply that either $Fr_\theta(B) \subset Fr_\theta(A)$ or $Fr_\theta(A) \subset Fr_\theta(B)$. The reader can be verify this readily.

Definition 5. $Ext_\theta(A) = Int_\theta(X\backslash A)$ is said to be be a θ-exterior of A.

Theorem 2.12. *For a subset A of a space X, the following statements hold:*
(1) $Ext_\theta(A) \subset Ext(A)$ *where* $Ext(A)$ *denotes the exterior of* A.
(2) $Ext_\theta(A)$ *is open.*
(3) $Ext_\theta(A) = Int_\theta(X\backslash A) = X\backslash Cl_\theta(A)$.
(4) $Ext_\theta(Ext_\theta(A)) = Int_\theta(Cl_\theta(A))$.
(5) If $A \subset B$, *then* $Ext_\theta(A) \supset Ext_\theta(B)$.
(6) $Ext_\theta(A \cup B) = Ext_\theta(A) \cup Ext_\theta(B)$.
(7) $Ext_\theta(A \cap B) \supset Ext_\theta(A) \cap Ext_\theta(B)$.
(8) $Ext_\theta(X) = \emptyset$.
(9) $Ext_\theta(\emptyset) = X$.
(10) $Ext_\theta(X\backslash Ext_\theta(A)) \subset Ext_\theta(A)$.
(11) $Int_\theta(A) \subset Ext_\theta(Ext_\theta(A))$.
(12) $X = Int_\theta(A) \cup Ext_\theta(A) \cup Fr_\theta(A)$.

Proof. (4) $Ext_\theta(Ext_\theta(A)) = Ext_\theta(X\backslash Cl_\theta(A)) = Int_\theta(X\backslash(X\backslash Cl_\theta(A))) = Int_\theta(Cl_\theta(A))$.
(10) $Ext_\theta(X\backslash Ext_\theta(A)) = Ext_\theta(X\backslash Int_\theta(X\backslash A)) = Int_\theta(X\backslash(X\backslash Int_\theta(X\backslash A))) = Int_\theta(Int_\theta(X\backslash A)) \subset Int_\theta(X\backslash A) = Ext_\theta(A)$.
(11) $Int_\theta(A) \subset .Int_\theta(Cl_\theta(A)) = Int_\theta(X\backslash Int_\theta(X\backslash A))) = Int_\theta(X\backslash Ext_\theta(A)) = Ext_\theta(Ext_\theta(A))$.

3 Aplications of θ-open Sets

Definition 6. Let X be a topological space. A set $A \subset X$ is said to be θ-saturated if for every $x \in A$ it follows $Cl_\theta(\{x\}) \subset A$. The set of all θ-saturated sets in X we denote by $B_\theta(X)$.

Theorem 3.1. *Let X be a topological space. Then $B_\theta(X)$ is a complete Boolean set algebra.*

Proof. We will prove that all the unions and complements of elements of $B_\theta(X)$ are members of $B_\theta(X)$. Obviously, only the proof regarding the complements is not trivial. Let $A \in B_\theta(X)$ and suppose that $Cl_\theta(\{x\}) \not\subset X\backslash A$ for some $x \in X\backslash A$. Then there exists $y \in A$ such that $y \in Cl_\theta(\{x\})$. It follows that x, y have no disjoint neighbourhoods. Then $x \in Cl_\theta(\{y\})$. But this is a contradiction, because by the definition of $B_\theta(X)$ we have $Cl_\theta(\{y\}) \subset A$. Hence, $Cl_\theta(\{x\}) \subset X\backslash A$ for every $x \in X\backslash A$, which implies $X\backslash A \in B_\theta(X)$.

Corollary 3.2. *$B_\theta(X)$ contains every union and every intersection of θ-closed and θ-open sets in X.*

A filter base Φ in X has a θ-cluster point $x \in X$ if $x \in \cap\{Cl_\theta(F) \mid F \in \Phi\}$. The filter base Φ θ-converges to its θ-limit x if for every closed neighbourhood H of x there is $F \in \Phi$ such that $F \subset H$. A net $f(B, \geq)$ has a θ-cluster point (a θ-limit) $x \in X$ if x is a θ-cluster point (a θ-limit) of the derived filter base $\{f(\alpha) \mid \alpha \geq \beta \mid \beta \in B\}$.

Recall that a topological space X is said to be (countably) θ-regular [5], [7] if every (countable) filter base in X with a θ-cluster point has a cluster point. Obviously, a space X is θ-regular if and only if every θ-convergent net in X has a cluster point.

Theorem 3.3. *Let X be a θ-regular topological space. Then every element of $B_\theta(X)$ is θ-regular.*

Proof. Let $f(B, \geq)$ be a net in $Y \in B_\theta(X)$, which θ-converges to $y \in Y$ in the topology of Y. Then $f(B, \geq)$ θ-converges to y in X and hence, $f(B, \geq)$ has a cluster point $x \in X$. One can easily check that x, y have no disjoint neighbourhoods in X, which implies that $x \in Cl_\theta(\{y\})$ and hence $x \in Y$. Then every θ-convergent net in Y has a cluster point in Y, which implies that Y is θ-regular.

Recall that a subspace of a topological space is θF_σ if it is a union of countably many θ-closed sets. A subspace of a topological space called θG_δ if it is an intersection of countably many θ-open sets.

Example 3.4. There is a compact topological space X containing an F_σ-subspace Y which even is not countably θ-regular.

Proof. Let $Y = \{2, 3, \ldots\}$, $U_x = \{n \cdot x | \, n = 1, 2, \ldots\}$ for every $x \in Y$. The family $S = \{U_x : x \in Y\}$ defines a topology (as its base) on Y. Since $U_x \cap U_y \neq \emptyset$ for every $x, y \in Y$, every open non-empty set $U \subset Y$ has $Cl_Y U = Y$. It follows that the net $id(P, \geq)$, where P is the set of all prime numbers with their natural order $\geq$, is clearly θ-convergent, but with no cluster point in Y. It follows that Y is not countably θ-regular. Let $X = \{1\} \cup Y$ and take on X the topology of Alexandroff's compactification of Y. To see that Y is an F_σ-subspace of X, let $K_x = Y \setminus \bigcup_{y>x} U_y$ for every $x \in Y$. Every K_x is closed, finite, and hence compact in topology of Y. It follows that K_x is closed in X. Since $x \in K_x$, $Y = \bigcup_{x=2}^{\infty} K_x$.

Corollary 3.5. *In contrast to F_σ-subspaces, every θF_σ-subspace of a θ-regular space is θ-regular.*

Corollary 3.6. *Every θG_δ-subspace of a θ-regular space is θ-regular.*

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