A New Table of Constant Weight Codes of Length Greater than 28

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Abstract

Existing tables of constant weight codes are mainly confined to codes of length $n \leq 28$. This paper presents tables of codes of lengths $29 \leq n \leq 63$. The motivation for creating these tables was their application to the generation of good sets of frequency hopping lists in radio networks. The complete generation of all relevant cases by a small number of algorithms is augmented in individual cases by miscellaneous constructions. These sometimes give a larger number of codewords than the algorithms.

1 Introduction

$A(n, d, w)$ is the maximum possible number of binary vectors of length n, weight w and pairwise Hamming distance no less than d [10]. Such a set of vectors is known as a *constant weight code* and the vectors are referred to as *codewords*. Tables of constant weight codes are given in [6] for $n \leq 28$. These tables are extended to $n \leq 65$ and sometimes above in [13], but the results are very sparse for larger values of n. Improved results for upper bounds are given in [2] and corresponding tables for $n \leq 28$ can be found at [14].

In this paper tables of constant weight codes are given for $29 \leq n \leq 63$. The motivation for this work was the generation of frequency hopping lists for use in assignment problems in radio networks. Large distance between codewords gives smaller overlap between lists. This leads to fewer clashes on the same frequency and so less interference. Similarly, a larger number of codewords allows larger list re-use distances in the network and again leads to lower interference. More information on the work can be found in [12] and an evaluation of assignments of the lists generated can be found in [11]. The tables given here are significantly more complete than those given in [12] and include many improvements.

The restriction $n \leq 63$ was selected as 63 is the maximum number of frequencies possible in GSM mobile telephone systems when frequency hopping is used. The range

$3 \leq w \leq 8$ was selected for the work described in [12] as lists of length 2 are known to give no advantages when hopping and the maximum gains from frequency diversity (mitigating frequency selective fading) and interference diversity (averaging interference to ensure that the error-control coding is effective) are achieved when $w = 8$. Disjoint lists lead to unsatisfactorily small list re-use distances, so the cases $d = 2w - 2$ (overlap 1), $d = 2w - 4$ (overlap 2) and $d = 2w - 6$ (overlap 3) were considered. However, the tables in [13] are complete for $w = 3$ and $w = 4$ when $n \leq 63$, so the tables for these values (which were included in [12]) will not be presented here.

Within these ranges it was required that a constant weight code could be generated with a large number of codewords (without necessarily achieving $A(n, d, w)$) in all cases, using either (i) one of only a small number of algorithms suitable for Engineering application (ii) individual mathematical constructions which give further increases in the number of codewords in specific cases. Thus the tables allow a general comparison of the merits of the chosen algorithms. They also demonstrate further improvements by detailed constructions in some individual cases.

Let(q_1, q_2, n_1, n_2, d) denote a mixed error-correcting code, of length $n = n_1 + n_2$, where the first n_1 entries of any codeword take values from 0 to $q_1 - 1$, the next n_2 entries take values from 0 to $q_2 - 1$, and the minimal Hamming distance between different codewords is no less than d. In one of the construction methods in Section 2.2 the following simple result is used:

Proposition 1 *Suppose that C is a (q_1, q_2, n_1, n_2, d) code. Then there exists a constant weight binary code $\widetilde{C}$ with $|C|$ words of length $\widetilde{n} = q_1 n_1 + q_2 n_2$, minimum distance $2d$, and weight $w = n_1 + n_2$. This code is constructed in the following way: take each word $X = [x_1\ x_2\ \ldots\ x_n]$ of the initial (q_1, q_2, n_1, n_2, d) code and substitute each x_i in it by a row of q_1 entries if $i \leq n_1$ and by a row of q_2 entries if $i > n_1$. Of those entries (numbering the entries starting from 1) the entry with number $x_i + 1$ is equal to 1 and all other entries are 0.*

2 Constructions

In this section the constructions used to create the tables are described:

2.1 Construction from permutation groups

This construction of codes from a permutation group G generated by a single permutation is taken from [6]. Initially all orbits of G are determined, starting from orbits of codewords of weight 1. Consider a complete set of binary vectors of length n and weight i, each of which is a lexicographically maximal representative of its orbit. Suppose also that these are arranged in decreasing lexicographic order. From each vector, new vectors of weight $i + 1$ are generated by converting a single 0 to a 1 in all possible ways. For each vector generated, determine whether it is lexicographically maximal over its orbit. If it is, record the vector, otherwise discard it.

For $t = w - d/2 + 1$, a matrix B with columns indexed by orbits of weight w and with rows indexed by orbits of weight t can be formed. B specifies how often a representative vector of weight t is covered by the vectors in a given orbit of weight w. Orbits of weight w for which the corresponding row of B contains an entry greater than 1 can be discarded (as two elements of the constant weight code will have overlap greater than $w - d/2$).

The remaining orbits of weight w are represented by the vertices of a weighted graph, with the vertex labelled by the number of codewords in the set. Two vertices are joined if the pair does not conflict with the minimum distance condition. A maximum clique algorithm is then used to find the maximum weighted clique in this graph which, from the orbits represented by its vertices, gives the maximum constant weight code for this G.

Cyclic cases (with the permutation a cycle of length n), extended cyclic cases (with the permutation a cycle of length $n - 1$) and (for $n = 2s$) quasi-cyclic cases (with a permutation $(1\ 2 \ldots s)(s+1\ \ldots 2s)$) were considered. For $29 \leq n \leq 63$ these provided a challenging set of computations. The maximum clique software used was based on the algorithm in [7]. The computation could fail for reasons of either memory or run time. In some cases (marked *) only an incomplete clique search was possible. Sometimes these incomplete searches gave new best results. A poor result from an incomplete search is a reflection of the infeasibility of the algorithm rather than its ineffectiveness. If the clique search will not terminate it is often useful to apply the maximum clique algorithm with several different orderings by vertex degrees. This sometimes leads to a larger clique being found quickly. Sometimes a clique of size 1 or 2 gives a good result and the maximum clique algorithm is unnecessary.

Cyclic cases are denoted CC in the tables; extended cyclic cases are denoted EC in the tables and quasi-cyclic cases are denoted QC in the tables.

2.2 Lexicographic search for mixed or non-binary codes

In this method parameters for a suitable non-binary or mixed code (so that $n = n_1 q_1$ or $n = n_1 q_1 + n_2 q_2$) are determined and a lexicographic search [9] for such a code is performed. The code found by the search which has the maximum number of codewords is then used in Proposition 1. Results are given in the columns marked $NB - Mix$. Constant weight codes constructed by this method cannot be expected to be particularly good; in just four cases this method gave the best result. However, the method is easily the fastest of those used (finding an example for every case $9 \leq n \leq 63$ in a single run of under 24 hours on a 400MHz Pentium PC with 128Mb of memory). In the frequency hopping application it may prove useful that the 1's in the codewords constructed appear once in every q_1 or q_2 consecutive positions.

2.3 Binary lexicographic search

Several variations of binary lexicographic search [9] are possible. Here binary vectors of length n and weight w are arranged in either forward or reverse lexicographic order. A single vector is used as a seed vector and the other vectors are selected in turn and added to the code if they satisfy the necessary distance condition. This search is usually faster than the permutation group construction, but may take over a day of computation in the largest cases. However, it always proved possible to complete both the forward and the reverse search. The better of the two results is given in the tables in the column marked $B - Lex$, and annotated (R) if it is obtained by reverse search.

2.4 Random search

If the run time for lexicographic search is unsatisfactory, codes can be constructed randomly. Words of weight w are chosen randomly and tested to see whether they meet the required distance condition with previously chosen codewords. If they do, they are added to the code. The ultimate number of codewords selected is smaller but more codewords can be obtained in a limited search. No results are presented as they are almost always much worse than binary lexicographic search.

2.5 Miscellaneous constructions

Miscellaneous constructions of four types were considered and lead to codes as follows:

- codes with $29 \leq n \leq 63$ constructed by application of some method described in [6],
- other codes taken from [13] where a construction is given,
- other codes taken from the literature,
- codes constructed by the authors.

In all cases the method is indicated in the Key. All methods in [6] were considered when attempting to construct a new best code. Of course it was impossible to be comprehensive in selecting a permutation group or code to be the basis of some of these constructions.

3 A table of constant weight codes

The results are given in Tables 1– 12. The tables also display the Johnson upper bound [6] (in the column marked UB in the tables). The reader is referred to [2] for possible improvements to this bound. The best result is indicated in bold in the tables. The column *NewBest* indicates a new best result. The meaning of the annotations is given in the Key.

The tables show the relative merits of the general algorithms in terms of the number of codewords generated, with the permutation group construction being generally the best of the algorithms used, followed by binary lexicographic search. Non-binary or mixed lexicographic search is certainly the fastest method, but binary lexicographic search will generally be preferred if the permutation group construction is too slow.

KEY

- A_n – From a code above (or from [6]) of length n.
- B – Using $A(65,8,8) = 65520$ [15], equation 5(ii) of [6] gives $A(64,8,8) = 57456$.
  Comment. $A(64,8,8) = 57456$, $A(65,8,8) = 65520$. The code realising $A(65,8,8) = 65520$ consists of two orbits of length 32760 under $PSL_2(64)$. Taken from [13].
- BE – via Baker's elliptic semi-plane, a $\{7\}$-GDD of type 3^{15}, [4], p.191 [8], [13].
  Comment. $A(45,12,7) = 45$. Given a partition of the 45 points into 15 groups of size 3 then in the GDD two of the 45 points are either in a (single) block (of 7 points) or in a single group, but not both. Thus the overlap is at most 1. Counting we have $\binom{45}{2} = b.\binom{7}{2} + 13.\binom{3}{2}$ so $b = 45$. Taken from [13].
- C – Theorem 1 of [3].
  Comment. $A(33,8,5) = 44$, $A(34,8,5) = 47$.
- D – Construction I from [1].
  Comment. $A(55,8,5) = 121$, $A(42,8,6) = 343$. In the first case $p = 11$, $n = 5$, $k = 2$ and in the second case $p = 7$, $n = 6$, $k = 3$.
- EH – Words of weight 5 in a translate of the $(32,26)$ Extended Hamming Code, using a vector of weight 1.
  Comment. $A(32,4,5) = 6293$.
- Eq. x – Equation x of [6].
- H_n – Adding words to a code above (or from [6]) of length n.
- M – Manual construction.
- NB – Construct the code realising $A_5(8,7) = 10$ [5] and apply Proposition 1.
  Comment. $A(40,14,8) = 44$.
- P – Words of weight 6 in the Preparata code of length 64.
- R – Reverse binary lexicographic search.
- Th. y – Theorem y of [6].
- S – Completing a $(28,4,1)$ RBIBD (p. 90, [8]) gives a partially balanced design with 63 blocks of size 5 and one of size 9. This last block can be replaced by two blocks of size 5, p90 [8], [13].
  Comment. $A(37,8,5) = 65$. A parallel class of a $(28,4,1)$ design consists of 7 blocks forming a partition of the 28 points. The 63 blocks form 9 parallel classes. Add a point 29 to each block of the first parallel class, 30 to each block of the second,. . .37 to each block of the ninth. Finally, add two extra blocks 29,30,31,32,33 and 33,34,35,36,37. Taken from [13].
- SS – A Steiner system.
- * – Clique search was incomplete.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	3770, *3591, –	816	3731 (R)	**4095** (Eq. 31)	4750	
30		976	4459 (R)	**4751** (Eq. 31)	5262	
31		1136	5313 (R)	**5481** (Eq. 31)	6274	
32		1324	**6293** (R)	**6293** (EH)	6944	
33		1544	6503	**7192** (Eq. 31)	8184	
34		1801	7051	**8184** (Eq. 31)	8976	
35		2101	7159	**9276** (Eq. 31)	10472	
36		2401	7881	**10472** (Eq. 31)	11397	
37		2744	8353 (R)	**11781** (Eq. 31)	13186	
38		3136	9259	**13209** (Eq. 31)	14341	
39		3584	10168	**14763** (Eq. 31)	16450	
40		4096	11334	**16451** (Eq. 31)	17784	
41		4096	12598	**18278** (Eq. 31)	20254	
42		4160	14156	**20254** (Eq. 31)	21781	
43		4288	15831	**22386** (Eq. 5(ii))	24647	
44		4481	17635	**25256** (Eq. 5(ii))	26488	
45		4741	19657	**28413** (Eq. 5(ii))	29799	
46		5001	21940	**31878** (Eq. 5(ii))	31878	
47		5328	24488	**35673** (SS)	35673	
48		5724	27273 (R)	**35674** (Th. 18)	38006	
49		6192	30348 (R)	**38916** (Eq. 31)	42336	
50		6736	33667	**42376** (Eq. 31)	45080	
51		7280	37092 (R)	**46060** (Eq. 31)	49980	
52		7900	40928 (R)	**49980** (Eq. 31)	53040	
53		8600	45101 (R)	**54145** (Eq. 31)	58565	
54		9385	49633 (R)	**58565** (Eq. 31)	61959	
55		10000	54547 (R)	**63251** (Eq. 31)	68156	
56		10000	59867 (R)	**68211** (Eq. 31)	72072	
57		10000	65618 (R)	**73458** (Eq. 31)	79002	
58		10000	71722 (R)	**79002** (Eq. 31)	83311	
59		10000	78302 (R)	**84854** (Eq. 31)	91025	
60		10000	85386 (R)	**91026** (Eq. 31)	95748	
61		10000	93003 (R)	**97527** (Eq. 31)	104310	
62		10000	101123 (R)	**104371** (Eq. 31)	109678	
63		10000	109833 (R)	**111569** (Eq. 31)	119133	

Table 1: Comparison of Results $d = 4, w = 5$.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	**290**,*287,–	100	244		365	Y
30	306,***319**,*315	112	271 (R)		390	Y
31	341,***366**,–	127	311 (R)		415	Y
32	384,***403**,*384	137	340		492	Y
33	**429**,*424,–	150	379 (R)		528	Y
34	476,***495**,*459	162	413		557	Y
35	**532**,*510,–	182	456 (R)		651	Y
36	**576**,*567,*504	202	496		691	Y
37	**629**,*621,–	219	542		732	Y
38	**684**,*666,–	200	591 (R)		843	Y
39	**741**,*722,–	224	638		889	Y
40		256	694 (R)	**741** (A_{39})	936	Y
41		288	**755** (R)		1066	Y
42		288	**817**		1117	Y
43		321	**874**		1169	Y
44		346	**941** (R)		1320	Y
45		372	**1009**		1386	Y
46		405	**1097** (R)		1444	Y
47		430	**1172**		1616	Y
48		464	**1254** (R)		1689	Y
49		504	**1343** (R)		1764	Y
50		532	**1429**		1960	Y
51		619	**1517** (R)		2040	Y
52		668	**1617**		2121	Y
53		731	**1719**		2342	Y
54		776	**1822**		2430	Y
55		824	1924 (R)	**1936** (Eq. 5(ii))	2519	
56		736	2036	**2125** (Eq. 5(ii))	2766	
57		674	2162	**2329** (Eq. 5(ii))	2872	
58		576	2280 (R)	**2548** (Eq. 5(ii))	2969	
59		576	2397	**2783** (Eq. 5(ii))	3245	
60		576	2531	**3036** (Eq. 5(ii))	3360	
61		898	2665	**3306** (Eq. 5(ii))	3477	
62		1017	2801 (R)	**3596** (Eq. 5(ii))	3782	
63		1122	2952 (R)	**3906** (Eq. 5(i)P)	3906	

Table 2: Comparison of Results $d = 6, w = 5$.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	899,*882,–	226	853 (R)	**1170** (A_{27})	1459	
30		265	1005 (R)	**1179** (H_{27})	1825	Y
31		303	1163 (R)	**1205** (H_{27})	2015	Y
32		353	**1331** (R)		2213	
33		412	**1528** (R)		2706	Y
34		468	**1740**		2992	Y
35		538	**1973** (R)		3249	Y
36		618	**2240**		3906	Y
37		676	**2539** (R)		4261	Y
38		762	**2836** (R)		4636	Y
39		842	**3167**		5479	Y
40		944	**3545**		5926	Y
41		1049	**3964**		6396	Y
42		1175	**4397**		7462	Y
43		1284	**4860**		8005	Y
44		1402	**5378**		8572	Y
45		1368	**5933** (R)		9900	Y
46		1568	**6521**		10626	Y
47		1792	**7160** (R)		11311	Y
48		2048	**7845**		12928	Y
49		2190	**8568**		13793	Y
50		2366	**9348** (R)		14700	Y
51		2577	**10175**		16660	Y
52		2807	11064	**11316** (Eq. 5(ii))	17680	
53		3055	12025	**12760** (Eq. 5(ii))	18735	
54		3346	13017 (R)	**14355** (Eq. 5(ii))	21078	
55		3605	14091	**16112** (Eq. 5(ii))	22275	
56		3881	15221 (R)	**18045** (Eq. 5(ii))	23510	
57		4210	16422	**20167** (Eq. 5(ii))	26277	
58		4532	17683	**22493** (Eq. 5(ii))	27762	
59		4854	19028 (R)	**25039** (Eq. 5(ii))	29195	
60		5258	20431	**27821** (Eq. 5(ii))	32450	
61		5638	21940	**30856** (Eq. 5(ii))	34160	
62		6026	23493	**34162** (Eq. 5(ii))	35929	
63		6473	25185 (R)	**37758** (Eq. 5(ii)P)	39711	

Table 3: Comparison of Results $d = 6, w = 6$.

n	CC, EC, QC	$NB-Mix$	$B-Lex$	$Misc$	UB	$NewBest$
29	29, **35**, –	18	27		40	Y
30	**36**, 29, **36**	18	29		42	
31	31, **36**, –	20	32		43	
32	32, 31, 32	23	34	**38** (Eq. 5(ii))	44	
33	33, 40, –	24	36	**44** (C)	52	
34	34, 33, 34	25	38	**47** (C)	54	
35	42, 34, –	27	41	**50** (Eq. 5(ii))	56	
36	36, 42, 54	31	44	**57** (Eq. 5(ii))	57	
37	37, 45, –	31	47	**65** (S)	66	
38	38, 37, 57	32	50	**65** (A_{37})	68	
39	39, 38, –	32	52	**65** (A_{37})	70	
40	48, 39, 64	32	57	**72** (Eq. 5(ii))	72	
41	**82**, 58, –	32	60	**82** (SS)	82	
42	42, **82**, *63	33	62		84	
43	**86**, 42, –	35	67		86	
44	**88**, 86, ***88**	33	68		88	
45	54, 55, –	33	73	**99** (SS)	99	
46	92, 54, *92	36	76	**99** (A_{45})	101	
47	94, 92, –	40	80	**99** (A_{45})	103	
48	96, 94, *96	40	81	**99** (A_{45})	105	
49	98, **108**, –	46	85		117	Y
50	**110**, 98, *80	46	89		120	
51	102, **110**, –	52	91		122	
52	104, 102, *54	54	96	**110** (A_{51})	124	
53	106, **117**, –	57	100		137	
54	108, *106, *54	65	105	**117** (A_{53})	140	
55	110, *108, –	68	107	**121** (D)	143	
56	112, ***121**, –	65	112		145	
57	114, *70, –	60	117	**129** (Eq. 5(ii))	159	
58	116, *114, –	56	119	**141** (Eq. 5(ii))	162	
59	118, *116, –	48	123	**154** (Eq. 5(ii))	165	
60	120, *118, –	48	122	**168** (Eq. 5(ii))	168	
61	61, *75, –	56	131	**183** (SS)	183	
62	*124, *122, –	63	136	**183** (A_{61})	186	
63	*126, *124, –	71	137	**183** (A_{61})	189	

Table 4: Comparison of Results $d = 8, w = 5$.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	116, 112, –	65	99	**130** (A26)	159	
30	125, 116, *105	67	115 (R)	**131** (H26)	200	Y
31	155, 155, –	69	125 (R)	**156** (Eq. 5(ii))	217	Y
32	**192**, 155, *104	73	131 (R)		229	Y
33	–, 160, –	76	139 (R)	**192** (A32)	242	Y
34		76	152 (R)	**192** (A32)	294	Y
35		80	168	**192** (A32)	315	Y
36		88	184 (R)	**193** (H32)	336	
37		95	**199**		351	
38		109	**222**		418	
39		118	**244** (R)		442	
40		128	**275** (R)		466	
41		137	285 (R)	**294** (Eq. 5(ii))	492	
42		147	307	**343** (D)	574	
43		160	332	**343** (H42)	602	
44		164	**355** (R)		630	
45		178	**381**		660	Y
46		200	**411** (R)		759	
47		224	**440** (R)		791	Y
48		256	**477** (R)		824	Y
49		256	**501**		857	Y
50		264	**542**		975	Y
51		253	**576**		1020	Y
52		271	**609** (R)		1057	Y
53		289	**650**		1095	Y
54		314	**682**		1233	Y
55		334	**729**		1283	Y
56		347	**766**		1334	Y
57		376	**830**		1377	Y
58		402	**872**		1537	Y
59		420	**935**		1593	Y
60		446	**982**		1650	Y
61		471	**1028**		1708	Y
62		505	**1079**		1891	Y
63		531	**1143**		1953	Y

Table 5: Comparison of Results $d = 8, w = 6$.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	**319**, 308, –	64	300		617	Y
30		76	**327** (R)		681	
31		86	**363**		885	Y
32		104	**403** (R)		992	
33		122	**444**		1079	Y
34		150	**498**		1175	Y
35		165	**555** (R)		1470	
36		187	**622** (R)		1620	
37		214	**696**		1776	
38		241	**785** (R)		1905	
39		278	**869** (R)		2328	
40		310	**977**		2525	Y
41		350	**1095** (R)		2729	
42		390	**1206** (R)		2952	
43		425	**1347**		3526	Y
44		474	**1478**		3784	Y
45		522	**1639**		4050	Y
46		578	**1795**		4337	
47		631	**1987**		5096	Y
48		699	**2173** (R)		5424	Y
49		766	**2376**		5768	Y
50		835	**2603** (R)		6121	Y
51		896	**2839**		7103	Y
52		970	**3101**		7577	Y
53		1072	**3376** (R)		8003	Y
54		1376	**3651** (R)		8447	Y
55		1792	**3941** (R)		9687	Y
56		2048	**4270**		10264	Y
57		2048	**4625**		10862	Y
58		1557	**4971** (R)		11409	Y
59		1675	**5384** (R)		12954	Y
60		1780	**5770**		13654	Y
61		1910	**6223** (R)		14378	Y
62		2078	**6693**		15128	Y
63		2227	7171	**7182** (Eq. 5(i)B)	17019	

Table 6: Comparison of Results $d = 8, w = 7$.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	0, 0, –	12	15	**20** (Eq. 5(ii))	24	
30	5, 0, 15	13	17 (R)	**25** (Eq. 5(ii))	25	
31	**31**, 11, –	14	19	**31** (SS)	31	
32	0, 31, 16	15	23	**31** (A_{31})	32	
33	0, 0, –	17	23	**31** (A_{31})	33	
34	0, 0, 17	19	26	**31** (A_{31})	34	
35	**35**, 0, –	22	26		35	
36	**36**, 35, **36**	22	27		42	Y
37	**37**, 36, –	22	30		43	Y
38	**38**, 37, **38**	25	31		44	Y
39	**39**, 38, –	26	32		45	Y
40	**40**, 39, **40**	28	35		46	Y
41	41, 40, –	31	34	**42** (Eq. 5(ii))	54	Y
42	**49**, 41, **49**	32	38		56	Y
43	43, **49**, –	34	39		57	Y
44	44, 43, *44	29	43	**49** (A_{43})	58	Y
45	45, 44, –	31	44	**49** (A_{43})	60	Y
46	46, **54**, 23	33	46		69	Y
47	47, 46, –	32	48	**56** (Th. 21)	70	
48	**56**, 47, 24	32	47		72	
49	49, **56**, –	36	50		73	
50	50, 49, 25	32	50	**56** (A_{49})	75	
51	51, **60**, –	36	55		85	Y
52	52, 51, 26	40	59	**60** (A_{51})	86	Y
53	53, 52, –	45	**63**		88	Y
54	63, 53, –	48	**65**		90	Y
55	55, 63, –	49	**68**		91	Y
56	56, 66, –	48	**70**		102	Y
57	57, 56, –	52	69	**70** (A_{56})	104	Y
58	58, 57, –	58	**72**		106	Y
59	59, 58, –	60	**77**		108	Y
60	70, 59, –	62	**79**		110	Y
61	61, 72, –	65	**83**		122	Y
62	62, 61, –	67	**84**		124	Y
63	**126**, 62, –	71	85		126	Y

Table 7: Comparison of Results $d = 10, w = 6$.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	29, 32, –	22	34	**37** (Eq. 5(ii))	95	Y
30	30, 29, *45	25	38 (R)	**48** (Eq. 5(ii))	102	Y
31	**62**, 35, –	28	43		110	Y
32	32, **62**, *32	30	47		141	Y
33	**66**, 32, –	34	54		150	Y
34	**68**, 66, *34	36	60 (R)		160	Y
35	**75**, 68, –	39	66		170	Y
36	72, ***75**, *36	44	**75**		180	Y
37	*74, *78, –	46	83	**91** (Eq. 5(ii))	222	Y
38	–, ***111**, –	53	92 (R)		233	Y
39	–, ***114**, –	57	102 (R)		245	Y
40	–, ***117**, –	64	113 (R)		257	Y
41		68	**126**		269	Y
42		72	**133** (R)		324	Y
43		79	142	**155** (Eq. 5(ii))	344	
44		83	155 (R)	**184** (Eq. 5(ii))	358	
45		90	169 (R)	**217** (Eq. 5(ii))	372	
46		94	181 (R)	**255** (Eq. 5(ii))	394	
47		105	191 (R)	**299** (Eq. 5(ii))	463	
48		112	209 (R)	**350** (Th. 21)	480	
49		118	224	**350** (A_{48})	504	
50		126	241 (R)	**350** (A_{48})	521	
51		137	255	**350** (A_{48})	546	
52		146	272	**350** (A_{48})	631	
53		154	287	**350** (A_{48})	651	
54		192	308 (R)	**351** (Th. 21)	678	
55		224	327 (R)	**351** (A_{54})	707	
56		256	348 (R)	**351** (A_{54})	728	
57		196	**366** (R)		830	Y
58		210	**394** (R)		861	Y
59		220	**414** (R)		893	Y
60		237	**431**		925	Y
61		251	**458**		958	Y
62		266	**486**		1080	Y
63		277	**514**		1116	Y

Table 8: Comparison of Results $d = 10, w = 7$.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	**87**, *84, –	28	75 (R)		319	Y
30	90, *87, *45	36	89 (R)	**92** (Eq. 5(ii))	356	Y
31	**124**, *90, –	48	104		395	Y
32		64	119	**124** (A_{31})	440	Y
33		64	**134**		581	Y
34		69	**156**		637	Y
35		78	**176** (R)		700	Y
36		86	**198** (R)		765	Y
37		98	**223**		832	Y
38		109	**249**		1054	Y
39		123	**285**		1135	Y
40		136	**318** (R)		1225	Y
41		150	**353**		1317	Y
42		168	**390**		1412	Y
43		179	**432** (R)		1741	Y
44		198	**484** (R)		1892	Y
45		218	**532**		2013	Y
46		242	**590**		2139	Y
47		268	**642** (R)		2314	Y
48		294	**711** (R)		2778	Y
49		312	**776** (R)		2940	Y
50		338	**852**		3150	Y
51		370	**929** (R)		3321	Y
52		391	**1007** (R)		3549	Y
53		433	**1095** (R)		4180	Y
54		469	**1194** (R)		4394	Y
55		508	**1289**		4661	Y
56		544	**1405**		4949	Y
57		579	**1517**		5187	Y
58		631	**1633**		6017	Y
59		672	**1767**		6349	Y
60		724	**1908**		6697	Y
61		792	**2043**		7053	Y
62		992	**2189** (R)		7424	Y
63		1024	**2352**		8505	Y

Table 9: Comparison of Results $d = 10, w = 8$.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	0, 4, –	4	5	**8** (M)	16	Y
30	0, 0, 0	5	6	**8** (A_{29})	17	Y
31	0, 5, –	7	6	**9** (M)	22	
32	0, 0, 0	8	**9**		22	
33	0, 0, –	**10**	6		23	Y
34	0, 0, 0	5	8	**12** (Eq. 5(ii))	24	
35	5, 0, –	5	11	**15** (Eq. 14)	25	
36	0, 5, 0	9	12	**15** (A_{35})	25	
37	0, 6, –	11	15	**16** (Eq. 5(ii))	31	Y
38	0, 0, **19**	13	12		32	Y
39	0, 0, –	17	18	**19** (A_{38})	33	Y
40	0, 0, **20**	14	18		34	Y
41	0, 0, –	12	**24** (R)		35	Y
42	6, 0, 24	12	22	**27** (Eq. 5(ii))	36	
43	0, 13, –	18	19	**32** (Eq. 5(ii))	43	
44	0, 0, 22	19	27	**38** (Eq. 5(ii))	44	
45	0, 0, –	21	28	**45** (BE)	45	
46	0, 0, 23	19	30	**45** (A_{45})	46	
47	0, 0, –	23	31	**45** (A_{45})	47	
48	**48**, 0, **48**	25	33 (R)		48	
49	49, 56, –	25	36 (R)	**56** (SS)	56	
50	50, 49, 50	27	35	**56** (A_{49})	57	
51	51, 50, –	28	38 (R)	**56** (A_{49})	58	
52	52, 51, 52	31	40 (R)	**56** (A_{49})	59	
53	53, 52, –	33	40 (R)	**56** (A_{49})	60	
54	54, 53, 54	34	43	**56** (A_{49})	61	
55	55, 54, –	32	44	**57** (H_{49})	70	
56	56, 55, 56	32	47 (R)	**57** (A_{55})	72	
57	**57**, 56, –	32	48 (R)		73	
58	**58**, 57, **58**	34	47		74	Y
59	**59**, 58, –	32	51		75	Y
60	**60**, 59, **60**	36	53		77	Y
61	**61**, 70, –	40	54 (R)		87	Y
62	62, 61, 62	33	55	**64** (Eq. 5(ii))	88	Y
63	**72**, 62, –	33	57		90	Y

Table 10: Comparison of Results $d = 12, w = 7$.

n	CC, EC, QC	$NB - Mix$	$B - Lex$	$Misc$	UB	$NewBest$
29	0, 14, –	12	16	**22** (Eq. 5(ii))	58	
30	15, 0, **30**	12	17		60	Y
31	**31**, 15, –	12	21 (R)		65	Y
32	**32**, 31, **32**	16	23		88	Y
33	**33**, 32, –	16	24 (R)		90	Y
34	**34**, 33, ***34**	18	28 (R)		97	Y
35	**35**, 34, –	22	31		105	Y
36	36, **40**, *36	24	36		112	Y
37	37, 36, –	27	38 (R)	**40** (A_{36})	115	Y
38	38, 37, *38	26	39	**40** (A_{36})	147	Y
39	39, 38, –	29	46 (R)	**48** (Eq. 5(ii))	156	Y
40	**60**, 39, *40	33	50 (R)		165	Y
41		35	53	**64** (Eq. 5(ii))	174	
42		37	58	**79** (Eq. 5(ii))	183	
43		40	64 (R)	**96** (Eq. 5(ii))	193	
44		44	66	**117** (Eq. 5(ii))	236	
45		47	72	**142** (Eq. 5(ii))	247	
46		52	78	**171** (Eq. 5(ii))	258	
47		56	85	**205** (Eq. 5(ii))	270	
48		58	91 (R)	**246** (Eq. 5(ii))	282	
49		61	99 (R)	**294** (Eq. 5(ii))	294	
50		65	105	**350** (SS)	350	
51		71	114	**350** (A_{50})	363	
52		74	122 (R)	**350** (A_{50})	377	
53		81	131	**350** (A_{50})	390	
54		85	141 (R)	**350** (A_{50})	405	
55		91	152 (R)	**350** (A_{50})	419	
56		95	160 (R)	**351** (H_{50})	490	
57		99	168	**351** (A_{56})	513	
58		106	178	**351** (A_{56})	529	
59		110	193	**351** (A_{56})	545	
60		118	204 (R)	**352** (H_{56})	562	
61		123	216 (R)	**352** (A_{60})	587	
62		200	226	**352** (A_{60})	674	
63		224	244	**352** (A_{60})	693	

Table 11: Comparison of Results $d = 12, w = 8$.

n	CC, EC, QC	$NB-Mix$	$B-Lex$	$Misc$	UB	$NewBest$
29	0, **4**, –	**4**	**4**		14	
30	0, 0, 0	4	4	**5** (M)	15	
31	0, 0, –	4	**5**		15	
32	4, 0, 4	4	**5**		16	
33	0, 4, –	4	5	**6** (M)	16	
34	0, 0, 0	4	5	**6** (A_{33})	17	
35	0, 0, –	5	5	**7** (M)	17	
36	0, 5, 0	5	6	**9** (M)	22	
37	0, 0, –	6	6	**9** (A_{36})	23	
38	0, 0, 0	6	6	**9** (A_{36})	23	
39	0, 0, –	5	6	**9** (A_{36})	24	
40	0, 0, 5	5	6	**10** (NB)	25	
41	0, 5, –	5	7	**10** (A_{40})	25	
42	0, 0, 0	6	7	**10** (A_{40})	26	
43	0, 6, –	8	**10**		32	
44	0, 0, 0	9	**12**		33	Y
45	0, 0, –	**12**	7		33	Y
46	0, 0, 0	**13**	9		34	Y
47	0, 0, –	6	12	**15** (Eq. 5(ii))	35	
48	6, 0, 6	6	14 (R)	**18** (Eq. 5(ii))	36	
49	0, 6, –	11	15	**21** (Eq. 5(ii))	36	
50	0, 7, 0	14	19	**25** (Eq. 14)	43	
51	0, 0, –	15	14	**25** (A_{50})	44	
52	0, 0, 26	18	21	**27** (Eq. 5(ii))	45	
53	0, 0, –	19	22	**31** (Eq. 5(ii))	46	
54	0, 0, 27	18	25	**36** (Eq. 5(ii))	47	
55	0, 0, –	15	27 (R)	**42** (Eq. 5(ii))	48	
56	7, 0, 28	15	28	**49** (Eq. 5(ii))	49	
57	**57**, 15, –	24	23	**57** (SS)	57	
58	0, **57**, 29	23	33		58	
59	0, 0, –	25	33	**57** (A_{58})	59	
60	0, 0, 30	25	35 (R)	**57** (A_{58})	60	
61	0, 0, –	27	37 (R)	**57** (A_{58})	61	
62	0, 0, 31	28	37 (R)	**58** (H_{58})	62	
63	**63**, 0, –	56	41 (R)		63	

Table 12: Comparison of Results $d = 14, w = 8$.

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