New Optimal Constant Weight Codes

I. Gashkov*, D. Taub

Department of Mathematics
Karlstad University, Sweden
igor.gachkov@kau.se,taub.math@gmail.com

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Abstract

In 2006, Smith *et al.* published a new table of constant weight codes, updating existing tables originally created by Brouwer *et al.* This paper improves upon these results by filling in 9 missing constant weight codes, all of which are optimal by the second Johnson bound. This completes the tables for $A(n, 16, 9)$ and $A(n, 18, 10)$ up to $n = 63$ and corrects some $A(n, 14, 8)$.

Introduction

A *binary constant weight code* is any subset of $\mathbb{B}^n$ such that all elements, *codewords*, have the same weight. An important problem in coding theory is finding $A(n, d, w)$, the maximum possible number of codewords in a constant weight code with length n, minimum distance d, and weight w.

A large table of lower bounds on these numbers was published by Brouwer *et al.* [1], and later added to by Smith *et al.* [4]. However, these tables are still far from complete, and many of the existing values can be improved upon.

When the lower bound for a code matches the upper bound, the code is said to be *optimal*. There are a number of different methods for determining upper bounds for constant weight codes, but for the codes presented in this paper the most useful is the one given by theorem 1.

Theorem 1 (The second Johnson bound) *There can be a code with parameters n, d and w and size M only if*

$$(n-b)a(a-1) + ba(a+1) \leq \left(w - \left\lceil \frac{d}{2} \right\rceil\right) M(M-1)$$

holds, where a and b are the unique integers such that $wM = an + b$ and $0 \leq b < n$.

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Proof See [2], p. 526. ■

The second Johnson bound $J_2(n, d, w)$ is the largest M such that the inequality in theorem 1 holds. (It is possible that the inequality holds for all M, in which case $J_2(n, d, w) = \infty$.)

All of the codes presented in this paper are optimal by the second Johnson bound.

New Lexicographic Methods

In general, lexicographic methods, or *lexicodes*, rarely achieve good lower bounds. However, with some simple modifications, standard lexicodes can be used to obtain a number of useful results.

By adding a degree of randomness to a standard lexicode we were able to obtain a number of new optimal constant weight codes. Additional codes were found by using a genetic algorithm based on randomized lexicodes. These new codes complete the tables for $A(n, 16, 9)$ and $A(n, 18, 10)$ up to $n = 63$.

A complete list of all new lower bounds, as well as the actual codes, is presented below. We presented binary vector $v = (b_1, b_2, \ldots, b_n)$ in support form $\{i \mid b_i = 1\}$

Results

The results are given in two tables. All codes presented in this section are optimal by the second Johnson bound.

Table 1

n:	39	40	41	42	43	44	45
A(n,14,8):	10	10	11	12	12	13	14

$A(39, 14, 8) = 10$,$A(41, 14, 8) = 11$,$A(42, 14, 8) = 12$ (See [6])

$A(40, 14, 8) = 10$ the same as $A(39, 14, 8)$.

$A(42, 14, 8) = 12$ the same as $A(41, 14, 8)$.

$A(45, 14, 8) = 14$

1:	1	2	3	4	5	6	7	8
2:	1	9	10	11	12	13	14	15
3:	1	16	17	18	19	20	21	22
4:	23	2	9	16	32	33	34	35
5:	23	3	10	17	28	29	30	31
6:	23	4	11	18	24	25	26	27
7:	5	12	19	24	28	32	36	37
8:	6	13	20	33	29	25	36	38
9:	7	14	21	34	30	26	37	39
10:	8	15	22	35	31	27	39	38
11:	38	30	12	4	16	40	41	42
12:	36	31	7	18	9	40	43	44
13:	39	32	20	3	11	41	43	45
14:	37	25	22	2	10	42	44	45

Table 2

n:	45	46	47	48	48	50	51	52	53	54	55	56	57	58	59	60	61	62	63
A(n,16,9):	10	10	10	11	11	12	12	13	13	14	15	16	19	19	20	21	22	24	28

$A(45, 16, 9) = 10$ (See [3])
$A(46, 16, 9) = 10$ and
$A(47, 16, 9) = 10$ the same as $A(45, 16, 9)$.
$A(48, 16, 9) = 11$ (See [5], p. 912-915.)
$A(49, 16, 9) = 11$ the same as $A(48, 16, 9)$.

$A(50, 16, 9) = 12$

1:	1	2	3	4	5	6	7	8	9
2:	1	10	11	12	13	14	15	16	17
3:	2	10	18	19	20	21	22	23	24
4:	3	11	18	25	26	27	28	29	30
5:	4	12	19	25	31	32	33	34	35
6:	5	13	20	26	31	36	37	38	39
7:	6	14	21	27	32	36	40	45	50
8:	7	15	18	31	40	41	42	43	44
9:	8	16	22	25	37	42	45	46	47
10:	5	17	23	29	33	43	45	48	49
11:	1	24	28	34	38	44	46	48	50
12:	9	10	30	35	39	41	47	49	50

$A(51, 16, 9) = 12$ the same as $A(50, 16, 9)$. See also [5], p. 912-915.

$A(52, 16, 9) = 13$

1:	1	2	3	4	5	6	7	8	9
2:	1	10	11	12	13	14	15	16	17
3:	2	10	18	19	20	21	22	23	24
4:	3	11	18	25	26	27	28	29	30
5:	4	12	19	25	31	32	33	34	35
6:	5	13	20	26	31	36	37	38	39
7:	6	14	21	27	32	36	40	41	42
8:	7	15	22	28	33	37	40	47	52
9:	8	16	18	31	40	43	44	45	46
10:	9	17	23	25	36	44	47	48	49
11:	1	24	26	34	41	45	47	50	51
12:	2	14	29	35	38	46	48	50	52
13:	4	10	30	39	42	43	49	51	52

$A(53, 16, 9) = 13$ the same as $A(52, 16, 9)$.

$A(54, 16, 9) = 14$

1:	1	2	3	4	5	6	7	8	9
2:	1	10	11	12	13	14	15	16	17
3:	2	10	18	19	20	21	22	23	24
4:	3	11	18	25	26	27	28	29	30
5:	4	12	19	25	31	32	33	34	35
6:	5	13	20	26	31	36	37	38	39
7:	6	14	21	27	32	36	40	41	42
8:	7	15	22	28	33	37	40	43	44
9:	8	16	23	29	34	38	41	43	46
10:	1	18	31	42	44	45	46	47	48
11:	9	17	19	26	40	46	49	50	51
12:	3	13	24	32	43	47	49	52	53
13:	2	15	25	39	41	48	50	52	54
14:	7	10	27	35	38	45	51	53	54

$A(55, 16, 9) = 15$

1:	1	2	3	4	5	6	7	8	9
2:	1	10	11	12	13	14	15	16	17
3:	2	10	18	19	20	21	22	23	24
4:	3	11	18	25	26	27	28	29	30
5:	4	12	19	25	31	32	33	34	35
6:	5	13	20	26	31	36	37	38	39
7:	6	14	21	27	32	36	40	41	42
8:	7	15	22	28	33	37	40	43	44
9:	8	16	23	29	34	38	41	43	45
10:	9	17	24	30	35	39	42	44	45
11:	1	18	31	40	45	46	47	48	49
12:	2	11	32	38	44	47	50	51	52
13:	3	10	35	36	43	48	50	53	54
14:	4	16	22	26	42	49	51	53	55
15:	6	12	23	28	39	46	52	54	55

$A(56, 16, 9) = 16$ (See [3])

$A(57, 16, 9) = 19$ (See [3])

$A(58, 16, 9) = 19$ the same as $A(57, 16, 9)$.

$A(59, 16, 9) = 20$

1:	1	2	3	4	5	6	7	8	9
2:	1	10	11	12	13	14	15	16	17
3:	1	18	19	20	21	22	23	24	25
4:	1	26	27	28	29	30	31	32	33
5:	34	6	14	22	30	35	36	37	38
6:	34	7	15	23	31	39	40	41	42
7:	34	8	16	24	32	43	44	45	46
8:	34	9	17	25	33	47	48	49	50
9:	51	2	10	18	26	38	42	46	50
10:	51	3	11	19	27	37	41	45	49
11:	51	4	12	20	28	36	40	44	48
12:	51	5	13	21	29	35	39	43	47
13:	5	12	45	50	23	30	52	53	54
14:	19	26	35	40	9	16	52	55	56
15:	4	13	38	41	25	32	52	57	58
16:	18	27	44	47	7	14	53	56	58
17:	2	20	37	43	15	33	53	57	59
18:	11	29	42	48	6	24	54	57	55
19:	3	21	36	46	17	31	54	56	59
20:	10	28	39	49	8	22	55	58	59

$A(60, 16, 9) = 21$

1:	1	2	3	4	5	6	7	8	9
2:	1	10	11	12	13	14	15	16	17
3:	2	11	18	19	20	21	22	23	24
4:	2	10	25	26	27	28	29	30	31
5:	3	10	18	32	33	34	35	36	37
6:	1	18	25	38	39	40	41	42	43
7:	4	11	25	33	44	45	46	47	48
8:	2	13	32	39	44	49	50	51	52
9:	3	12	19	26	38	44	53	54	55
10:	3	11	29	39	56	57	58	59	60
11:	5	14	22	27	32	40	46	53	56
12:	5	12	21	28	34	41	47	50	57
13:	9	12	20	27	36	42	45	49	58
14:	6	14	20	26	34	43	48	51	59
15:	6	15	22	28	33	42	52	54	60
16:	8	13	24	28	35	40	45	55	59
17:	9	16	19	30	35	41	46	51	60
18:	4	17	21	30	36	43	52	55	56
19:	7	16	24	27	37	38	48	52	57
20:	7	15	23	29	35	43	47	49	53
21:	8	17	23	31	37	46	50	54	58

$A(61, 16, 9) = 22$

1:	1	2	3	4	5	6	7	8	9
2:	1	10	11	12	13	14	15	16	17
3:	2	10	18	19	20	21	22	23	24
4:	1	20	25	26	27	28	29	30	31
5:	1	18	32	33	34	35	36	37	38
6:	2	12	25	32	39	40	41	42	43
7:	4	11	18	26	39	44	45	46	47
8:	5	10	26	34	40	48	49	50	51
9:	3	10	25	33	46	52	53	54	55
10:	2	11	28	33	48	56	57	58	59
11:	3	13	19	27	34	39	56	60	61
12:	9	12	19	29	36	45	49	52	57
13:	5	15	22	27	35	41	44	52	59
14:	9	13	20	35	42	47	51	53	58
15:	17	22	28	38	42	45	50	54	61
16:	8	12	21	30	37	44	48	53	60
17:	7	17	23	29	32	44	51	55	56
18:	4	14	20	36	43	50	55	59	60
19:	8	14	23	31	33	41	47	49	61
20:	6	16	21	27	38	43	46	51	57
21:	6	15	24	30	36	40	47	54	56
22:	7	16	24	31	37	39	50	52	58

$A(62, 16, 9) = 24$ and $A(63, 16, 9) = 28$ (See [3]).

Table 2

n:	55	56	57	58	59	60	61	62	63
A(n,18,10):	11	11	11	12	12	12	13	13	14

$A(55, 18, 10) = 11$ (See [3])
$A(56, 18, 10) = 11$ and
$A(57, 18, 10) = 11$ the same as $A(55, 18, 10)$.
$A(58, 18, 10) = 12$ (See [5], p. 912-915.)
$A(59, 18, 10) = 12$ and
$A(60, 18, 10) = 12$ the same as $A(58, 18, 10)$.

$A(61, 18, 10) = 13$

1:	1	2	3	4	5	6	7	8	9	10
2:	3	11	12	13	14	15	16	17	18	19
3:	5	15	20	21	22	23	24	25	26	27
4:	4	12	21	28	29	30	31	32	33	34
5:	6	11	20	28	35	36	37	38	39	40
6:	7	11	22	31	41	42	43	44	45	46
7:	7	13	23	29	35	47	48	49	50	51
8:	9	14	23	30	37	41	52	53	54	55
9:	9	16	25	32	36	42	50	56	57	58
10:	10	15	30	38	44	47	56	59	60	61
11:	2	17	24	33	39	43	48	52	57	60
12:	8	18	26	33	40	45	49	53	58	59
13:	1	19	27	34	39	46	51	54	58	61

$A(62, 18, 10) = 13$ the same as $A(61, 18, 10)$.

$A(63,18,10)=14$

1:	1	2	3	4	5	6	7	8	9	10
2:	1	11	12	13	14	15	16	17	18	19
3:	2	11	20	21	22	23	24	25	26	27
4:	3	12	20	28	29	30	31	32	33	34
5:	4	13	21	29	35	36	37	38	39	40
6:	5	15	23	28	35	41	42	43	44	45
7:	6	14	22	30	35	46	47	48	49	50
8:	1	24	31	36	41	46	51	52	53	54
9:	7	13	22	32	42	51	55	56	57	58
10:	7	16	24	28	37	47	59	60	61	62
11:	8	12	26	38	44	49	52	55	59	63
12:	8	17	25	33	39	43	48	53	56	60
13:	9	18	27	31	40	45	48	58	61	63
14:	10	19	26	34	39	45	50	54	57	62

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