Constructing Hypohamiltonian Snarks with Cyclic Connectivity 5 and 6

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Abstract

A graph is hypohamiltonian if it is not hamiltonian but every vertex-deleted subgraph is. In this paper we study hypohamiltonian snarks – cubic hypohamiltonian graphs with chromatic index 4. We describe a method, based on superposition of snarks, which produces new hypohamiltonian snarks from old ones. By choosing suitable ingredients we can achieve that the resulting graphs are cyclically 5-connected or 6-connected. Previously, only three sporadic hypohamiltonian snarks with cyclic connectivity 5 had been found, while the flower snarks of Isaacs were the only known family of cyclically 6-connected hypohamiltonian snarks. Our method produces hypohamiltonian snarks with cyclic connectivity 5 and 6 for all but finitely many even orders.

1 Introduction

Deciding whether a graph is hamiltonian, that is to say, whether it contains a cycle through all the vertices, is a notoriously known difficult problem which remains NP-complete problem even in the class of cubic graphs [6]. As with other hard problems in mathematics, it is useful to focus on objects that are critical with respect to the property that defies characterisation. Much attention has been therefore paid to non-hamiltonian graphs which are in some sense close to being hamiltonian. A significant place among such graphs is held by two families – graphs where any two non-adjacent vertices are connected by a hamiltonian path (known as *maximally non-hamiltonian* graphs) and those where the

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removal of every vertex results in a hamiltonian graph (called *hypohamiltonian* graphs). The latter family is the main object this paper.

There exists a variety of known constructions which produce infinite families of hypohamiltonian graphs. One particularly elegant is due Thomassen [15] which sometimes produces graphs that are not only hypohamiltonian but also maximally non-hamiltonian. That hypohamiltonian graphs constitute a relatively rich family was proved by Collier and Schmeichel [3] who bounded the number of hypohamiltonian graphs of order n from below by a certain exponential function.

Among hypohamiltonian graphs, cubic graphs have a special place, since they have the smallest number of edges on a given number of vertices. Indeed, removing any vertex from a hypohamiltonian graph leads to a graph with a hamiltonian cycle, and this graph must be 2-connected. Therefore every hypohamiltonian graph is 3-connected; in particular, its minimum vertex valency is at least 3.

Since hamiltonian cubic graphs are 3-edge-colourable, it is natural to search for hypohamiltonian cubic graphs among cubic graphs with chromatic index 4. Non-trivial examples of such graphs are commonly known as *snarks*. It has been generally accepted that the term 'non-trivial' requires a snark to have girth at least 5 and to be cyclically 4-connected (see [5] for example). Recall that a cubic graph G is *cyclically k-connected* if no set of fewer than k edges separates two cycles. The largest integer k for which G is cyclically k-connected is the *cyclic connectivity* of G. (There are three exceptional graphs in which no two cycles can be separated, namely $K_{3,3}$, K_4, and the graph consisting of two vertices joined by three parallel edges. For these, the cyclic connectivity is defined to be their cycle rank $|E(G)| - |V(G)| + 1$; see [11] for more information.)

The smallest hypohamiltonian snark is the Petersen graph. In 1983, Fiorini [4] proved that the well known Isaacs flower snarks I_k of order $4k$ are hypohamiltonian for each odd $k \geq 5$. In fact, as early as in 1977, a larger class of hypohamiltonian graphs was found by Gutt [7], but only later it was noticed that it includes the Isaacs snarks. Fiorini [4] also established a sufficient condition for a dot-product of two hypohamiltonian snarks to be hypohamiltonian. By using this condition, Steffen [14] proved that there exist hypohamiltonian snarks of each even order greater than 90.

Steffen[13] also proved that each hypohamiltonian cubic graph with chromatic index 4 is bicritical. This means that the graph itself is not 3-edge-colourable but the removal of any two distinct vertices results in a 3-edge-colourable graph. Furthermore, Nedela and Škoviera [12] showed that each bicritical cubic graph is cyclically 4-edge-connected and has girth at least 5. Therefore *each hypohamiltonian cubic graph with chromatic index* 4 *has girth at least* 5 *and cyclic connectivity at least* 4, and thus is a snark in the usual sense. Since the removal of a single vertex from a cubic graph with no 3-edge-colouring cannot give rise to a 3-edge-colourable graph, hypohamiltonian snarks lie on the border between cubic graphs which are 3-edge colourable and those which are not.

On the other hand, Jaeger and Swart [9] conjectured that each snark has cyclic connectivity at most 6. If this conjecture is true, the cyclic connectivity of a hypohamiltonian snark can take one of only three possible values – 4, 5, and 6. Thomassen went even further to conjecture that there exists a constant k (possibly $k = 8$) such that every cyclically k-connected cubic graph is hamiltonian. The value $k = 8$ is certainly best possible because the well known Coxeter graph of order 28 is hypohamiltonian and has cyclic connectivity 7 [2].

The situation with known hypohamiltonian snarks regarding their cyclic connectivity can be summarised as follows. The snarks constructed by Steffen in [14] have cyclic connectivity 4. There are three sporadic hypohamiltonian snarks with cyclic connectivity 5 – the Petersen graph, the Isaacs flower snark I_5 and the double-star snark (see [4]). The flower snarks I_k, where $k \geq 7$ is odd, have cyclic connectivity 6 ([4]).

In the present paper we develop a method, based on superposition [10], which produces hypohamiltonian snarks from smaller ones. By employing suitable ingredients we show that for each sufficiently large even integer there exist hypohamiltonian snarks with cyclic connectivity 5 and 6. A slight modification of the method can also provide snarks with cyclic connectivity 4.

2 Preliminaries

It is often convenient to compose cubic graphs from smaller building blocks that contain 'dangling' edges. Such structures are called multipoles. Formally, a *multipole* is a pair $M = (V(M), E(M))$ of disjoint finite sets, the *vertex-set* $V(M)$ and the *edge-set* $E(M)$. Every edge $e \in E(M)$ has two *ends* and every end of e can, but need not, be incident with a vertex. An end of an edge that is not incident with a vertex is called a *semiedge*. Semiedges are usually grouped into non-empty pairwise disjoint *connectors*. Each connector is endowed with a linear order of its semiedges.

The reason for the existence of semiedges is that a pair of distinct semiedges x and y can be identified to produce a new proper edge $x * y$. The ends of $x * y$ are the other end of the edge supporting x and the other end of the edge supporting y. This operation is called *junction*. The *junction* of two connectors S_1 and S_2 of size n identifies the i-th semiedge of S_1 with the i-th semiedge of S_2 for each $1 \leq i \leq n$, decreasing the total number of semiedges by $2n$.

A multipole whose semiedges are split into two connectors of equal size is called a *dipole*. The connectors of a dipole are referred to as the *input*, $\mathrm{In}(M)$, and the *output*, $\mathrm{Out}(M)$. The common size of the input and the output connector is the *width* of M. Let M and N be dipoles with the same width n. The *serial junction* $M \circ N$ of M and N is a dipole which arises by the junction of $\mathrm{Out}(M)$ with $\mathrm{In}(N)$. The *n-th power* M^n of M is the serial junction of n copies of M, that is $M \circ\ M \circ \cdots \circ M$ (n times). Another useful operation is the *closure* $\overline{M}$ of a dipole M which arises from M by the junction of $\mathrm{In}(M)$ with $\mathrm{Out}(M)$.

For illustration consider the dipole Y of width 3 with $\mathrm{In}(Y) = (e_1, e_2, e_3)$ and $\mathrm{Out}(Y) = (f_1, f_2, f_3)$ displayed in Fig. 1. The closure of the serial junction of an odd number of copies of Y, that is $\overline{Y^k}$ where $k \geq 5$ is odd, is in fact the Isaacs flower snark I_k introduced in [8]; see Fig. 5 left.

As another example consider the flower snark I_5 with its unique 5-cycle removed to obtain a multipole M of order 15 having a single connector of size 5. Order the semiedges

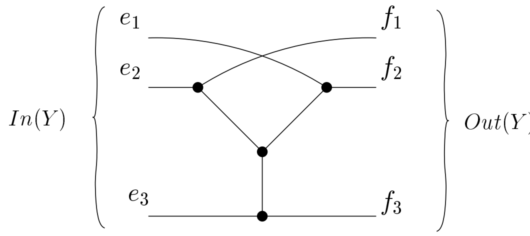


Figure 1: Dipole Y

consistently with a cyclic orientation of the removed cycle. Let M' be a copy of M but with semiedges reordered: the new ordering will be derived from the same cyclic orientation by taking every second semiedge in the order. The cubic graph resulting from the junction of M and M' is the double-star snark constructed by Isaacs in [8], see Fig. 7 left.

A Tait colouring of a multipole is a proper 3-edge-colouring which uses non-zero elements of the group $\mathbb{Z}_2 \times \mathbb{Z}_2$ as colours. The fact that any two adjacent edges receive distinct colours is easily seen to be equivalent to the condition that the colours meeting at any vertex sum to zero in $\mathbb{Z}_2 \times \mathbb{Z}_2$. This in turn means that a Tait colouring is in fact a nowhere-zero $\mathbb{Z}_2 \times \mathbb{Z}_2$-flow on the multipole.

We say that a dipole M is *proper* if for every Tait colouring of M the sum of colours on the input semiedges is different from zero. A straightforward flow argument (or equivalently, the well-known Parity Lemma [8]) implies that the same must be true for the output semiedges.

Proper dipoles are a substantial ingredient of an important construction of snarks called superposition [10]. Let G be a cubic graph. Let $U_1, U_2, \ldots, U_l$ be multipoles, with three connectors each, called *supervertices*, and let $X_1, X_2 \ldots, X_k$ be dipoles, called *superedges*. Take a function $f : V(G) \cup E(G) \to \{U_1, U_2, \ldots, U_l, X_1, X_2 \ldots, X_k\}$, called the *substitution function*, which associates with each vertex of G one of the multipoles U_i and with each edge of G one of the dipoles X_j in such a way that the connectors which correspond to an incidence between a vertex and an edge in G have the same size. We make an additional agreement that if $f(v)$ is not specified, then it is meant to be the multipole consisting of a single vertex and three dangling edges having three connectors of one semiedge each. Similarly, if $f(e)$ is not specified, it is meant to be the dipole consisting of a single isolated edge having one semiedge in each connector. We now construct a new cubic graph $\tilde{G}$ as follows. For each vertex v of G we take a copy $\tilde{v}$ of $f(v)$ (isomorphic to some U_i), for each edge e we take a copy $\tilde{e}$ of $f(e)$ (isomorphic to some X_j), and perform all junctions of pairs of connectors corresponding to the incidences in G. The resulting graph $\tilde{G}$ is called a *superposition* of G. In the rest of the paper, the symbols $\tilde{G}$, $\tilde{v}$ and $\tilde{e}$ will refer to a superposition of G, the supervertex substituting a vertex v and the superedge substituting and edge e of G, respectively.

If all superedges used in a superposition of a snark are proper dipoles, the resulting graph is again a snark. This fact was proved by Kochol in [10, Theorem 4] and will be used in our construction.

3 Main result

Let G be a cubic graph and let $\mathcal{C}$ be a collection of disjoint cycles in G. A $\mathcal{C}$-*superposition* of G is a graph $\tilde{G}$ created by a substitution function which sends each edge in $\mathcal{C}$ to a dipole of width three and each vertex in $\mathcal{C}$ to a copy of the multipole V with three connectors (e_t, e_m, e_b), (x), and (f_t, f_m, f_b) shown in Fig. 2. (For easier reference, the subscripts of semiedges refer to 'top', 'middle' and 'bottom'.)

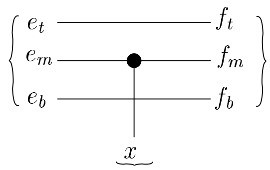


Figure 2: Multipole V

The purpose of this section is to present a sufficient condition under which a $\mathcal{C}$-superposition $\tilde{G}$ of a hypohamiltonian snark G is again a hypohamiltonian snark. Such a condition must guarantee the existence of a *hypohamiltonian* cycle for each vertex of the larger graph (that is, a cycle containing all but that one vertex). Therefore a number of paths through superedges employed in the superposition has to be specified.

Let E be a dipole of width three and let $\mathrm{In}(E) = (e_t, e_m, e_b)$ and $\mathrm{Out}(E) = (f_t, f_m, f_b)$. Let us enumerate the vertices of E as $1, 2, \ldots, n$ in such a way that the vertex incident with e_m will have label 1, and the vertex incident with f_m will have label 2 (see Fig. 3). We introduce the following notation for paths through E corresponding to different ways of traversal of E (see Fig. 4):

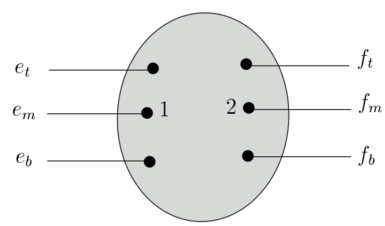


Figure 3: Dipole E

- **Type O:** One path through E, denoted by $x(E, \mathrm{O})y$, covering all the vertices of E and ending with dangling edges x and y, where $x, y \in \{e_t, e_m, e_b, f_t, f_m, f_b\}$.
- **Type A:** Two disjoint paths $e_\alpha(E, \mathrm{A})^1 e_\beta$ and $f_\gamma(E, \mathrm{A})^2 f_\delta$ through E which together cover all the vertices of E, the former ending with e_α and e_β, the latter one ending with f_γ and f_δ, where $\alpha, \beta, \gamma, \delta \in \{t, m, b\}$.
- **Type B:** Two disjoint paths $e_\alpha(E, \mathrm{B})^1 f_\beta$ and $\kappa_\gamma(E, \mathrm{B})^2 \kappa_\delta$ through E which together cover all the vertices of E, the former ending with e_α and f_β, the latter ending with κ_γ and κ_δ, where $\alpha, \beta, \gamma, \delta \in \{t, m, b\}$ and $\kappa \in \{e, f\}$.

- **Type Z:** Two disjoint paths $e_\alpha(E-i,\mathrm{Z})^1 f_\beta$ and $e_\gamma(E-i,\mathrm{Z})^2 f_\delta$ through E covering all the vertices of E except for the vertex labelled i, the former ending with e_α and f_β, the latter ending with e_γ and f_δ, where $\alpha,\beta,\gamma,\delta \in \{t,m,b\}$.

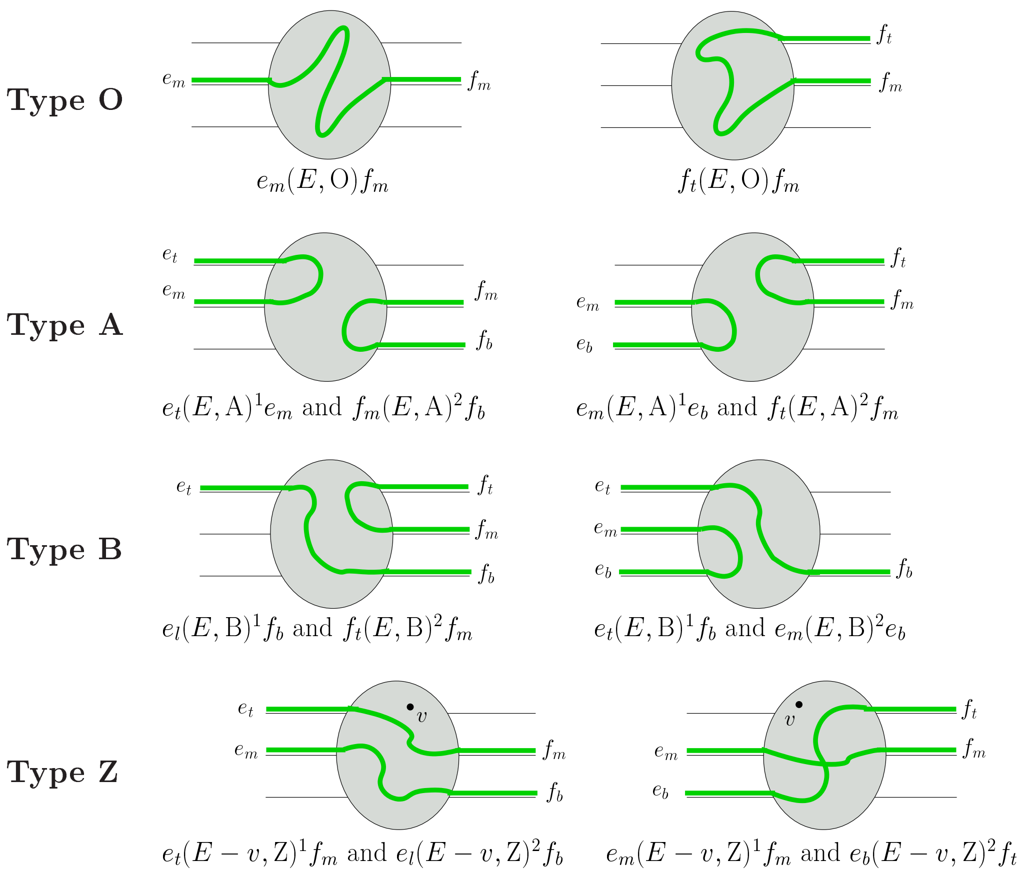


Figure 4: Paths through E

Transitions through supervertices are unambiguous, since each supervertex contains only one vertex. Thus we can denote the path which enters a supervertex U with a dangling edge x and leaves it with a dangling edge y simply by xUy.

A dipole E of width three will be called *feasible* if it has all the following paths and pairs of paths:

(1) $x(E,\mathrm{O})y$ for any $(x,y) \in \{(e_m,f_m),(e_b,f_m),(e_m,f_b),(e_b,f_b),(e_t,f_t),(f_m,f_b),(e_m,e_b)\}$;

(2) $e_\alpha(E,\mathrm{A})^1 e_\beta$ and $f_\gamma(E,\mathrm{A})^2 f_\delta$ for any $(\alpha,\beta,\gamma,\delta) \in \{(m,b,m,t),(m,t,m,b)\}$;

(3) $e_\alpha(E,\mathrm{B})^1 f_\beta$ and $\kappa_\gamma(E,\mathrm{B})^2 \kappa_\delta$ for any $(\alpha,\beta,\gamma,\delta,\kappa) \in \{(t,b,m,t,f),(m,t,m,b,f),(t,m,m,b,e)\}$;

(4a) $e_\alpha(E-1,\mathrm{Z})^1 f_\beta$ and $e_\gamma(E-1,\mathrm{Z})^2 f_\delta$ such that $\{\beta,\delta\}=\{b,m\}$, for suitable α and γ;

(4b) $e_m(E-2,\mathrm{Z})^1 f_\beta$ and $e_b(E-2,\mathrm{Z})^2 f_\delta$ such that $\{\beta,\delta\}=\{t,b\}$, for suitable β and δ;

and for every $i \in V(E)-\{1,2\}$,

(4c) $e_\alpha(E-i,\mathrm{Z})^1 f_\beta$ and $e_\gamma(E-i,\mathrm{Z})^2 f_\delta$ such that both $\{\alpha,\gamma\}$ and $\{\beta,\delta\}$ contain m, for suitable α, β, γ, and δ.

Accordingly, a $\mathcal{C}$-superposition will be called *feasible* if all the dipoles replacing the edges of $\mathcal{C}$ are feasible.

Our main result is the following theorem.

Theorem 3.1 *Let G be a hypohamiltonian snark and let $\tilde{G}$ be a feasible $\mathcal{C}$-superposition of G with respect to a set $\mathcal{C}$ of disjoint cycles in G. Then $\tilde{G}$ is a hypohamiltonian snark.*

We prove the theorem in the next section, but now we show that feasible dipoles indeed exist. To see this, take the Isaacs snark I_k, k odd, remove two vertices u and v shown in Fig. 5 and group the semiedges formerly incident with u into the input connector and those formerly incident with v into the output connector. Let J_k be the resulting dipole with $\mathrm{In}(J_k)$ and $\mathrm{Out}(J_k)$ as indicated in Fig. 5. Fig. 6 displays the dipole J_7 together with a numbering of its vertices.

Another feasible dipole can be created from the double-star snark by removing two vertices u and v and grouping the resulting semiedges into connectors as shown in Fig. 7. We denote it by D.

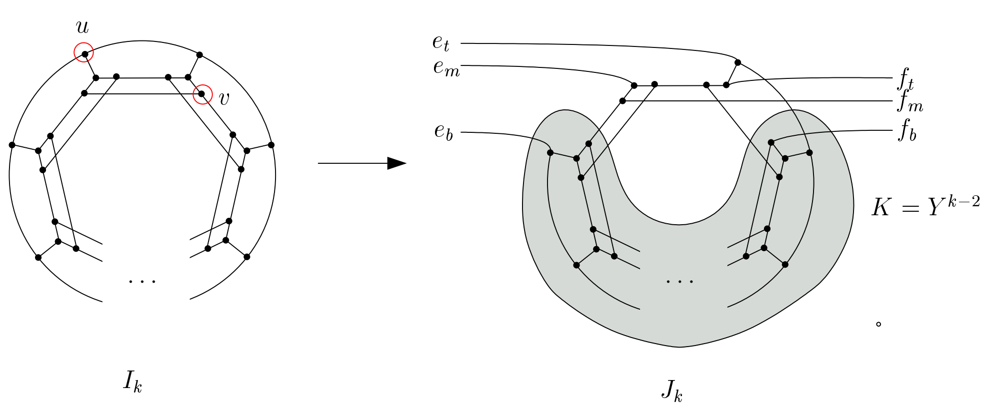


Figure 5: Constructing J_k from I_k

Proposition 3.2 *The dipoles J_{7+4i}, $i \geq 0$, and the dipole D are feasible.*

Proof. Tables 1-8 show that the dipoles J_7 and D have all the required paths and therefore are feasible. To prove that J_{7+4i} is feasible for each $i \geq 0$ we employ induction on i. As the base step has already been done above, we proceed to the induction step.

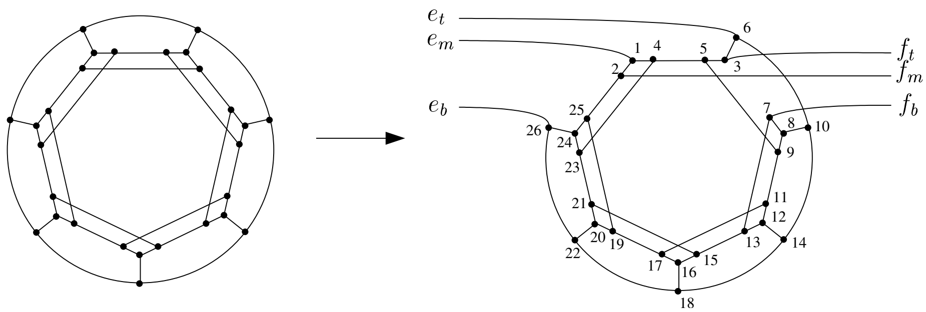


Figure 6: Dipole J_7 with a vertex labelling

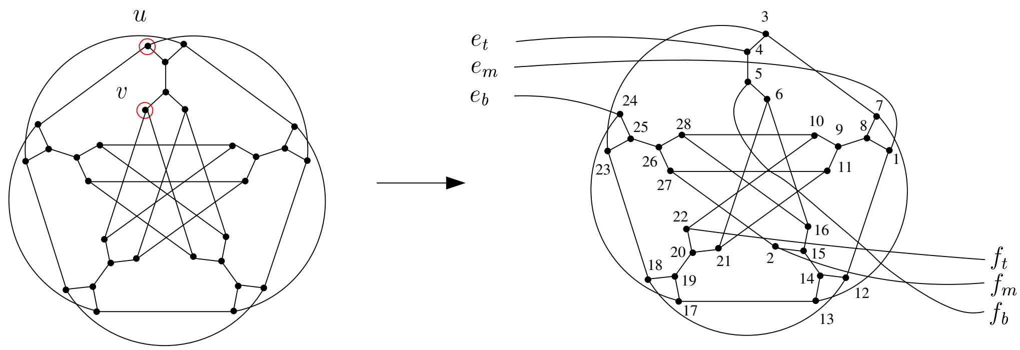


Figure 7: Constructing D from the double-star

Assume that J_{7+4i} is feasible for some $i \geq 0$. In order to show that $J_{7+4(i+1)}$ has all the required paths to be feasible we extend the paths guaranteed in J_{7+4i} to paths in $J_{7+4(i+1)}$.

To construct paths or pairs of paths which cover all the vertices of $J_{7+4(i+1)}$ (Types O, A, and B) we proceed as follows. Since the vertices u and v removed from I_{7+4i} to create J_{7+4i} belong to two neighbouring copies of Y, the dipole J_{7+4i} contains a dipole isomorphic to Y^{7+4i-2} which we denote by K_{7+4i} (see Fig. 5). It is easy to see that Y does not contain a collection of paths covering all its vertices and at the same time containing all the dangling edges. Therefore, in the dipole $K_{7+4i} \subseteq J_{7+4i}$ there exists a copy Y' of Y whose output connector has at most two semiedges covered by the paths. Replace Y' in J_{7+4i} with $Y' \circ Y^4$ to obtain $J_{7+4(i+1)}$. Now extend the paths through Y' to paths through $Y' \circ Y^4$ by using paths in Y^4 indicated in Fig. 8 in such a way that the covered semiedges in $\mathrm{Out}(Y')$ and in $\mathrm{In}(Y^4)$ match. It is easy to see that such an extension is always possible.

To finish the proof we construct paths which leave an arbitrary single vertex v of $J_{7+4(i+1)}$ not covered, that is, $e_\alpha(J_{7+4(i+1)} - v, \mathrm{Z})^1 f_\beta$ and $e_\gamma(J_{7+4(i+1)} - v, \mathrm{Z})^2 f_\delta$. If v is not contained in $K_{7+4(i+1)}$, we can proceed as in the previous case. If v is in $K_{7+4(i+1)}$, we observe that $K_{7+4(i+1)}$ contains at least nine subsequent copies of Y. Therefore $J_{7+4(i+1)} - v$ contains a copy Y'' of $Y - v$ connected to a copy of Y^4 in at least one of two possible ways, either $Y'' \circ Y^4$ or $Y^4 \circ Y''$. It is easy to see that $Y - v$, too, does not contain a collection of paths covering all its vertices and at the same time containing all the dangling edges.

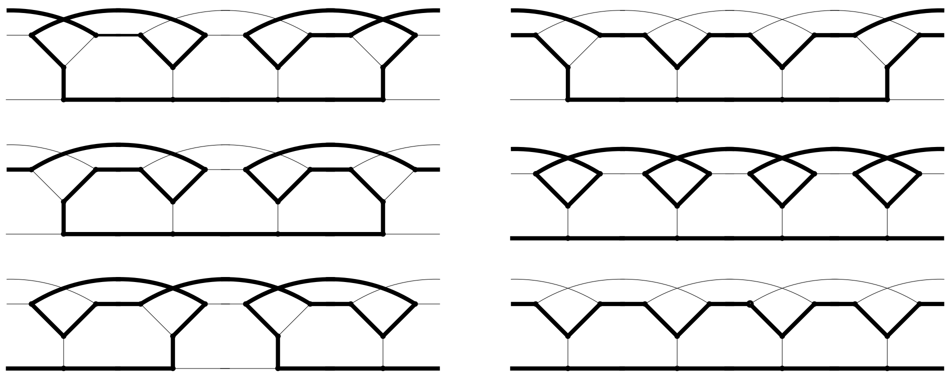


Figure 8: Paths in Y^4

By replacing this copy of Y^4 with three parallel edges we obtain $J_{7+4i} - v$ which, by the induction hypothesis, contains paths $e_\alpha(J_{7+4i} - v, \mathrm{Z})^1 f_\beta$ and $e_\gamma(J_{7+4i} - v, \mathrm{Z})^2 f_\delta$ covering all the vertices but v. Since the original Y^4 in $J_{7+4(i+1)}$ is connected to $Y - v$, at most two of the three parallel edges are covered by these paths. Therefore it is possible to extend these paths to the required paths in $J_{7+4(i+1)} - v$ by using paths in Y^4 shown in Fig. 8. □

One can easily check that the smallest cyclically 5-connected snark and the smallest cyclically 6-connected snark which can be composed from the dipoles described in Proposition 3.2 are of order 140 and 166 respectively. With a little bit more care the following result can be obtained.

Corollary 3.3
(a) *There exists a hypohamiltonian snark with cyclic connectivity* 5 *and order* n *for each even* $n \geq 140$.
(b) *There exists a hypohamiltonian snark with cyclic connectivity* 6 *and order* n *for each even* $n \geq 166$.

Proof. Let G be the Petersen graph and let C be any 5-cycle in G. Substitute the edges of C by i copies of J_7, $(4-i)$ copies of D and one copy of J_{7+4j}, $0 \leq i \leq 4$, $j \geq 0$. These graphs cover five of the eight even residue classes modulo 16. For the remaining three even residue classes, let $G = I_5$ and let C be its unique 5-cycle. Replace i edges of C by a copy of J_7, $(4-i)$ edges by a copy of D and the last edge by J_{7+4j}, $i = 2, 3, 4$, $j \geq 0$. Altogether this yields cyclically 5-connected hypohamiltonian snarks of any even order greater than 138. This proves (a).

To construct snarks with cyclic connectivity 6, let us start again with G the Petersen graph, but take C to be a 6-cycle in G. Substitute the edges of C by i copies of J_7, $(5-i)$ copies of D and one copy of J_{7+4j}, $0 \leq i \leq 5$, $j \geq 0$. The graphs now cover six of the eight even residue classed modulo 16. For the remaining two even residue classes take $G = I_5$ and C a 6-cycle intersecting the unique 5-edge-cut in I_5. Replace i edges of C by J_7, $(5-i)$ edges by a copy of D and the last edge by J_{7+4j}, $i = 3, 4$, $j \geq 0$. The resulting graphs are cyclically 6-connected hypohamiltonian snarks of any even order greater than 164. This proves (b). □

Call a snark *irreducible* if the removal of every edge-cut different from the three edges incident with a vertex yields a 3-edge-colourable graph. It was shown in [12] that a snark is irreducible if and only if it is bicritical, that is, if the removal of any two distinct vertices produces a 3-edge-colourable graph. It is not difficult to see that every hypohamiltonian snark is bicritical and hence irreducible (see [13]). Thus the following result is true.

Corollary 3.4
(a) *There exists an irreducible snark of cyclic connectivity* 5 *and order* n *for each even* $n \geq 140$.
(b) *There exists an irreducible snark of cyclic connectivity* 6 *and order* n *for each even* $n \geq 166$.

4 Proof

In this section we prove Theorem 3.1. Let G be a hypohamiltonian snark and let $\mathcal{C}$ be a collection of disjoint cycles in G. We want to show that every feasible $\mathcal{C}$-superposition $\tilde{G}$ of G is a hypohamiltonian snark. Since any feasible dipole is proper, the result of Kochol [10] implies that $\tilde{G}$ is a snark, and hence a non-hamiltonian graph. It remains to prove that every vertex-deleted subgraph of $\tilde{G}$ is hamiltonian.

Without loss of generality we may assume that $\mathcal{C}$ consists of a single cycle $C = (w_0 f_0 w_1 f_1 \ldots f_{k-1} w_{k-1})$, for otherwise we can repeat the whole procedure with other cycles of $\mathcal{C}$. Recall that this superposition substitutes each vertex w_i with a copy V_i of the multipole V exhibited in Fig. 2, and each edge f_j with a copy E_j of a feasible dipole. In order to show that for each vertex v of $\tilde{G}$ the subgraph $\tilde{G} - v$ contains a hamiltonian cycle we proceed as follows. We find a suitable vertex v' in G, take a hamiltonian cycle in $G - v'$, say, $H = (v_0 e_0 v_1 e_1 \ldots e_{n-1} v_{n-1})$, and expand it into a hamiltonian cycle $\tilde{H} = (P_0 Q_0 P_1 Q_1 \ldots P_{n-1} Q_{n-1})$ of $\tilde{G} - v$ by replacing each vertex v_i in H with a path P_i intersecting the corresponding supervertex V_i, and by replacing each edge e_j in H with a path Q_j intersecting the corresponding superedge E_j. Each of the paths P_i and Q_j will be referred to as a *vertex-section* and an *edge-section* of $\tilde{H}$, respectively. Note, however, that the required hypohamiltonian cycle $\tilde{H}$ must traverse all the vertices of $\tilde{G}$ but one, including the vertices in superedges corresponding to edges outside H. Therefore certain vertex-sections of $\tilde{H}$ have to make 'detours' into such superedges.

Let S denote the subgraph of $\tilde{G}$ corresponding to $G - C$. From the way how $\tilde{G}$ was constructed from G it is clear that the vertices and edges of H contained in $G - C$ can be substituted by their identical copies. Thus the corresponding vertex-sections and edge-sections of $\tilde{H}$ are either a single vertex or a single edge. In other words, we set $\tilde{H} \cap S = H \cap (G - C)$.

We now describe the remaining vertex-sections and edge-sections of $\tilde{H}$. In fact, each edge-section Q_i can easily be derived from the vertex-sections P_i and P_{i+1} (indices taken modulo n): it is either a single edge or a path of Type O with a suitable initial and terminal semiedge guaranteed by feasibility. Thus we only need to describe vertex-sections. Our description will depend on the position of the vertex v avoided by $\tilde{H}$ and will split in a number of cases and subcases according to the type of a multipole containing the vertex v. It may happen, however, that a vertex-section displayed in a certain direction will actually have to be traversed in the opposite direction. Throughout the rest of the proof, the copy of a vertex u of in E_j will be denoted by u_j.

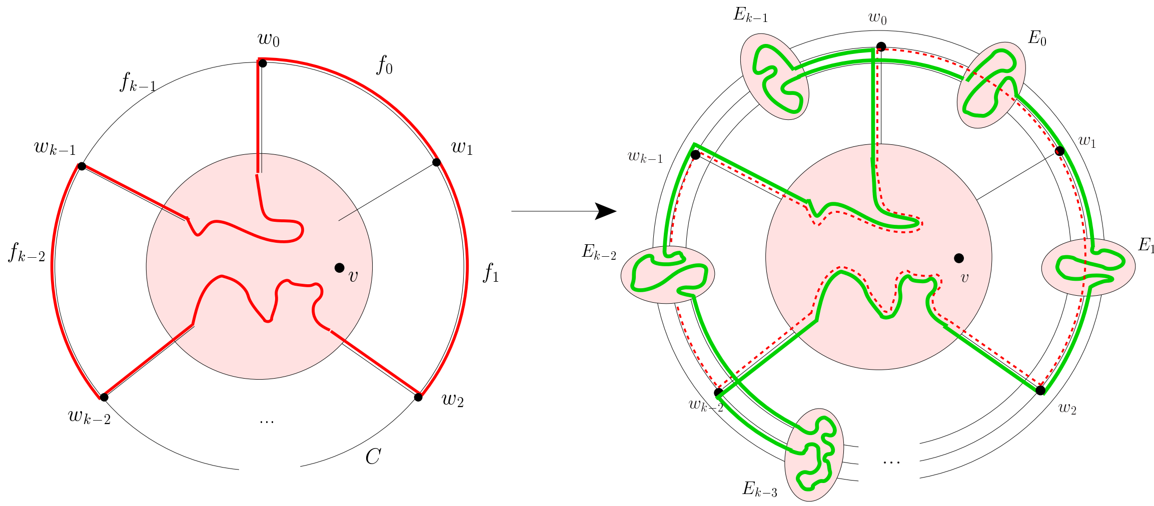


Figure 9: Case I: $v \in S$

Case I: $v \in S$**.** In this case, every superedge E_j of $\tilde{G}$ which substitutes an edge in $C \cap H$ will be traversed by a path of Type O. A superedge E_j corresponding to an edge in $C - H$ will be traversed by a path which ends in the neighbouring supervertex V_{j+1} (indices taken modulo k), its traversal through E_j being also of Type O (see Fig. 9 and Scheme $\boxed{\text{i}}$).

We now give an exact description of all vertex-sections P_i of $\tilde{H}$. There will always be two cases, each case can have two or more subcases, and so on. For the case analysis a nested numeration will be employed. The cases on the same level will always be mutually excluding.

To get the definition of P_i, proceed as follows. Start on the topmost level. Then, on the current level, choose either the branch with one of conditions following the **if** word or, if there is no such condition, choose the **else** branch, and move to the next level. Once the **then** word is reached, the definition of P_i is completed; a boxed number after it denotes the corresponding traversal scheme which is shown in the figure with the same number.

Vertex-sections P_i*.*
1. **if** $v_i \in S$ **then** $P_i = v_i$
2. **if** $v_i \notin S(v_i = w_j)$
    2.1 **if** $e'_j \in H$
        2.1.1 **if** $v_{i+1} = w_{j+1}$ or $v_{i-1} = w_{j+1}$ **then** $P_i = xV_j e_m * f_m(E_{j-1}, \mathrm{O}) f_b * e_b V_j f_b$ $\boxed{\text{i}}$
        2.1.2 **if** $v_{i+1} = w_{j-1}$ or $v_{i-1} = w_{j-1}$ **then** $P_i = xV_j e_m$ $\boxed{\text{ii}}$
    2.2 **if** $e'_j \notin H$ **then** $P_i = e_m V_j f_m$ $\boxed{\text{iii}}$

Used path types: O.

Case II: $v = V_j$**.** We proceed similarly as in the previous case, but since the vertex in V_j is missing, the vertices of E_{j-1} will be covered by a 'detour' ending in V_{i-1} (cf. Scheme $\boxed{\text{iv}}$).

Vertex-sections P_i.

1. **if** $v_i \in S$ **then** $P_i = v_i$
2. **if** $v_i \notin S(v_i = w_j)$
   2.1 **if** $e'_j \in H$
      2.1.1 **if** $v_{i+1} = w_{j+1}$ or $v_{i-1} = w_{j+1}$ **then** $P_i = xV_je_m * f_m(E_{j-1}, \mathrm{O})f_b * e_bV_jf_b$ $\boxed{\text{i}}$
      2.1.2 **if** $v_{i+1} = w_{j-1}$ or $v_{i-1} = w_{j-1}$
         2.1.2.1 **if** $w_{j+1} = v$ **then** $P_i = xV_jf_m * e_m(E_{j+1}, \mathrm{O})e_b * f_bV_jf_b$ $\boxed{\text{iv}}$
         2.1.2.2 **if** $w_{j+1} \neq v$ **then** $P_i = xV_je_m$ $\boxed{\text{ii}}$
   2.2 **if** $e'_j \notin H$ **then** $P_i = e_mV_jf_m$ $\boxed{\text{iii}}$

Used path types: O.

Case III: $v \in E_a$**.** Since E_a is feasible, there exists a pair of paths $e_\alpha(E - v, \mathrm{Z})^1 f_\beta$ and $e_\gamma(E - v, \mathrm{Z})^2 f_\delta$. Let $L = \{\alpha, \gamma\}$ and $R = \{\beta, \delta\}$. Vertices in E_a will be covered by paths $e_\alpha(E - v, \mathrm{Z})^1 f_\beta$ and $e_\gamma(E - v, \mathrm{Z})^2 f_\delta$. The remaining portions of $\tilde{H}$ will be constructed in dependence on L and R.

(a) $v \neq 1_a$ and $v \neq 2_a$.

1. **if** $v_i \in S$ **then** $P_i = v_i$
2. **if** $v_i \notin S$ (we have $v_i = w_j$)
   2.1. **if** $e'_j \in H$
      2.1.1 **if** $v_{i+1} = w_{j+1}$ or $v_{i-1} = w_{j+1}$
         2.1.1.1 **if** $v_i = w_{a+1}$
            2.1.1.1.1 **if** $L = \{m, b\}$ **then** $P_i = xV_{a+1}e_m * f_m(E_a - v, \mathrm{Z})^1 e_z * f_zV_ae_z * f_z(E_{a-1}, \mathrm{O})f_y *$
            $* e_yV_af_y * e_y(E_a - v, \mathrm{Z})^2 f_w * e_wV_{a+1}f_w$,
            where $\{y, z\} = \{m, b\}$ and $w \in \{t, b\}$ $\boxed{\text{v}}$
            2.1.1.1.2 **if** $L = \{t, m\}$ **then** $P_i = xV_{a+1}e_m * f_m(E_a - v, \mathrm{Z})^1 e_z * f_zV_ae_z * f_z(E_{a-1}, \mathrm{A})^2 f_y *$
            $* e_yV_af_y * e_y(E_a - v, \mathrm{Z})^2 f_w * e_wV_{a+1}f_w$,
            where $\{y, z\} = \{m, t\}$ and $w \in \{t, b\}$ $\boxed{\text{vi}}$
         2.1.1.2 **if** $v_i = w_{a+3}$ and $R = \{t, m\}$ **then** $P_i = xV_{a+3}e_m * f_m(E_{a+2}, \mathrm{A})^2 f_b * e_bV_{a+3}f_b$ $\boxed{\text{vii}}$
         2.1.1.3 **else** $P_i = xV_je_m * f_m(E_{j-1}, \mathrm{O})f_b * e_bV_jf_b$ $\boxed{\text{i}}$
      2.1.2 **if** $v_{i+1} = w_{j-1}$ or $v_{i-1} = w_{j-1}$
         2.1.2.1 **if** $v_i = w_{a-1}$ and $L = \{t, m\}$ **then** $P_i = xV_{a-1}f_m * e_m(E_{a-1}, \mathrm{A})^1 e_b * f_bV_{a-1}e_b$ $\boxed{\text{viii}}$
         2.1.2.2 **if** $v_i = w_{a+2}$ and $R = \{t, m\}$ **then** $P_i = xV_{a+2}f_m * e_m(E_{a+2}, \mathrm{A})^1 e_t * f_tV_{a+2}e_t$ $\boxed{\text{ix}}$
         2.1.2.3 **else** $P_i = xV_je_m$ $\boxed{\text{ii}}$
   2.2 **if** $e'_j \notin H$
      2.2.1 **if** $v_i = w_{a+2}$ and $e'_{a+2} \notin H$ **then** $P_i = e_bV_{a+2}f_b * e_b(E_{a+2}, \mathrm{B})^2 e_m * f_mV_{a+2}e_m *$
      $* f_m(E_{a+1}, \mathrm{B})^2 f_m * e_tV_{a+2}f_t$ $\boxed{\text{x}}$
      2.2.2 **else** $P_i = e_mV_jf_m$ $\boxed{\text{iii}}$

Used path types: O, A, B, Z.

(b) $v = 1_a$

1. **if** $v_i \in S$ **then** $P_i = v_i$
2. **if** $v_i \notin S$ $(v_i = w_j)$
   - 2.1 **if** $e'_j \in H$
     - 2.1.1 **if** $v_{i+1} = w_{j+1}$ or $v_{i-1} = w_{j+1}$
       - 2.1.1.1 **if** $v_i = w_{a+1}$ **then** $P_i = xV_{a+1}f_m$ $\boxed{\text{x}}$
       - 2.1.1.2 **if** $v_i \neq w_{a+1}$ **then** $P_i = xV_je_m * f_m(E_{j-1}, \mathrm{O})f_b * e_bV_jf_b$ $\boxed{\text{i}}$
     - 2.1.2 **if** $v_{i+1} = w_{j-1}$ or $v_{i-1} = w_{j-1}$
       - 2.1.2.1 **if** $v_i = w_{a+1}$ **then** $P_i = xV_{a-1}e_m * f_m(E_{a-2}, \mathrm{B})^2 f_b * e_b(E_{a-1} - 1, \mathrm{Z})^2 f_b * e_bV_af_b *$ $* e_b(E_a, \mathrm{O})e_m * f_mV_ae_m * f_m(E_{a-1} - 1, \mathrm{Z})^1 e_t * f_tV_{a-1}e_t * f_t(E_{a-2}, \mathrm{O})e_m$ $\boxed{\text{xi}}$
       - 2.1.2.2 **else** $P_i = xV_je_m$ $\boxed{\text{ii}}$
   - 2.2 **if** $e'_j \notin H$ **then** $P_i = e_mV_jf_m$ $\boxed{\text{iii}}$

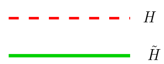
Used path types: O, B, Z.

(c) $v = 2_a$

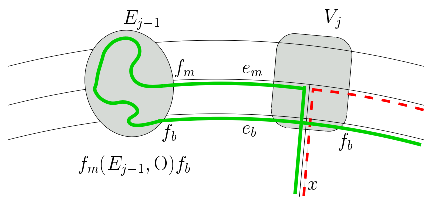
1. **if** $v_i \in S$ **then** $P_i = v_i$
2. **if** $v_i \notin S$ $(v_i = w_j)$
   - 2.1. **if** $e'_j \in H$
     - 2.1.1 **if** $v_{i+1} = w_{j+1}$ or $v_{i-1} = w_{j+1}$
       - 2.1.1.1 **if** $v_i = w_{a+1}$ **then** $P_i = xV_{a+1}f_m * e_m(E_{a+1}, \mathrm{B})^2 e_b * f_bV_{a+1}e_b *$ $f_b(E_a - 2, \mathrm{Z})^2 e_m * f_mV_ae_m * f_m(E_{a-1}, \mathrm{O})f_b * e_bV_af_b * e_b(E_a - 2, \mathrm{Z})^1 f_t *$ $* e_tV_{a+1}f_t * e_t(E_{a+1}, \mathrm{B})^1 f_m$ $\boxed{\text{xii}}$
       - 2.1.1.2 **else** $P_i = xV_je_m * f_m(E_{j-1}, \mathrm{O})f_b * e_bV_jf_b$ $\boxed{\text{i}}$
     - 2.1.2 **if** $v_{i+1} = w_{j-1}$ or $v_{i-1} = w_{j-1}$ **then** $P_i = xV_je_m$ $\boxed{\text{ii}}$
   - 2.2 **if** $e'_j \notin H$ **then** $P_i = e_mV_jf_m$ $\boxed{\text{iii}}$

Used path types: O, B, Z.

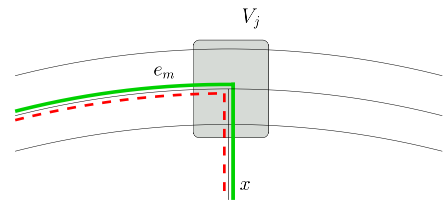
It is a routine matter to verify that the case analysis is complete, and that in each case the required hypohamiltonian cycle has been constructed. The proof is finished. □



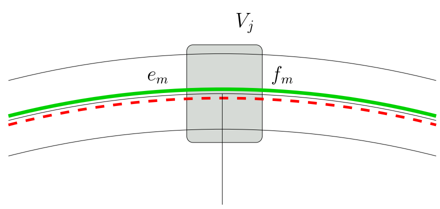
(a)



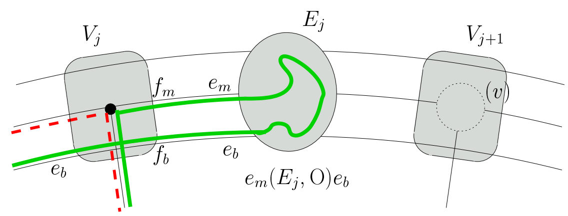
(b) Scheme $\boxed{\text{i}}$



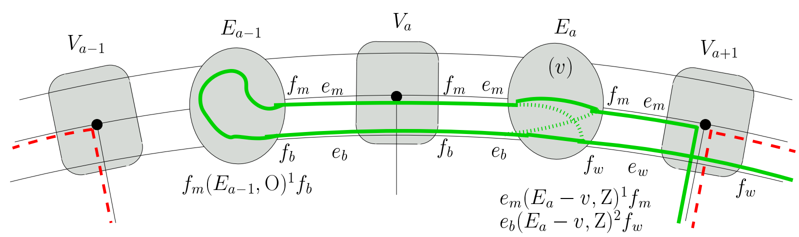
(c) Scheme $\boxed{\text{ii}}$



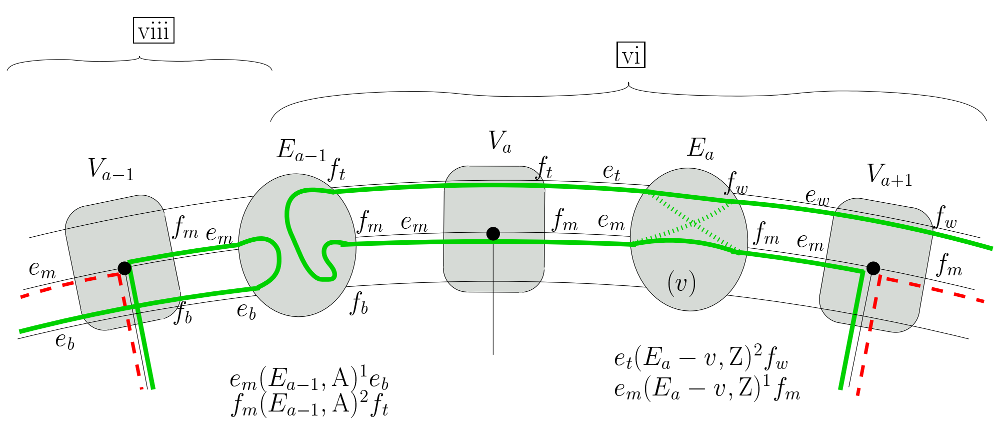
(d) Scheme $\boxed{\text{iii}}$



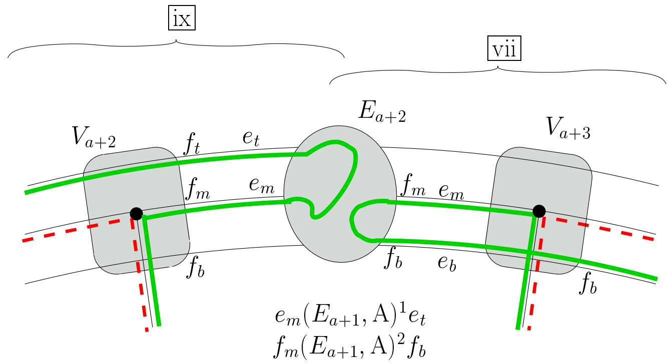
(e) Scheme $\boxed{\text{iv}}$



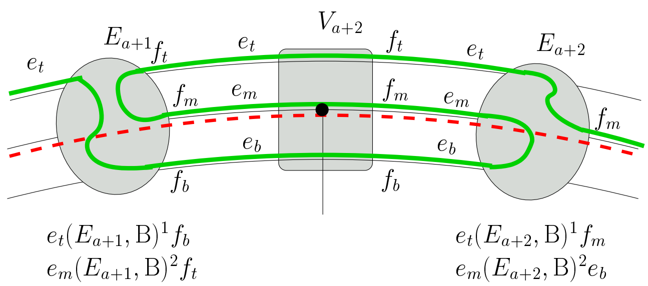
(f) Scheme $\boxed{\text{v}}$



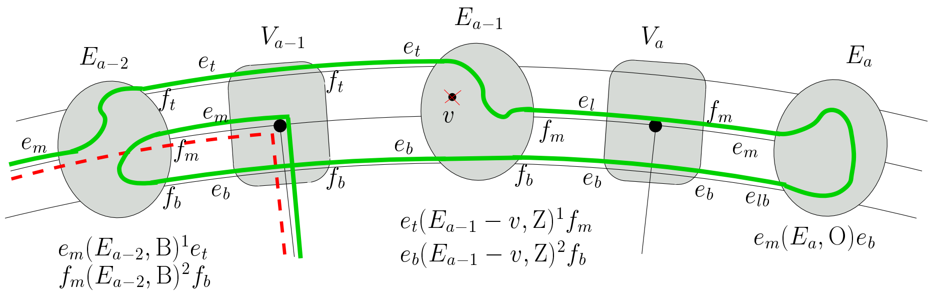
(g) Schemes $\boxed{\text{vi}}$ and $\boxed{\text{viii}}$



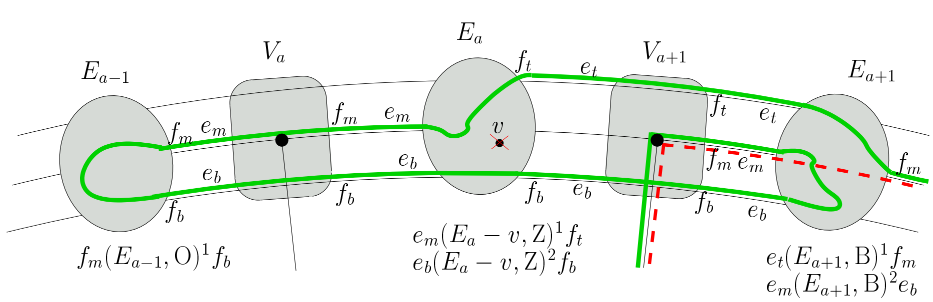
(h) Schemes $\boxed{\text{vii}}$ and $\boxed{\text{ix}}$



(i) Scheme $\boxed{\text{x}}$



(j) Scheme $\boxed{\text{xi}}$



(k) Scheme $\boxed{\text{xii}}$

Tables

Superedge $E = J_7$

Table 1: $x(E, \mathrm{O})y$

x	y	$x(E, \mathrm{O})y$
e_m	f_m	1 4 23 21 15 13 7 8 10 6 3 5 9 11 12 14 18 16 17 19 20 22 26 24 25 2
e_b	f_m	26 22 18 14 10 6 3 5 9 8 7 13 12 11 17 16 15 21 20 19 25 24 23 4 1 2
e_m	f_b	1 2 25 19 17 11 9 8 10 6 3 5 4 23 24 26 22 20 21 15 16 18 14 12 13 7
e_b	f_b	26 22 18 14 10 6 3 5 4 1 2 25 24 23 21 20 19 17 16 15 13 12 11 9 8 7
e_t	f_t	6 10 8 7 13 12 14 18 16 15 21 20 22 26 24 23 4 1 2 25 19 17 11 9 5 3
f_m	f_b	2 1 4 23 21 20 19 25 24 26 22 18 14 12 13 15 16 17 11 9 5 3 6 10 8 7
e_m	e_b	1 2 25 19 17 16 18 22 20 21 15 13 7 8 9 11 12 14 10 6 3 5 4 23 24 26

Table 2: $e_\alpha(E, \mathrm{A})^1 e_\beta$ and $f_\gamma(E, \mathrm{A})^2 f_\delta$

α	β	γ	δ	$e_\alpha(E, \mathrm{A})^1 e_\beta$	$f_\gamma(E, \mathrm{A})^2 f_\delta$
m	b	m	t	1 4 5 9 8 7 13 12 11 17 16 15 21 23 24 26	2 25 19 20 22 18 14 10 6 3
m	t	m	b	1 4 23 21 15 13 12 11 9 5 3 6	2 25 24 26 22 20 19 17 16 18 14 10 8 7

Table 3: $e_\alpha(E, \mathrm{B})^1 f_\beta$ and $\kappa_\gamma(E, \mathrm{B})^2 \kappa_\delta$

α	β	γ	δ	κ	$e_\alpha(E, \mathrm{B})^1 f_\beta$	$\kappa_\gamma(E, \mathrm{B})^2 \kappa_\delta$
t	b	m	t	r	6 10 8 7	2 1 4 23 21 20 19 25 24 26 22 18 14 12 13 15 16 17 11 9 5 3
m	t	m	b	r	1 4 5 9 8 10 6 3	2 25 19 20 21 23 24 26 22 18 14 12 11 17 16 15 13 7
t	m	m	b	l	6 3 5 9 11 12 13 7 8 10 14 18 22 20 21 15 16 17 19 25 2	1 4 23 24 26

Table 4: $e_\alpha(E-i,\mathrm{Z})^1 f_\beta$ and $e_\gamma(E-i,\mathrm{Z})^2 f_\delta$

i	$e_\alpha(E-i,Z)^1 f_\beta$	$e_\gamma(E-i,Z)^2 f_\delta$
1	6 3 5 4 23 21 15 16 17 19 20 22 18 14 10 8 9 11 12 13 7	26 24 25 2
2	1 4 5 9 8 7	26 22 20 21 23 24 25 19 17 11 12 13 15 16 18 14 10 6 3
3	1 2	6 10 14 18 16 15 13 12 11 17 19 25 24 26 22 20 21 23 4 5 9 8 7
4	1 2	26 22 20 21 23 24 25 19 17 11 12 13 15 16 18 14 10 6 3 5 9 8 7
5	1 4 23 21 15 16 18 22 20 19 17 11 9 8 7 13 12 14 10 6 3	26 24 25 2
6	1 4 23 21 15 16 17 19 20 22 18 14 10 8 7 13 12 11 9 5 3	26 24 25 2
7	1 4 5 3	6 10 8 9 11 17 16 15 13 12 14 18 22 26 24 23 21 20 19 25 2
8	1 4 23 24 25 2	26 22 18 16 15 21 20 19 17 11 9 5 3 6 10 14 12 13 7
9	1 2	6 10 8 7 13 15 16 17 11 12 14 18 22 26 24 25 19 20 21 23 4 5 3
10	1 2	6 3 5 4 23 21 20 19 25 24 26 22 18 14 12 13 15 16 17 11 9 8 7
11	1 4 23 24 25 2	26 22 18 16 17 19 20 21 15 13 12 14 10 6 3 5 9 8 7
12	1 2	6 10 14 18 16 15 13 7 8 9 11 17 19 25 24 26 22 20 21 23 4 5 3
13	1 4 23 21 15 16 17 19 20 22 18 14 12 11 9 5 3 6 10 8 7	26 24 25 2
14	1 4 23 24 25 2	26 22 18 16 17 19 20 21 15 13 12 11 9 5 3 6 10 8 7
15	1 2	6 3 5 4 23 21 20 22 26 24 25 19 17 16 18 14 10 8 9 11 12 13 7
16	1 4 23 21 15 13 12 11 17 19 20 22 18 14 10 6 3 5 9 8 7	26 24 25 2
17	1 2	6 10 14 12 11 9 8 7 13 15 16 18 22 26 24 25 19 20 21 23 4 5 3
18	1 4 5 3	6 10 14 12 13 7 8 9 11 17 16 15 21 23 24 26 22 20 19 25 2
19	1 4 23 24 25 2	26 22 20 21 15 13 12 11 17 16 18 14 10 6 3 5 9 8 7

Table 4

i	$e_\alpha(E-i,Z)^1 f_\beta$	$e_\gamma(E-i,Z)^2 f_\delta$
20	1 2	6 3 5 4 23 21 15 16 17 19 25 24 26 22 18 14 10 8 9 11 12 13 7
21	1 4 23 24 25 2	26 22 20 19 17 11 12 13 15 16 18 14 10 6 3 5 9 8 7
22	1 4 23 21 20 19 17 11 12 13 15 16 18 14 10 6 3 5 9 8 7	26 24 25 2
23	1 4 5 3	6 10 14 12 13 7 8 9 11 17 19 20 21 15 16 18 22 26 24 25 2
24	1 4 23 21 15 13 12 11 17 16 18 14 10 6 3 5 9 8 7	26 22 20 19 25 2
25	1 2	6 10 14 12 13 7 8 9 11 17 19 20 21 15 16 18 22 26 24 23 4 5 3
26	1 2	6 3 5 4 23 24 25 19 17 16 15 21 20 22 18 14 10 8 9 11 12 13 7

Superedge $E = D$

Table 5: $x(E,\mathrm{O})y$

x	y	$x(E,\mathrm{O})y$
e_m	f_m	1 8 7 13 14 12 18 19 17 24 25 23 3 4 5 6 21 20 22 10 9 11 27 26 28 16 15 2
e_b	f_m	24 17 13 7 3 4 5 6 21 11 9 8 1 12 14 15 16 28 10 22 20 19 18 23 25 26 27 2
e_m	f_b	1 12 14 13 7 8 9 11 27 2 15 16 6 21 20 22 10 28 26 25 24 17 19 18 23 3 4 5
e_b	f_b	24 17 19 18 12 1 8 7 13 14 15 2 27 11 9 10 22 20 21 6 16 28 26 25 23 3 4 5
e_t	f_t	4 5 6 16 28 26 25 24 17 13 7 3 23 18 19 20 21 11 27 2 15 14 12 1 8 9 10 22
f_m	f_b	2 27 26 28 10 22 20 19 17 24 25 23 18 12 1 8 9 11 21 6 16 15 14 13 7 3 4 5
e_m	e_b	1 8 9 11 21 6 5 4 3 7 13 14 12 18 23 25 26 27 2 15 16 28 10 22 20 19 17 24

Table 6: $e_\alpha(E,\mathrm{A})^1 e_\beta$ and $f_\gamma(E,\mathrm{A})^2 f_\delta$

α	β	γ	δ	$e_\alpha(E,\mathrm{A})^1 e_\beta$	$f_\gamma(E,\mathrm{A})^2 f_\delta$
m	b	m	t	1 8 7 3 4 5 6 21 20 19 17 13 14 12 18 23 25 24	2 15 16 28 26 27 11 9 10 22
m	t	m	b	1 8 7 13 14 12 18 19 17 24 25 23 3 4	2 15 16 28 26 27 11 9 10 22 20 21 6 5

Table 7: $e_\alpha(E,\mathrm{B})^1 f_\beta$ and $\kappa_\gamma(E,\mathrm{B})^2\kappa_\delta$

α	β	γ	δ	κ	$e_\alpha(E,\mathrm{B})^1 f_\beta$	$\kappa_\gamma(E,\mathrm{B})^2\kappa_\delta$
t	b	m	t	r	4 3 7 8 1 12 14 13 17 24 25 23 18 19 20 21 6 5	2 15 16 28 26 27 11 9 10 22
m	t	m	b	r	1 12 14 13 17 24 25 23 18 19 20 22	2 15 16 6 21 11 27 26 28 10 9 8 7 3 4 5
t	m	m	b	l	4 5 6 21 20 22 10 28 16 15 2	1 8 9 11 27 26 25 23 3 7 13 14 12 18 19 17 24

Table 8: $e_\alpha(E-i,\mathrm{Z})^1 f_\beta$ and $e_\gamma(E-i,\mathrm{Z})^2 f_\delta$

i	$e_\alpha(E-i,Z)^1 f_\beta$	$e_\gamma(E-i,Z)^2 f_\delta$
1	4 3 23 18 12 14 15 16 28 10 22 20 19 17 13 7 8 9 11 21 6 5	24 25 26 27 2
2	1 8 9 11 27 26 28 10 22	24 25 23 18 12 14 15 16 6 21 20 19 17 13 7 3 4 5
3	1 12 14 15 2	4 5 6 16 28 26 27 11 21 20 19 18 23 25 24 17 13 7 8 9 10 22
4	1 8 9 10 22 20 21 11 27 26 28 16 6 5	24 25 23 3 7 13 17 19 18 12 14 15 2
5	1 8 9 10 28 26 27 11 21 6 16 15 2	4 3 7 13 14 12 18 23 25 24 17 19 20 22
6	1 12 14 15 16 28 10 22 20 21 11 9 8 7 13 17 19 18 23 3 4 5	24 25 26 27 2
7	1 8 9 10 28 16 15 2	24 25 26 27 11 21 6 5 4 3 23 18 12 14 13 17 19 20 22
8	1 12 14 13 7 3 4 5 6 21 20 22	24 17 19 18 23 25 26 27 11 9 10 28 16 15 2
9	1 8 7 3 4 5	24 25 23 18 12 14 13 17 19 20 22 10 28 26 27 11 21 6 16 15 2
10	1 8 9 11 21 6 5 4 3 7 13 17 19 20 22	24 25 23 18 12 14 15 16 28 26 27 2
11	1 12 14 15 16 28 10 9 8 7 13 17 19 18 23 3 4 5 6 21 20 22	24 25 26 27 2
12	1 8 9 10 28 16 6 5 4 3 7 13 14 15 2	24 17 19 18 23 25 26 27 11 21 20 22
13	1 8 7 3 4 5 6 21 11 9 10 28 16 15 14 12 18 23 25 26 27 2	24 17 19 20 22
14	1 12 18 19 17 13 7 8 9 11 27 26 28 10 22 20 21 6 16 15 2	24 25 23 3 4 5
15	1 12 14 13 7 8 9 10 22 20 21 11 27 2	4 3 23 18 19 17 24 25 26 28 16 6 5

Table 8

16	1 8 9 10 28 26 27 11 21 6 5 4 3 7 13 17 19 20 22	24 25 23 18 12 14 15 2
17	1 12 14 13 7 8 9 10 28 16 15 2	24 25 26 27 11 21 6 5 4 3 23 18 19 20 22
18	1 12 14 15 16 6 21 11 9 8 7 13 17 19 20 22 10 28 26 27 2	24 25 23 3 4 5
19	1 8 7 3 4 5 6 21 20 22	24 17 13 14 12 18 23 25 26 27 11 9 10 28 16 15 2
20	1 8 7 13 17 19 18 12 14 15 16 28 26 27 2	24 25 23 3 4 5 6 21 11 9 10 22
21	1 8 7 13 17 19 20 22	24 25 26 27 11 9 10 28 16 6 5 4 3 23 18 12 14 15 2
22	1 8 7 13 17 19 20 21 6 16 15 14 12 18 23 3 4 5	24 25 26 28 10 9 11 27 2
23	1 8 9 11 27 2	24 25 26 28 10 22 20 21 6 16 15 14 12 18 19 17 13 7 3 4 5
24	1 8 9 11 21 6 5	4 3 7 13 17 19 20 22 10 28 16 15 14 12 18 23 25 26 27 2
25	1 12 14 13 7 8 9 11 27 26 28 10 22 20 21 6 16 15 2	24 17 19 18 23 3 4 5
26	1 8 7 13 17 19 18 12 14 15 16 28 10 9 11 27 2	24 25 23 3 4 5 6 21 20 22
27	1 8 7 13 17 19 20 21 11 9 10 22	24 25 26 28 16 6 5 4 3 23 18 12 14 15 2
28	1 8 7 13 17 19 20 22 10 9 11 21 6 16 15 14 12 18 23 3 4 5	24 25 26 27 2

References

[1] V. Chvátal, *Flip-flops in hypohamiltonian graphs*, Canad. Math. Bull. **16** (1973), 33–41.

[2] L. Clark and R. Entringer, *Smallest maximally nonhamiltonian graphs*, Period. Math. Hung. **14** (1983), 57–68.

[3] J. B. Collier and E. F. Schmeichel, *New flip-flop constructions for hypohamiltonian graphs*, Discrete Math. **18** (1977), 265–271.

[4] S. Fiorini, *Hypohamiltonian snarks*, in: Graphs and Other Combinatorial Topics, Proc. 3rd Czechoslovak Symp. on Graph Theory, Prague, Aug. 24–27, 1982 (M. Fiedler, Ed.), Teubner-Texte zur Math., Bd. 59, Teubner, Leipzig, 1983, 70–75.

[5] S. Fiorini and R. J. Wilson, *Edge-colorings of graphs*, in: Selected Topics in Graph Theory Vol. 1 (L.W. Beineke and R.J. Wilson, eds.), Academic Press, London, 1978, 71–95.

[6] M. R. Garey, D. S. Johnson, and R. E. Tarjan, *The planar Hamiltonian circuit problem is NP-complete*, SIAM J. Comput. **5** (1976), 704–714.

[7] S. Gutt, *Infinite families of hypohamiltonian graphs*, Acad. Roy. Belg. Bull. Cl. Sci. V. Sér. **63** (1977), 432–440.

[8] R. Isaacs, *Infinite families of non-trivial trivalent graphs which are not Tait colorable*, Am. Math. Monthly, **82** (1975), 221–239.

[9] F. Jaeger and T. Swart, *Problem session*, Ann. Discrete Math. **9** (1980), 305.

[10] M. Kochol, *Snarks without small cycles*, J. Combin. Theory Ser. B **67** (1996), 34–47.

[11] R. Nedela and M. Škoviera, *Atoms of cyclic connectivity in cubic graphs*, Math. Slovaca **45** (1995), 481–499.

[12] R. Nedela and M. Škoviera, *Decompositions and reductions of snarks*, J. Graph Theory, **22** (1996), 253–279.

[13] E. Steffen, *Classifications and characterizations of snarks*, Discrete Math. **188** (1998), 183–203.

[14] E. Steffen, *On bicritical snarks*, Math. Slovaca **51** (2001), 141–150.

[15] C. Thomassen, *Hypohamiltonian and hypotraceable graphs*, Discrete Math **9** (1974), 91–96.