Corrigendum to: "A note on the non-colorability threshold of a random graph"

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Fountoulakis and McDiarmid [3] brought to our attention an error in [2]. Specifically in paragraph B of the Appendix of [2], it is claimed that the function $r\ln\left(\beta/(\beta y)^y\right)$ is (*upwards*) convex as a function of $\beta, x \in (0,1)$ (r is constant > 0). This is not correct, as it is not hard to show by a numerical example that the Hessian of this function is not negative semi-definite in $(0,1)^2$. The erroneous statement that $r\ln\left(\beta/(\beta y)^y\right)$ is convex is used in [2] for the sole purpose of showing that the function

$$r\ln\left(\frac{\alpha\beta(1-\alpha-\beta)}{(\alpha(1-x-y))^{(1-x-y)}(\beta y)^y((1-\alpha-\beta)x)^x}\right)$$

is convex in the region

$$\mathcal{D} = \{(\alpha,\beta,x,y) \in (0,1)^4, \alpha+\beta<1, x+y<1, x \geq \alpha/r, y \geq \alpha/r, 1-x-y \geq \beta/r\}.$$

Fortunately, the latter function *is* convex (despite the non-convexity of $rln\left(\beta/(\beta y)^y\right)$), as it immediately follows from the proposition below. This corrects the error in [2]. It should be pointed out that Fountoulakis and McDiarmid [3] have independently obtained the results in [2] by similar methods, but they used a different approach for the numerical computation of the upper bound to the threshold (expectedly, they get the same value).

Proposition. *The Hessian of the function* $-\ln\left(\frac{\alpha\beta(1-\alpha-\beta)}{(\alpha(1-x-y))^{(1-x-y)}(\beta y)^y((1-\alpha-\beta)x)^x}\right)$ *is positive definite for all points in the region* $\mathcal{E} = \{(\alpha,\beta,x,y) \in (0,1)^4, \alpha+\beta<1, x+y<1\}$.

Proof. It is well known (see e.g. [1, Theorem 7.2.5]) that it suffices to prove that at all points of $\mathcal{E}$, the leading principal minor determinants (LPMD) of the Hessian of this function are (strictly) positive. By elementary and straightforward computations, it can be verified that the LPMDs *when the variables of the function are ordered as* x, y, β, α are:

$$
\begin{aligned}
A_1 &= \frac{1-y}{x(1-x-y)}, \\
A_2 &= \frac{1}{xy(1-x-y)}, \\
A_3 &= \frac{\left((1-\alpha-\beta)(1-y)-\beta(1-x)\right)^2 + 2(1-\alpha-\beta)\beta(1-x-y)}{\beta^2 xy(1-x-y)(1-\alpha-\beta)^2}, \\
A_4 &= \frac{\left(\alpha(1-y)^2 - x(1-\beta-y) - \beta y(1-x-y)\right)^2 + 2xy(1-x-y)\left((1-\beta-y)^2+\beta^2\right)}{\alpha^2\beta^2 xy(1-x-y)(1-\alpha-\beta)^2(1-y)^2}.
\end{aligned}
$$

Obviously, A_1, A_2, A_3 and A_4 are positive at all points of the region $\mathcal{E}$. ☐

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References

[1] R.A. Horn and C.R. Johnson, *Matrix Analysis,* Cambridge University Press, 1999.

[2] A.C. Kaporis, L.M. Kirousis, and Y.C. Stamatiou, "A note on the non-colorability threshold of a random graph," *Electronic Journal of Combinatorics* 7:R29, 2000.

[3] N. Fountoulakis and C. McDiarmid, personal communication.