# COMMENT ON JASON BANDLOW AND KENDRA KILLPATRICK: "AN AREA-TO-INV BIJECTION BETWEEN DYCK PATHS AND 312-AVOIDING PERMUTATIONS"

COMMENT BY MARKUS FULMEK AND CHRISTIAN KRATTENTHALER, FEBRUARY 6, 2002

The bijection given in the paper [1] is the same (modulo the trivial transformation that converts 312-avoiding permutations to 132-avoiding permutations, i.e., the transformation which sends every letter $l$ in a permutation of $1, 2, \dots, n$ to $n-l+1$) as the bijection in [4, Section 2], where it is described in a different, but equivalent, way. An alternative and simpler description of this bijection is contained in [3, Section 3.1] from which the "area–to–inversions" property is transparent.

Furthermore, Lemma 4 in [1] is identical with [4, (3.2)] (again, modulo the same transformation), from which then a continued fraction result due to Mansour and Vainshtein [5], and some further generating function results are derived. It should also be mentioned that the continued fraction result has been recently refined by Brändén, Claesson, and Steingrímsson in [2].

## References

[1] J. Bandlow and K. Killpatrick, An area–to–inv bijection between Dyck paths and 312–avoiding permutations, *Electron. J. Combin.* **8** (2001), Article R40, 16 pp.

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[3] M. Fulmek, Enumeration of permutations containing a prescribed number of occurrences of a pattern of length 3, preprint, math.CO/0112092, 2001.

[4] C. Krattenthaler, Permutations with restricted patterns and Dyck paths, *Adv. Appl. Math.* **27** (2001), 510–530; math.CO/0002200.

[5] T. Mansour and A. Vainshtein, Restricted permutations, continued fractions, and Chebyshev polynomials, *Electron. J. Combin.* **7** (2000), Article R17, 9 pp.

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