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ON A HIGHER ORDER TWO DIMENSIONAL THERMOELASTIC SYSTEM COMBINING A LOCAL AND NONLOCAL BOUNDARY CONDITIONS

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Abstract. Due to their importance and numerous applications, evolution mixed problems with nonlocal constraints in the boundary conditions have been extensively studied during the two last decades. In this paper, we consider an initial boundary value problem for a higher order thermoelastic system arising in linear thermoelasticity which combines some Dirichlet and weighted integral boundary conditions. The studied system modelizes in general a Kirchoff plate. We prove the well posedness of the given problem. Our proofs are mainly based on some a priori bounds in some Sobolev type space functions and on some density arguments.

1. Introduction

During the few last decades, many researchers have studied linear and nonlinear systems of thermoelastic equations and many results have been published. Most of these results were dealing with the study of existence, asymptotic behavior, regularity, controllability, propagation of singularities and blow up of solutions. For example Assila [1], has studied global existence and asymptotic behavior of solutions for a purely linear multidimensional system of nonhomogeneous and anisotropic thermoelasticity, associated with nonlinear boundary conditions. Dafermos and Hsiao [4], Hrusa and Messaoudi [6], Munoz-Rivera [15], Racke [16], Racke and Shibata [17] and Slemrod[19] have studied and obtained some results about the existence, regularity, controllability and long-time behavior of some systems of thermoelasticity. Also Racke and Wang [18] have considered some linear and semilinear Cauchy problems and described the propagation of singularities. Mixed problems for thermoelastic systems become very hard to handle in case of the presence of nonlocal constraints in the boundary (such

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as an integral condition instead of a Dirichlet or Neumann classical condition). There are two types of nonlocal problems, spatial nonlocal problems and time nonlocal problems. These type of mixed problems, arise mainly when the data on the boundary cannot be measured directly, but it can be replaced by a nonlocal condition such as an integral condition. Physically, this kind of condition can represent a mean, a total energy or a total mass. Boundary value problems with nonlocal constraints have many important applications such as in underground-water flow, population dynamics, chemical diffusion, thermoelasticity, heat conduction processes, certain biological processes, nuclear reactor dynamics, control theory, medical science, biochemistry, and transmission theory. Nonlocal problems were first investigated by using the method of separation of variables and the corresponding eigenvalues and eigenfunctions were considered. Later on, other methods such as the functional analysis method, the energetic method and the method of singular integral equations were applied to mixed nonlocal problems but with great difficulties. For some nonlocal mixed problems for parabolic and hyperbolic equations the reader should refer to Mesloub [7,8,9,10], Mesloub and Mesloub.F [12], Mesloub and Messaoudi [13] and Mesloub and Bouziani [11].

Motivated by the previous studies, we consider the following initial boundary value problem for a fourth order two dimensional linear thermoelastic system with Dirichlet and nonlocal constraints of integral type:

$$\left\{\begin{array}{c} \mathcal{L}_1 u = \frac{\partial^2 u}{\partial t^2} + \triangle^2 u - \alpha \triangle \theta + c_1 u + c_2 u_t = f(x,\ y,\ t),\ \ (x,\ y,\ t) \in Q, \\ \mathcal{L}_2 \theta = \beta \frac{\partial \theta}{\partial t} - \eta \triangle \theta + \sigma \theta + \alpha \triangle u_t = g(x,\ y,\ t),\ (x,\ y,\ t) \in Q, \\ u(x,\ y,\ 0) = u_\circ(x, y),\ u_t(x,\ y,\ 0) = u_1(x,\ y),\ \theta(x,\ y,\ 0) = \theta_\circ(x,\ y), \\ u(0,\ y,\ t) = 0,\ \ u(a,\ y,\ t) = 0,\ \ 0 < y < b,\ \ 0 < t < T, \\ u(x,\ 0,\ t) = 0,\ \ u(x,\ b,\ t) = 0,\ \ 0 < x < a,\ \ 0 < t < T, \\ \int_0^a x^k u dx = 0,\ \int_0^b y^k u dy = 0,\ \int_0^a x^k \theta dx = 0,\ \int_0^b y^k \theta dy = 0,\ k = 0, 1. \end{array}\right. \quad (1)$$

where $Q = \Omega \times [0,\ T]$, with $\Omega = (0,\ a) \times (0,\ b)$, $T < \infty$, $a < \infty$ and $b < \infty$.

The given data satisfy the consistency conditions

$$\begin{aligned} u_\circ(0,\ y) &= u_\circ(a,\ y) = u_\circ(x,\ 0) = u_\circ(x,\ b) = 0, \quad (2) \\ u_1(0,\ y) &= u_1(a,\ y) = u_1(x,\ 0) = u_1(x,\ b) = 0, \end{aligned}$$

and

$$\begin{aligned} \int_0^a x^k u_\circ dx &= 0, \quad \int_0^b y^k u_\circ dy = 0, \int_0^a x^k u_1 dx = 0, \qquad (3) \\ \int_0^b y^k u_1 dy &= 0, \int_0^a x^k \theta_\circ dx = 0, \quad \int_0^b y^k \theta_\circ dy = 0, \quad k = 0, 1. \end{aligned}$$

Problem (1) arises in linear thermoelasticity. It modelizes a Kirchoff plate, where u is the displacement, θ is the thermal damping. The integral conditions may be interpreted as the average and the weighted average of the displacement and the thermal damping, and α, β, η, σ, c_1 and c_2 are positive constants.

On the basis of some a priori bounds, energy estimates and some density arguments, we prove the existence, uniqueness and the continuous dependence of the solution on the data of the given problem (1).The paper is organized as follows: In section 1, we start by an introduction about previous and related results concerning the subject. In section 2, we introduce some notations, function spaces and reformulation of the studied problem. Section 3 is devoted to the study of uniqueness of the solution of the stated problem. In the fourth and last section, we establish the existence of the weak solution of our problem and give some remarks. At the end of the paper, we give a list of some used references.

2. Notations, functional frame, some auxiliary inequalities and reformulation of the problem

We denote by $L^2(Q)$ the usual square integrable functions space, and by$W_2^{1,0}(Q)$ the Sobolev space having the inner product

$$(u,\ v)_{W^{1,0}(Q)} = (u,\ v)_{L^2(Q)} + (u_x,\ v_x)_{L^2(Q)}\,.$$

Let $B_2^m(0,a)$ (See [2,3]) be the space constituted of functions $u \in L^2(0,a)$, if $m = 0$ and of functions u such that $\Im_x^m u \in L^2(0,a)$, if

$m \geq 1$, where

$$\Im_x^m V = \frac{1}{(m-1)!}\int_0^x (x-\xi)^{m-1} V(\xi,t)d\xi = \int_0^x \int_0^{\xi_1} \cdots \int_0^{\xi_{m-1}} V(\eta,t)d\eta d\xi_{m-1} \ldots d\xi_1,$$

with inner product $(u,v)_{B_2^m(0,a)} = \int_0^a \Im_x^m u.\Im_x^m v dx$, and associated norm $\|u\|_{B_2^m(0,a)} = \|\Im_x^m u\|_{L^2(0,a)}$, for $m \geq 1$.

We also use the function spaces $C(I, B_2^m(\Omega))$, $C(I, B_2^{1,x}(\Omega))$, $C(I, B_2^{1,y}(\Omega))$ and $C(I, B_2^{1,x,y}(\Omega))$ of continuous mappings from $I = [0,T]$ onto the Hilbert spaces $B_2^m(\Omega)$, $B_2^{1,x}(\Omega)$, $B_2^{1,y}(\Omega)$, and $B_2^{1,x,y}(\Omega)$ respectively, and with inner products (respectively) given by

$$\begin{aligned}
(u,v)_{B_2^m(\Omega)} &= \int_\Omega \Im_x^m u.\Im_x^m v dx dy, \quad (u,v)_{B_2^{1,x}(\Omega)} = \int_\Omega \Im_x u.\Im_x v dx dy, \\
(u,v)_{B_2^{1,y}(\Omega)} &= \int_\Omega \Im_y u.\Im_y v dx dy, \quad (u,v)_{B_2^{1,x,y}(\Omega)} = \int_\Omega \Im_{xy} u.\Im_{xy} v dy dx.
\end{aligned}$$

The following inequalities are used in our paper:

A) For every $u \in L^2(\Lambda)$ (Λ is either $(0,d)$ or Ω) and all $m \in IN^*$, we have the inequalities

$$\|h\|^2_{B_2^m(0,d)} \leq \frac{d^2}{2} \, \|h\|^2_{B_2^{m-1}(0,d)} \tag{1*}$$

and

$$\|h\|^2_{B_2^m(0,d)} \leq \left(\frac{d^2}{2}\right)^m . \|h\|^2_{L^2(0,d)}, \tag{2*}$$

$$\|\Im_x u\|^2_{L^2(\Omega)} \leq \frac{a^2}{2} \|u\|^2_{L^2(\Omega)} \tag{3*}$$

$$\|h\|^2_{B_2^{1,x,y}(\Omega)} = \|\Im_{xy} h\|^2_{L^2(\Omega)} \leq \frac{(ab)^2}{4} \|h\|^2_{L^2(\Omega)}, \tag{4*}$$

B) Growall's Lemma [5, Lemma 7.1]. If $f_1(s)$, $f_2(s)$ and $f_3(s)$ are nonnegative functions on $(0,\ T)$, $f_1(s)$ and $f_2(s)$ integrable functions, and $f_3(s)$ is nondecreasing on $(0,\ T)$, then if $\int_0^s f_1(t)\,dt + f_2(s) \leq c\int_0^s f_2(t)\,dt + f_3(s)$, then $\int_0^s f_1(t)\,dt + f_2(s) \leq \exp(cs).f_3(s)$.

C) Cauchy $\varepsilon-$inequality: For all $\varepsilon > 0$, and for arbitrary a, b in $\mathbb{R}$, we have

$$|ab| \leq \frac{\varepsilon}{2}|a|^2 + \frac{1}{2\varepsilon}|b|^2.$$

Let us now formulate problem (1) in its operator form. Problem (1) can be viewed as the problem of solving the operator equation: $JU = G$, with $U = (u,\ \theta)$, $JU = (J_1 u,\ J_2\theta)$, and $G = (G_1,\ G_2) = (\{f,\ u_\circ,\ u_1\}, \{g,\ \theta_\circ\})$, where

$$J_1u = \{\mathcal{L}_1u,\ \ell_1u,\ \ell_2u\},\qquad J_2\theta = \{\mathcal{L}_2\theta,\ \ell_3\theta\}.$$

The unbounded operator A is considered on the Banach space B into a Hilbert space H with domain $D(J)$ defined by

$$D(J) = \left\{ \begin{array}{c} U = (u,\ \theta) \in \left(L^2(Q)\right)^2 \text{ such that } u_t,\ \theta_t,\ u_{tt},\ \frac{\partial^i u}{\partial x^i},\ \frac{\partial^i u}{\partial y^i},\ u_{tx}, \\ u_{txx},\ u_{ty},\ u_{tyy} \in L^2(Q), \qquad i = \overline{1.4} \end{array} \right. \tag{4}$$

and the functions $U = (u,\ \theta)$ satisfy boundary conditions in (1). Here B is the Banach space obtained by enclosing $D(J)$ with respect to the finite norm

$$\begin{aligned} \|U\|_B &= \Big(\|u_t(.,\ .,\tau)\|^2_{C(I,B_2^{1,x,y}(\Omega))} + \|u(.,\ y,\ \tau)\|^2_{C(I,B_2^{1,x}(\Omega))} \\ &\quad + \|u(x,\ .,\ \tau)\|^2_{C(I,B_2^{1,y}(\Omega))} + \|\theta(.,\ .,\tau)\|^2_{C(I,B_2^{1,x,y}(\Omega))}\Big)^{1/2}, \end{aligned} \tag{5}$$

where

$$\Im_x u = \int_0^x u(\xi, y, t)d\xi,\quad \Im_{xy} u = \int_0^x \int_0^y u(\xi, \eta, t)d\eta d\xi.$$

The elements $U = (u,\ \theta) \in B$ are the set of continuous functions u and θ on $I = [0, T]$, where functions u have values in $B_2^{1,x}(\Omega)$, $B_2^{1,y}(\Omega)$ and have derivatives u_t which are continuous on I with values in $B_2^{1,x,y}(\Omega)$ and θ have values in $B_2^{1,x,y}(\Omega)$.

Let $H = H_1 \times H_2$ be the Hilbert space $\{L^2(Q) \times W_2^1(\Omega) \times L^2(\Omega)\} \times \{L^2(Q) \times L^2(\Omega)\}$ having the finite norm

$$\|G\|^2_H = \|f\|^2_{L^2(Q)} + \|u_\circ\|^2_{W_2^1(\Omega)} + \|u_1\|^2_{L^2(\Omega)} + \|g\|^2_{L^2(Q)} + \|\theta_\circ\|^2_{L^2(\Omega)}. \tag{6}$$

3. Uniqueness of solution

We now state the first result concerning the uniqueness of solution of problem (1).

Theorem 1. For any function $U = (u,\ \theta) \in D(J)$, there exists a positive constant $C > 0$ independent of U, such that

$$\|U\|^2_B - C\,\|JU\|^2_H \leq 0 \tag{7}$$

Proof. Taking the inner product in $L^2(Q_\tau)$ of equations $\mathcal{L}_1 u = f$, $\mathcal{L}_2 u = g$ and the intgrodifferential operators $\Im^2_{xy} u_t$ and $\Im^2_{xy}\theta$, respectively, where $Q^\tau = (0,\ \tau)\times\Omega$ with $0 \le \tau \le T$, we have

$$\begin{aligned}
& c_1\left(u,\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} + c_2\left(u_t,\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} + \left(u_{tt},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} \\
& + \left(\frac{\partial^4 u}{\partial x^4},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} + 2\left(\frac{\partial^4 u}{\partial x^2\partial y^2},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} + \left(\frac{\partial^4 u}{\partial y^4},\ \Im^2_{xy}u_t\right)_{L^2(Q)} \\
& -\alpha\left(\frac{\partial^2\theta}{\partial x^2},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} - \alpha\left(\frac{\partial^2\theta}{\partial y^2},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} + \beta\left(\theta_t,\ \Im^2_{xy}\theta\right)_{L^2(Q^\tau)} \\
& -\eta\left(\frac{\partial^2\theta}{\partial x^2},\ \Im^2_{xy}\theta\right)_{L^2(Q)} - \eta\left(\frac{\partial^2\theta}{\partial y^2},\ \Im^2_{xy}\theta\right)_{L^2(Q)} \\
& +\sigma\left(\theta,\ \Im^2_{xy}\theta\right)_{L^2(Q^\tau)} + \alpha\left(\frac{\partial^3 u}{\partial x^2\partial t},\ \Im^2_{xy}\theta\right)_{L^2(Q^\tau)} + \alpha\left(\frac{\partial^3 u}{\partial y^2\partial t},\ \Im^2_{x}\theta\right)_{L^2(Q^\tau)} \\
= & \left(f,\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} + \left(g,\ \Im^2_{xy}\theta\right)_{L^2(Q^\tau)}. \qquad (8)
\end{aligned}$$

By successive integration by parts of each term of (8), and using boundary conditions in (1),we derive the following equalities:

$$c_1\left(u,\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} = \frac{c_1}{2}\left\|\Im_{xy}u(.,\ .,\tau)\right\|^2_{L^2(\Omega)} - \frac{c_2}{2}\left\|\Im_{xy}u_0\right\|^2_{L^2(\Omega)}, \quad (9)$$

$$c_2\left(u_t,\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} = c_2\left\|\Im_{xy}u_t\right\|^2_{L^2(Q^\tau)}, \quad (10)$$

$$\left(u_{tt},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} = \frac{1}{2}\left\|\Im_{xy}u_t(.,\ .,\tau)\right\|^2_{L^2(\Omega)} - \frac{1}{2}\left\|\Im_{xy}u_1\right\|^2_{L^2(\Omega)}, \quad (11)$$

$$\left(\frac{\partial^4 u}{\partial x^4},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} = \frac{1}{2}\left\|\Im_y u_x(x,\ .,\tau)\right\|^2_{L^2(\Omega)} - \frac{1}{2}\left\|\Im_y\frac{\partial u_\circ}{\partial x}\right\|^2_{L^2(\Omega)}, \quad (12)$$

$$2\left(\frac{\partial^4 u}{\partial x^2\partial y^2},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} = \left\|u(x,\ y,\ \tau)\right\|^2_{L^2(\Omega)} - \left\|u_\circ\right\|^2_{L^2(\Omega)}, \quad (13)$$

$$\left(\frac{\partial^4 u}{\partial y^4},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} = \frac{1}{2}\left\|\Im_x u_y(.,\ y,\ \tau)\right\|^2_{L^2(\Omega)} - \frac{1}{2}\left\|\Im_x\frac{\partial u_\circ}{\partial y}\right\|^2_{L^2(\Omega)}, \quad (14)$$

$$-\alpha\left(\frac{\partial^2\theta}{\partial x^2},\ \Im^2_{xy}u_t\right)_{L^2(Q^\tau)} = \alpha\left(\Im_y\theta,\ \Im_y u_t\right)_{L^2(Q^\tau)}, \quad (15)$$

$$-\alpha\left(\frac{\partial^2\theta}{\partial y^2},\ \Im_{xy}^2 u_t\right)_{L^2(Q^\tau)} = \alpha\left(\Im_x\theta,\ \Im_x u_t\right)_{L^2(Q^\tau)}, \tag{16}$$

$$\beta\left(\theta_t,\ \Im_{xy}^2\theta\right)_{L^2(Q^\tau)} = \frac{\beta}{2}\left\|\Im_{xy}\theta\left(.,\ .,\tau\right)\right\|_{L^2(\Omega)}^2 - \frac{\beta}{2}\left\|\Im_{xy}\theta_\circ\right\|_{L^2(\Omega)}^2, \tag{17}$$

$$-\eta\left(\frac{\partial^2\theta}{\partial x^2},\ \Im_{xy}^2\theta\right)_{L^2(Q^\tau)} = \eta\left\|\Im_y\theta\right\|_{L^2(Q^\tau)}^2, \tag{18}$$

$$-\eta\left(\frac{\partial^2\theta}{\partial y^2},\ \Im_{xy}^2\theta\right)_{L^2(Q^\tau)} = \eta\left\|\Im_x\theta\right\|_{L^2(Q^\tau)}^2, \tag{19}$$

$$\sigma\left(\theta,\ \Im_{xy}^2\theta\right)_{L^2(Q^\tau)} = \sigma\left\|\Im_{xy}\theta\right\|_{L^2(Q^\tau)}^2, \tag{20}$$

$$\alpha\left(\frac{\partial^3 u}{\partial x^2\partial t},\ \Im_{xy}^2\theta\right)_{L^2(Q^\tau)} = -\alpha\left(\Im_y u_t,\ \Im_y\theta\right)_{L^2(Q^\tau)}, \tag{21}$$

$$\alpha\left(\frac{\partial^3 u}{\partial y^2\partial t},\ \Im_{xy}^2\theta\right)_{L^2(Q^\tau)} = -\alpha\left(\Im_x u_t,\ \Im_x\theta\right)_{L^2(Q^\tau)}, \tag{22}$$

$$\left(f,\ \Im_{xy}^2 u_t\right)_{L^2(Q^\tau)} = \left(\Im_{xy}f,\ \Im_{xy}u_t\right)_{L^2(Q^\tau)}, \tag{23}$$

$$\left(g,\ \Im_{xy}^2\theta\right)_{L^2(Q^\tau)} = \left(\Im_{xy}g,\ \Im_{xy}\theta\right)_{L^2(Q^\tau)}. \tag{24}$$

If we use Cauchy-ε- inequality given in **C)**, Poincare' inequality of type (4*), and equalities (9) − (24), then equation (8) reduces to

$$\begin{aligned}
&\left\|\Im_{xy}u(.,\ .,\tau)\right\|_{L^2(\Omega)}^2 + \left\|\Im_{xy}u_t\left(.,\ .,\tau\right)\right\|_{L^2(\Omega)}^2 + \left\|\Im_x u\left(.,\ y,\ \tau\right)\right\|_{L^2(\Omega)}^2 \\
&+\left\|\Im_x u_y\left(.,\ y,\ \tau\right)\right\|_{L^2(\Omega)}^2 + \left\|\Im_y u\left(x,\ .,\ \tau\right)\right\|_{L^2(\Omega)}^2 + \left\|\Im_{xy}u_t\right\|_{L^2(Q^\tau)}^2 \\
&+\left\|\Im_y u_x\left(x,\ .,\tau\right)\right\|_{L^2(\Omega)}^2 + \left\|\Im_{xy}\theta\left(.,\ .,\tau\right)\right\|_{L^2(\Omega)}^2 \\
\leq\ & C_1\begin{pmatrix} \left\|u_1\right\|_{L^2(\Omega)}^2 + \left\|u_\circ\right\|_{L^2(\Omega)}^2 + \left\|\frac{\partial u_\circ}{\partial x}\right\|_{L^2(\Omega)}^2 + \left\|\Im_{xy}u_0\right\|_{L^2(\Omega)}^2 \\ +\left\|\frac{\partial u_\circ}{\partial y}\right\|_{L^2(\Omega)}^2 + \left\|\theta_\circ\right\|_{L^2(\Omega)}^2 + \left\|f\right\|_{L^2(Q^\tau)}^2 \\ +\left\|g\right\|_{L^2(Q^\tau)}^2 + \left\|\Im_{xy}u_t\right\|_{L^2(Q^\tau)}^2 + \left\|\Im_{xy}\theta\right\|_{L^2(Q^\tau)}^2 \end{pmatrix}
\end{aligned} \tag{25}$$

where

$$C_1 = \frac{\max\left\{1,\ \frac{a^2}{4},\ \frac{b^2}{4},\ \frac{(ab)^2}{8},\ \frac{\beta(ab)^2}{8},\ \frac{c_1}{2}\right\}}{\min\left\{\frac{a^2}{4},\ \frac{b^2}{4},\frac{c_2}{2}\right\}}.$$

Applying the Gronwall's lemma given in **B)** to inequality (25) and discarding the fourth, sixth and seventh term of its left-hand, we obtain

$$\begin{aligned}
&\left\|\Im_{xy}u(.,\ .,\tau)\right\|^2_{L^2(\Omega)}+\left\|\Im_{xy}u_t\,(.,\ .,\tau)\right\|^2_{L^2(\Omega)}+\left\|\Im_{x}u\,(.,\ y,\ \tau)\right\|^2_{L^2(\Omega)}\\
&+\left\|\Im_{y}u\,(x,\ .,\ \tau)\right\|^2_{L^2(\Omega)}+\left\|\Im_{xy}\theta\,(.,\ .,\tau)\right\|^2_{L^2(\Omega)}\\
\le\ & C_1e^{C_1T}\left(\begin{array}{c}\|u_1\|^2_{L^2(\Omega)}+\|u_\circ\|^2_{W_2^1(\Omega)}+\|\theta_\circ\|^2_{L^2(\Omega)}\\ +\|f\|^2_{L^2(Q)}+\|g\|^2_{L^2(Q)}\end{array}\right). \qquad (26)
\end{aligned}$$

Each term on the left-hand side of (26) is bounded and since the right-hand of the above inequality (26) is independent of τ, we can take the least upper bound of each term of the left-hand side with respect to τ over $[0,\ T]$, we get the desired estimate (7) with $C=C_1e^{C_1T}$. This completes the proof of Theorem 1.

It can be proved in a standard way that the operator $J\colon B\longrightarrow H=H_1\times H_2$ is closable.

Proposition 2. The operator $J\colon B\rightarrow H=H_1\times H_2$ has a closure See [15].

Let $\overline{J}$ be the closure of J and $D(\overline{J})$ its domain of definition. We define the strong generalized solution of problem (1) as the solution of the operator equation $\overline{J}U=(J_1u,\ J_2\theta)$, with $J_1u=\{\mathcal{L}_1u,\ \ell_1u,\ \ell_2u\}$, $J_2\theta=\{\mathcal{L}_2\theta,\ \ell_3\theta\}$, $(J_1u,\ J_2\theta)\in H$. If we pass to the limit in (7) we have the result

Corollary 3. There exists a positive constant C such that

$$\|U\|^2_B-C\left\|\overline{J}U\right\|^2_H\le 0,\qquad \forall U\in D\left(\overline{J}\right). \qquad (27)$$

We deduce from the a priori estimate (27) that a strong generalized solution of (1) if it exists is unique and depends continuously on $G=(G_1,G_2)=(\{f,\ u_\circ,\ u_1\},\{g,\ \theta_\circ\})\in H$, and that the range $R\left(\overline{J}\right)$ of $\overline{J}$ is closed in H and $R\left(\overline{J}\right)=\overline{R\left(J\right)}$.

Existence of the solution of the stated problem.

Theorem 4. Problem (1) has a unique strong solution verifying

$$u\in C(I,B_2^{1,x}(\Omega)),\ u\in C(I,B_2^{1,y}(\Omega)),\ \theta\in C(I,B_2^{1,x,y}(\Omega)),\ \frac{\partial u}{\partial t}\in C(I,B_2^{1,x,y}(\Omega)).$$

Moreover, the functions $\Im_x u$, $\Im_y u$, , $\Im_{xy}\theta$, $\Im_{xy}\theta$ depend continuously on the free terms $f \in L^2(Q)$, $g \in L^2(Q)$, and on the initial data $u_\circ \in W_2^1(\Omega)$, $u_1 \in L^2(\Omega)$, $\theta_\circ \in L^2(\Omega)$, that is

$$\begin{aligned}
\|u(.,\ .,\tau)\|^2_{C(I,B_2^{1,x,y}(\Omega))} &\leq C\,\|JU\|_H\,,\\
\|u(x,\ .,\ \tau)\|_{C(I,B_2^{1,y}(\Omega))} &\leq C\,\|JU\|_H\,,\\
\|u(.,\ y,\ \tau)\|_{C(I,B_2^{1,x}(\Omega))} &\leq C\,\|JU\|_H\,,\\
\|u_t(.,\ .,\tau)\|_{C(I,B_2^{1,x,y}(\Omega))} &\leq C\,\|JU\|_H\,,\\
\|\theta(.,\ .,\tau)\|^2_{C(I,B_2^{1,x,y}(\Omega))} &\leq C\,\|JU\|_H\,. \qquad (28)
\end{aligned}$$

Proof. To establish the existence of the solution of problem (1), it is sufficient to prove that the image of the operator J is dense in H.

General case for density: Since H is a Hilbert space, $\overline{R(J)} = H$ is equivalent to the orthogonality of a vector $\Phi = (\mathcal{G}_1,\ \mathcal{G}_2) = (\{\sigma_1,\ \sigma_2,\ \sigma_3\},\ \{\sigma_4,\ \sigma_5\}) \in H$ to the range $R(J)$, that is the equality

$$\begin{aligned}
&+ (\ell_2 u,\ \sigma_4)_{L^2(\Omega)} + (\ell_3\theta,\ \sigma_5)_{L^2(\Omega)}\\
&+ (\mathcal{L}_1 u,\ \sigma_1)_{L^2(Q)} + (\mathcal{L}_2\theta,\ \sigma_2)_{L^2(Q)} + (\ell_1 u,\ \sigma_3)_{W_2^1(\Omega)}\\
=\ & 0, \qquad (29)
\end{aligned}$$

for all $U = (u,\ \theta) \in D(J)$, implies that $\Phi = 0$. Let $D_0(J)$ be a subset of $D(J)$ for which $\ell_1 u = 0$, $\ell_2 u = 0$, $\ell_3\theta = 0$. If $U \in D_0(J)$, then (29) reduces to

$$(\mathcal{L}_1 u,\ \sigma_1)_{L^2(Q)} + (\mathcal{L}_2\theta,\ \sigma_2)_{L^2(Q)} = 0. \qquad (30)$$

We have to prove that $\Psi = (\sigma_1,\ \sigma_2) = 0$ everywhere in Q. Thus we must prove the following special case (of density) and then go back to the general case.

Proposition 5. If, for some function $\Psi = (\sigma_1,\ \sigma_2) \in (L^2(Q))^2$ and for all elements $U \in D_\circ(J)$, we have

$$(\mathcal{L}_1 u,\ \sigma_1)_{L^2(Q)} + (\mathcal{L}_2\theta,\ \sigma_2)_{L^2(Q)} = 0, \qquad (31)$$

then Ψ vanishes almost everywhere in Q.

Proof. Since relation (31) holds for any element of $D_\circ(J)$, we then take an element $U = (u,\ \theta)$ with a special form given by

$$U = \begin{cases} (0,\ 0)\,, & 0 \leq t \leq s,\\ \left(\int_s^t (\tau - t)\, u_{\tau\tau} d\tau,\ \int_s^t \theta_\tau d\tau\right), & s \leq t \leq T\,, \end{cases} \qquad (32)$$

such that $(u_{tt},\ \theta_t)$ is a solution of the system

$$\Im_{xy}^2 u_{tt} = E_1(r,\ t) = \int_t^T \sigma_1(r,\ \tau)\,d\tau, \qquad \Im_{xy}^2 \theta_t = E_2(r,\ t) = \int_t^T \sigma_2(r,\ \tau)\,d\tau, \tag{33}$$

where $E_1(x,\ t) = \int_t^T \sigma_1(r,\ \tau)\,d\tau,$ and $E_2(x,\ t) = \int_t^T \sigma_2(r,\ \tau)\,d\tau$. It is clear that

$$\sigma_1 = -\Im_{xy}^2 u_{ttt},\ \sigma_2 = -\Im_{xy}^2 \theta_{tt}. \tag{34}$$

Proposition 6. The function $\Psi = (\sigma_1,\ \sigma_2) \in \left(L^2(Q)\right)^2$ defined in (34) is in $\left(L^2(Q)\right)^2$.

Proof. it can be carried out as in [10].

Now, replacing the functions $\sigma_1$and σ_2 given by (34) in (31), we obtain

$$\begin{aligned}
&-c_1\left(u,\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} - c_2\left(u_t,\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} - \left(u_{tt},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} \\
&-\left(\frac{\partial^4 u}{\partial x^4},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} - 2\left(\frac{\partial^4 u}{\partial x^2 \partial y^2},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} - \left(\frac{\partial^4 u}{\partial y^4},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} \\
&+\alpha\left(\frac{\partial^2 \theta}{\partial x^2},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} + \alpha\left(\frac{\partial^2 \theta}{\partial y^2},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} \\
&-\beta\left(\theta_t,\ \Im_{xy}^2 \theta_{tt}\right)_{L^2(Q)} + \eta\left(\frac{\partial^2 \theta}{\partial x^2},\ \Im_{xy}^2 \theta_{tt}\right)_{L^2(Q)} \\
&+\eta\left(\frac{\partial^2 \theta}{\partial y^2},\ \Im_{xy}^2 \theta_{tt}\right)_{L^2(Q)} + -\sigma\left(\theta,\ \Im_{xy}^2 \theta_{tt}\right)_{L^2(Q)} \\
&-\alpha\left(\frac{\partial^3 u}{\partial x^2 \partial t},\ \Im_{xy}^2 \theta_{tt}\right)_{L^2(Q)} - \alpha\left(\frac{\partial^3 u}{\partial y^2 \partial t},\ \Im_{xy}^2 \theta_{tt}\right)_{L^2(Q)} \\
=&\ 0
\end{aligned} \tag{35}$$

Taking into account the special form of U given by (32) and (33), using boundary conditions in (1) and integrating by parts each terms of (35), gives

$$\begin{aligned}
-c_1\left(u,\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} &= c_1 \int_{Q_s} \Im_x u.\Im_{xyy} u_{ttt}\,dxdydt \\
&= -c_1 \int_{Q_s} \Im_{xy} u.\Im_{xy} u_{ttt}\,dxdydt \\
&= \frac{c_1}{2}\left\|\Im_{xy} u_t(.,\ .,\ T)\right\|_{L^2(\Omega)}^2,
\end{aligned} \tag{36}$$

$$
\begin{aligned}
-c_2\left(u_t,\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} &= c_2\int_{Q_s}\Im_x u_t.\Im_{xyy}u_{ttt}dxdydt \\
&= -c_2\int_{Q_s}\Im_{xy}u_t.\Im_{xy}u_{ttt}dxdydt \\
&= c_2\left\|\Im_{xy}u_{tt}\right\|_{L^2(\Omega)}^2, \qquad (37)
\end{aligned}
$$

$$
\begin{aligned}
-\left(u_{tt},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} &= \int_{Q_s}\Im_x u_{tt}.\Im_{xyy}u_{ttt}dxdydt \\
&= -\int_{Q_s}\Im_{xy}u_{tt}.\Im_{xy}u_{ttt}dxdydt \\
&= \frac{1}{2}\left\|\Im_{xy}u_{tt}\left(.,\ .,\ s\right)\right\|_{L^2(\Omega)}^2, \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
-\left(\frac{\partial^4 u}{\partial x^4},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} &= \int_{Q_s}\frac{\partial^3 u}{\partial x^3}.\Im_{xyy}u_{ttt}dxdydt \\
&= -\int_{Q_s}\frac{\partial^2 u}{\partial x^2}.\Im_y^2 u_{ttt}dxdydt \\
&= \int_{Q_s}u_x.\Im_y^2 u_{tttx}dxdydt \\
&= -\int_{Q_s}\Im_y u_x.\Im_y u_{tttx}dxdydt \\
&= \int_{Q_s}\Im_y u_{tx}.\Im_y u_{ttx}dxdydt \\
&= \frac{1}{2}\left\|\Im_y u_{xt}\left(.,\ .,T\right)\right\|_{L^2(\Omega)}^2, \qquad (39)
\end{aligned}
$$

$$
\begin{aligned}
-2\left(\frac{\partial^4 u}{\partial x^2 \partial y^2},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} &= 2\int_{Q_s} \frac{\partial^3 u}{\partial x \partial y^2}.\Im_{xyy} u_{ttt}\,dxdydt \\
&= -2\int_{Q_s} \frac{\partial^2 u}{\partial y^2}.\Im_{y}^2 u_{ttt}\,dxdydt \\
&= 2\int_{Q_s} u_y.\Im_y u_{ttt}\,dxdydt \\
&= -2\int_{Q_s} u.u_{ttt}\,dxdydt \\
&= 2\int_{Q_s} u_t.u_{tt}\,dxdydt \\
&= \left\| u_t\left(x,\ y,\ T\right)\right\|_{L^2(\Omega)}^2, \qquad (40)
\end{aligned}
$$

$$
\begin{aligned}
-\left(\frac{\partial^4 u}{\partial y^4},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} &= \int_{Q_s} \frac{\partial^3 u}{\partial y^3}.\Im_{xxy} u_{ttt}\,dxdydt \\
&= \int_{Q_s} u_y.\Im_x^2 u_{ttty}\,dxdydt \\
&= -\int_{Q_s} \Im_x u_y.\Im_x u_{ttty}\,dxdydt \\
&= \int_{Q_s} \Im_x u_{ty}.\Im_x u_{tty}\,dxdydt \\
&= \frac{1}{2}\left\|\Im_x u_{ty}\left(.,\ y,\ T\right)\right\|_{L^2(\Omega)}^2, \qquad (41)
\end{aligned}
$$

$$
\begin{aligned}
\alpha\left(\frac{\partial^2 \theta}{\partial x^2},\ \ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} &= -\alpha\int_{Q_s} \theta_x.\Im_{xyy} u_{ttt}\,dxdydt \\
&= \alpha\int_{Q_s} \theta.\Im_y^2 u_{ttt}\,dxdydt \\
&= -\alpha\int_{Q_s} \Im_y\theta.\Im_y u_{ttt}\,dxdydt \\
&= \alpha\left(\Im_y\theta_t,\ \Im_y u_{tt}\right)_{L^2(Q_s)}, \qquad (42)
\end{aligned}
$$

$$\alpha\left(\frac{\partial^2\theta}{\partial y^2},\ \Im_{xy}^2 u_{ttt}\right)_{L^2(Q)} = -\alpha\int_{Q_s}\theta_y.\Im_{xxy}u_{ttt}dxdydt$$
$$= \alpha\int_{Q_s}\theta.\Im_x^2 u_{ttt}dxdydt$$
$$= -\alpha\int_{Q_s}\Im_x\theta.\Im_x u_{ttt}dxdydt$$
$$= \alpha\left(\Im_x\theta_t,\ \Im_x u_{tt}\right)_{L^2(Q_s)}, \qquad (43)$$

$$-\beta\left(\theta_t,\ \Im_{xy}^2\theta_{tt}\right)_{L^2(Q)} = \beta\int_{Q_s}\Im_x\theta_t.\Im_{xyy}\theta_{tt}dxdydt$$
$$= -\beta\int_{Q_s}\Im_{xy}\theta_t.\Im_{xy}\theta_{tt}dxdydt$$
$$= \frac{\beta}{2}\left\|\Im_{xy}\theta_t\left(.,\ .,\ s\right)\right\|^2_{L^2(\Omega)}, \qquad (44)$$

$$\eta\left(\frac{\partial^2\theta}{\partial x^2},\ \Im_{xy}^2\theta_{tt}\right)_{L^2(Q)} = -\eta\int_{Q_s}\theta_x.\Im_{xyy}\theta_{tt}dxdydt$$
$$= \eta\int_{Q_s}\theta.\Im_y^2\theta_{tt}dxdydt$$
$$= -\eta\int_{Q_s}\Im_y\theta.\Im_y\theta_{tt}dxdydt$$
$$= \eta\left\|\Im_y\theta_t\right\|^2_{L^2(Q_s)}, \qquad (45)$$

$$\eta\left(\frac{\partial^2\theta}{\partial y^2},\ \Im_{xy}^2\theta_{tt}\right)_{L^2(Q)} = \eta\left\|\Im_x\theta_t\right\|^2_{L^2(Q_s)}, \qquad (46)$$

$$-\sigma\left(\theta,\ \Im_{xy}^2\theta_{tt}\right)_{L^2(Q)} = \sigma\int_{Q_s}\Im_x\theta.\Im_{xyy}\theta_{tt}dxdydt$$
$$= -\sigma\int_{Q_s}\Im_{xy}\theta.\Im_{xy}\theta_{tt}dxdydt$$
$$= \sigma\left\|\Im_{xy}\theta_t\right\|^2_{L^2(Q_s)}, \qquad (47)$$

$$-\alpha\left(\frac{\partial^3 u}{\partial x^2\partial t},\ \Im_{xy}^2\theta_{tt}\right)_{L^2(Q)} = \alpha\int_{Q_s}\frac{\partial^2 u}{\partial x\partial t}.\Im_{xyy}\theta_{tt}dxdydt$$
$$= -\alpha\int_{Q_s} u_t.\Im_y^2\theta_{tt}dxdydt$$
$$= \alpha\int_{Q_s}\Im_y u_t.\Im_y\theta_{tt}dxdydt$$
$$= -\alpha\left(\Im_y u_{tt},\ \Im_y\theta_t\right)_{L^2(Q_s)}, \qquad (48)$$

$$-\alpha\left(\frac{\partial^3 u}{\partial y^2\partial t},\ \Im_{xy}^2\theta_{tt}\right)_{L^2(Q)} = \alpha\int_{Q_s}\frac{\partial^2 u}{\partial y\partial t}.\Im_{xxy}\theta_{tt}dxdydt$$
$$= -\alpha\int_{Q_s} u_t.\Im_x^2\theta_{tt}dxdydt$$
$$= \alpha\int_{Q_s}\Im_x u_t.\Im_x\theta_{tt}dxdydt$$
$$= -\alpha\left(\Im_x u_{tt},\ \Im_x\theta_t\right)_{L^2(Q_s)}, \qquad (49)$$

Combining equalities $(36)-(49)$ and (35) we obtain

$$\frac{1}{2}\left\|\Im_{xy}u_{tt}(.,\ .,\ s)\right\|^2_{L^2(\Omega)} + \frac{1}{2}\left\|\Im_y u_{xt}(.,\ .,T)\right\|^2_{L^2(\Omega)}$$
$$+\left\|u_t(x,\ y,\ T)\right\|^2_{L^2(\Omega)} + \frac{1}{2}\left\|\Im_x u_{ty}(.,\ y,\ T)\right\|^2_{L^2(\Omega)}$$
$$+\frac{\beta}{2}\left\|\Im_{xy}\theta_t(.,\ .,\ s)\right\|^2_{L^2(\Omega)} + \eta\left\|\Im_y\theta_t\right\|^2_{L^2(Q_s)}$$
$$+\eta\left\|\Im_x\theta_t\right\|^2_{L^2(Q_s)} + \sigma\left\|\Im_{xy}\theta_t\right\|^2_{L^2(Q_s)} + \frac{c_1}{2}\left\|\Im_{xy}u_t(.,\ .,\ T)\right\|^2_{L^2(\Omega)}$$
$$+c_2\left\|\Im_{xy}u_{tt}\right\|^2_{L^2(\Omega)}$$
$$= 0, \qquad (50)$$

where $Q_s = \Omega\times(s,\ T)$.

Equality (50) implies that $\Im_{xy}u_{tt}(\zeta,\ \varsigma,\ s) = 0$ on Ω, and $\Im_{xy}\theta_t = 0$ onQ_s, hence we deduce that $\Psi = (\sigma_1,\ \sigma_2) = (0,\ 0)$ almost everywhere in Q_s.

Proceeding in this way step by step, we prove that $\Psi = 0$ almost every where in Q.

Now back to the general case: Since $\Psi = (\sigma_1,\ \sigma_2) = 0$ everywhere in Q, therefore equality (29) becomes

$$(\ell_1 u,\ \sigma_3)_{W_2^1(\Omega)} + (\ell_2 u,\ \sigma_4)_{L^2(\Omega)} + (\ell_3 \theta,\ \sigma_5)_{L^2(\Omega)} = 0. \tag{51}$$

Since the three quantities in (51) vanish independently and since the ranges of the trace operators $\ell_1,\ \ell_2$, and ℓ_3 are respectively everywhere dense in the spaces $W_2^1(\Omega),\ L^2(\Omega)$, and $L^2(\Omega)$, therefore it follows, from (51), that $\sigma_3 = \sigma_4 = \sigma_5 = 0$. Hence $\overline{R(J)} = H$. This achieves the proof of Theorem 4.

Remark: The following larger class of problems may handled by using the previous same techniques

$$\begin{cases} \mathcal{L}_1 u = \frac{\partial^2 u}{\partial t^2} + \triangle^2 u - \alpha \triangle \theta + (c_1 u + c_2 u_t) = f(x,\ y, t\ ,\theta, u),\ \ (x,\ y,\ t) \in Q, \\ \mathcal{L}_2 \theta = \beta \frac{\partial \theta}{\partial t} - \eta \triangle \theta + \sigma \theta + \alpha \triangle u_t - c_3 \Delta u = g(x,\ y,\ t\ ,\theta, u),\ (x,\ y,\ t) \in Q, \\ u(x,\ y,\ 0) = u_\circ(x, y),\ u_t(x,\ y,\ 0) = u_1(x,\ y),\ \theta(x,\ y,\ 0) = \theta_\circ(x,\ y), \\ u(0,\ y,\ t) = 0,\ \ u(a,\ y,\ t) = 0,\ \ 0 < y < b,\ \ 0 < t < T, \\ u(x,\ 0,\ t) = 0,\ \ u(x,\ b,\ t) = 0,\ \ 0 < x < a,\ \ 0 < t < T, \\ \int_0^a x^k u dx = 0, \int_0^b y^k u dy = 0,\ \int_0^a x^k \theta dx = 0, \int_0^b y^k \theta dy = 0,\ k = 0, 1, \end{cases} \tag{47}$$

where the functions f and g satisfy the conditions

$$\begin{aligned} |f(x,\ y,t\ ,\theta_1, u_1) - f(x,\ y,t\ ,\theta_2, u_2)| &\leq \mu\left(|\theta_1 - \theta_2| + |u_1 - u_2|\right), \\ |g(x,\ y,t\ ,\theta_1, u_1) - g(x,\ y,t\ ,\theta_2, u_2)| &\leq \mu\left(|\theta_1 - \theta_2| + |u_1 - u_2|\right). \end{aligned}$$

and $\alpha,\ \beta,\ \eta,\ \sigma, c_1$ and c_2 are positive constants.

We first deal with the associated linear problem, that is when $f(x, y,t\ ,\theta, u) = f(x,\ y,t\ )$ and $g(x,\ y,\ t\ ,\theta, u) = g(x,\ y,\ t\ )$ and then, on the basis of the obtained results of the linear problem, we apply an iterative process to establish the existence and uniqueness of the nonlinear problem.

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