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HOMOTOPY INVARIANCE OF RELATIVE ETA-INVARIANTS AND C^*-ALGEBRA K-THEORY

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ABSTRACT. We prove a close cousin of a theorem of Weinberger about the homotopy invariance of certain relative eta-invariants by placing the problem in operator K-theory. The main idea is to use a homotopy equivalence $h: M \to M'$ to construct a loop of invertible operators whose "winding number" is related to eta-invariants. The Baum-Connes conjecture and a technique motivated by the Atiyah-Singer index theorem provides us with the invariance of this winding number under twistings by finite-dimensional unitary representations of $\pi_1(M)$.

1. Introduction

Eta-invariants arose in the work of Atiyah, Patodi, and Singer [APS1] as the contribution from the boundary in their formula for the signature of a manifold with boundary.

Definition. Let N be a smooth, compact, Riemannian manifold, and let D be a self-adjoint, first-order elliptic operator on N. Define

$$\eta_D(s) = \sum_{\lambda \in sp(D), \lambda \neq 0} \operatorname{sign}(\lambda)|\lambda|^{-s},$$

where $sp(D)$ is the spectrum of D.

The sum converges for $\operatorname{Re} s \gg 0$. It is a deep result that $\eta_D(s)$ has an analytic continuation that is regular at $s = 0$ — see [G], for instance. The quantity $\eta_D(0)$ measures the "spectral asymmetry" of D, in the sense that if D is an operator whose spectrum is symmetric with respect to 0, then $\eta_D(0) = 0$.

Let N be a $4k$-dimensional, oriented, Riemannian manifold with boundary M. If N is locally a (Riemannian) product near the boundary, the Atiyah-Patodi-Singer signature theorem [APS1] states that

$$Sign(N, M) = \int_N \mathcal{L}(TN) + \eta_D(0),$$

where D is the signature operator on M, and $\mathcal{L}$ is the Hirzebruch $\mathcal{L}$-class of N.

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In [APS2], $\eta_D(0)$ is examined for signature operators on manifolds M which are not necessarily boundaries. Suppose M has fundamental group Γ. Let $\alpha, \beta : \Gamma \to U(n)$ be unitary, finite-dimensional representations of Γ of the same dimension. Let L_α be the flat vector bundle over M associated to α. Let D_α denote the signature operator with coefficients in L_α.

Define

$$\eta_\alpha(s) = \eta_{D_\alpha}(s), \qquad \eta_\beta(s) = \eta_{D_\beta}(s),$$

and set

$$\rho_{\alpha,\beta}(s) = \eta_\alpha(s) - \eta_\beta(s).$$

In [APS2] it is proved that $\rho_{\alpha,\beta}(0)$ is a *differential invariant* of M (it does not depend on the Riemannian structure on M). We denote $\rho_{\alpha,\beta}(0)$ by $\rho_{\alpha,\beta}(M)$; it is called the *relative eta-invariant* of M associated to the representations α, β [APS2].

The following question may be posed: Is $\rho_{\alpha,\beta}(M)$ a homotopy invariant of M? In other words, if $h : M \to M'$ is an orientation-preserving homotopy equivalence between oriented compact manifolds without boundary, is $\rho_{\alpha,\beta}(M) = \rho_{\alpha\circ h_*,\beta\circ h_*}(M')$?

The answer to this question is no in general, as is illustrated by the example of the lens spaces $L(7,1)$ and $L(7,2)$. These are homotopy equivalent manifolds ([deR], Theorem 1, p. 97) which have different relative eta-invariants [Kes].

On the other hand, Neumann has shown in [N] that $\rho_{\alpha,\beta}(M)$ is a homotopy invariant for manifolds M with free abelian fundamental group. Mathai reproved this result using index theory [M]. In 1988, Shmuel Weinberger [W] noticed a connection between the homotopy invariance problem for eta-invariants and the assembly map in surgery theory. He extended Neumann's result, proving the homotopy invariance of $\rho_{\alpha,\beta}(M)$ for a larger class of manifolds M, those for which the Borel conjecture is known for the fundamental group Γ.

Motivated by the close analogy (see [KM] and [HR]) between the assembly map in surgery theory and the index map of Kasparov [K2] and Baum-Connes [BCH], our goal is to give an analytic proof of a result parallel to Weinberger's, with the Borel conjecture replaced by its operator-theoretic analogue, the Baum-Connes conjecture.

Let $C^*_{\max}\Gamma$ and $C^*_{\text{red}}\Gamma$ denote the full and reduced group C^*-algebras of Γ, respectively. Denote by $B\Gamma$ the classifying space for principal Γ-bundles. For torsion-free discrete groups Γ, the Baum-Connes conjecture (see [BCH]) states that a certain index map $\mu_{\text{red}} : K_*(B\Gamma) \to K_*(C^*_{\text{red}}\Gamma)$ is an isomorphism. At the present time the groups Γ for which this conjecture is known to be true have the property that $K_*(C^*_{\max}\Gamma) \cong K_*(C^*_{\text{red}}\Gamma)$. Thus, for the torsion-free discrete groups for which the Baum-Connes conjecture is currently known, there is an index isomorphism $\mu_{\max} : K_*(B\Gamma) \to K_*(C^*_{\max}\Gamma)$. In our work we will need to use the maximal group C^*-algebra since we require that finite-dimensional unitary representations of Γ induce a representation of the C^*-algebra involved.

Theorem 1.1. *Let M be a closed, smooth, oriented, odd-dimensional, Riemannian manifold. Suppose that $\pi_1(M) = \Gamma$ is torsion-free and the Baum-Connes index map $\mu_{\max}$ is an isomorphism for Γ. Let α, β be finite-dimensional unitary representations of Γ of the same dimension. Then the relative eta-invariant $\rho_{\alpha,\beta}(M)$ is an oriented homotopy invariant of M.*

We will prove this theorem by means of the following association of ideas:

$$\text{Eta-invariants} \longleftrightarrow \text{Winding numbers} \longleftrightarrow K\text{-theory}.$$

In Section 2 we establish a link between eta-invariants and the winding numbers of (open) paths of unitary operators. Using these paths and a homotopy equivalence $h : M \to M'$ we construct a *loop* of unitary operators describing an element of the K-theory of $C^*_{\max}\Gamma$. At this point we will bring the Baum-Connes machinery to bear upon the problem of understanding the invariance of the winding number of this loop under twistings by finite-dimensional unitary representations of Γ.

Most of the known groups covered by our theorem are covered by Weinberger's and vice versa. There are however, two examples of groups for which we are able to extend Weinberger's result, namely, amenable groups and other groups which act metrically properly on Hilbert spaces [HK].

Some of the techniques developed in the proof may be of interest for other purposes. In particular, this applies to our observation that K-homological equivalences between elliptic operators on manifolds are realized through paths of operators with certain controlled analytic properties.

2. Winding numbers

First we review an integral formula for the eta-invariant. Notice that

$$\int_0^\infty t^{\frac{s-1}{2}} \lambda e^{-\lambda^2 t} dt = \Gamma(\tfrac{s+1}{2}) \mathrm{sign}(\lambda)|\lambda|^{-s} \qquad \text{for } \mathrm{Re}\, s > -1.$$

Here $\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt$ is the gamma function. Let D be the signature operator on a smooth, closed manifold M. Summing over all $\lambda \in sp(D)$, $\lambda \neq 0$, we get for $\mathrm{Re}\, s \gg 0$,

$$\int_0^\infty t^{\frac{s-1}{2}} \mathrm{trace}(De^{-tD^2}) dt = \Gamma(\tfrac{s+1}{2}) \eta_D(s).$$

Analytically continuing to $s = 0$ and noting that $\Gamma(1/2) = \sqrt{\pi}$, we obtain

$$\frac{1}{\sqrt{\pi}} \int_0^\infty t^{-\frac{1}{2}} \mathrm{trace}(De^{-tD^2}) dt = \eta_D(0).$$

From the substitution $t \to t^2$ we get

$$\frac{2}{\sqrt{\pi}} \int_0^\infty \mathrm{trace}(De^{-t^2 D^2}) dt = \eta_D(0). \tag{2.1}$$

For a detailed derivation see [G]. The convergence of this integral is a delicate matter which is also treated in [G]. Notice though that according to this formula,

$$\rho_{\alpha,\beta}(0) = \frac{2}{\sqrt{\pi}} \int_0^\infty \mathrm{trace}(D_\alpha e^{-t^2 D_\alpha^2}) - \mathrm{trace}(D_\beta e^{-t^2 D_\beta^2}) dt,$$

where $\alpha, \beta : \Gamma \to U(n)$ are finite-dimensional unitary representations of Γ of the same dimension. The right-hand side of the above equation exists for much easier reasons than the integral appearing in (2.1) since it involves a *difference* of integrands. For small t, the Schwartz kernel of $D_\alpha e^{-t^2 D_\alpha^2}$ is concentrated near the diagonal and (up to a small error which vanishes as $t \to 0$) depends only on the local geometry of the manifold and the bundle L_α. Since L_α is *flat*, it has no local geometry. It follows that as $t \to 0$ the difference of local traces

$$\mathrm{trace}_x(D_\alpha e^{-t^2 D_\alpha^2}) - \mathrm{trace}_x(D_\beta e^{-t^2 D_\beta^2})$$

(where $x \in M$) converges uniformly to 0.

Now, let U_t, $a \leq t \leq b$, be a norm continuous path of unitary operators on a Hilbert space H satisfying:

1. $U_t = I + \text{Trace Class}$, for $a \leq t \leq b$; and
2. the path U_t is smooth in the trace norm.

Define the *winding number* of $\mathcal{U} = \{U_t\}_{a \leq t \leq b}$:

$$w(\mathcal{U}) = \frac{1}{2\pi i} \int_a^b \mathrm{trace}(U_t^{-1} \frac{dU_t}{dt}) dt$$

(compare [HS]). Let $\varphi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. Define a path of unitary operators by

$$V_t = -\exp(i\pi\varphi(tD)).$$

Since $1+\exp(i\pi\varphi(x))$ and the derivative of $\exp(i\pi\varphi(x))$ are Schwartz class functions, the path V_t satisfies conditions 1 and 2 above (except that it is defined on the open interval $0 < t < \infty$). Formally calculating the winding number of $\mathcal{V} = \{V_t\}_{t>0}$, we get

$$\begin{aligned} w(\mathcal{V}) &= \frac{1}{2\pi i} \int_0^\infty \mathrm{trace}\big(V_t^{-1} \frac{dV_t}{dt}\big) dt \\ &= \frac{i\pi}{2\pi i} \int_0^\infty \mathrm{trace}\big(\exp(-i\pi\varphi(tD)) \exp(i\pi\varphi(tD))\varphi'(tD)D\big)\, dt \\ &= \frac{1}{\sqrt{\pi}} \int_0^\infty \mathrm{trace}\big(De^{-t^2D^2}\big) dt \\ &= \frac{1}{2}\eta_D(0). \end{aligned}$$

Twist V_t by α to obtain the path V_t^α as follows:

$$V_t^\alpha = -\exp(i\pi\varphi(tD_\alpha)).$$

Then

$$\begin{aligned} 2\lim_{\epsilon\to 0}(w(\mathcal{V}_\epsilon^\alpha) - w(\mathcal{V}_\epsilon^\beta)) &= \lim_{\epsilon \to 0} \frac{2}{\sqrt{\pi}} \int_\epsilon^{1/\epsilon} \mathrm{trace}(D_\alpha e^{-t^2 D_\alpha^2}) - \mathrm{trace}(D_\beta e^{t^2 D_\beta^2}) dt \\ &= \rho_{\alpha,\beta}(M). \end{aligned}$$

3. The Baum-Connes conjecture

Let $h : M \to M'$ be a homotopy equivalence between manifolds M, M'. Let Γ be the fundamental group of M and let D and D' denote the signature operators on M and M' respectively. Let $L_{\max}$ denote the Miscenko bundle [Ros1]; this is a flat bundle over $B\Gamma$ whose fibers are projective $C^*_{\max}\Gamma$-modules. Let $D_{\max}, D'_{\max}$ denote the signature operators with coefficients in the pullback of $L_{\max}$ over M, M' respectively. Let

$$V_t = -\exp(i\pi\varphi(tD_{\max})), \qquad V_t^{'} = -\exp(i\pi\varphi(tD'_{\max})).$$

If $\epsilon > 0$, then we shall obtain, from the homotopy equivalence $h : M \to M'$, a loop $[h]$ of operators on Hilbert $C^*_{\max}\Gamma$-modules which is comprised of four segments, two

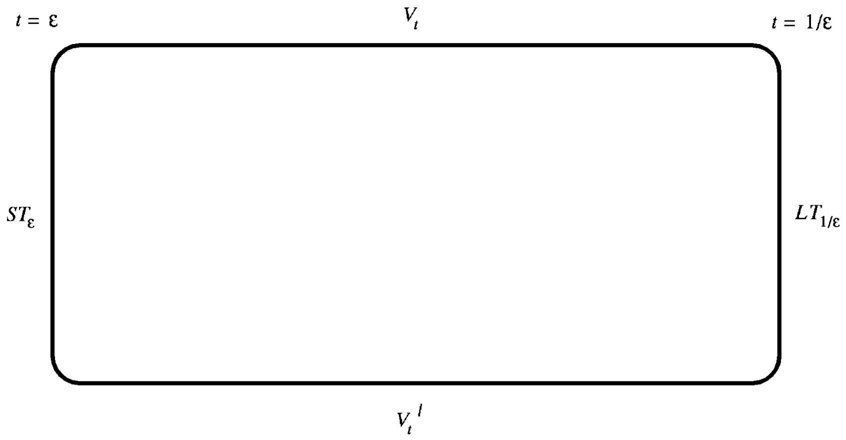

FIGURE 1. The loop $[h]$.

of which are the closed paths $\{V_t\}_{t=\epsilon}^{1/\epsilon}$ and $\{V_t^{'}\}_{t=\epsilon}^{1/\epsilon}$. The remaining two segments are constructed as follows: we use the hypothesis of *injectivity* of $\mu_{\max}$ to connect V_ϵ to $V_\epsilon^{'}$ through what we shall call the small time path (denoted ST_ϵ) of unitary operators. This will be done in Section 5. In Section 4 we shall connect $V_{1/\epsilon}$ to $V_{1/\epsilon}^{'}$ through a large time path (denoted $LT_{1/\epsilon}$) of unitary operators (see Figure 1).

Note that the large and small time paths actually depend on $\epsilon > 0$. However, the homotopy class of $[h]$ is independent of ϵ. Putting the constructions of these paths aside for a moment, we shall outline in this section our proof of Theorem 1.1.

Notice that $[h]$ is a loop of unitary operators, each of which is a compact perturbation of the identity. So $[h]$ corresponds to an element of $\pi_1(GL(C^*_{\max}\Gamma))$, which by Bott periodicity is the same as $K_0(C^*_{\max}\Gamma)$; see [B].

Now every finite-dimensional unitary representation $\alpha : \Gamma \to U(n)$ determines a trace map $\mathrm{trace}_\alpha : K_0(C^*_{\max}\Gamma) \to \mathbb{C}$.

Lemma 3.1. *Let the twisted index map* $\mathrm{Index}_\alpha : K_0(B\Gamma) \to \mathbb{Z}$ *be given by*

$$\mathrm{Index}_\alpha([\mathcal{H}, F]) = \mathrm{Index}(F_\alpha).$$

(Here $[\mathcal{H}, F]$ *is an abstract elliptic operator on* $B\Gamma$*; these form the basic cycles of Kasparov's model for K-homology* [K1]*.) Then, the following diagram commutes:*

$$\begin{array}{ccc} K_0(B\Gamma) & \xrightarrow{\mu_{\max}} & K_0(C^*_{\max}\Gamma) \\ {\scriptstyle \mathrm{Index}_\alpha}\big\downarrow & & \big\downarrow{\scriptstyle \mathrm{trace}_\alpha} \\ \mathbb{Z} & \longrightarrow & \mathbb{C} \end{array}$$

Proposition 3.2. *If the Baum-Connes assembly map*

$$\mu_{\max} : K_0(B\Gamma) \to K_0(C^*_{\max}\Gamma)$$

is surjective, and if $\alpha, \beta : \Gamma \to U(n)$ *are finite-dimensional unitary representations of* Γ *of the same dimension, then* $\mathrm{trace}_\alpha = \mathrm{trace}_\beta$ *on* $K_0(C^*_{\max}\Gamma)$*. In particular,*

$$\mathrm{trace}_\alpha[h] - \mathrm{trace}_\beta[h] = 0.$$

Proof. Let $[P] \in K_0(C^*_{\max}\Gamma)$. Since $\mu_{\max}$ is surjective, there is a preimage $[\mathcal{H}, F] \in K_0(B\Gamma)$ to $[P]$. By Lemma 3.1, all we need to check is that $\mathrm{Index}(F_\alpha) = \mathrm{Index}(F_\beta)$.

To simplify the argument slightly we will assume for now that $B\Gamma$ is a manifold. By using Chern-Weil theory and the Chern isomorphism, one can show that in topological K-theory, flat bundles are of the following form [Ros2]:

$$[L_\alpha] = \dim(\alpha).1 + \text{torsion} \in K^0(B\Gamma).$$

Thus, since $\dim(\alpha) = \dim(\beta)$, we have that

$$[L_\alpha] - [L_\beta] = \text{torsion} \in K^0(B\Gamma).$$

Now we know that there is a canonical pairing

$$K^0(X) \otimes K_0(X) \to \mathbb{Z}$$

given by

$$[E] \otimes [F] \to \mathrm{Index}(F_E).$$

And thus,

$$\begin{aligned} \mathrm{Index}(F_\alpha) - \mathrm{Index}(F_\beta) &= ([L_\alpha] - [L_\beta]) \otimes [F] \\ &= \text{torsion} \otimes [F] \\ &= 0. \end{aligned}$$

□

To calculate $\mathrm{trace}_\alpha[h]$ we note that tensor product with α constructs from a loop of unitary operators on a Hilbert $C^*_{\max}\Gamma$-module a loop of Hilbert space unitary operators. Let us denote by $[h]_\alpha$ the loop so obtained from $[h]$. Like $[h]$ it is composed of four segments: the paths $\{V_{\alpha,t}\}_{t=\epsilon}^{1/\epsilon}$ and $\{V'_{\alpha,t}\}_{t=\epsilon}^{1/\epsilon}$ together with paths $ST_{\epsilon,\alpha}$ and $LT_{1/\epsilon,\alpha}$. We shall prove in Sections 4 and 5 that:

Theorem 3.3. *If $\alpha, \beta : \Gamma \to U(n)$ are finite-dimensional unitary representations of Γ of the same dimension, then in the limit as $\epsilon \to 0$, the winding numbers of $ST_{\epsilon,\alpha}$ and $ST_{\epsilon,\beta}$ are equal, whereas the winding numbers of $LT_{1/\epsilon,\alpha}$ and $LT_{1/\epsilon,\beta}$ are zero.*

Now,

Lemma 3.4. *Let α be a finite-dimensional, unitary representation of Γ. Then*

$$\mathrm{trace}_\alpha([h]) = w([h]_\alpha).$$

From Theorem 3.3 and Lemma 3.4 we obtain:

Theorem 3.5. *For $\alpha, \beta : \Gamma \to U(n)$ finite-dimensional unitary representations of Γ of the same dimension and $[h]$ as above,*

$$\mathrm{trace}_\alpha[h] - \mathrm{trace}_\beta[h] = \rho_{\alpha,\beta}(M) - \rho_{\alpha,\beta}(M').$$

Proof. By Lemma 3.4 the traces of $[h]$ are winding numbers. Decomposing the winding numbers into contributions from the four segments of $[h]$, we see from Theorem 3.3 that the α and β contributions from the large and small time paths cancel one another as $\epsilon \to 0$, whereas the remaining contributions converge to $\rho_{\alpha,\beta}(M) - \rho_{\alpha,\beta}(M')$. □

Proof of Theorem 1.1. This is immediate from Proposition 3.2 and Theorem 3.5. □

4. The large time path

In Theorem 3.18 in [HR], Higson and Roe write down an explicit path that realizes the equality in $K_*(C^*_{\max}\Gamma)$ of the indices of the signature operators of homotopy equivalent manifolds having fundamental group Γ (compare also [KM]). We use this path to obtain the large time path. One checks by an explicit calculation that for any finite-dimensional unitary representation α of Γ,

Lemma 4.1. *As $t \to \infty$, the winding number of $LT_{t,\alpha}$ converges to 0.*

5. The small time path

The small time path is obtained from the equivalence relation defined in K-homology and a result of Kasparov.

Lemma 5.1 ([KM]). *If the Baum-Connes assembly map is injective, then $[D] = [D']$ in $K_1(B\Gamma)$.*

Sketch of Proof. It is first established that due to the homotopy equivalence $h : M \to M'$, D and D' have the same index in $K_1(C^*_{\max}\Gamma)$; see Theorem 3.18 in [HR]. The result then follows from the injectivity of $\mu_{\max}$. □

Definition. Let $\epsilon > 0$. An *ϵ-compression* of a bounded operator F is an operator F_ϵ satisfying the following conditions:

1. F_ϵ is a trace class perturbation of F; and
2. the propagation of F_ϵ is no more than ϵ.

Definition. An operator F is said to have *polynomial growth* if there is a polynomial p such that for each $\epsilon > 0$, there is an ϵ-compression of F, F_ϵ, satisfying

$$||F - F_\epsilon||_1 < p\left(\frac{1}{\epsilon}\right).$$

Definition. Let Y be a metric space. A path F_t of bounded operators on a Hilbert space $\mathcal{H}$ equipped with an action of $C(Y)$ is called a *controlled path* provided the following are true:

1. the path F_t has polynomial growth; and
2. the paths $F_t^2 - 1$ and $F_t(F_t^2 - 1)$ are paths made up of trace class operators and are trace-norm continuous and piecewise continuously differentiable in the trace norm.

Note that this definition is modelled on the equivalence relation in Kasparov's realization of K-homology [K1]. In particular, a controlled path is a homotopy of abstract elliptic operators in the sense of [K1].

Definition. A chopping function is a continuous function f on $\mathbb{R}$ which satisfies the following:

1. $|f| \leq 1$;
2. $\lim_{x\to\pm\infty} f(x) = \pm 1$.

In the following theorem we use the (M, E, ϕ) description of K-homology due to P. Baum [BD]. We also use the notion of a degenerate operator as defined by Kasparov in his formulation of K-homology [K1].

Definition. Let Y be a compact Riemannian manifold (possibly with boundary). Let H be a Hilbert space equipped with an action of $C(Y)$. A *degenerate operator* is a bounded self-adjoint operator F on H satisfying:

1. $F^2 - I = 0$;
2. $Ff - fF = 0$ for all $f \in C(Y)$.

Theorem 5.2. *Let Y be a compact Riemannian manifold with boundary. Let (M, E, ϕ) and (M', E', ϕ') be two Baum K-cycles on Y and suppose that $\phi : M \to Y$ and $\phi' : M' \to Y$ are Lipschitz maps. Let $\chi(x)$ be a chopping function such that:*

1. *the derivative of χ is Schwartz class;*
2. *the Fourier transform of χ is smooth and is supported in $[-1, 1]$; and*
3. *the functions $\chi^2 - 1$ and $\chi(\chi^2 - 1)$ are Schwartz class and their Fourier transforms are supported in $[-1, 1]$.*

Let $D_E, D'_{E'}$ be the Dirac operators on M, M' respectively, with coefficients in E, E' respectively. If $\big[(M, E, \phi)\big] = \big[(M', E', \phi')\big] \in K_*(Y)$, then there are degenerate operators A, A' such that

1. *$\chi(D_E) \oplus A$ and $A' \oplus \chi(D'_{E'})$ are defined on the same Hilbert space $\mathcal{H}$;*
2. *the Hilbert space $\mathcal{H}$ has an action of $C(Y)$;*
3. *$\chi(D_E) \oplus A$ is connected to $A' \oplus \chi(D'_{E'})$ by a controlled path.*

A consequence of Lemma 5.1 and Theorem 5.2 is

Corollary 5.3. *There is a controlled path $F_{s,\epsilon}$ connecting $\varphi(\epsilon D) \oplus I$ and $\varphi(\epsilon D') \oplus I$.*

Let $ST_\epsilon = \{-\exp(i\pi F_{s,\epsilon}) | 0 \le s \le 1\}$.

If α and β are finite-dimensional unitary representations of Γ of the same dimension, then

Lemma 5.4. *As $\epsilon \to 0$, the difference of the winding numbers of $ST_{\epsilon,\alpha}$ and $ST_{\epsilon,\beta}$ converges to zero.*

Sketch of Proof. Using standard techniques (see [R], Proposition 5.11), one writes

$$ST_{\epsilon,\alpha}(s) = F_{s,\epsilon} + G_{s,\epsilon}$$

where $F_{s,\epsilon}$ has small propagation and $tr(G_{s,\epsilon}) \to 0$ as $\epsilon \to 0$. Now the Schwartz kernels of $F_{s,\epsilon}$ are localized near the diagonal and hence depend only on the local geometry of the manifold and the bundle L_α. Since L_α is flat, it has no local geometry and thus the kernel of $F_{s,\epsilon}$ does not detect it. Thus $w(F_{s,\epsilon})$ is independent of L_α and $w(G_{s,\epsilon}) \to 0$ as $\epsilon \to 0$. □

References

[APS1] M. Atiyah, V. K. Patodi, and I. Singer, *Spectral asymmetry and Riemannian geometry.* I, Math. Proc. Camb. Phil. Soc. **77** (1975), 43–69. MR **53:**1655a

[APS2] ______, *Spectral asymmetry and Riemannian geometry.* II, Math. Proc. Camb. Phil. Soc. **78** (1975), 405–432. MR **53:**1655b

[B] B. Blackadar, *K-theory for operator algebras*, MSRI Publications, vol. 5, Springer-Verlag, New York, 1986. MR **88g:**46082

[BCH] P. Baum, A. Connes, and N. Higson, *Classifying space for proper G-actions and K-theory of group C^*-algebras*, Proceedings of a Special Session on C^*-algebras, Contemporary Mathematics, volume 167, American Mathematical Society, Providence, RI, 1994, pp. 241–291. MR **96c:**46070

[BD] P. Baum and R. Douglas, *K-homology and index theory*, Proc. Symp. Pure Math. **38** (1982), 117–173. MR **84d:**58075

[G] P. Gilkey, *Invariance theory, the heat equation, and the Atiyah-Singer index theorem*, 2nd ed., CRC Press, 1995. MR **98b:**58156

[HS] P. de la Harpe and G. Skandalis, *Déterminant associé à une trace sur une algèbre de Banach*, Ann. Inst. Fourier (Grenoble) **34** (1984), 241–260. MR **87i:**46146a

[HK] N. Higson and G. Kasparov, *Operator K-theory for groups which act properly and isometrically on Hilbert space*, ERA Amer. Math. Soc. **3** (1997), 131–142. CMP 98:05

[HR] N. Higson and J. Roe, *Mapping surgery to analysis*, Preprint.

[K1] G. Kasparov, *Topological invariants of elliptic operators. I: K-homology*, Math. USSR Izv. **9** (1975), 751–792. MR **58:**7603

[K2] ———, *Equivariant KK-theory and the Novikov conjecture*, Inven. Math. **91** (1988), 147–201. MR **88j:**58123

[KM] J. Kaminker and J. Miller, *Homotopy invariance of the analytic index of signature operators over C^*-algebras*, J. Operator Theory **14** (1985), 113–127. MR **87b:**58082

[Kes] N. Keswani, *Homotopy invariance of relative eta-invariants and C^*-algebra K-theory*, Ph.D. Thesis, 1997, University of Maryland at College Park.

[M] V. Mathai, *On the homotopy invariance of reduced eta and other signature type invariants*, Preprint.

[N] W. Neumann, *Signature related invariants of manifolds. I: monodromy and γ-invariants*, Topology **18** (1979), 147–172. MR **80k:**57048

[deR] G. de Rham, S. Maumary, and M. A. Kervaire, *Torsion et type simple d'homotopie*, Lecture Notes in Mathematics, vol. 48, Springer-Verlag, 1967. MR **36:**5943

[R] J. Roe, *Elliptic operators, topology, and asymptotic methods*, Longman Scientific & Technical, Harlow, Essex, England, and Wiley, New York, 1988. MR **89j:**58126

[Ros1] J. Rosenberg, *Analytic Novikov for topologists*, Novikov Conjectures, Index Theorems and Rigidity, vol.1 (S. Ferry, A. Ranicki, and J. Rosenberg, eds.), London Math. Soc. Lecture Notes, vol. 226, Cambrige Univ. Press, Cambridge, 1995, pp. 338–372. MR **97b:**58138

[Ros2] ———, *Recent progress in algebraic K-theory and its relationship with topology and analysis*, Lecture notes prepared for the Joint Summer Research Conference on Algebraic K-theory, Seattle, July 1997, K-theory preprint server.

[W] S. Weinberger, *Homotopy invariance of eta invariants*, Proc. Nat. Acad. Sci. **85** (1988), 5362–5365. MR **90a:**58168

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