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REVIEW of the book
Making Mathematics Come to Life
A Guide for Teachers and Students
by
O. A. Ivanov
Translated by R. Burns
The book was published by American Mathematical Society in 2009

This work was originally published in Russian by MUHMO in 2009. The present translation was created under license for the A.M.S. and is published by permission. This is a book about "elementary" mathematics being, in the same time, a problem book, a textbook, an expository book. Most of all, it furnishes a captivated (and beautiful) reading about real mathematics, mathematical chefs d'oeuvre, computers and mathematics, teaching, etc. As a personal opinion, this book should be studied in connection with another Russian book namely: "Non-elementary Problems Treated Elementary" by A.M. Iaglom and I.M. Iaglom published in different languages (1967, in English, Golden Day, San Francisco). The present book is primary intended for students in mathematical departments of institutes of teacher-education but also addresses to: professors-especially those working with lower-year students-as illustrating how to introduce new concepts to "newcomers" in this world, math teachers, high-school students, math students and even "non-math people" since the math the book deals with should be understood by a motivated high-school student. The book consists of ten mathematical novels united by recurring ideas. We have therefore ten chapters-after an Introduction, Foreword, In memory of a teacher-Conclusion, Solutions of the Supplementary Problems, Bibliography and Index. Before going into the brief exposition of the chapters let us reveal some important ideas and facts the reader could see in this work.

1. The new concepts are introduced with examples and some of them with proofs.

2. Most of the chapters are provided with "Comments of a pedagogical nature".

3. Each chapter ends with "Supplementary problems" whose solutions are presented at the end of the book; there are over 300 exercises most of which are intended to be solved independently by the reader. The exercises presented throughout each chapter have the solutions at the end of the chapter.

4. The modern way to teach mathematics-making it less abstract, more "applied"-is present in the book and is illustrated in Chapter 10. Mathematics and the Computer.

5. There are proved or presented the proofs of some famous chefs d'oeuvre such as:

- Euler's theorem on the number of partitions of a natural numbers;

- Euler's theorem on the distribution of prime numbers;

- Euler's formula for planar graphs and Hilbert's proof of the Jordan curve theorem;

- Ramsey's theorem, Minkowski's lemma, Euler and Lagrange's theorems on the four squares;

- Gauss's proof of the constructability of a regular 17-gon by means of straightedge and compass;

- d'Alembert's proof of the "Fundamental Theorem of Algebra";

- Euler's celebrated formula ;

- Connes' proof of Morley's theorem.

6. The Index is very useful making the book easy to read. The first chapter is devoted to the induction seen as providing a solid foundation for the next chapters such as that about natural numbers combinatorics, etc. In the second chapter, about combinatorics, is presented the method of generating functions and, as applications, Euler's theorem about the number of partitions of a natural number, Catalan's numbers, the number of cells of n-spaces. The third chapter "The Whole Numbers" is founded on Peano's axiomatic system. Here the reader has the opportunity to discover mathematical gems: the theorems of Fermat and Euler on the congruencies, the distribution of primes (Euler and Chebyshew, Hadamard, de la Vallee Poussin), arithmetic functions, public-key codes, etc. The next chapter is dedicated to geometry that is geometric transformations. It is prepared the setting for the Chapter 8, more precisely for Connes' gem and also for the Napoleon's triangle. The fifth chapter deals with inequalities. It is stressed, by giving examples, on the importance of the inequalities in any field of mathematics. There are presented at least five proofs of the a.m.-g.m. inequality. The chapter 6 is devoted to graphs but not only. The reader can see-via Euler's theorem and (coming from) Euler-Poincare characteristic-the

implications in Topology and other fields. This is made more relevant by the Hilbert's proof of the Jordan curve theorem in the case of polygons. In the seventh chapter entitled "The pigeonhole Principle" it can be read the proofs of the Minkowski lemma, Euler and Lagrange theorems about sums of two and four squares. Chapter 8. Complex Numbers and Polynomials is a real mathematical jewel in itself; one can see how different fields are connected in the proof of a theorem or a theory. The proof of A. Connes of the Morley theorem with the aid of complex numbers and rotations is brilliant. The same "philosophy" can be detected in the each and any proof of the Poncelet's theorem: projective geometry, complex numbers, mathematical billiards, dynamical systems, etc. The historical part of Conne's story including Napoleon Bonaparte's theorem (proposed as a supplementary problem, and very easy to prove, inspired by Connes) is also very interesting. This is why it's core is 8.4. Complex numbers and geometry. In the ninth chapter we get closer to computers by examining some problems on rational approximation: Farey sequences, continued fractions, quadratic irrationals, algebraic and transcendental numbers, etc. In the last chapter one can see how to solve problems with the computer, at least how to be provided with some hints. As for examples, very instructive chosen, are implemented in MATHEMATICA from Wolfram. The pedagogical comments should mandatory be studied by anyone interested in teaching mathematics. The conclusion's title is: Teaching How to Look for Solutions of Problems, or Fantasy in the Manner of Polya and needs no more explanations.

As a conclusion of the review: "...what is sometimes studied at school as "mathematics" resembles real mathematics not any closer than a plucked flower gathering dust in a herbarium or pressed between the pages of a book resembles the same flower in the meadow besprinkled with dewdrops sparkling in the light of the rising sun" (O. Ivanov). This book clearly can make a difference in the unfortunate above image, it can, for sure, improve the pedagogical "style" of the reader.

Adrian Girjoaba
Department of Mathematics,
Faculty of Sciences,
"Lucian Blaga" University of Sibiu,
Romania