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Research Article

On Bloch-Type Functions with Hadamard Gaps

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We give some sufficient and necessary conditions for an analytic function f on the unit ball B with Hadamard gaps, that is, for $f(z)=\sum_{k=1}^{\infty}P_{n_k}(z)$ (the homogeneous polynomial expansion of f) satisfying $n_{k+1}/n_k \geq \lambda > 1$ for all $k \in \mathbb{N}$, to belong to the space $\mathcal{B}_p^{\alpha}(B) = \{f \mid \sup_{0<r<1}(1-r^2)^{\alpha}\|\mathcal{R}f_r\|_p < \infty, f \in H(B)\}$, $p = 1, 2, \infty$ as well as to the corresponding little space. A remark on analytic functions with Hadamard gaps on mixed norm space on the unit disk is also given.

1. Introduction

Let $B=\{z\in\mathbb{C}^n : |z|<1\}$ be the open unit ball of $\mathbb{C}^n$, $\partial B=\{z\in\mathbb{C}^n : |z|=1\}$ its boundary, $\mathbb{D}$ the unit disk in $\mathbb{C}$, dv the normalized Lebesgue measure of B (i.e., $v(B)=1$), and $d\sigma$ the normalized rotation invariant Lebesgue measure of S satisfying $\sigma(\partial B)=1$. We denote the class of all holomorphic functions on the unit ball by $H(B)$.

For $f\in H(B)$ with the Taylor expansion $f(z)=\sum_{|\beta|\geq 0}a_\beta z^\beta$, let $\mathcal{R}f(z)=\sum_{|\beta|\geq 0}|\beta|a_\beta z^\beta$ be the radial derivative of f, where $\beta=(\beta_1,\beta_2,\dots,\beta_n)$ is a multi-index and $z^\beta=z_1^{\beta_1}\cdots z_n^{\beta_n}$. It is well known that $\mathcal{R}f(z)=\sum_{j=1}^{n}z_j(\partial f/\partial z_j)(z)=\sum_{k=0}^{\infty}kP_k(z)$, if $f(z)=\sum_{k=0}^{\infty}P_k(z)$.

As usual, we write

$$||f_r||_p=\left(\int_S |f(r\zeta)|^p d\sigma(\zeta)\right)^{1/p} \tag{1.1}$$

if $p\in(0,\infty)$, and where $f_r(\zeta)=f(r\zeta)$. If $p=\infty$, then $\|f\|_\infty=\sup_{z\in B}|f(z)|$.

Let $\alpha > 0$. The α-Bloch space $\mathcal{B}^\alpha = \mathcal{B}^\alpha(B)$ is the space of all holomorphic functions f on B such that

$$b_\alpha(f) = \sup_{z\in B}\left(1-|z|^2\right)^\alpha \left|\mathcal{R}f(z)\right| < \infty. \tag{1.2}$$

It is clear that $\mathcal{B}^\alpha$ is a normed space under the norm $\|f\|_{\mathcal{B}^\alpha} = |f(0)| + b_\alpha(f)$, and $\mathcal{B}^{\alpha_1} \subset \mathcal{B}^{\alpha_2}$ for $\alpha_1 < \alpha_2$. Let $\mathcal{B}^\alpha_0$ denote the subspace of $\mathcal{B}^\alpha$ consisting of those $f \in \mathcal{B}^\alpha$ for which $(1-|z|^2)^\alpha|\mathcal{R}f(z)| \to 0$ as $|z| \to 1$. This space is called the little α-Bloch space. For $\alpha = 1$, the α-Bloch space and the little α-Bloch space become Bloch space $\mathcal{B}$ and the little Bloch space $\mathcal{B}_0$. Some characterizations of these spaces can be found, for example, in the following papers [1–6].

We say that an analytic function f on the unit disk $\mathbb{D}$ has Hadamard gaps if $f(z) = \sum_{k=1}^\infty a_k z^{n_k}$ where $n_{k+1}/n_k \ge \lambda > 1$, for all $k \in \mathbb{N}$.

In [7], Yamashita proved the following result.

Theorem 1.1. *Assume that f is an analytic function on $\mathbb{D}$ with Hadamard gaps. Then for $\alpha > 0$, the following two propositions hold:*

(a) $f \in \mathcal{B}^\alpha(\mathbb{D})$ *if and only if* $\limsup_{k\to\infty} |a_k| n_k^{1-\alpha} < \infty$;

(b) $f \in \mathcal{B}^\alpha_0(\mathbb{D})$ *if and only if* $\lim_{k\to\infty} |a_k| n_k^{1-\alpha} = 0$.

An analytic function on B with the homogeneous expansion $f(z) = \sum_{k=1}^\infty P_{n_k}(z)$ (here, P_{n_k} is a homogeneous polynomial of degree n_k) is said to have Hadamard gaps if $n_{k+1}/n_k \ge \lambda > 1$, for all $k \in \mathbb{N}$. In [8], among others, Choa generalizes the main result in [9], proving the following result.

Theorem 1.2. *Assume that $p \in (0,\infty)$ and $f(z) = \sum_{k=1}^\infty P_{n_k}(z)$ is an analytic function on B with Hadamard gaps. Then the following statements are equivalent:*

(a) $\|f\|_{X_p} = \left(\int_B |\mathcal{R}f(z)|^p (1-|z|^2)^{p-1} dv(z)\right)^{1/p} < \infty$;

(b) $\sum_{k=1}^\infty \|P_{n_k}\|_p^p < \infty$.

This result motivates us to find some characterizations for certain function spaces of analytic functions on the unit ball, in terms of the sequence $(\|P_{n_k}\|_p)_{k\in\mathbb{N}}$.

Now note that the quantity b_α in the definition of the α-Bloch spaces can be written in the following form:

$$b_\alpha(f) = \sup_{0<r<1}\left(1-r^2\right)^\alpha \sup_{\zeta\in S}\left|\mathcal{R}f(r\zeta)\right| = \sup_{0<r<1}\left(1-r^2\right)^\alpha M_\infty(\mathcal{R}f, r). \tag{1.3}$$

On the other hand, the quantity b_α can be considered as the limit case of the following quantities:

$$\|f\|_{\mathcal{B}^\alpha_p} = \sup_{0<r<1}\left(1-r^2\right)^\alpha \left|\left|\mathcal{R}f_r\right|\right|_p, \tag{1.4}$$

as $p \to \infty$. Note that for every $f \in H(B)$ and $p \in (0,\infty)$,

$$\sup_{0<r<1}\left(1-r^2\right)^\alpha \left|\left|\mathcal{R}f_r\right|\right|_p \le \sup_{0<r<1}\left(1-r^2\right)^\alpha \left|\left|\mathcal{R}f_r\right|\right|_\infty. \tag{1.5}$$

Hence, in this paper we also consider analytic functions with Hadamard gaps on the following spaces:

$$\mathcal{B}_p^\alpha = \Big\{f \mid \sup_{0<r<1} (1-r^2)^\alpha \|\mathcal{R}f_r\|_p < \infty,\ f \in H(B)\Big\},$$
$$\mathcal{B}_{p,0}^\alpha = \Big\{f \mid \lim_{r\to 1}(1-r^2)^\alpha \|\mathcal{R}f_r\|_p = 0,\ f \in H(B)\Big\}. \tag{1.6}$$

Motivated by Theorem 1.1 in this paper, we study analytic functions with Hadamard gaps, which belong to $\mathcal{B}_p^\alpha$ or $\mathcal{B}_{p,0}^\alpha$ space when $p = 1,2,\infty$. Some characterizations for these classes of functions on the unit ball are given in terms of the sequence $(\|P_{n_k}\|_p)_{k\in\mathbb{N}}$. The following are the main results.

Theorem 1.3. *Assume that $\alpha > 0$, $p = 1,2,\infty$, and $f(z) = \sum_{k=1}^\infty P_{n_k}(z)$ is an analytic function on B with Hadamard gaps. Then the following statements are equivalent:*
(a) $f \in \mathcal{B}_p^\alpha$;
(b) $\limsup_{k\to\infty} \|P_{n_k}\|_p n_k^{1-\alpha} < \infty$.

Theorem 1.4. *Assume that $\alpha > 0$, $p = 1,2,\infty$, and $f(z) = \sum_{k=1}^\infty P_{n_k}(z)$ is an analytic function on B with Hadamard gaps. Then the following statements are equivalent:*
(a) $f \in \mathcal{B}_{p,0}^\alpha$;
(b) $\lim_{k\to\infty} \|P_{n_k}\|_p n_k^{1-\alpha} = 0$.

Throughout this paper, constants are denoted by C, they are positive and may differ from one occurrence to the other. The notation $A \asymp B$ means that there is a positive constant C such that $B/C \le A \le CB$.

2. Proof of main results

Before proving the main results of this paper we quote two auxiliary results which are incorporated in the lemmas which follow (see [9, 10]).

Lemma 2.1. *Assume that $p \in (0,\infty)$. If (n_k) is an increasing sequence of positive integers satisfying $n_{k+1}/n_k \ge \lambda > 1$, for all k, then there is a positive constant A depending only on p and λ such that*

$$\frac{1}{A}\left(\sum_{k=1}^\infty |a_k|^2\right)^{1/2} \le \left(\frac{1}{2\pi}\int_0^{2\pi}\left|\sum_{k=1}^\infty a_k e^{in_k\theta}\right|^p d\theta\right)^{1/p} \le A\left(\sum_{k=1}^\infty |a_k|^2\right)^{1/2} \tag{2.1}$$

for any number a_k, $k \in \mathbb{N}$.

Lemma 2.2. *Assume that $\alpha > 0$, $p > 0$, $n \in \mathbb{N}_0$, $(a_n)_{n\in\mathbb{N}_0}$ is the sequence of nonnegative numbers, $I_n = \{k \mid 2^n \le k < 2^{n+1},\ k \in \mathbb{N}\}$, $t_n = \sum_{k\in I_n} a_k$, and $g(x) = \sum_{n=1}^\infty a_n x^n$. Then there is a positive constant K depending only on p and α such that*

$$\frac{1}{K}\sum_{n=0}^\infty \frac{t_n^p}{2^{n\alpha}} \le \int_0^1 (1-x)^{\alpha-1} g^p(x)\,dx \le K\sum_{n=0}^\infty \frac{t_n^p}{2^{n\alpha}}. \tag{2.2}$$

Proof of Theorem 1.3. (a)⇒(b) (Case $p = 1$). Let $f \in \mathcal{B}_1^\alpha$. Let $f_\zeta(w) = f(\zeta w)$, $\zeta \in S$, where ζ is fixed and $w \in \mathbb{D}$, be a slice function. By some calculation we see that

$$f_\zeta'(w) = \zeta_1 \frac{\partial f}{\partial z_1}(w\zeta) + \cdots + \zeta_n \frac{\partial f}{\partial z_n}(w\zeta) = \frac{1}{w}\mathcal{R}f(w\zeta). \tag{2.3}$$

From (2.3) and since $f_\zeta'(w) = \sum_{k=1}^\infty n_k P_{n_k}(\zeta) w^{n_k-1}$, we have that

$$\begin{aligned}\int_S n_k |P_{n_k}(\zeta)|\, d\sigma(\zeta) &= \int_S \left| \frac{1}{2\pi i}\int_{\partial r\mathbb{D}} \frac{\eta f_\zeta'(\eta)}{\eta^{n_k+1}} d\eta \right| d\sigma(\zeta) \\ &\le \frac{1}{2\pi}\int_{\partial r\mathbb{D}}\int_S \frac{|\mathcal{R}f(\zeta\eta)|}{|\eta^{n_k+1}|} d\sigma(\zeta)|d\eta| \\ &\le \frac{\|f_r\|_{\mathcal{B}_1^\alpha}}{(1-r)^\alpha r^{n_k}},\end{aligned} \tag{2.4}$$

which implies that

$$n_k r^{n_k} \|P_{n_k}\|_1 \le \frac{\|f\|_{\mathcal{B}_1^\alpha}}{(1-r)^\alpha}, \tag{2.5}$$

for every $k \in \mathbb{N}$ and $r \in (0,1)$. Choosing $r = 1 - (1/n_k)$, we obtain $n_k^{1-\alpha}\|P_{n_k}\|_1 \le C$, as desired.

(b)⇒(a) (Case $p = 1$). Assume $\limsup_{k\to\infty} \|P_{n_k}\|_1 n_k^{1-\alpha} < \infty$. We have that

$$\begin{aligned}\|f\|_{\mathcal{B}_1^\alpha} &= \sup_{0<r<1} (1-r^2)^\alpha \int_S |\mathcal{R}f(r\zeta)|\, d\sigma(\zeta) \\ &= \sup_{0<r<1} (1-r^2)^\alpha \int_S \left| \sum_{k=1}^\infty n_k P_{n_k}(\zeta) r^{n_k} \right| d\sigma(\zeta) \\ &\le \sup_{0<r<1} (1-r^2)^\alpha \sum_{k=1}^\infty n_k \|P_{n_k}\|_1 r^{n_k} \\ &\le \sup_{0<r<1} (1-r^2)^{\alpha+1} \sum_{n=1}^\infty \left(\sum_{n_k \le n} n_k \|P_{n_k}\|_1 \right) r^n \\ &\le C \sup_{0<r<1} (1-r^2)^{\alpha+1} \sum_{n=1}^\infty \left(\sum_{n_k\le n} n_k^\alpha \right) r^n \\ &\le C \sup_{0<r<1} (1-r^2)^{\alpha+1} \sum_{n=1}^\infty n^\alpha r^n \le C,\end{aligned} \tag{2.6}$$

where we have used the fact that there is a positive constant C independent of n such that $\sum_{n_k\le n} n_k^\alpha \le C n^\alpha$ (here is used the assumption that $n_{k+1}/n_k \ge \lambda > 1$) and the following well-known estimate:

$$\sum_{n=1}^\infty n^\alpha r^n \le C(1-r)^{-(\alpha+1)}, \tag{2.7}$$

$\alpha > 0$, $r \in [0,1)$; see, for example, [11].

Case $p = 2$. Since

$$\|f\|_{\mathcal{B}_2^\alpha} = \sup_{0<r<1}(1-r^2)^\alpha\left(\sum_{k=1}^\infty n_k^2\|P_{n_k}\|_2^2 r^{2n_k}\right)^{1/2} \tag{2.8}$$

we have that

$$\sup_{0<r<1}(1-r^2)^\alpha n_k\|P_{n_k}\|_2 r^{n_k} \le \|f\|_{\mathcal{B}_2^\alpha} \le \sup_{0<r<1}(1-r^2)^\alpha\sum_{k=1}^\infty n_k\|P_{n_k}\|_2 r^{n_k}, \tag{2.9}$$

from which the result follows similar to the case $p = 1$.

Now we show that $(a)\Leftrightarrow(b)$ for case $p = \infty$. As above, the function $f_\zeta(w) = \sum_{k=1}^\infty P_{n_k}(\zeta)w^{n_k}$, where $w = re^{i\theta}$, is a lacunary series in $\mathbb{D}$ and

$$(1-r^2)^\alpha \mathcal{R}f(r\zeta) = re^{i\theta}(1-r^2)^\alpha f'_{\zeta e^{-i\theta}}(re^{i\theta}), \tag{2.10}$$

from which by Theorem 1.1 the equivalence follows. □

Proof of Theorem 1.4. (a)⇒(b) (Case $p = 1$). Let $f \in \mathcal{B}_{1,0}^\alpha$, then for every $\varepsilon > 0$ there is a $\delta > 0$ such that

$$(1-r^2)^\alpha\int_S|\mathcal{R}f(r\zeta)|\,d\sigma(\zeta) < \varepsilon, \tag{2.11}$$

whenever $\delta < r < 1$. From (2.4), (2.11), and rotational invariance of $d\sigma$, we have that

$$\begin{aligned}\int_S n_k|P_{n_k}(\zeta)|\,d\sigma(\zeta) &\le \frac{1}{2\pi}\int_{\partial r\mathbb{D}}\int_S\frac{|\mathcal{R}f(\zeta\eta)|}{|\eta^{n_k+1}|}d\sigma(\zeta)|d\eta| \\ &\le \frac{1}{2\pi}\int_{\partial r\mathbb{D}}\int_S\frac{(1-r^2)^\alpha|\mathcal{R}f(\zeta\eta)|}{(1-r^2)^\alpha r^{n_k+1}}d\sigma(\zeta)|d\eta| \\ &\le \frac{\varepsilon}{(1-r)^\alpha r^{n_k}},\end{aligned} \tag{2.12}$$

which implies that

$$n_k r^{n_k}\|P_{n_k}\|_1 \le \frac{\varepsilon}{(1-r)^\alpha} \tag{2.13}$$

for every $k \in \mathbb{N}$ and $r \in (\delta, 1)$. Choosing $r = 1-(1/n_k)$, we obtain

$$n_k\|P_{n_k}\|_1 \le C\varepsilon n_k^\alpha, \tag{2.14}$$

from which (b) follows in this case.

(b)⇒(a) (Case $p = 1$). Assume that $\lim_{k\to\infty}\|P_{n_k}\|_1 n_k^{1-\alpha} = 0$, then for every $\varepsilon > 0$ there is a $k_0 \in \mathbb{N}$ such that

$$\|P_{n_k}\|_1 \le \varepsilon n_k^{\alpha-1}, \quad \text{for } k \ge k_0. \tag{2.15}$$

We may assume that $k_0 = 1$. From this and by the proof of Theorem 1.3, (b)⇒(a) (Case $p = 1$), we have that

$$\begin{aligned}(1-r^2)^{\alpha}\|\mathscr{R}f_r\|_1 &\le \sup_{0<r<1}(1-r^2)^{\alpha+1}\sum_{n=1}^{\infty}\left(\sum_{n_k\le n} n_k\|P_{n_k}\|_1\right)r^n \\ &\le C\varepsilon \sup_{0<r<1}(1-r^2)^{\alpha+1}\sum_{n=1}^{\infty}\left(\sum_{n_k\le n} n_k^{\alpha}\right)r^n \\ &\le C\varepsilon \sup_{0<r<1}(1-r^2)^{\alpha+1}\sum_{n=1}^{\infty} n^{\alpha}r^n \le C\varepsilon,\end{aligned} \tag{2.16}$$

from which the implication follows.

Case $p = 2$. By using (2.9) the result follows similar to the Case $p = 1$. The proof is omitted.

Finally, in view of (2.10) and employing Theorem 1.1(b) it is easy to see that $(a)\Leftrightarrow(b)$ for case $p = \infty$. □

3. The case of mixed norm space

In this section, we give a note concerning analytic functions with Hadamard gaps on the mixed norm space. The mixed norm space $H_{p,q,\alpha}(B)$, $p,q > 0$, and $\alpha \in (-1,\infty)$, consists of all $f \in H(B)$ such that

$$\|f\|_{p,q,\alpha} = \left(\int_0^1 \|f(r\zeta)\|_p^q (1-r)^{\alpha} dr\right)^{1/q} < \infty. \tag{3.1}$$

From [12, Theorem 4] the following result holds.

Theorem 3.1. *Assume that $p \in (0,\infty)$, $\alpha > -1$ and $f(z) = \sum_{k=1}^{\infty} a_k z^{n_k}$ is an analytic function on $\mathbb{D}$ with Hadamard gaps. Then $f^{(m)} \in H_{p,q,\alpha}(\mathbb{D})$ if and only if $\sum_{k=0}^{\infty} n_k^{qm-\alpha-1}|a_k|^q < \infty$.*

Proof. First we consider the case $m = 0$. Similar to the proof of [12, Theorem 4] and by Lemmas 2.1 and 2.2, we have that

$$\begin{aligned}\|f\|_{H_{p,q,\alpha}}^q &= \int_0^1 \left(\frac{1}{2\pi}\int_0^{2\pi}\left|\sum_{k=1}^{\infty} a_k r^{n_k} e^{in_k\theta}\right|^p d\theta\right)^{q/p}(1-r)^{\alpha}dr \\ &\asymp \int_0^1 \left(\sum_{k=1}^{\infty}|a_k|^2 r^{2n_k}\right)^{q/2}(1-r)^{\alpha}dr \\ &\asymp \int_0^1 \left(\sum_{k=1}^{\infty}|a_k|^2 \rho^{n_k}\right)^{q/2}(1-\rho)^{\alpha}d\rho \\ &\asymp \sum_{k=0}^{\infty}\frac{1}{2^{(\alpha+1)k}}\left(\sum_{m\in I_k}|a_m|^2\right)^{q/2} \asymp \sum_{k=0}^{\infty}\frac{|a_k|^q}{n_k^{\alpha+1}},\end{aligned} \tag{3.2}$$

from which the result follows in this case.

Since f has Hadamard gaps and $f^{(m)}(z)=\sum_{k=1}^{\infty}a_k n_k(n_k-1)\cdots(n_k-m+1)z^{n_k-m}$, it follows that $f^{(m)}$ has Hadamard gaps too. Applying the just proved result to the function $f^{(m)}$, we obtain that $f^{(m)}\in H_{p,q,\alpha}(\mathbb{D})$ if and only if

$$\sum_{k=0}^{\infty}\frac{|n_k(n_k-1)\cdots(n_k-m+1)a_k|^q}{n_k^{\alpha+1}}\asymp\sum_{k=0}^{\infty}\frac{|a_k|^q}{n_k^{\alpha+1-mq}}<\infty, \tag{3.3}$$

finishing the proof. □

Remark 3.2. Motivated by [12, Theorems 3 and 4], we can conjecture that if $p\in(0,\infty)$, $\alpha>-1$, and $f(z)=\sum_{k=1}^{\infty}P_{n_k}(z)$ is an analytic function on B with Hadamard gaps, then $\mathscr{R}^{(m)}f\in H_{p,q,\alpha}(B)$ if and only if $\sum_{k=0}^{\infty}n_k^{qm-\alpha-1}\|P_{n_k}\|_p^q<\infty$. Note that the result is true for the case of the weighted Bergman space, that is, when $p=q$, see [12, Corollary 1]. It is also expected that Theorems 1.3 and 1.4 hold for every $p\in[1;\infty]$ (for the case $n=1$, see [13]).

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