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*Research Article*

# Product of Extended Cesàro Operator and Composition Operator from Lipschitz Space to $F(p,q,s)$ Space on the Unit Ball

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This paper characterizes the boundedness and compactness of the product of extended Cesàro operator and composition operator from Lipschitz space to $F(p,q,s)$ space on the unit ball of $\mathbb{C}^n$.

## 1. Introduction

Let $\mathbb{B}$ be the unit ball in the $n$-dimensional complex space $\mathbb{C}^n$, the closure of $\mathbb{B}$ will be written as $\overline{\mathbb{B}}$. By $dv$ we denote the Lebesgue measure on $\mathbb{B}$ normalized so that $v(\mathbb{B}) = 1$ and by $d\sigma$ the normalized rotation invariant measure on the boundary $S = \partial\mathbb{B}$ of $\mathbb{B}$. Let $H(\mathbb{B})$ be the class of all holomorphic functions on $\mathbb{B}$ and $S(\mathbb{B})$ the collection of all the holomorphic self-mappings of $\mathbb{B}$. Denote by $A(\mathbb{B})$ the unit ball algebra of all continuous functions on $\overline{\mathbb{B}}$ that are holomorphic on $\mathbb{B}$.

For $f \in H(\mathbb{B})$, let

$$\Re f(z) = \sum_{j=1}^{n} z_j \frac{\partial f}{\partial z_j}(z) \tag{1.1}$$

be the radial derivative of $f$.

We recall that the $\alpha$-Bloch space $\mathcal{B}^\alpha$ for $\alpha \geq 0$ consists of all $f \in H(\mathbb{B})$ such that

$$\mathcal{B}_\alpha(f) = \sup_{z\in\mathbb{B}} \left(1 - |z|^2\right)^\alpha \left|\Re f(z)\right| < \infty. \tag{1.2}$$

The expression $\mathcal{B}_\alpha(f)$ defines a seminorm while the natural norm is given by $\|f\|_{\mathcal{B}^\alpha} = |f(0)| + \mathcal{B}_\alpha(f)$. This norm makes $\mathcal{B}^\alpha$ into a Banach space. When $\alpha = 1$, $\mathcal{B}_1 = \mathcal{B}$ is the well known Bloch space.

For $\alpha \in (0,1)$, $\mathcal{L}_\alpha(\mathbb{B})$ denotes the holomorphic Lipschitz space of order $\alpha$ which is the set of all $f \in H(\mathbb{B})$ such that, for some $C > 0$,

$$\left|f(z) - f(w)\right| \leq C|z - w|^\alpha \tag{1.3}$$

for every $z, w \in \mathbb{B}$. It is clear that each space $\mathcal{L}_\alpha(\mathbb{B})$ contains the polynomials and is contained in the ball algebra $A(\mathbb{B})$. It is well known that $\mathcal{L}_\alpha(\mathbb{B})$ is endowed with a complete norm $\|\cdot\|_{\mathcal{L}_\alpha}$ that is given by

$$\|f\|_{\mathcal{L}_\alpha} = \left|f(0)\right| + \sup_{z \neq w; z, w \in \mathbb{B}} \left\{ \frac{\left|f(z) - f(w)\right|}{|z - w|^\alpha} \right\}. \tag{1.4}$$

See [1, 2] for more information of the Lipschitz spaces on $\mathbb{B}$.

For $a \in \mathbb{B}$, let $g(z, a) = \log|\varphi_a(z)|^{-1}$ be Green's function on $\mathbb{B}$ with logarithmic singularity at $a$, where $\varphi_a$ is the Möbius transformation of $\mathbb{B}$ with $\varphi_a(0) = a$, $\varphi_a(a) = 0$, and $\varphi_a = \varphi_a^{-1}$.

Let $0 < p, s < \infty$, $-n - 1 < q < \infty$, a function $f \in H(\mathbb{B})$ is said to belong to $F(p, q, s)$ if (see, e.g., [3–5])

$$\|f\|_{F(p,q,s)}^p = \left|f(0)\right|^p + \sup_{a \in \mathbb{B}} \int_{\mathbb{B}} \left|\Re f(z)\right|^p \left(1 - |z|^2\right)^q g^s(z, a)\, dv(z) < \infty. \tag{1.5}$$

If $X$ is a Banach space of holomorphic functions on a domain $\Omega$ and if $\varphi$ is a (holomorphic) self-map of $\Omega$, the composition operator of symbol $\varphi$ is defined by $C_\varphi(f) = f \circ \varphi$. The study of composition operators consists in the comparison of the properties of the operator $C_\varphi$ with that of the function $\varphi$ itself, which is called the symbol of $C_\varphi$. One can characterize boundedness and compactness of $C_\varphi$ and many other properties. We refer to the books in [6, 7] and to some recent papers in [4, 5, 8] to learn much more on this subject.

Let $h \in H(\mathbb{B})$, the following integral-type operator was first introduced in [9]

$$T_h f(z) = \int_0^1 f(tz) \Re h(tz) \frac{dt}{t}, \quad f \in H(\mathbb{B}),\ z \in \mathbb{B}. \tag{1.6}$$

This operator is called generalized Cesàro operator. It has been well studied in many papers, see, for example, [3, 9–24] as well as the related references therein.

It is natural to discuss the product of extended Cesàro operator and composition operator. For $h \in H(\mathbb{B})$ and $\varphi \in S(\mathbb{B})$, the product can be expressed as

$$T_h C_\varphi f(z) = \int_0^1 f(\varphi(tz)) \Re h(tz) \frac{dt}{t}, \quad f \in H(\mathbb{B}),\ z \in \mathbb{B}. \tag{1.7}$$

It is interesting to characterize the boundedness and compactness of the product operator on all kinds of function spaces. Even on the disk of $\mathbb{C}$, some properties are not easily managed; see some recent papers in [18, 25–28].

Building on those foundations, the present paper continues this line of research and discusses the operator in high dimension. The remainder is assembled as follows: in Section 2, we state a couple of lemmas. In Section 3, we characterize the boundedness and compactness of the product $T_h C_\varphi$ of extended Cesàro operator and composition operator from Lipschitz spaces to $F(p,q,s)$ spaces on the unit ball of $\mathbb{C}^n$.

Throughout the remainder of this paper, $C$ will denote a positive constant, the exact value of which will vary from one appearance to the next. The notation $A \asymp B$ means that there is a positive constant $C$ such that $B/C \le A \le CB$.

## 2. Some Lemmas

To begin the discussion, let us state a couple of lemmas, which are used in the proofs of the main results.

**Lemma 2.1.** *Suppose that $f, h \in H(\mathbb{B})$. Then,*

$$\Re\left[T_h C_\varphi(f)\right](z) = f(\varphi(z))\Re h(z). \tag{2.1}$$

*Proof.* The proof of this Lemma follows by standard arguments (see, e.g., [9, 29, 30]). □

**Lemma 2.2** (see [2, 31]). *If $0 < \alpha < 1$, then $\mathcal{B}^{1-\alpha} = \mathcal{L}_\alpha(\mathbb{B})$; furthermore,*

$$\|f\|_{B^{1-\alpha}} \asymp \|f\|_{\mathcal{L}_\alpha} \tag{2.2}$$

*as $f$ varies through $\mathcal{L}_\alpha(\mathbb{B})$.*

The following criterion for compactness follows from standard arguments similar to the corresponding lemma in [6]. Hence, we omit the details.

**Lemma 2.3.** *Assume that $h \in H(\mathbb{B})$ and $\varphi \in S(\mathbb{B})$. Suppose that $X$ or $Y$ is one of the following spaces $\mathcal{L}_\alpha(\mathbb{B})$, $F(p,q,s)$. Then, $T_h C_\varphi : X \to Y$ is compact if and only if $T_h C_\varphi : X \to Y$ is bounded, and for any bounded sequence $\{f_k\}_{k \in N}$ in $X$ which converges to zero uniformly on compact subsets of $\mathbb{B}$ as $k \to \infty$, one has $\|T_h C_\varphi f_k\|_Y \to 0$ as $k \to \infty$.*

**Lemma 2.4** (see [4, 5]). *If $f \in \mathcal{B}^\alpha$, then*

$$\left|f(z)\right| \le C\|f\|_{B^\alpha}, \quad 0 < \alpha < 1, \tag{2.3}$$

$$\left|f(z)\right| \le C\|f\|_{B^\alpha} \ln \frac{e}{1 - |z|^2}, \quad \alpha = 1, \tag{2.3$'$}$$

$$\left|f(z)\right| \le C \frac{\|f\|_{B^\alpha}}{\left(1 - |z|^2\right)^{\alpha - 1}}, \qquad \alpha > 1. \tag{2.3$''$}$$

The next lemma was obtained in [32].

**Lemma 2.5.** *If $a > 0$, $b > 0$, then the elementary inequality holds*

$$(a+b)^p \le \begin{cases} a^p + b^p, & 0 < p < 1, \\ 2^{p-1}(a^p + b^p), & p \ge 1. \end{cases} \tag{2.4}$$

It is obvious that Lemma 2.5 holds for the sum of finite number $k$, that is,

$$(a_1 + \cdots + a_k)^p \le C\left(a_1^p + \cdots + a_k^p\right), \tag{2.5}$$

where $a_1, \ldots, a_k > 0$ and $C$ is a positive constant.

**Lemma 2.6** (see [4, 5]). *For $0 < p$, $s < +\infty$, $-n-1 < q < +\infty$, $q + s > -1$, there exists $C > 0$ such that*

$$\sup_{a\in\mathbb{B}} \int_{\mathbb{B}} \frac{\left(1-|w|^2\right)^p}{|1-\langle z, w\rangle|^{n+1+q+p}} \left(1-|z|^2\right)^q g^s(z,a)\,dv(z) \le C \tag{2.6}$$

*for every $\omega \in \mathbb{B}$.*

**Lemma 2.7** (see [4]). *There is a constant $C > 0$ so that, for all $t > -1$ and $z \in \mathbb{B}$, one has*

$$\int_{\mathbb{B}} \left|\ln \frac{1}{1-\langle z, w\rangle}\right|^2 \frac{\left(1-|w|^2\right)^t}{|1-\langle z, w\rangle|^{n+1+t}}\,dv(z) \le C\left(\ln \frac{1}{1-|z|^2}\right)^2. \tag{2.7}$$

**Lemma 2.8** (see [4, 5]). *Suppose that $0 < p$, $s < \infty$, $-n-1 < q < \infty$, and $q+s > -1$. If $f \in F(p,q,s)$, then $f \in \mathcal{B}^{(n+1+q)/p}$, and $\|f\|_{\mathcal{B}^{(n+1+q)/p}} \le C\|f\|_{F(p,q,s)}$.*

**Lemma 2.9.** *Let $\{f_k\}_{k\in N}$ be a bounded sequence in $F(p,q,s)$ which converges to zero uniformly on compact subsets of the unit ball $\mathbb{B}$, where $(n+1+q)/p < 1$. Then, $\lim_{k\to\infty}\sup_{z\in\mathbb{B}}|f_k(z)| = 0$.*

*Proof.* It follows from Lemma 2.8 that $F(p,q,s) \subseteq \mathcal{B}^{(n+1+q)/p}$ and $\|f\|_{\mathcal{B}^{(n+1+q)/p}} \le C\|f\|_{F(p,q,s)}$ for any $f \in F(p,q,s)$. So, when $(n+1+q)/p < 1$, the proof of this lemma is similar to that of Lemma 3.6 of [33], hence the proof is omitted. □

## 3. The Boundedness and Compactness of the Operator $T_hC_\varphi : \mathcal{L}_\alpha(\mathbb{B}) \to F(p,q,s)$

**Theorem 3.1.** *Assume that $\alpha \in (0,1)$, $0 < p$, $s < \infty$, $-n-1 < q < \infty$, $q+s > -1$, $\varphi \in S(\mathbb{B})$, and $h \in H(\mathbb{B})$. Then, $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is bounded if and only if $h \in F(p,q,s)$.*

*Proof.* Assume that $h \in F(p,q,s)$. Since $0 < 1-\alpha < 1$, by Lemmas 2.2 and 2.4, for any $f \in \mathcal{L}_\alpha$, we have

$$|f(z)| \le C\|f\|_{\mathcal{B}^{1-\alpha}} \le C\|f\|_{\mathcal{L}_\alpha}. \tag{3.1}$$

Since $|T_hC_\varphi f(0)| = 0$, by using Lemma 2.1 and relations (2.3) and (3.1), we have

$$\begin{aligned}
\|T_hC_\varphi f\|^p_{F(p,q,s)} &= \sup_{a\in\mathbb{B}}\int_{\mathbb{B}} |f(\varphi(z))\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)\,dv(z) \\
&\le C\sup_{a\in\mathbb{B}}\int_{\mathbb{B}} |\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)\,dv(z)\|f\|^p_{\mathcal{B}^{1-\alpha}} \\
&\le C\|h\|^p_{F(p,q,s)}\|f\|^p_{\mathcal{L}^\alpha} < \infty.
\end{aligned} \tag{3.2}$$

Thus $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is bounded.

Conversely, suppose that $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is bounded. Taking the function $f(z) = 1 \in \mathcal{L}_\alpha$, then

$$\begin{aligned}
\|T_hC_\varphi f\|^p_{F(p,q,s)} &= |T_hC_\varphi f(0)|^p + \sup_{a\in\mathbb{B}}\int_{\mathbb{B}} |\Re(T_hC_\varphi f)(z)|^p\left(1-|z|^2\right)^q g^s(z,a)\,dv(z) \\
&= \sup_{a\in\mathbb{B}}\int_{\mathbb{B}} |f(\varphi(z))\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)\,dv(z) \\
&= \sup_{a\in\mathbb{B}}\int_{\mathbb{B}} |\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)\,dv(z) = \|h\|^p_{F(p,q,s)}.
\end{aligned} \tag{3.3}$$

From which, the boundedness of $T_hC_\varphi$ implies that $h \in F(p,q,s)$. This completes the proof of this theorem. □

Next, we characterize the compactness of $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$.

**Theorem 3.2.** *Assume that $\alpha \in (0,1)$, $0 < p, s < \infty$, $-n-1 < q < \infty$, $q + s > -1$, $\varphi \in S(\mathbb{B})$, and $h \in H(\mathbb{B})$. Then, $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is compact if and only if $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is bounded, and*

$$\lim_{r\to 1}\sup_{a\in\mathbb{B}}\int_{\{|\varphi(z)|>r\}} |\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)\,dv(z) = 0. \tag{3.4}$$

*Proof.* Assume that $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is bounded and (3.4) holds. It follows from Theorem 3.1 that $h \in F(p,q,s)$.

Now, let $\{f_j\}_{j\in N}$ be a bounded sequence of functions in $\mathcal{L}_\alpha$ such that $f_j \to 0$ uniformly on the compact subsets of $\mathbb{B}$ as $j \to \infty$. Suppose that $\sup_{j\in N}\|f_j\|_{\mathcal{L}_\alpha} \le L$. It follows from (3.4) that, for any $\varepsilon > 0$, there exists $r_0 \in (0,1)$ such that, for every $r_0 < r < 1$,

$$\sup_{a\in\mathbb{B}}\int_{\{|\varphi(z)|>r\}} |\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)\,dv(z) < \varepsilon. \tag{3.5}$$

Set $r_0 < r < 1$, then

$$\begin{aligned}\left\|T_hC_\varphi f_j\right\|^p_{F(p,q,s)} &= \sup_{a\in\mathbb{B}}\int_{\mathbb{B}}\left|f_j(\varphi(z))\right|^p|\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)d\nu(z)\\ &\le \sup_{a\in\mathbb{B}}\int_{\{|\varphi(z)|\le r\}}\left|f_j(\varphi(z))\right|^p|\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)d\nu(z)\\ &\quad+\sup_{a\in\mathbb{B}}\int_{\{|\varphi(z)|> r\}}\left|f_j(\varphi(z))\right|^p|\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)d\nu(z)\\ &= I_1+I_2,\end{aligned} \tag{3.6}$$

where

$$\begin{aligned}I_1 &:= \sup_{a\in\mathbb{B}}\int_{\{|\varphi(z)|\le r\}}\left|f_j(\varphi(z))\right|^p|\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)d\nu(z),\\ I_2 &:= \sup_{a\in\mathbb{B}}\int_{\{|\varphi(z)|> r\}}\left|f_j(\varphi(z))\right|^p|\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)d\nu(z).\end{aligned} \tag{3.7}$$

Let $K = \{w : |w| \le r\}$, then $K$ is a compact subset of $\mathbb{B}$. Since $f_j \to 0$ uniformly on compact subsets of $\mathbb{B}$ as $j \to \infty$ and $h \in F(p,q,s)$, we get

$$\begin{aligned}I_1 &\le \sup_{w\in K}\left|f_j(w)\right|^p\sup_{a\in\mathbb{B}}\int_{\{|\varphi(z)|\le r\}}|\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)d\nu(z)\\ &\le \|h\|^p_{F(p,q,s)}\sup_{w\in K}\left|f_j(w)\right|^p \le C\sup_{w\in K}\left|f_j(w)\right|^p \longrightarrow 0,\quad j\longrightarrow\infty.\end{aligned} \tag{3.8}$$

On the other hand, by (3.5) and Lemmas 2.2 and 2.4, it follows that

$$\begin{aligned}I_2 &\le C\left\|f_j\right\|^p_{\mathcal{B}^{1-\alpha}}\sup_{a\in\mathbb{B}}\int_{\{|\varphi(z)|> r\}}|\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a)d\nu(z)\\ &\le C\|f_j\|^p_{\mathcal{L}_\alpha}\varepsilon \le CL^p\varepsilon.\end{aligned} \tag{3.9}$$

Since $\varepsilon$ is arbitrary, from the above inequalities, we get

$$\lim_{j\to\infty}\left\|T_hC_\varphi f_j\right\|_{F(p,q,s)} = 0. \tag{3.10}$$

Hence, by (3.10) and Lemma 2.3, we conclude that $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is compact.

For the converse direction, we suppose that $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is compact. It is obvious that $T_hC_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is bounded.

Now, we prove (3.4). Setting the test functions $f_l^{(m)}(z) = z_l^m$ for fixed $l \in \{1,\dots,n\}$, where $z = (z_1,\dots,z_n)$ and $m = 1,2,\dots$. It is easy to check that $\|f_l^{(m)}\|_{\mathcal{L}_\alpha} \le C$, and $f_l^{(m)} \to 0$

uniformly on the compact subsets of $\mathbb{B}$ as $m \to \infty$. Write $\varphi = (\varphi_1, \ldots, \varphi_n)$, since $T_h C_\varphi : \mathcal{L}_\alpha \to F(p,q,s)$ is compact, by Lemma 2.3, it follows that, as $m \to \infty$,

$$\left\| T_h C_\varphi f_l^{(m)} \right\|_{F(p,q,s)}^p = \sup_{a\in\mathbb{B}} \int_{\mathbb{B}} \left|\varphi_l(z)\right|^{mp} |\Re h(z)|^p \left(1-|z|^2\right)^q g^s(z,a)\, dv(z) \longrightarrow 0. \tag{3.11}$$

Note that $|\varphi(z)|^2 = |\varphi_1(z)|^2 + \cdots + |\varphi_n(z)|^2 \leq (|\varphi_1(z)| + \cdots + |\varphi_n(z)|)^2$; by the relation (3.11) and Lemma 2.5, we have

$$\begin{aligned} &\sup_{a\in\mathbb{B}} \int_{\mathbb{B}} \left|\varphi(z)\right|^{mp} |\Re h(z)|^p \left(1-|z|^2\right)^q g^s(z,a)\, dv(z) \\ &\quad\leq \sup_{a\in\mathbb{B}} \int_{\mathbb{B}} \left(\sum_{l=1}^n \left|\varphi_l(z)\right|\right)^{mp} |\Re h(z)|^p \left(1-|z|^2\right)^q g^s(z,a)\, dv(z) \\ &\quad\leq C \sup_{a\in\mathbb{B}} \int_{\mathbb{B}} \left(\sum_{l=1}^n \left|\varphi_l(z)\right|^{mp}\right) |\Re h(z)|^p \left(1-|z|^2\right)^q g^s(z,a)\, dv(z) \longrightarrow 0, \quad m \longrightarrow \infty. \end{aligned} \tag{3.12}$$

This means that, for every $\varepsilon > 0$, there is $m_0 \in N$ such that, for every $r \in (0,1)$,

$$\begin{aligned} &r^{m_0 p} \sup_{a\in\mathbb{B}} \int_{\{|\varphi(z)|>r\}} |\Re h(z)|^p \left(1-|z|^2\right)^q g^s(z,a)\, dv(z) \\ &\quad= \sup_{a\in\mathbb{B}} \int_{\{|\varphi(z)|>r\}} r^{m_0 p} |\Re h(z)|^p \left(1-|z|^2\right)^q g^s(z,a)\, dv(z) \\ &\quad\leq \sup_{a\in\mathbb{B}} \int_{\{|\varphi(z)|>r\}} \left|\varphi(z)\right|^{m_0 p} |\Re h(z)|^p \left(1-|z|^2\right)^q g^s(z,a)\, dv(z) \\ &\quad\leq \sup_{a\in\mathbb{B}} \int_{\mathbb{B}} \left|\varphi(z)\right|^{m_0 p} |\Re h(z)|^p \left(1-|z|^2\right)^q g^s(z,a)\, dv(z) \\ &\quad< \varepsilon. \end{aligned} \tag{3.13}$$

Thus, when $r > 2^{-(1/m_0 p)}$, by the above inequality, we obtain

$$\sup_{a\in\mathbb{B}} \int_{\{|\varphi(z)|>r\}} |\Re h(z)|^p \left(1-|z|^2\right)^q g^s(z,a)\, dv(z) < 2\varepsilon. \tag{3.14}$$

From which, the desired result (3.4) holds. This completes the proof of this theorem. □

*Remark 3.3.* When $\varphi(z) = z$, the product of extended Cesàro operator $T_h C_\varphi$ is the generalized extended Cesàro operator $T_h$; thus, by Theorems 3.1 and 3.2, we have the following two corollaries.

**Corollary 3.4.** *Assume that $\alpha \in (0,1)$, $0 < p,\ s < \infty$, $-n-1 < q < \infty$, $q+s > -1$, and $h \in H(\mathbb{B})$. Then, $T_h : \mathcal{L}_\alpha \to F(p,q,s)$ is bounded if and only if $h \in F(p,q,s)$.*

**Corollary 3.5.** *Assume that $\alpha \in (0,1)$, $0 < p$, $s < \infty$, $-n-1 < q < \infty$, $q+s > -1$, and $h \in H(\mathbb{B})$. Then, $T_h : \mathcal{L}_\alpha \to F(p,q,s)$ is compact if and only if $T_h : \mathcal{L}_\alpha \to F(p,q,s)$ is bounded, and*

$$\lim_{r\to 1}\sup_{a\in\mathbb{B}}\int_{|z|>r}|\Re h(z)|^p\left(1-|z|^2\right)^q g^s(z,a) = 0. \tag{3.15}$$

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