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Research Article

On Value Distribution of Difference Polynomials of Meromorphic Functions

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We study the value distribution of the difference counterpart $\Delta f(z) - af(z)^n$ of $f'(z) - af(z)^n$ and obtain an almost direct difference analogue of results of Hayman.

1. Introduction and Results

Hayman proved the following Theorem A.

Theorem A (see [1]). *If $f(z)$ is a transcendental entire function, $n \geq 3$ is an integer, and $a(\neq 0)$ is a constant, then $f'(z) - af(z)^n$ assumes all finite values infinitely often.*

Recently, many papers (see [2–7]) have focused on complex differences, giving many difference analogues in value distribution theory of meromorphic functions.

It is well known that $\Delta f(z) = f(z+c) - f(z)$ (where $c \in \mathbb{C} \setminus \{0\}$ is a constant satisfying $f(z+c) - f(z) \not\equiv 0$) is regarded as the difference counterpart of $f'(z)$, so that $\Delta f(z) - af(z)^n$ is regarded as the difference counterpart of $f'(z) - af(z)^n$, where $a \in \mathbb{C} \setminus \{0\}$ is a constant.

Liu and Laine [7] obtain the following

Theorem B. *Let f be a transcendental entire function of finite order ρ, not of period c, where c is a nonzero complex constant, and let $s(z)$ be a nonzero function, small compared to f. Then the difference polynomial $f(z)^n + f(z+c) - f(z) - s(z)$ has infinitely many zeros in the complex plane, provided that $n \geq 3$.*

We use the basic notions of Nevanlinna's theory (see [8, 9]) and in addition use $\sigma(f)$ to denote the order of growth of the meromorphic $f(z)$ and $\lambda(f)$ to denote the exponent of convergence of the zeros of $f(z)$.

In this paper, we consider the difference counterpart of Theorem A. When $n \geq 3$ is an integer, we prove the following Theorem 1.1. Compared with Theorem B, Theorem 1.1 is an almost direct difference analogue of of Theorem A and gives an estimate of numbers of b-points, namely, $\lambda(\Psi_n(z) - b) = \sigma(f)$ for every $b \in \mathbb{C}$. Our method of the proof is also different from the method of the proof in Theorem B.

Theorem 1.1. *Let $f(z)$ be a transcendental entire function of finite order, and let $a, c \in \mathbb{C} \setminus \{0\}$ be constants, with c such that $f(z+c) \not\equiv f(z)$. Set $\Psi_n(z) = \Delta f(z) - af(z)^n$, where $\Delta f(z) = f(z+c) - f(z)$ and $n \geq 3$ is an integer. Then $\Psi_n(z)$ assumes all finite values infinitely often, and for every $b \in \mathbb{C}$ one has $\lambda(\Psi_n(z) - b) = \sigma(f)$.*

Example 1.2. For $f(z) = \exp\{\exp\{z\}\}$, $c = \log 3$ and $a = 1$, we have

$$\Psi_3(z) = \Delta f(z) - af(z)^3 = -\exp\{\exp\{z\}\}. \tag{1.1}$$

Here $\Psi_3(z) \neq 0$, which shows that Theorem 1.1 may fail for entire functions of infinite order.

Example 1.3. For $f(z) = \exp\{z\} + 1, c = \log 3, a = 1$, we have $\Psi_2(z) = \Delta f(z) - af(z)^2 = -\exp\{2z\} - 1$. Here $\Psi_2(z) \neq -1$, which shows that Theorem 1.1 may fail for $n = 2$ and that the condition $n \geq 3$ in Theorem 1.1 is sharp.

Example 1.4. For $f(z) = \exp\{z\}, c = \log 3, a = 1$, we have $\Psi_2(z) = \Delta f(z) - af(z)^2 = \exp\{z\}(2 - \exp\{z\})$, which assumes all finite values infinitely often.

What can we say about $\Psi_2(z)$ when $n = 2$? We consider this question and obtain the following Theorems 1.5 and 1.6.

Theorem 1.5. *Let $f(z)$ be a transcendental entire function of finite order with a Borel exceptional value 0, and let $a, c \in \mathbb{C} \setminus \{0\}$ be constants, with c such that $f(z+c) \not\equiv f(z)$. Then $\Psi_2(z)$ assumes all finite values infinitely often, and for every $b \in \mathbb{C}$ one has $\lambda(\Psi_2(z) - b) = \sigma(f)$.*

Theorem 1.6. *Let $f(z)$ be a transcendental entire function of finite order with a finite nonzero Borel exceptional value d, and let $a, c \in \mathbb{C} \setminus \{0\}$ be constants, with c such that $f(z+c) \not\equiv f(z)$. Then for every $b \in \mathbb{C}$ with $b \neq -ad^2$, $\Psi_2(z)$ assumes the value b infinitely often, and $\lambda(\Psi_2(z) - b) = \sigma(f)$.*

Remark 1.7. From Theorems 1.5 and 1.6, we see that if $f(z)$ has the Borel exceptional value 0, then $\Psi_2(z)$ has not any finite Borel exceptional value, but if $f(z)$ has a nonzero Borel exceptional value, then $\Psi_2(z)$ may have a finite Borel exceptional value. From Theorem 1.6, this possible Borel exceptional value is $-ad^2$. Example 1.3 shows that this Borel exceptional value $-ad^2 (= -1)$ may arise, and thus the conclusion of Theorem 1.6 is sharp.

2. Proof of Theorem 1.1

We need the following lemmas.

Lemma 2.1 (see [3, 4]). *Let $f(z)$ be a meromorphic function of finite order, and let $c \in \mathbb{C} \setminus \{0\}$. Then*

$$m\left(r, \frac{f(z+c)}{f(z)}\right) = S(r, f), \tag{2.1}$$

where $S(r, f) = o\{T(r, f)\}$.

Lemma 2.2 (see [3]). *Let $f(z)$ be a meromorphic function with order $\sigma(f) = \sigma < \infty$, and let c be a nonzero constant. Then, for each $\varepsilon > 0$, one has*

$$T(r, f(z+c)) = T(r, f) + O\left(r^{\sigma-1+\varepsilon}\right) + O(\log r). \tag{2.2}$$

Lemma 2.3. *Suppose that n, a, c, $f(z)$, $\Psi_n(z)$ satisfy the conditions of Theorem 1.1. If $b \in \mathbb{C}$, then $\Psi_n(z) - b$ is transcendental.*

Proof. Suppose that $\Psi_n(z) - b = p(z)$, where $p(z)$ is a polynomial. Then

$$-af(z)^n = b - \Delta f(z) + p(z). \tag{2.3}$$

By Lemma 2.2, for each $\varepsilon > 0$, we have

$$T(r, \Delta f(z)) \leq 2T(r, f) + S(r, f) + O\left(r^{\sigma-1+\varepsilon}\right), \tag{2.4}$$

where $\sigma = \sigma(f)$. By an identity due to Valiron-Mohon'ko (see [10, 11]), we have

$$\begin{aligned} T(r, af(z)^n) &= nT(r, f) + S(r, f) \\ &= T(r, b - \Delta f(z) + p(z)) \\ &\leq 2T(r, f) + S(r, f) + O\left(r^{\sigma-1+\varepsilon}\right). \end{aligned} \tag{2.5}$$

This contradicts the fact that $n \geq 3$. Hence $\Psi_n(z) - b$ is transcendental. □

Lemma 2.4. *Suppose that n, a, c, $f(z)$, $\Psi_n(z)$ satisfy the conditions of Theorem 1.1. Suppose also that $b \in \mathbb{C}$, $q(z) \not\equiv 0$ is a polynomial, and $p(z) \not\equiv 0$ is an entire function with $\sigma(p) < \sigma(f)$. If*

$$\Psi_n(z) - b = p(z) \exp\{q(z)\}, \tag{2.6}$$

then

$$P(z, f) = np(z)f'(z) - \left(p'(z) + q'(z)p(z)\right)f(z) \not\equiv 0. \tag{2.7}$$

Proof. Suppose that

$$np(z)f'(z) - \left(p'(z) + q'(z)p(z)\right)f(z) \equiv 0. \tag{2.8}$$

Integrating (2.8) results in

$$f(z)^n = dp(z) \exp\{q(z)\}, \tag{2.9}$$

where $d(\neq 0)$ is a constant. Therefore, by (2.6), (2.9), and the definition of $\Psi_n(z)$, we obtain

$$\Psi_n(z) - b = f(z+c) - f(z) - af(z)^n - b = \frac{1}{d} f(z)^n, \tag{2.10}$$

and so

$$d\big(f(z+c) - f(z)\big) = (ad+1)f(z)^n + bd. \tag{2.11}$$

We must have $ad + 1 \neq 0$. In fact, if $ad + 1 = 0$, then by (2.11) and $d \neq 0$, we have that

$$f(z+c) - f(z) = b, \tag{2.12}$$

so $f'(z)$ is periodic. Then, write (2.8) as

$$f'(z) = R(z)f(z), \tag{2.13}$$

where

$$R(z) = \frac{p'(z) + q'(z)p(z)}{np(z)}. \tag{2.14}$$

Clearly, $R(z) \not\equiv 0$ and $\sigma(R) \leq \sigma(p) < \sigma(f)$. We obtain from (2.12) and (2.13)

$$(R(z+c) - R(z))f(z) = -bR(z+c). \tag{2.15}$$

If $R(z+c)-R(z) \equiv 0$, then $b = 0$ by (2.15) and $R(z) \not\equiv 0$. Thus, by (2.12), we have $f(z+c) \equiv f(z)$, which contradicts our condition. If $R(z+c) - R(z) \not\equiv 0$, then by (2.15), we have

$$\sigma(f) = \sigma\left(\frac{-bR(z+c)}{R(z+c) - R(z)}\right) \leq \sigma(R) < \sigma(f). \tag{2.16}$$

This is also a contradiction. Hence $ad + 1 \neq 0$.

Differentiating (2.11), and then dividing by $f'(z)$ result in

$$d\left(\frac{f'(z+c)}{f'(z)} - 1\right) = n(ad+1)f(z)^{n-1}. \tag{2.17}$$

Therefore, by Lemma 2.1, we get that

$$(n-1)T(r,f) = (n-1)m(r,f) = S(r,f') = S(r,f), \tag{2.18}$$

a contradiction for $n \geq 2$. Hence $P(z,f) \not\equiv 0$. □

Halburd and Korhonen obtained the following difference analogue of the Clunie lemma [4, Corollary 3.3].

Lemma 2.5. *Let $f(z)$ be a nonconstant, finite order meromorphic solution of*

$$f^n P_1(z,f) = Q_1(z,f), \tag{2.19}$$

where $P_1(z,f)$, $Q_1(z,f)$ are difference polynomials in $f(z)$ with small meromorphic coefficients, and let $\delta < 1$. If the degree of $Q_1(r,f)$ as a polynomial in $f(z)$ and its shifts is at most n, then

$$m(r, P_1(z,f)) = o\left(\frac{T(r+|c|,f)}{r^\delta}\right) + o(T(r,f)) \tag{2.20}$$

for all r outside an exceptional set of finite logarithmic measure.

Remark 2.6. If coefficients of P_1, Q_1 are $a_j(z)$ $(j = 1,\ldots,s)$ satisfying $\sigma(a_j) < \sigma(f)$, then using the same method as in the proof of Lemma 2.5 (see [4]), we have

$$m(r, P_1(z,f)) = o\left(\frac{T(r+|c|,f)}{r^\delta}\right) + o(T(r,f)) + O\left(\sum_{j=1}^{s} m(r,a_j)\right) \tag{2.21}$$

for all r outside an exceptional set of finite logarithmic measure.

We are now able to prove Theorem 1.1. We only prove the case $\sigma(f) > 0$. For the case $\sigma(f) = 0$, we can use the same method in the proof. Suppose that $b \in \mathbb{C}$ and $\lambda(\Psi_n(z) - b) < \sigma(f)$. Then, by Lemma 2.3, we see that $\Psi_n(z) - b$ is transcendental. Thus, $\Psi_n(z) - b$ can be written as

$$\Psi_n(z) - b = p(z)\exp\{q(z)\}, \tag{2.22}$$

where $q(z) \not\equiv 0$ is a polynomial, $p(z) \not\equiv 0$ is an entire function with $\sigma(p) < \sigma(f)$.

Differentiating (2.22) and eliminating $\exp\{q(z)\}$, we obtain

$$f(z)^{n-1} P(z,f) = Q(z,f), \tag{2.23}$$

where

$$P(z,f) = anp(z)f'(z) - a(p'(z) + q'(z)p(z))f(z)$$
$$Q(z,f) = p(z)f'(z) - p(z)f'(z+c) + \Delta f(z)(p'(z) + q'(z)p(z)) - b(p'(z) + q'(z)p(z)). \tag{2.24}$$

By Lemma 2.4, we see that $P(z,f)\not\equiv 0$. Since $n\geq 3$ and the total degree of $Q(z,f)$ as a polynomial in $f(z)$ and its shifts, $\deg_f Q(z,f)=1$, by (2.23), Lemma 2.5, and Remark 2.6, we obtain that for $\delta<1$

$$T(r,P(z,f))=m(r,P(z,f))=o\left(\frac{T(r+|c|,f)}{r^{\delta}}\right)+o(T(r,f))+O(m(r,p)),$$
$$T(r,fP(z,f))=m(r,fP(z,f))=o\left(\frac{T(r+|c|,f)}{r^{\delta}}\right)+o(T(r,f))+O(m(r,p)) \tag{2.25}$$

for all r outside of an exceptional set of finite logarithmic measure.

Thus, (2.25) give that

$$T(r,f)=o\left(\frac{T(r+|c|,\ f)}{r^{\delta}}\right)+o(T(r,f))+O(m(r,p)) \tag{2.26}$$

for all r outside of an exceptional set of finite logarithmic measure. This is a contradiction. Hence $\Psi_n(z)-b$ has infinitely many zeros and $\lambda(\Psi_n(z)-b)=\sigma(f)$, which proves Theorem 1.1.

3. Proof of Theorem 1.5

We need the following lemma.

Lemma 3.1 (see [12, page 69–70], [13, page 79–80], or [14]). *Suppose that $n\geq 2$, and let $f_j(z)$, $j=1,\ldots,n$, be meromorphic functions and $g_j(z)$, $j=1,\ldots,n$, entire functions such that*

(i) $\sum_{j=1}^{n} f_j(z)\exp\{g_j(z)\}\equiv 0$;

(ii) *when* $1\leq j<k\leq n$, $g_j(z)-g_k(z)$ *is not constant;*

(iii) *when*$1\leq j\leq n$, $1\leq h<k\leq n$,

$$T(r,f_j)=o\{T(r,\exp\{g_h-g_k\})\}\quad (r\longrightarrow\infty, r\notin E), \tag{3.1}$$

where $E\subset(1,\infty)$ is of finite linear measure or finite logarithmic measure. Then $f_j(z)\equiv 0$, $j=1,\ldots,n$.

To prove Theorem 1.5, note first that $f(z)$ has a Borel exceptional value 0, we can write $f(z)$ as

$$f(z)=g(z)\exp\left\{\alpha z^k\right\},\qquad f(z+c)=g(z+c)g_1(z)\exp\left\{\alpha z^k\right\}, \tag{3.2}$$

where $\alpha(\neq 0)$ is a constant, $k(\geq 1)$ is an integer satisfying $\sigma(f)=k$, and $g(z), g_1(z)$ are entire functions such that $g(z)g_1(z)\not\equiv 0$, $\sigma(g)<k$, $\sigma(g_1)=k-1$.

First, we prove $\Psi_2(z) - b = \Delta f(z) - af(z)^2 - b$ is transcendental. If $\Psi_2(z) - b = p(z)$, where $p(z)$ is a polynomial, then

$$af(z)^2 = \Delta f(z) - b + p(z). \tag{3.3}$$

Thus by Lemma 2.1 and an identity due to Valiron-Mohon'ko (see [10, 11]), we have

$$\begin{aligned} T\left(r, af^2\right) &= 2T(r,f) + S(r,f) \\ T(r, \Delta f(z) - b + p(z)) &= m(r, \Delta f(z) - b + p(z)) \\ &\leq m(r,f) + m\left(r, \frac{f(z+c)}{f(z)} - 1\right) + O(\log r) \\ &\leq T(r,f) + S(r,f). \end{aligned} \tag{3.4}$$

By (3.4), we see that (3.3) is a contradiction.

Secondly, we prove $\sigma(\Psi_2 - b) = \sigma(f) = k \geq 1$. By the expression of $\Psi_2(z)$, we have $\sigma(\Psi_2 - b) \leq k$. Set $G(z) = \Psi_2(z) - b$. If $\sigma(G) = k_1 < k$, then by (3.2), we have

$$(g(z+c)g_1(z) - g(z)) \exp\left\{\alpha z^k\right\} - ag(z)^2 \exp\left\{2\alpha z^k\right\} - (b + G(z)) = 0. \tag{3.5}$$

Since $\sigma(G) = k_1 < k$ and $\sigma(g) < k$, we see that the left hand side of (3.5) is of order $= k$ by applying the general form of the Valiron-Mohon'ko lemma in [10], a contradiction. So, $\sigma(G) = k$.

Thirdly, we prove $\lambda(G) = k(\geq 1)$. If $\lambda(G) < k$, then $G(z)$ can be written as

$$G(z) = g^*(z) \exp\left\{\beta z^k\right\}, \tag{3.6}$$

where $\beta(\neq 0)$ is a constant, $g^*(z)(\not\equiv 0)$ is an entire function satisfying $\sigma(g^*) < k$. Thus by (3.2), (3.6), and $G(z) = \Psi_2(z) - b$, we have

$$(g(z+c)g_1(z) - g(z)) \exp\left\{\alpha z^k\right\} - ag(z)^2 \exp\left\{2\alpha z^k\right\} - b - g^* \exp\left\{\beta z^k\right\} = 0. \tag{3.7}$$

In (3.7), there are three cases for β:

(i) $\beta \neq \alpha$ and $\beta \neq 2\alpha$;

(ii) $\beta = \alpha$;

(iii) $\beta = 2\alpha$.

Applying Lemma 3.1 to (3.7), we have in case (i)

$$g(z+c)g_1(z) - g(z) \equiv ag(z)^2 \equiv g^*(z) \equiv 0; \tag{3.8}$$

in case (ii), we have

$$ag(z)^2 \equiv 0; \tag{3.9}$$

in case (iii), we have

$$g(z+c)g_1(z) - g(z) \equiv 0. \tag{3.10}$$

Since $f(z+c) - f(z) \not\equiv 0$ and $f(z)$ is transcendental, we see that any one of (3.8)–(3.10) is a contradiction. Hence $\lambda(\Psi_2 - b) = \sigma(f)$.

4. Proof of Theorem 1.6

Since $f(z)$ has a nonzero Borel exceptional value d, we can write $f(z)$ as

$$f(z) = d + \varphi(z)\exp\left\{\alpha z^k\right\}, \qquad f(z+c) = d + \varphi(z+c)\varphi_1(z)\exp\left\{\alpha z^k\right\}, \tag{4.1}$$

where $\alpha(\neq 0)$ is a constant, $k(\geq 1)$ is an integer satisfying $\sigma(f) = k$, and $\varphi(z)$, $\varphi_1(z)$ are entire functions such that $\varphi(z)\varphi_1(z) \not\equiv 0$, $\sigma(\varphi) < k$, $\sigma(\varphi_1) = k-1$.

Using the same method as in the proof of Theorem 1.5, we can show that

$$\Psi_2(z) - b = \Delta f(z) - af(z)^2 - b \tag{4.2}$$

is transcendental and $\sigma(\Psi_2 - b) = \sigma(f) = k \geq 1$.

Now we show that $\lambda(\Psi_2(z) - b) = k(\geq 1)$. Set $G(z) = \Psi_2(z) - b$. If $\lambda(G) < k$, then G can be written as

$$G(z) = \varphi^*(z)\exp\left\{sz^k\right\}, \tag{4.3}$$

where $s(\neq 0)$ is a constant and $\varphi^*(z)(\not\equiv 0)$ is an entire function satisfying $\sigma(\varphi^*) < k$. Thus by (4.1) and (4.3), we have

$$\begin{aligned}&\left(\varphi(z+c)\varphi_1(z) - \varphi(z) - 2ad\varphi(z)\right)\exp\left\{\alpha z^k\right\} - a\varphi(z)^2\exp\left\{2\alpha z^k\right\} - \varphi^*(z)\exp\left\{sz^k\right\}\\ &\quad - \left(ad^2 + b\right) = 0.\end{aligned} \tag{4.4}$$

In (4.4), there are three cases for s:

(i) $s \neq \alpha$ and $s \neq 2\alpha$;

(ii) $s = \alpha$;

(iii) $s = 2\alpha$.

Applying the same method as in the proof of Theorem 1.5 to these three cases, we obtain

$$ad^2 + b = 0, \tag{4.5}$$

which contradicts our supposition that $b \neq -ad^2$. Hence $\lambda(\Psi_2 - b) = \sigma(f)$.

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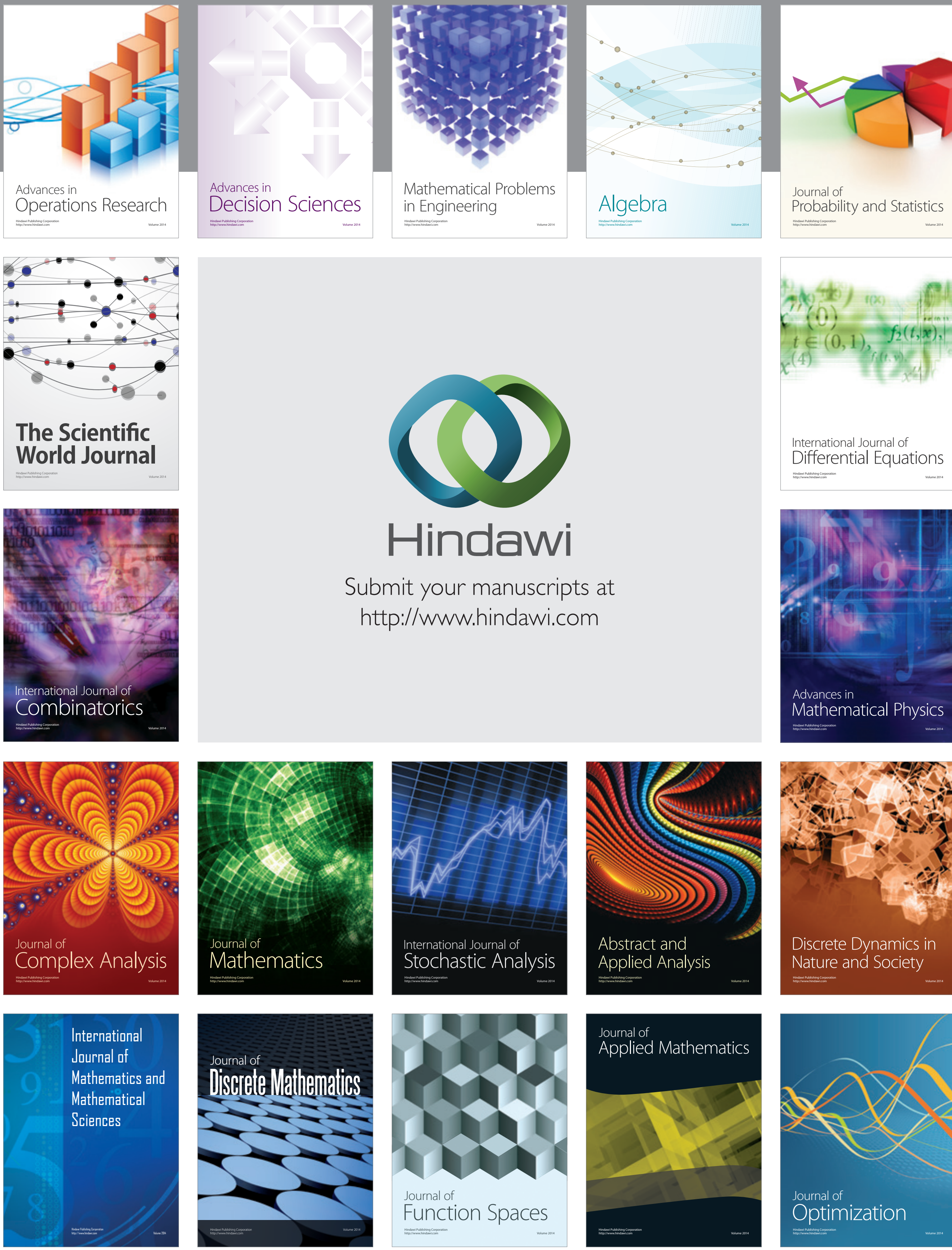
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