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Research Article

The Equivalence of Convergence Results between Mann and Multistep Iterations with Errors for Uniformly Continuous Generalized Weak Φ-Pseudocontractive Mappings in Banach Spaces

Guiwen Lv[1] and Haiyun Zhou[2]

[1] *Department of Mathematics and Physics, Shijiazhuang Tiedao University, Shijiazhuang 050043, China*
[2] *Department of Mathematics, Shijiazhuang Mechanical Engineering College, Shijiazhuang 050003, China*

Correspondence should be addressed to Guiwen Lv, lvguiwenyy@126.com

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We prove the equivalence of the convergence of the Mann and multistep iterations with errors for uniformly continuous generalized weak Φ-pseudocontractive mappings in Banach spaces. We also obtain the convergence results of Mann and multistep iterations with errors. Our results extend and improve the corresponding results.

1. Introduction

Let E be a real Banach space, E^* be its dual space, and $J : E \to 2^{E^*}$ be the normalized duality mapping defined by

$$J(x) = \left\{ f \in E^* : \langle x, f \rangle = \|x\| \cdot \|f\| = \|f\|^2 \right\}, \tag{1.1}$$

where $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. The single-valued normalized duality mapping is denoted by j.

Definition 1.1. A mapping $T : E \to E$ is said to be

(1) strongly accretive if for all $x, y \in E$, there exist a constant $k \in (0,1)$ and $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y) \rangle \ge k\|x-y\|^2; \tag{1.2}$$

(2) ϕ-strongly accretive if there exist $j(x-y) \in J(x-y)$ and a strictly increasing function $\phi : [0,+\infty) \to [0,+\infty)$ with $\phi(0) = 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \geq \phi(\|x-y\|)\|x-y\|, \quad \forall x, y \in E; \tag{1.3}$$

(3) generalized Φ-accretive if, for any $x, y \in E$, there exist $j(x-y) \in J(x-y)$ and a strictly increasing function $\Phi : [0,+\infty) \to [0,+\infty)$ with $\Phi(0) = 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \geq \Phi(\|x-y\|). \tag{1.4}$$

Remark 1.2. Let $N(T) = \{x \in E : Tx = 0\} \neq \emptyset$. If $x, y \in E$ in the formulas of Definition 1.1 is replaced by $x \in E$, $q \in N(T)$, then T is called strongly quasi-accretive, ϕ-strongly quasi-accretive, generalized Φ-quasi-accretive mapping, respectively.

Closely related to the class of accretive-type mappings are those of pseudocontractive type mappings.

Definition 1.3. A mapping T with domain $D(T)$ and range $R(T)$ is said to be

(1) strongly pseudocontractive if there exist a constant $k \in (0,1)$ and $j(x-y) \in J(x-y)$ such that for each $x, y \in D(T)$,

$$\langle Tx - Ty, j(x-y)\rangle \leq k\|x-y\|^2; \tag{1.5}$$

(2) ϕ-strongly pseudocontractive if there exist $j(x-y) \in J(x-y)$ and a strictly increasing function $\phi : [0,+\infty) \to [0,+\infty)$ with $\phi(0) = 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \phi(\|x-y\|)\|x-y\|, \quad \forall x, y \in D(T); \tag{1.6}$$

(3) generalized Φ-pseudocontractive if, for any $x, y \in D(T)$, there exist $j(x-y) \in J(x-y)$ and a strictly increasing function $\Phi : [0,+\infty) \to [0,+\infty)$ with $\Phi(0) = 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \Phi(\|x-y\|). \tag{1.7}$$

Definition 1.4. Let $F(T) = \{x \in E : Tx = x\} \neq \emptyset$. The mapping T is called Φ-strongly pseudocontractive, generalized Φ-pseudocontractive, if, for all $x \in D(T)$, $q \in F(T)$, the formula (2), (3) in the above Definition 1.3 hold.

Definition 1.5. A mapping T is said to be

(1) generalized weak Φ-accretive if, for all $x, y \in E$, there exist $j(x-y) \in J(x-y)$ and a strictly increasing function $\Phi : [0,+\infty) \to [0,+\infty)$ with $\Phi(0) = 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \geq \frac{\Phi(\|x-y\|)}{1 + \|x-y\|^2 + \Phi(\|x-y\|)}; \tag{1.8}$$

(2) generalized weak Φ-quasi-accretive if, for all $x \in E$, $q \in N(T)$, there exist $j(x-q) \in J(x-q)$ and a strictly increasing function $\Phi : [0,+\infty) \to [0,+\infty)$ with $\Phi(0) = 0$ such that

$$\langle Tx - q, j(x-q)\rangle \geq \frac{\Phi(\|x-q\|)}{1 + \|x-q\|^2 + \Phi(\|x-q\|)}; \tag{1.9}$$

(3) generalized weak Φ-pseudocontractive if, for any $x, y \in D(T)$, there exist $j(x-y) \in J(x-y)$ and a strictly increasing function $\Phi : [0,+\infty) \to [0,+\infty)$ with $\Phi(0) = 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \frac{\Phi(\|x-y\|)}{1 + \|x-y\|^2 + \Phi(\|x-y\|)}; \tag{1.10}$$

(4) generalized weak Φ-hemicontractive if, for any $x \in K$, $q \in F(T)$, there exist $j(x-q) \in J(x-q)$ and a strictly increasing function $\Phi : [0,+\infty) \to [0,+\infty)$ with $\Phi(0) = 0$ such that

$$\langle Tx - q, j(x-q)\rangle \leq \|x-q\|^2 - \frac{\Phi(\|x-q\|)}{1 + \|x-q\|^2 + \Phi(\|x-q\|)}. \tag{1.11}$$

It is very well known that a mapping T is strongly pseudocontractive (hemicontractive), ϕ-strongly pseudocontractive (ϕ-strongly hemicontractive), generalized Φ-pseudocontractive (generalized Φ-hemicontractive), generalized weak Φ-pseudocontractive (generalized weak Φ-hemicontractive) if and only if $(I-T)$ are strongly accretive (quasi-accretive), ϕ-strongly accretive (ϕ-strongly quasi-accretive), $(I-T)$ is generalized Φ-accretive (generalized Φ-quasi-accretive), generalized weak Φ-accretive (weak Φ-quasi-accretive), respectively.

It is shown in [1] that the class of strongly pseudocontractive mappings is a proper subclass of ϕ-strongly pseudocontractive mappings. Furthermore, an example in [2] shows that the class of ϕ-strongly hemicontractive mappings with the nonempty fixed point set is a proper subclass of generalized Φ-hemicontractive mappings. Obviously, generalized Φ-hemicontractive mapping must be generalized weak Φ-hemicontractive, but, on the contrary, it is not true. We have the following example.

Example 1.6. Let $E = (-\infty, +\infty)$ be real number space with usual norm and $K = [0, +\infty)$. $T : K \to E$ defined by

$$Tx = \frac{x + x^3 + x^2\sqrt{x} - \sqrt{x}}{1 + x\sqrt{x} + x^2}, \quad \forall x \in K. \tag{1.12}$$

Then T has a fixed point $0 \in F(T)$. $\Phi : [0,+\infty) \to [0,+\infty)$ defined by $\Phi(t) = t^{3/2}$ is a strictly increasing function with $\Phi(0) = 0$. For all $x \in K$ and $0 \in F(T)$, we have

$$\begin{aligned}
\langle Tx - T0, j(x-0)\rangle &= \left\langle \frac{x + x^3 + x^2\sqrt{x} - \sqrt{x}}{1 + x\sqrt{x} + x^2} - 0, x - 0 \right\rangle \\
&= \frac{x^2 + x^4 + x^3\sqrt{x} - x\sqrt{x}}{1 + x\sqrt{x} + x^2} = x^2 - \frac{x^{3/2}}{1 + x^{3/2} + x^2} \\
&= |x-0|^2 - \frac{\Phi(x)}{1 + \Phi(x) + x^2} = |x-0|^2 - \sigma(x) \\
&\geq |x-0|^2 - \Phi(x).
\end{aligned} \tag{1.13}$$

Then T is a generalized weak Φ-hemicontractive map, but it is not a generalized Φ-hemicontractive map; that is, the class of generalized weak Φ-hemicontractive maps properly contains the class of generalized Φ-hemicontractive maps. Hence the class of generalized weak Φ-hemicontractive mappings is the most general among those defined above.

Definition 1.7. The mapping $T : E \to E$ is called Lipschitz, if there exists a constant $L > 0$ such that

$$\|Tx - Ty\| \leq L\|x - y\|, \quad \forall x, y \in E. \tag{1.14}$$

It is clear that if T is Lipschitz, then it must be uniformly continuous. Otherwise, it is not true. For example, the function $f(x) = \sqrt{x}, x \in [0,+\infty)$ is uniformly continuous but it is not Lipschitz.

Now let us consider the multi-step iteration with errors. Let K be a nonempty convex subset of E, and let $\{T_i\}_{i=1}^{M}$ be a finite family of self-maps of K. For $x_0 \in K$, the sequence $\{x_n\}$ is generated as follows:

$$\begin{aligned}
x_{n+1} &= (1 - \alpha_n - \delta_n)x_n + \alpha_n T_n y_n^1 + \delta_n \nu_n, \\
y_n^i &= \left(1 - \beta_n^i - \eta_n^i\right)x_n + \beta_n^i T_n y_n^{i+1} + \eta_n^i \omega_n^i, \quad i = 1, \ldots, p-2, \\
y_n^{p-1} &= \left(1 - \beta_n^{p-1} - \eta_n^{p-1}\right)x_n + \beta_n^{p-1} T_n x_n + \eta_n^{p-1}\omega_n^{p-1}, \quad p \geq 2,
\end{aligned} \tag{1.15}$$

where $\{\nu_n\}, \{\omega_n^i\}$ are any bounded sequences in K and $\{\alpha_n\}$, $\{\delta_n\}$, $\{\beta_n^i\}$, $\{\eta_n^i\}$, $(i = 1, 2, \ldots, p-1)$ are sequences in $[0,1]$ satisfying certain conditions.

If $p = 2$, (1.15) becomes the Ishikawa iteration sequence with errors $\{x_n\}_{n=0}^{\infty}$ defined iteratively by

$$\begin{aligned}
x_{n+1} &= (1 - \alpha_n - \delta_n)x_n + \alpha_n T_n y_n^1 + \delta_n \nu_n, \\
y_n^1 &= (1 - \beta_n - \eta_n)x_n + \beta_n T_n x_n + \eta_n \omega_n, \quad \forall n \geq 0.
\end{aligned} \tag{1.16}$$

If $\beta_n = \eta_n = 0$, for all $n \geq 0$, then from (1.16), we get the Mann iteration sequence with errors $\{u_n\}_{n=0}^{\infty}$ defined by

$$u_{n+1} = (1 - \alpha_n - \delta_n)u_n + \alpha_n T_n u_n + \delta_n \mu_n, \quad \forall n \geq 0, \tag{1.17}$$

where $\{\mu_n\} \subset K$ is bounded.

Recently, many authors have researched the iteration approximation of fixed points by Lipschitz pseudocontractive (accretive) type nonlinear mappings and have obtained some excellent results [3–12]. In this paper we prove the equivalence between the Mann and multistep iterations with errors for uniformly continuous generalized weak Φ-pseudocontractive mappings in Banach spaces. Our results extend and improve the corresponding results [3–12].

Lemma 1.8 (see [13]). *Let E be a real normed space. Then, for all $x, y \in E$, the following inequality holds:*

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x + y)\rangle, \quad \forall j(x + y) \in J(x + y). \tag{1.18}$$

Lemma 1.9 (see [14]). *Let $\{\rho_n\}$ be nonnegative sequence which satisfies the following inequality:*

$$\rho_{n+1} \leq (1 - \lambda_n)\rho_n + \sigma_n, \quad n \geq N, \tag{1.19}$$

where $\lambda_n \in (0, 1), \lim_{n\to\infty}\lambda_n = 0$ and $\sum_{n=0}^{\infty} \lambda_n = \infty$, $\sigma_n = o(\lambda_n)$. Then $\rho_n \to 0$ as $n \to \infty$.

Lemma 1.10. *Let $\{\theta_n\}, \{c_n\}, \{e_n\}$ and $\{t_n\}$ be four nonnegative real sequences satisfying the following conditions:* (i) $\lim_{n\to\infty} t_n = 0$; (ii) $\sum_{n=0}^{\infty} t_n = \infty$; (iii) $c_n = o(t_n), e_n = o(t_n)$. *Let $\Phi : [0, +\infty) \to [0, +\infty)$ be a strictly increasing and continuous function with $\Phi(0) = 0$ such that*

$$\theta_{n+1}^2 \leq (1 + c_n)\theta_n^2 - t_n \frac{\Phi(\theta_{n+1})}{1 + \Phi(\theta_{n+1}) + \theta_{n+1}^2} + e_n, \quad n \geq 0. \tag{1.20}$$

If $\{\theta_n\}$ is bounded, then $\theta_n \to 0$ as $n \to \infty$.

Proof. Since $\lim_{n\to\infty} t_n = 0, \{\theta_n\}$ is bounded, we set $R = \max\{\sup_{n\geq 0} t_n, \sup_{n\geq 0}\theta_n\}$, $\gamma = \liminf_{n\to\infty}(\Phi(\theta_{n+1})/(1 + \theta_{n+1}^2)[1 + \Phi(R) + R^2])$, then $\gamma = 0$. Otherwise, we assume that $\gamma > 0$, then there exists a constant $\delta > 0$ with $\delta = \min\{1, \gamma\}$ and a natural number N_1 such that

$$\Phi(\theta_{n+1}) > \left(\delta + \delta\theta_{n+1}^2\right)\left[1 + \Phi(R) + R^2\right] > \delta\theta_{n+1}^2\left[1 + \Phi(R) + R^2\right], \tag{1.21}$$

for $n > N_1$.

Then, from (1.20), we get

$$\theta_{n+1}^2 \leq \frac{1 + c_n}{1 + \delta t_n}\theta_n^2 + e_n. \tag{1.22}$$

Since $c_n = o(t_n)$, there exists a nature number $N_2 > N_1$, such that $c_n < (\delta/2)t_n$, $n > N_2$. Hence $(1 + c_n)/(1 + \delta t_n) < 1 - (\delta/2)t_n$ and (1.22) becomes

$$\theta_{n+1}^2 \leq \left(1 - \frac{\delta}{2}t_n\right)\theta_n^2 + e_n. \tag{1.23}$$

By Lemma 1.9, we obtain that $\theta_n \to 0$ as $n \to \infty$. Since Φ is strictly increasing and continuous with $\Phi(0) = 0$. Hence $\gamma = 0$, which is contradicting with the assumption $\gamma > 0$. Then $\gamma = 0$, there exists a subsequence $\{\theta_{n_j}\}$ of $\{\theta_n\}$ such that $\theta_{n_j} \to 0$ as $j \to \infty$. Let $0 < \varepsilon < 1$ be any given. Since $c_n = o(t_n)$, $e_n = o(t_n)$, then there exists a natural number $N_3 > N_2$, such that

$$\theta_{n_j} < \varepsilon, \qquad c_{n_j} < \frac{\Phi(\varepsilon)}{2M^2(1 + R^2 + \Phi(R))}t_{n_j}, \qquad e_{n_j} < \frac{\Phi(\varepsilon)}{2(1 + R^2 + \Phi(R))}t_{n_j} \tag{1.24}$$

for all $j > N_3$. Next, we will show that $\theta_{n_j+m} < \varepsilon$ for all $m = 1, 2, 3, \ldots$. First, we want to prove that $\theta_{n_j+1} < \varepsilon$. Suppose that it is not the case, then $\theta_{n_j+1} \geq \varepsilon$. Since Φ is strictly increasing,

$$\Phi\left(\theta_{n_j+1}\right) \geq \Phi(\varepsilon). \tag{1.25}$$

From (1.24) and (1.25), we obtain that

$$\begin{aligned}\theta_{n_j+1}^2 &\leq \left(1 + c_{n_j}\right)\theta_{n_j}^2 - 2t_{n_j}\frac{\Phi(\varepsilon)}{1 + R^2 + \Phi(R)} + e_{n_j} \\ &< \theta_{n_j}^2 - \frac{\Phi(\varepsilon)}{(1 + R^2 + \Phi(R))}t_{n_j} < \theta_{n_j}^2 < \varepsilon^2.\end{aligned} \tag{1.26}$$

That is $\theta_{n_j+1} < \varepsilon$, which is a contradiction. Hence $\theta_{n_j+1} < \varepsilon$. Now we assume that $\theta_{n_j+m} < \varepsilon$ holds. Using the similar way, it follows that $\theta_{n_j+m+1} < \varepsilon$. Therefore, this shows that $\theta_n \to 0$ as $n \to \infty$. □

2. Main Results

Theorem 2.1. *Let K be a nonempty closed convex subset of a Banach space E. Suppose that $T_n = T_{n(\bmod M)}$, and $T_i : K \to K, i \in I = \{1, 2, \ldots, M\}$ are M uniformly continuous generalized weak Φ-hemicontractive mappings with $F = \bigcap_{i=1}^{M} F(T_i) \neq \emptyset$. Let $\{u_n\}$ be a sequence in K defined iteratively from some $u_0 \in K$ by* (1.17)*, where $\{\mu_n\}$ is an arbitrary bounded sequence in K and $\{\alpha_n\}, \{\delta_n\}$ are two sequences in* $[0,1]$ *satisfying the following conditions:* (i) $\alpha_n + \delta_n \leq 1$, (ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$, (iii) $\lim_{n\to\infty}\alpha_n = 0$, (iv) $\delta_n = o(\alpha_n)$. *Then the iteration sequence $\{u_n\}$ converges strongly to the unique fixed point of T.*

Proof. Since $F = \bigcap_{i=1}^{M} F(T_i) \neq \emptyset$, set $q \in F$. Since the mapping T_n are generalized weak Φ-hemicontractive mappings, there exist strictly increasing functions $\Phi_l : [0,+\infty) \to [0,+\infty)$ with $\Phi_l(0) = 0$ and $j(x-y) \in J(x-y)$ such that

$$\langle T_l x - T_l y, j(x-y)\rangle \leq \|x-y\|^2 - \frac{\Phi_l(\|x-y\|)}{1+\|x-y\|^2+\Phi_l(\|x-y\|)}, \quad \forall x,y \in K,\ l \in I. \tag{2.1}$$

Firstly, we claim that there exists $u_0 \in K$ with $u_0 \neq Tu_0$ such that $t_0 = \|u_0 - Tu_0\| \cdot \|u_0 - q\| \cdot [1 + \|u_0 - q\|^2 + \Phi_1(\|u_0 - q\|)] \in R(\Phi_1)$. In fact, if $u_0 = Tu_0$, then we have done. Otherwise, there exists the smallest positive integer $n_0 \in N$ such that $u_{n_0} \neq Tu_{n_0}$. We denote $u_{n_0} = u_0$, then we will obtain that $t_0 \in R(\Phi_1)$. Indeed, if $R(\Phi_1) = [0,+\infty)$, then $t_0 \in R(\Phi)$. If $R(\Phi_1) = [0, A]$ with $0 < A < +\infty$, then for $q \in K$, there exists a sequence $\{w_n\} \subset K$ such that $w_n \to q$ as $n \to \infty$ with $w_n \neq q$, and we also obtain that the sequence $\{w_n - Tw_n\}$ is bounded. So there exists $n_0 \in N$ such that $\|w_n - Tw_n\| \cdot \|w_n - q\| \cdot [1 + \|w_n - q\|^2 + \Phi_1(\|w_n - q\|)] \in R(\Phi_1)$ for $n \geq n_0$, then we redefine $u_0 = w_{n_0}$, let $\omega_0 = \Phi_1^{-1}(t_0) > 0$.

Next we shall prove $\|u_n - q\| \leq \omega_0$ for $n \geq 0$. Clearly, $\|u_0 - q\| \leq \omega_0$ holds. Suppose that $\|u_n - q\| \leq \omega_0$, for some n, then we want to prove $\|u_{n+1} - q\| \leq \omega_0$. If it is not the case, then $\|u_{n+1} - q\| > \omega_0$. Since T is a uniformly continuous mapping, setting $\epsilon_0 = \Phi_l(\omega_0)/12\omega_0[1 + \Phi_l((3/2)\omega_0) + ((3/2)\omega_0)^2]$, there exists $\delta > 0$ such that $\|T_n x - T_n y\| < \epsilon_0$, whenever $\|x - y\| < \delta$; and T_n are bounded operators, set $M = \sup\{\|T_n x\| : \|x - q\| \leq \omega_0\} + \sup_n \|\omega_n\|$. Since $\lim_{n\to\infty}\alpha_n = 0$, $\delta_n = o(\alpha_n)$, without loss of generality, let

$$\begin{aligned} \alpha_n, \frac{\delta_n}{\alpha_n} < \min\Bigg\{ &\frac{1}{4}, \frac{\omega_0}{4M}, \frac{\delta}{4(M+\omega_0)}, \frac{\Phi_l(\omega_0)}{4\omega_0\left[1+\Phi_l((3/2)\omega_0)+((3/2)\omega_0)^2\right]}, \\ &\frac{\Phi_l(\omega_0)}{12\left[1+\Phi_l((3/2)\omega_0)+((3/2)\omega_0)^2\right]M\omega_0}\Bigg\}, \quad n \geq 0. \end{aligned} \tag{2.2}$$

From (1.17), we have

$$\begin{aligned} \|u_{n+1} - q\| &= \|(1-\alpha_n-\delta_n)(u_n - q) + \alpha_n(T_n u_n - q) + \delta_n(\omega_n - q)\| \\ &\leq \|u_n - q\| + \alpha_n\|T_n u_n - q\| + \delta\|\omega_n - q\| \\ &\leq \omega_0 + \alpha_n\|T_n u_n - q\| + \delta_n\|\omega_n - q\| \\ &\leq \omega_0 + M(\alpha_n + \delta_n) \leq \omega_0 + 2M\alpha_n \leq \frac{3}{2}\omega_0, \end{aligned} \tag{2.3}$$

$$\begin{aligned} \|u_{n+1} - u_n\| &= \|\alpha_n T_n u_n + \delta_n \omega_n - (\alpha_n + \delta_n)u_n\| \\ &\leq \alpha_n\|T_n u_n - q\| + \delta_n\|\omega_n - q\| + (\alpha_n + \delta_n)\|u_n - q\| \\ &\leq (\alpha_n + \delta_n)(M + \omega_0) < \delta. \end{aligned} \tag{2.4}$$

Since T_n are uniformly continuous mappings, so $\|T_n u_{n+1} - T_n u_n\| < \varepsilon_0$.

Applying Lemma 1.8, the recursion (1.17), and the above inequalities, we obtain

$$\begin{aligned}
\|u_{n+1}-q\|^2 &= \|(1-\alpha_n-\delta_n)(u_n-q)+\alpha_n(T_nu_n-q)+\delta_n(\omega_n-q)\|^2 \\
&\le (1-\alpha_n-\delta_n)^2\|u_n-q\|^2+2\alpha_n\langle T_nu_n-q, j(u_{n+1}-q)\rangle \\
&\quad + 2\delta_n\|\omega_n-q\|\cdot\|u_{n+1}-q\| \\
&\le (1-\alpha_n)^2\|u_n-q\|^2+2\alpha_n\langle T_nu_{n+1}-Tq, j(u_{n+1}-q)\rangle \\
&\quad + 2\alpha_n\|T_nu_n-T_nu_{n+1}\|\cdot\|u_{n+1}-q\|+2\delta_n\|\omega_n-q\|\cdot\|u_{n+1}-q\| \\
&\le (1-\alpha_n)^2\|u_n-q\|^2+2\alpha_n\left[\|u_{n+1}-q\|^2-\frac{\Phi_l(\|u_{n+1}-q\|)}{1+\Phi_l(\|u_{n+1}-q\|)+\|u_{n+1}-q\|^2}\right] \\
&\quad + 2\alpha_n\|T_nu_n-T_nu_{n+1}\|\cdot\|u_{n+1}-q\|+2\delta_n\|\omega_n-q\|\cdot\|u_{n+1}-q\|.
\end{aligned} \tag{2.5}$$

Inequality (2.5) implies

$$\begin{aligned}
\|u_{n+1}-q\|^2 &\le \|u_n-q\|^2-2\alpha_n\frac{\Phi_l(\|u_{n+1}-q\|)}{1+\Phi_l(\|u_{n+1}-q\|)+\|u_{n+1}-q\|^2}+\frac{\alpha_n^2}{1-2\alpha_n}\|u_n-q\|^2 \\
&\quad +\frac{2\alpha_n}{1-2\alpha_n}\|T_nu_n-T_nu_{n+1}\|\cdot\|u_{n+1}-q\|+\frac{2\delta_n}{1-2\alpha_n}\|\omega_n-q\|\cdot\|u_{n+1}-q\| \\
&\le \omega_0^2-2\alpha_n\frac{\Phi_l(\omega_0)}{1+\Phi_l((3/2)\omega_0)+((3/2)\omega_0)^2} \\
&\quad +2\alpha_n\frac{\Phi_l(\omega_0)}{4\left[1+\Phi_l((3/2)\omega_0)+((3/2)\omega_0)^2\right]\omega_0^2}\cdot\omega_0^2 \\
&\quad +4\alpha_n\frac{\Phi_l(\omega_0)}{12\left[1+\Phi_l((3/2)\omega_0)+((3/2)\omega_0)^2\right]\omega_0}\cdot\frac{3\omega_0}{2} \\
&\quad +4\alpha_n\frac{\Phi_l(\omega_0)}{12\left[1+\Phi_l((3/2)\omega_0)+((3/2)\omega_0)^2\right]M\omega_0}\cdot\frac{3M\omega_0}{2}<\omega_0^2,
\end{aligned} \tag{2.6}$$

which is a contradiction with the assumption $\|u_{n+1}-q\|>\omega_0$. Then $\|u_{n+1}-q\|\le\omega_0$; that is, the sequence $\{u_n\}$ is bounded. Let $N=\sup_n\|u_n-q\|$. From (2.4), we have

$$\|u_{n+1}-u_n\|\le(\alpha_n+\delta_n)(M+\omega_0)\longrightarrow 0,\quad n\longrightarrow\infty, \tag{2.7}$$

that is, $\lim_{n\to\infty}\|u_{n+1}-u_n\|=0$. Since T is on uniformly continuous, so

$$\lim_{n\to\infty}\|T_nu_{n+1}-T_nu_n\|=0. \tag{2.8}$$

Again using (2.5), we have

$$\|u_{n+1}-q\|^2 \le \|u_n-q\|^2 - 2\alpha_n \frac{\Phi_l(\|u_{n+1}-q\|)}{1+\Phi_l(\|u_{n+1}-q\|)+\|u_{n+1}-q\|^2}\Bigg] + A_n, \tag{2.9}$$

where

$$A_n = \alpha_n^2 N^2 + 2\alpha_n N\|T_n u_n - T_n u_{n+1}\| + 2\delta_n MN. \tag{2.10}$$

By (2.8), the conditions (iii) and (iv), we get $A_n = o(\alpha_n)$. So applying Lemma 1.10 on (2.9), we obtain $\lim_{n\to\infty}\|u_n - q\| = 0$. □

Theorem 2.2. *Let E be a Banach space and K be a nonempty closed convex subset of E, T_n are as in Theorem 2.1. For $x_0, u_0 \in K$, the sequence iterations $\{x_n\}, \{u_n\}$ are defined by* (1.15) *and* (1.17)*, respectively. $\{\alpha_n\}$, $\{\delta_n\}$, $\{\beta_n^i\}$, $\{\eta_n^i\}$, $i = 1, 2, \ldots, p-1$ are sequences in $[0,1]$ satisfying the following conditions:*

(i) $0 \le \alpha_n + \delta_n \le 1$, $0 \le \beta_n^i + \eta_n^i \le 1$, $1 \le i \le p-1$;

(ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$;

(iii) $\lim_{n\to\infty}\alpha_n = 0$;

(iv) $\lim_{n\to\infty}\beta_n^i = \lim_{n\to\infty}\eta_n^i = 0$, $i = 1, \ldots, p-1$;

(v) $\delta_n = o(\alpha_n)$.

Then the following two assertions are equivalent:

(I) *the iteration sequence $\{x_n\}$ strongly converges to the common point of $F(T_i)$, $i \in I$;*

(II) *the sequence iteration $\{u_n\}$ strongly converges to the common point of $F(T_i)$, $i \in I$.*

Proof. Since $F = \bigcap_{i=1}^{M} F(T_i) \ne \emptyset$, set $q \in F$. If the iteration sequence $\{x_n\}$ strongly converges to q, then setting $p = 2$, $\beta_n = \delta_n = 0$, we obtain the convergence of the iteration sequence $\{u_n\}$. Conversely, we only prove that (II)⇒(I). The proof is divided into two parts.

Step 1. We show that $\{x_n - u_n\}$ is bounded.

By the proof method of Theorem 2.1, there exists $x_0 \in K$ with $x_0 \ne T_1 x_0$ such that $r_0 = \|x_0 - T_1 x_0\| \cdot \|x_0 - q\| \cdot [1 + \|x_0 - q\|^2 + \Phi_1(\|x_0 - q\|)] \in R(\Phi)$. Setting $a_0 = \Phi_1^{-1}(r_0)$, we have $\|x_0 - q\| \le a_0$. Set $B_1 = \{\|x - q\| \le a_0 : x \in K\}$, $B_2 = \{\|x - q\| \le 2a_0 : x \in K\}$. Since T_l are bounded mappings and $\{\omega_n^i\}$ $(i = 1, \ldots, p-1)$, $\{v_n\}$ are some bounded sequences in K, we can set $M = \max\{\sup_{x\in B_2}\|T_n x - q\|; \sup_{n\in N}\|\omega_n^i - q\|; \sup_{n\in N}\|v_n - q\|\}$. Since T_i are uniformly continuous mappings, given $\epsilon_0 = \Phi_l(a_0)/4a_0[1 + (5a_0/4)^2 + \Phi_l(5a_0/4)]$, $\exists\delta > 0$, such that $\|Tx - Ty\| < \epsilon_0$ whenever $\|x - y\| < \delta$, for all $x, y \in B_2$. Now, we define $\tau_0 = \min\{1/2,\ a_0/8M,\ a_0/8(M + a_0),\ \delta/8(M + a_0),\ \Phi_l(a_0)/5a_0^2[1 + (5a_0/4)^2 + \Phi_l(5a_0/4)],\ \Phi_l(a_0)/5a_0 M[1 + (5a_0/4)^2 + \Phi_l(5a_0/4)]\}$. Since the control conditions (iii)-(iv), without loss of generality, we let $0 < \alpha_n, \delta_n/\alpha_n, \beta_n^i, \eta_n^i < \tau_0$, $n \ge 0$.

Now we claim that if $x_n \in B_1$, then $y_n^i \in B_2$, $1 \le i \le p-1$.

From (1.15), we obtain that

$$\begin{aligned}
\left\|y_n^{p-1}-q\right\| &\le \left(1-\beta_n^{p-1}-\eta_n^{p-1}\right)\|x_n-q\|+\beta_n^{p-1}\|T_nx_n-q\|+\eta_n^{p-1}\left\|\omega_n^{p-1}-q\right\| \\
&\le \|x_n-q\|+\left(\beta_n^{p-1}+\eta_n^{p-1}\right)M \\
&\le \|x_n-q\|+2\tau_0M\le 2a_0, \\
\left\|y_n^{p-2}-q\right\| &\le \left(1-\beta_n^{p-2}-\eta_n^{p-2}\right)\|x_n-q\|+\beta_n^{p-2}\left\|T_ny_n^{p-1}-q\right\|+\eta_n^{p-2}\left\|\omega_n^{p-2}-q\right\| \\
&\le \|x_n-q\|+\left(\beta_n^{p-2}+\eta_n^{p-2}\right)M \\
&\le \|x_n-q\|+2\tau_0M\le 2a_0,
\end{aligned} \tag{2.11}$$

we also obtain that

$$\left\|y_n^1-q\right\|\le 2a_0. \tag{2.12}$$

Now we suppose that $\|x_n-q\|\le a_0$ holds. We will prove that $\|x_{n+1}-q\|\le a_0$. If it is not the case, we assume that $\|x_{n+1}-q\|>a_0$. From (1.15), we obtain that

$$\begin{aligned}
\|x_{n+1}-q\| &= \left\|(1-\alpha_n-\delta_n)(x_n-q)+\alpha_n\left(T_ny_n^1-q\right)+\delta_n(\nu_n-q)\right\| \\
&\le \|x_n-q\|+\alpha_n\left\|T_ny_n^1-q\right\|+\delta_n\|\nu_n-q\| \\
&\le \|x_n-q\|+(\alpha_n+\delta_n)M \\
&\le \|x_n-q\|+2\tau_0M\le\|x_n-q\|+\frac{1}{4}a_0\le\frac{5}{4}a_0.
\end{aligned} \tag{2.13}$$

Consequently, by (2.11) and (2.12), we obtain

$$\begin{aligned}
\left\|x_{n+1}-y_n^1\right\| &= \Big\|\left[\left(\beta_n^1-\alpha_n\right)+\left(\eta_n^1-\delta_n\right)\right](x_n-q)+\alpha_n\left(T_ny_n^1-q\right) \\
&\quad -\beta_n^1\left(T_n^ny_n^2-q\right)+\delta_n(\nu_n-q)-\eta_n^1\left(\omega_n^1-q\right)\Big\| \\
&\le \left(\beta_n^1+\alpha_n+\eta_n^1+\delta_n\right)\|x_n-q\|+\alpha_n\left\|T_ny_n^1-q\right\| \\
&\quad +\beta_n^1\left\|T_ny_n^2-q\right\|+\delta_n\|\nu_n-q\|+\eta_n^1\left\|\omega_n^1-q\right\| \\
&\le 4\tau_0(\mu_0+M)\le\delta.
\end{aligned} \tag{2.14}$$

Since T_n are uniformly continuous mappings, we get

$$\left\|T_nx_{n+1}-T_ny_n^1\right\|\le\epsilon_0. \tag{2.15}$$

Using (2.1), Lemma 1.8, and the recursion formula (1.15), we have

$$\begin{aligned}
\|x_{n+1}-q\|^2 &= \left\|(1-\alpha_n-\delta_n)(x_n-q)+\alpha_n\left(T_ny_n^1-q\right)+\delta_n(v_n-q)\right\|^2 \\
&\le (1-\alpha_n)^2\|x_n-q\|^2+2\alpha_n\left\langle T_ny_n^1-q, j(x_{n+1}-q)\right\rangle+2\delta_n\|v_n-q\|\cdot\|x_{n+1}-q\| \\
&\le (1-\alpha_n)^2\|x_n-q\|^2+2\alpha_n\langle T_nx_{n+1}-q, j(x_{n+1}-q)\rangle \\
&\quad +2\alpha_n\left\langle T_ny_n^1-T_nx_{n+1}, j(x_{n+1}-q)\right\rangle+2\delta_n\|v_n-q\|\cdot\|x_{n+1}-q\| \\
&\le (1-\alpha_n)^2\|x_n-q\|^2+2\alpha_n\left[\|x_{n+1}-q\|^2-\frac{\Phi_l(\|x_{n+1}-q\|)}{1+\Phi_l(\|x_{n+1}-q\|)+\|x_{n+1}-q\|^2}\right] \\
&\quad +2\alpha_n\left\|T_ny_n^1-T_nx_{n+1}\right\|\cdot\|x_{n+1}-q\|+2\delta_nM\cdot\|x_{n+1}-q\|.
\end{aligned} \tag{2.16}$$

Which implies

$$\begin{aligned}
\|x_{n+1}-q\|^2 &\le \|x_n-q\|^2-\frac{2\alpha_n}{1-2\alpha_n}\frac{\Phi_l(\|x_{n+1}-q\|)}{1+\Phi_l(\|x_{n+1}-q\|)+\|x_{n+1}-q\|^2}+\frac{\alpha_n^2}{1-2\alpha_n}\|x_n-q\|^2 \\
&\quad +\frac{2\alpha_n}{1-2\alpha_n}\left\|T_ny_n^1-T_nx_{n+1}\right\|\cdot\|x_{n+1}-q\|+\frac{2\delta_n}{1-2\alpha_n}M\cdot\|x_{n+1}-q\| \\
&\le a_0^2-\frac{2\alpha_n}{1-2\alpha_n}\frac{\Phi_l(a_0)}{1+(5a_0/4)^2+\Phi_l(5a_0/4)} \\
&\quad +\frac{2\alpha_n}{1-2\alpha_n}\frac{\Phi_l(a_0)}{4a_0^2\left[1+(5a_0/4)^2+\Phi_l(5a_0/4)\right]}a_0^2 \\
&\quad +\frac{2\alpha_n}{1-2\alpha_n}\frac{\Phi_l(a_0)}{5a_0\left[1+(5a_0/4)^2+\Phi_l(5a_0/4)\right]}\cdot\frac{5a_0}{4} \\
&\quad +\frac{2\alpha_n}{1-2\alpha_n}\frac{\Phi_l(a_0)}{5a_0M\left[1+(5a_0/4)^2+\Phi_l(5a_0/4)\right]}\cdot\frac{5a_0M}{4}<a_0^2
\end{aligned} \tag{2.17}$$

which is a contradiction with the assumption $\|x_{n+1}-q\|>\mu_0$, then $\|x_{n+1}-q\|\le\mu_0$; that is, the sequence $\{x_n-q\}$ is bounded. Since $u_n\to q$, as $n\to\infty$, so the sequence $\{x_n-u_n\}$ is bounded.

Step 2. We prove $\lim_{n\to\infty}\|x_n-q\|=0$.

Since $\{x_n-u_n\}$ is bounded, again applying (2.11) and (2.12), we get the boundedness of $\{y_n^i-u_n\}$, $i=1,2,\ldots,p-1$. Since $T_n=T_{n(\bmod M)}$ are bounded mappings, set $L=$

$\max\{\sup_{n\geq 0}\|x_n - u_n\|, \sup_{n\geq 0}\|T_n x_n - u_n\|, \sup_{n\geq 0}\|T_n y_n^i - u_n\|, \sup_{n\geq 0}\|\mu_n - u_n\|, \sup_{n\geq 0}\|\nu_n - u_n\|, \sup_{n\geq 0}\|\omega_n^1 - u_n\|\}$, $(i = 1, 2, \ldots, p-1)$. From (1.15) and (1.17), we obtain

$$\begin{aligned}
&\|x_{n+1} - u_{n+1}\|^2 \\
&\quad = \left\|(1-\alpha_n-\delta_n)(x_n-u_n) + \alpha_n\left(T_n y_n^1 - T_n u_n\right) + \delta_n(\nu_n - \mu_n)\right\|^2 \\
&\quad \leq (1-\alpha_n)^2\|x_n-u_n\|^2 + 2\alpha_n\left\langle T_n y_n^1 - T_n u_n, j(x_{n+1}-u_{n+1})\right\rangle \\
&\qquad + 2\delta_n\|\nu_n-\mu_n\|\cdot\|x_{n+1}-u_{n+1}\| \\
&\quad \leq (1-\alpha_n)^2\|x_n-u_n\|^2 + 2\alpha_n\langle T_n x_{n+1} - T_n u_{n+1}, j(x_{n+1}-u_{n+1})\rangle \\
&\qquad\quad + 2\alpha_n\left\langle T_n y_n^1 - T_n x_{n+1} + T_n u_{n+1} - T_n u_n, j(x_{n+1}-u_{n+1})\right\rangle \\
&\qquad\qquad + 2\delta_n\|\nu_n-\mu_n\|\cdot\|x_{n+1}-u_{n+1}\| \\
&\quad \leq \left(1+\alpha_n^2\right)\|x_n-q\|^2 - 2\alpha_n\frac{\Phi_l(\|x_{n+1}-u_{n+1}\|)}{1+\Phi_l(\|x_{n+1}-u_{n+1}\|) + \|x_{n+1}-u_{n+1}\|^2} \\
&\qquad\quad + 2\alpha_n\left\|T_n y_n^1 - T_n x_{n+1}\right\|\cdot\|x_{n+1}-u_{n+1}\| + 2\alpha_n\|T_n u_{n+1} - T_n u_n\|\cdot\|x_{n+1}-u_{n+1}\| \\
&\qquad\quad + 2\delta_n M\cdot\|x_{n+1}-u_{n+1}\|
\end{aligned} \tag{2.18}$$

$$\begin{aligned}
\left\|x_{n+1} - y_n^1\right\| &\leq \left(\beta_n^1 + \alpha_n + \eta_n^1 + \delta_n\right)\|x_n - u_n\| + \alpha_n\left\|T_n^n y_n^1 - u_n\right\| \\
&\quad + \beta_n^1\left\|T_n^n y_n^2 - u_n\right\| + \delta_n\|\nu_n - u_n\| + \eta_n^1\left\|\omega_n^1 - u_n\right\| \\
&\leq 2\left(\beta_n^1 + \alpha_n + \eta_n^1 + \delta_n\right)L.
\end{aligned} \tag{2.19}$$

By the conditions (iii)–(v), we have

$$\lim_{n\to\infty}\left\|x_{n+1} - y_n^1\right\| = 0. \tag{2.20}$$

Since $\lim_{n\to\infty}\|u_n - q\| = 0$, so

$$\|u_{n+1} - u_n\| \leq \|u_n - q\| + \|u_{n+1} - q\|. \tag{2.21}$$

That is:

$$\lim_{n\to\infty}\|u_{n+1} - u_n\| = 0. \tag{2.22}$$

By the uniform continuity of T, we obtain

$$\lim_{n\to\infty}\left\|T_n x_{n+1} - T_n y_n^1\right\| = 0, \qquad \lim_{n\to\infty}\|T_n u_{n+1} - T_n u_n\| = 0. \tag{2.23}$$

From (2.23) and the conditions (iii) and (v), (2.18) becomes

$$\|x_{n+1} - u_{n+1}\|^2 \le \left(1+\alpha_n^2\right)\|x_n - u_n\|^2 - 2\alpha_n \frac{\Phi_l(\|x_{n+1} - u_{n+1}\|)}{1+\Phi_l(\|x_{n+1} - u_{n+1}\|) + \|x_{n+1} - u_{n+1}\|^2} + o(\alpha_n). \tag{2.24}$$

By Lemma 1.10, we get $\lim_{n\to\infty}\|x_n - u_n\| = 0$. Since $\lim_{n\to\infty}\|u_n - q\| = 0$, and the inequality $0 \le \|x_n - q\| \le \|x_n - u_n\| + \|u_n - q\|$, so $\lim_{n\to\infty}\|x_n - q\| = 0$. □

From Theorems 2.1 and 2.2, we can obtain the following corollary.

Corollary 2.3. *Let E be a Banach space and K be a nonempty closed convex subset of E, T_n are as in Theorem 2.1. For $x_0 \in K$, the sequence iterations $\{x_n\}$ is defined by (1.15). $\{\alpha_n\}$, $\{\delta_n\}$, $\{\beta_n^i\}$, $\{\eta_n^i\}$, $(i = 1, 2, \ldots, p-1)$ are sequences in $[0,1]$ satisfying the following conditions:*

(i) $0 \le \alpha_n + \delta_n \le 1$, $0 \le \beta_n^i + \eta_n^i \le 1$, $1 \le i \le p-1$;

(ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$;

(iii) $\lim_{n\to\infty}\alpha_n = 0$;

(iv) $\lim_{n\to\infty}\beta_n^i = \lim_{n\to\infty}\eta_n^i = 0$, $i = 1, \ldots, p-1$;

(v) $\delta_n = o(\alpha_n)$.

Then the iteration sequence $\{x_n\}$ strongly converges to the common point of $F(T_i)$, $i \in I$.

Corollary 2.4. *Let $T_n = S_{n(\bmod M)}$, $T_l : E \to E$, $l \in I = \{1, 2, \ldots, M\}$ are M uniformly continuous generalized weak Φ-quasi-accretive mappings. Suppose $N(F) = \bigcap_{i=1}^{M} N(F_i) \ne \emptyset$, that is, there exists $x^* \in N(F)$. Let $\{\alpha_n\}$, $\{\delta_n\}$, $\{\beta_n^i\}$, $\{\eta_n^i\}$, $(i = 1, 2, \ldots, p-1)$ be sequences in $[0,1]$ satisfying the following conditions:*

(i) $0 \le \alpha_n + \delta_n \le 1$, $0 \le \beta_n^i + \eta_n^i \le 1$, $1 \le i \le p-1$;

(ii) $\sum_{n=0}^{\infty} \alpha_n = \infty$;

(iii) $\lim_{n\to\infty}\alpha_n = 0$;

(iv) $\lim_{n\to\infty}\beta_n^i = \lim_{n\to\infty}\eta_n^i = 0$, $i = 1, \ldots, p-1$;

(v) $\delta_n = o(\alpha_n)$.

Let the sequence $\{x_n\}$ in E be generated iteratively from some $x_0 \in E$ by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n-\delta_n)x_n + \alpha_n S_n y_n^1 + \delta_n v_n, \\ y_n^i &= \left(1-\beta_n^i-\eta_n^i\right)x_n + \beta_n^i S_n y_n^{i+1} + \eta_n^i \omega_n^i, \quad i = 1, \ldots p-2, \\ y_n^{p-1} &= \left(1-\beta_n^{p-1}-\eta_n^{p-1}\right)x_n + \beta_n^{p-1} S_n x_n + \eta_n^{p-1}\omega_n^{p-1}, \quad p \ge 2, \end{aligned} \tag{2.25}$$

where $S_l x := x - T_l x$ for all $x \in E$ and $\{v_n\}$, $\{\omega_n^i\}$ are any bounded sequences in K.

Then $\{x_n\}$ *defined by* (2.25) *converges strongly to* x^*.

Proof. We simply observe that $S_l := I - T_l$, $l \in I$ are M uniformly continuous generalized weak Φ-hemicontractive mappings. The result follows from Corollary 2.3. □

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