

## Research Article

# Application of Sumudu Decomposition Method to Solve Nonlinear System of Partial Differential Equations

Hassan Eltayeb<sup>1</sup> and Adem Kılıçman<sup>2</sup>

<sup>1</sup> *Mathematics Department, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia*

<sup>2</sup> *Department of Mathematics and Institute for Mathematical Research, Universiti Putra Malaysia, Serdang, 43400 Selangor, Malaysia*

Correspondence should be addressed to Adem Kılıçman, [akilicman@putra.upm.edu.my](mailto:akilicman@putra.upm.edu.my)

Received 26 August 2012; Accepted 22 October 2012

Academic Editor: M. Mursaleen

Copyright © 2012 H. Eltayeb and A. Kılıçman. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We develop a method to obtain approximate solutions of nonlinear system of partial differential equations with the help of Sumudu decomposition method (SDM). The technique is based on the application of Sumudu transform to nonlinear coupled partial differential equations. The nonlinear term can easily be handled with the help of Adomian polynomials. We illustrate this technique with the help of three examples, and results of the present technique have close agreement with approximate solutions obtained with the help of Adomian decomposition method (ADM).

## 1. Introduction

Most of phenomena in nature are described by nonlinear differential equations. So scientists in different branches of science try to solve them. But because of nonlinear part of these groups of equations, finding an exact solution is not easy. Different analytical methods have been applied to find a solution to them. For example, Adomian has presented and developed a so-called decomposition method for solving algebraic, differential, integrodifferential, differential-delay and partial differential equations. In the nonlinear case for ordinary differential equations and partial differential equations, the method has the advantage of dealing directly with the problem [1, 2]. These equations are solved without transforming them to more simple ones. The method avoids linearization, perturbation, discretization, or any unrealistic assumptions [3, 4]. It was suggested in [5] that the noise terms appears always for inhomogeneous equations. Most recently, Wazwaz [6] established a necessary condition

that is essentially needed to ensure the appearance of “noise terms” in the inhomogeneous equations. In the present paper, the intimate connection between the Sumudu transform theory and decomposition method arises in the solution of nonlinear partial differential equations is demonstrated.

The Sumudu transform is defined over the set of the functions

$$A = \left\{ f(t) : \exists M, \tau_1, \tau_2 > 0, |f(t)| < M e^{t/\tau_j}, \text{ if } t \in (-1)^j \times [0, \infty) \right\} \quad (1.1)$$

by the following formula:

$$\begin{aligned} G(u) &= S[f(t); u] \\ &= \int_0^\infty f(ut) e^{-t} dt, \quad u \in (-\tau_1, \tau_2). \end{aligned} \quad (1.2)$$

The existence and the uniqueness were discussed in [7], for further details and properties of the Sumudu transform and its derivatives we refer to [8]. In [9], some fundamental properties of the Sumudu transform were established.

In [10], this new transform was applied to the one-dimensional neutron transport equation. In fact one can easily show that there is a strong relationship between double Sumudu and double Laplace transforms, see [7].

Further in [11], the Sumudu transform was extended to the distributions and some of their properties were also studied in [12]. Recently Kılıçman et al. applied this transform to solve the system of differential equations, see [13].

A very interesting fact about Sumudu transform is that the original function and its Sumudu transform have the same Taylor coefficients except a factor  $n!$ . Thus if

$$f(t) = \sum_{n=0}^{\infty} a_n t^n \quad (1.3)$$

then

$$F(u) = \sum_{n=0}^{\infty} n! a_n u^n, \quad (1.4)$$

see [14].

Similarly, the Sumudu transform sends combinations,  $C(m, n)$ , into permutations,  $P(m, n)$  and hence it will be useful in the discrete systems. Further

$$\begin{aligned} S(H(t)) &= \mathcal{L}(\delta(t)) = 1, \\ \mathcal{L}(H(t)) &= S(\delta(t)) = \frac{1}{u}. \end{aligned} \quad (1.5)$$

Thus we further note that since many practical engineering problems involve mechanical or electrical systems where action is defined by discontinuous or impulsive

forcing terms. Then the Sumudu transform can be effectively used to solve ordinary differential equations as well as partial differential equations and engineering problems. Recently, the Sumudu transform was introduced as a new integral transform on a time scale  $\mathbb{T}$  to solve a system of dynamic equations, see [15]. Then the results were applied on ordinary differential equations when  $\mathbb{T} = \mathbb{R}$ , difference equations when  $\mathbb{T} = \mathbb{N}_0$ , but also, for  $q$ -difference equations when  $\mathbb{T} = q^{\mathbb{N}_0}$ , where  $q^{\mathbb{N}_0} := \{q^t : t \in \mathbb{N}_0 \text{ for } q > 1\}$  or  $\mathbb{T} = q^{\mathbb{Z}} := q^{\mathbb{Z}} \cup \{0\}$  for  $q > 1$  which has important applications in quantum theory and on different types of time scales like  $\mathbb{T} = h\mathbb{N}_0$ ,  $\mathbb{T} = \mathbb{N}_0^2$ , and  $\mathbb{T} = \mathbb{T}_n$  the space of the harmonic numbers. During this study we use the following Sumudu transform of derivatives.

**Theorem 1.1.** *Let  $f(t)$  be in  $A$ , and let  $G^n(u)$  denote the Sumudu transform of the  $n$ th derivative,  $f^n(t)$  of  $f(t)$ , then for  $n \geq 1$*

$$G^n(u) = \frac{G(u)}{u^n} - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{u^{n-k}}. \tag{1.6}$$

For more details, see [16].

We consider the general inhomogeneous nonlinear equation with initial conditions given below:

$$LU + RU + NU = h(x, t), \tag{1.7}$$

where  $L$  is the highest order derivative which is assumed to be easily invertible,  $R$  is a linear differential operator of order less than  $L$ ,  $NU$  represents the nonlinear terms and  $h(x, t)$  is the source term. First we explain the main idea of SDM: the method consists of applying Sumudu transform

$$S[LU] + S[RU] + S[NU] = S[h(x, t)]. \tag{1.8}$$

Using the differential property of Laplace transform and initial conditions we get

$$\frac{1}{u^n} S[U(x, t)] - \frac{1}{u^n} U(x, 0) - \frac{1}{u^{n-1}} U'(x, 0) - \dots - \frac{U^{n-1}(x, 0)}{u} + S[RU] + S[NU] = S[h(x, t)]. \tag{1.9}$$

By arrangement we have

$$S[U(x, t)] = U(x, 0) + uU'(x, 0) + \dots + u^{n-1}U^{n-1}(x, 0) - u^n S[RU] - u^n S[NU] + u^n S[h(x, t)]. \tag{1.10}$$

The second step in Sumudu decomposition method is that we represent solution as an infinite series:

$$U(x, t) = \sum_{i=0}^{\infty} U_i(x, t) \tag{1.11}$$

and the nonlinear term can be decomposed as

$$NU(x, t) = \sum_{i=0}^{\infty} A_i, \quad (1.12)$$

where  $A_i$  are Adomian polynomials [6] of  $U_0, U_1, U_2, \dots, U_n$  and it can be calculated by formula

$$A_i = \frac{1}{i!} \frac{d^i}{d\lambda^i} \left[ N \sum_{i=0}^{\infty} \lambda^i U_i \right]_{\lambda=0}, \quad i = 0, 1, 2, \dots \quad (1.13)$$

Substitution of (1.11) and (1.12) into (1.10) yields

$$\begin{aligned} S \left[ \sum_{i=0}^{\infty} U_i(x, t) \right] &= U(x, 0) + uU'(x, 0) + \dots + u^{n-1}U^{n-1}(x, 0) - u^n S[RU(x, t)] \\ &\quad - u^n S \left[ \sum_{i=0}^{\infty} A_i \right] + u^n S[h(x, t)]. \end{aligned} \quad (1.14)$$

On comparing both sides of (1.14) and by using standard ADM we have:

$$S[U_0(x, t)] = U(x, 0) + uU'(x, 0) + \dots + u^{n-1}U^{n-1}(x, 0) + u^n S[h(x, t)] = Y(x, u) \quad (1.15)$$

then it follows that

$$\begin{aligned} S[U_1(x, t)] &= -u^n S[RU_0(x, t)] - u^n S[A_0], \\ S[U_2(x, t)] &= -u^n S[RU_1(x, t)] - u^n S[A_1]. \end{aligned} \quad (1.16)$$

In more general, we have

$$S[U_{i+1}(x, t)] = -u^n S[RU_i(x, t)] - u^n S[A_i], \quad i \geq 0. \quad (1.17)$$

On applying the inverse Sumudu transform to (1.15) and (1.17), we get

$$\begin{aligned} U_0(x, t) &= K(x, t), \\ U_{i+1}(x, t) &= -S^{-1}[u^n S[RU_i(x, t)] + u^n S[A_i]], \quad i \geq 0, \end{aligned} \quad (1.18)$$

where  $K(x, t)$  represents the term that is arising from source term and prescribed initial conditions. On using the inverse Sumudu transform to  $h(x, t)$  and using the given condition we get

$$\Psi = \Phi + S^{-1}[h(x, t)], \quad (1.19)$$

where the function  $\Psi$ , obtained from a term by using the initial condition is given by

$$\Psi = \Psi_0 + \Psi_1 + \Psi_2 + \Psi_3 + \cdots + \Psi_n, \quad (1.20)$$

the terms  $\Psi_0, \Psi_1, \Psi_2, \Psi_3, \dots, \Psi_n$  appears while applying the inverse Sumudu transform on the source term  $h(x, t)$  and using the given conditions. We define

$$U_0 = \Psi_k + \cdots + \Psi_{k+r}, \quad (1.21)$$

where  $k = 0, 1, \dots, n, r = 0, 1, \dots, n - k$ . Then we verify that  $U_0$  satisfies the original equation (1.7). We now consider the particular form of inhomogeneous nonlinear partial differential equations:

$$LU + RU + NU = h(x, t) \quad (1.22)$$

with the initial condition

$$U(x, 0) = f(x), \quad U_t(x, 0) = g(x), \quad (1.23)$$

where  $L = \partial^2/\partial t^2$  is second-order differential operator,  $NU$  represents a general non-linear differential operator where as  $h(x, t)$  is source term. The methodology consists of applying Sumudu transform first on both sides of (1.10) and (1.23),

$$S[U(x, t)] = f(x) + ug(x) - u^2S[RU] - u^2S[NU] + u^2S[h(x, t)]. \quad (1.24)$$

Then by the second step in Sumudu decomposition method and inverse transform as in the previous we have

$$U(x, t) = f(x) + tg(t) - S^{-1} \left[ u^2S[RU] - u^2S[NU] \right] + S^{-1} \left[ u^2S[h(x, t)] \right]. \quad (1.25)$$

## 2. Applications

Now in order to illustrate STDM we consider some examples. Consider a nonlinear partial differential equation

$$U_{tt} + U^2 - U_x^2 = 0, \quad t > 0 \quad (2.1)$$

with initial conditions

$$\begin{aligned} U(x, 0) &= 0, \\ U_t(x, 0) &= e^x. \end{aligned} \quad (2.2)$$

By taking Sumudu transform for (2.1) and (2.2) we obtain

$$S[U(x,t)] = ue^x + u^2 S[U_x^2 - U^2]. \quad (2.3)$$

By applying the inverse Sumudu transform for (2.3), we get

$$[U(x,t)] = te^x + S^{-1} \left[ u^2 S[U_x^2 - U^2] \right] \quad (2.4)$$

which assumes a series solution of the function  $U(x,t)$  and is given by

$$U(x,t) = \sum_{i=0}^{\infty} U_i(x,t). \quad (2.5)$$

Using (2.4) into (2.5) we get

$$\sum_{i=0}^{\infty} U_i(x,t) = te^x + S^{-1} \left[ u^2 S \left[ \sum_{i=0}^{\infty} A_i(U) - \sum_{i=0}^{\infty} B_i(U) \right] \right]. \quad (2.6)$$

In (2.6)  $A_i(u)$  and  $B_i(u)$  are Adomian polynomials that represents nonlinear terms. So Adomian polynomials are given as follows:

$$\begin{aligned} \sum_{i=0}^{\infty} A_i(U) &= U_x^2, \\ \sum_{i=0}^{\infty} A_i(U) &= U^2. \end{aligned} \quad (2.7)$$

The few components of the Adomian polynomials are given as follows:

$$\begin{aligned} A_0(U) &= U_{0x}^2, & A_1(U) &= 2U_{0x}U_{1x}, & A_i(U) &= \sum_{r=0}^i U_{rx}U_{i-rx}, \\ B_0(U) &= U_0^2, & B_1(U) &= 2U_0U_1, & B_i(U) &= \sum_{r=0}^i U_rU_{i-r}. \end{aligned} \quad (2.8)$$

From the above equations we obtain

$$\begin{aligned} U_0(x,t) &= te^x, \\ U_{i+1}(x,t) &= S^{-1} \left[ S \left[ \sum_{i=0}^{\infty} A_i(U) - \sum_{i=0}^{\infty} B_i(U) \right] \right], \quad n \geq 0. \end{aligned} \quad (2.9)$$

Then the first few terms of  $U_i(x, t)$  follow immediately upon setting

$$\begin{aligned}
 U_1(x, t) &= S^{-1} \left[ u^2 S \left[ \sum_{i=0}^{\infty} A_0(U) - \sum_{i=0}^{\infty} B_0(U) \right] \right] \\
 &= S^{-1} \left[ u^2 S \left[ U_{0x}^2 - U_0^2 \right] \right] = S^{-1} \left[ u^2 S \left[ t^2 e^{2x} - t^2 e^{2x} \right] \right] \\
 &= S^{-1} \left[ u^2 S[0] \right] = 0.
 \end{aligned}
 \tag{2.10}$$

Therefore the solution obtained by LDM is given as follows:

$$U(x, t) = \sum_{i=0}^{\infty} U_i(x, t) = te^x.
 \tag{2.11}$$

*Example 2.1.* Consider the system of nonlinear coupled partial differential equation

$$\begin{aligned}
 U_t(x, y, t) - V_x W_y &= 1, \\
 V_t(x, y, t) - W_x U_y &= 5, \\
 W_t(x, y, t) - U_x V_y &= 5
 \end{aligned}
 \tag{2.12}$$

with initial conditions

$$\begin{aligned}
 U(x, y, 0) &= x + 2y, \\
 V(x, y, 0) &= x - 2y, \\
 W(x, y, 0) &= -x + 2y.
 \end{aligned}
 \tag{2.13}$$

Applying the Sumudu transform (denoted by  $S$ ) we have

$$\begin{aligned}
 U(x, y, u) &= x + 2y + u + uS[V_x W_y], \\
 V(x, y, u) &= x - 2y + 5u + uS[W_x U_y], \\
 W(x, y, u) &= -x + 2y + 5u + uS[U_x V_y].
 \end{aligned}
 \tag{2.14}$$

On using inverse Sumudu transform in (2.14), our required recursive relation is given by

$$\begin{aligned}
 U(x, y, t) &= x + 2y + t + S^{-1} [uS[V_x W_y]], \\
 V(x, y, t) &= x - 2y + 5t + S^{-1} [uS[W_x U_y]], \\
 W(x, y, t) &= -x + 2y + 5t + S^{-1} [uS[U_x V_y]].
 \end{aligned}
 \tag{2.15}$$

The recursive relations are

$$\begin{aligned}
 U_0(x, y, t) &= t + x + 2y, \\
 U_{i+1}(x, y, t) &= S^{-1} \left[ uS \left[ \sum_{i=0}^{\infty} C_i(V, W) \right] \right], \quad i \geq 0, \\
 V_0(x, y, t) &= 5t + x - 2y, \\
 V_{i+1}(x, y, t) &= S^{-1} \left[ uS \left[ \sum_{i=0}^{\infty} D_i(U, W) \right] \right], \quad i \geq 0, \\
 W_0(x, y, t) &= 5t - x + 2y, \\
 W_{i+1}(x, y, t) &= S^{-1} \left[ uS \left[ \sum_{i=0}^{\infty} E_i(U, V) \right] \right], \quad i \geq 0,
 \end{aligned} \tag{2.16}$$

where  $C_i(V, W)$ ,  $D_i(U, W)$ , and  $E_i(U, V)$  are Adomian polynomials representing the nonlinear terms [1] in above equations. The few components of Adomian polynomials are given as follows

$$\begin{aligned}
 C_0(V, W) &= V_{0x}W_{0y}, \\
 C_1(V, W) &= V_{1x}W_{0y} + V_{0x}W_{1y}, \\
 &\vdots \\
 C_i(V, W) &= \sum_{r=0}^i V_{rx}W_{i-ry}, \\
 D_0(U, W) &= U_{0y}W_{0x}, \\
 D_1(U, W) &= U_{1y}W_{0x} + W_{1x}U_{0y}, \\
 &\vdots \\
 D_i(U, W) &= \sum_{r=0}^i W_{rx}U_{i-ry}, \\
 E_0(U, V) &= U_{0x}V_{0y}, \\
 E_1(U, V) &= U_{1x}V_{0y} + U_{0x}V_{1y}, \\
 &\vdots \\
 E_i(V, W) &= \sum_{r=0}^i U_{rx}V_{i-ry}.
 \end{aligned} \tag{2.17}$$



By this recursive relation we can find other components of the solution

$$\begin{aligned}
 U_1(x, y, t) &= S^{-1}[uS[C_0(V, W)]] = S^{-1}[uS[V_{0x}W_{0y}]] = S^{-1}[uS[(1)(2)]] = 2t, \\
 V_1(x, y, t) &= S^{-1}[uS[D_0(U, W)]] = S^{-1}[uS[W_{0x}U_{0y}]] = S^{-1}[uS[(-1)(2)]] = -2t, \\
 W_1(x, y, t) &= S^{-1}[uS[E_0(U, V)]] = S^{-1}[uS[U_{0x}V_{0y}]] = S^{-1}[uS[(1)(-2)]] = -2t, \\
 U_2(x, y, t) &= S^{-1}[uS[C_1(V, W)]] = S^{-1}[uS[V_{1x}W_{0y} + V_{0x}W_{1y}]] = 0, \\
 V_2(x, y, t) &= S^{-1}[uS[D_1(U, W)]] = S^{-1}[uS[U_{0y}W_{1x} + U_{1y}W_{0x}]] = 0, \\
 W_2(x, y, t) &= S^{-1}[uS[D_1(U, V)]] = S^{-1}[uS[U_{1x}V_{0y} + U_{0x}V_{1y}]] = 0.
 \end{aligned}
 \tag{2.18}$$

The solution of above system is given by

$$\begin{aligned}
 U(x, y, t) &= \sum_{i=0}^{\infty} U_i(x, y, t) = x + 2y + 3t, \\
 V(x, y, t) &= \sum_{i=0}^{\infty} V_i(x, y, t) = x - 2y + 3t, \\
 W(x, y, t) &= \sum_{i=0}^{\infty} W_i(x, y, t) = -x + 2y + 3t.
 \end{aligned}
 \tag{2.19}$$

*Example 2.2.* Consider the following homogeneous linear system of PDEs:

$$\begin{aligned}
 U_i(x, t) - V_x(x, t) - (U - V) &= 2, \\
 V_i(x, t) + U_x(x, t) - (U - V) &= 2,
 \end{aligned}
 \tag{2.20}$$

with initial conditions

$$U(x, 0) = 1 + e^x, \quad V(x, 0) = -1 + e^x.
 \tag{2.21}$$

Taking the Sumudu transform on both sides of (2.20), then by using the differentiation property of Sumudu transform and initial conditions, (2.21) gives

$$\begin{aligned}
 S[U(x, t)] &= 1 + e^x - 2u + uS[V_x] + uS[U - V], \\
 S[V(x, t)] &= -1 + e^x - 2u - uS[U_x] + uS[U - V],
 \end{aligned}
 \tag{2.22}$$

$$U_x(x, t) = \sum_{i=0}^{\infty} U_{xi}(x, t), \quad V_x(x, t) = \sum_{i=0}^{\infty} V_{xi}(x, t).
 \tag{2.23}$$

Using the decomposition series (2.23) for the linear terms  $U(x, t)$ ,  $V(x, t)$  and  $U_x$ ,  $V_x$ , we obtain

$$\begin{aligned} S\left[\sum_{i=0}^{\infty} U_i(x, t)\right] &= 1 + e^x - 2u + uS\left[\sum_{i=0}^{\infty} V_{ix}\right] + uS\left[\sum_{i=0}^{\infty} U_i - \sum_{i=0}^{\infty} V_i\right], \\ S\left[\sum_{i=0}^{\infty} V_i(x, t)\right] &= -1 + e^x - 2u - uS\left[\sum_{i=0}^{\infty} U_{ix}\right] + uS\left[\sum_{i=0}^{\infty} U_i - \sum_{i=0}^{\infty} V_i\right]. \end{aligned} \quad (2.24)$$

The SADM presents the recursive relations

$$\begin{aligned} S[U_0(x, t)] &= 1 + e^x - 2u, \\ S[V_0(x, t)] &= -1 + e^x - 2u, \\ S[U_{i+1}] &= uS[V_{ix}] + uS[U_i - V_i], \quad i \geq 0, \\ S[V_{i+1}] &= -uS[U_{ix}] + uS[U_i - V_i], \quad i \geq 0. \end{aligned} \quad (2.25)$$

Taking the inverse Sumudu transform of both sides of (2.25) we have

$$\begin{aligned} U_0(x, t) &= 1 + e^x - 2t, \\ V_0(x, t) &= -1 + e^x - 2t, \\ U_1 &= S^{-1}[uS[V_{0x}] + uS[U_0 - V_0]] = S^{-1}[ue^x + 2u] = te^x + 2t, \\ V_1 &= S^{-1}[-uS[U_{0x}] + uS[U_0 - V_0]] = S^{-1}[-ue^x + 2u] = -te^x + 2t, \\ U_2 &= S^{-1}[u^2 e^x] = \frac{t^2}{2!} e^x, \\ V_2 &= S^{-1}[u^2 e^x] = \frac{t^2}{2!} e^x, \end{aligned} \quad (2.26)$$

and so on for other components. Using (1.11), the series solutions are given by

$$\begin{aligned} U(x, t) &= 1 + e^x \left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} \cdots\right), \\ V(x, t) &= -1 + e^x \left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} \cdots\right). \end{aligned} \quad (2.27)$$

Then the solutions follows

$$\begin{aligned} U(x, t) &= 1 + e^{x+t}, \\ V(x, t) &= -1 + e^{x-t}. \end{aligned} \quad (2.28)$$

*Example 2.3.* Consider the system of nonlinear partial differential equations

$$\begin{aligned} U_t + VU_x + U &= 1, \\ V_t - UV_x - V &= 1 \end{aligned} \tag{2.29}$$

with initial conditions

$$U(x, 0) = e^x, \quad V(x, 0) = e^{-x}. \tag{2.30}$$

On using Sumudu transform on both sides of (2.29), and by taking Sumudu transform for the initial conditions of (2.30) we get

$$\begin{aligned} S[U(x, t)] &= e^x + u - uS[VU_x] - uS[U], \\ S[V(x, t)] &= e^{-x} + u + uS[UV_x] + uS[V]. \end{aligned} \tag{2.31}$$

Similar to the previous example, we rewrite  $U(x, t)$  and  $V(x, t)$  by the infinite series (1.11), then inserting these series into both sides of (2.31) yields

$$\begin{aligned} S\left[\sum_{i=0}^{\infty} U_i(x, t)\right] &= e^x + u - uS\left[\sum_{i=0}^{\infty} A_i\right] - uS\left[\sum_{i=0}^{\infty} U_i\right], \\ S\left[\sum_{i=0}^{\infty} V_i(x, t)\right] &= e^{-x} + u + uS\left[\sum_{i=0}^{\infty} B_i\right] - uS\left[\sum_{i=0}^{\infty} V_i\right], \end{aligned} \tag{2.32}$$

where the terms  $A_i$  and  $B_i$  are handled with the help of Adomian polynomials by (1.12) that represent the nonlinear terms  $VU_x$  and  $UV_x$ , respectively. We have a few terms of the Adomian polynomials for  $VU_x$  and  $UV_x$  which are given by

$$\begin{aligned} A_0 &= U_{0x}V_0, & A_1 &= U_{0x}V_1 + U_{1x}V_0, \\ A_2 &= U_{0x}V_2 + U_{1x}V_1 + U_{2x}V_0, \\ & \vdots \\ B_0 &= V_{0x}U_0, & B_1 &= V_{0x}U_1 + V_{1x}U_0, \\ B_2 &= V_{0x}U_2 + V_{1x}U_1 + V_{2x}U_0, \\ & \vdots \end{aligned} \tag{2.33}$$

By taking the inverse Sumudu transform we have

$$U_0 = e^x + t, \quad (2.34)$$

$$V_0 = e^{-x} + t,$$

$$U_{i+1} = S^{-1}[u] - S^{-1}[uS[A_i]] - S^{-1}[uS[U_i]], \quad (2.35)$$

$$V_{i+1} = S^{-1}[u] + S^{-1}[uS[B_i]] + S^{-1}[uS[V_i]].$$

Using the inverse Sumudu transform on (2.35) we have

$$U_1 = -t - \frac{t^2}{2!} - te^x - \frac{t^2}{2!}e^x, \quad (2.36)$$

$$V_1 = -t - \frac{t^2}{2!} + te^{-x} - \frac{t^2}{2!}e^{-x},$$

$$U_2 = \frac{t^2}{2!} + \frac{t^2}{2!}e^x \dots,$$

$$V_2 = \frac{t^2}{2!} + \frac{t^2}{2!}e^{-x} \dots.$$

The rest terms can be determined in the same way. Therefore, the series solutions are given by

$$U(x, t) = e^x \left( 1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} \dots \right), \quad (2.37)$$

$$V(x, t) = e^{-x} \left( 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} \dots \right).$$

Then the solution for the above system is as follows:

$$U(x, t) = e^{x-t}, \quad V(x, t) = e^{-x+t}. \quad (2.38)$$

### 3. Conclusion

The Sumudu transform-Adomian decomposition method has been applied to linear and nonlinear systems of partial differential equations. Three examples have been presented, this method shows that it is very useful and reliable for any nonlinear partial differential equation systems. Therefore, this method can be applied to many complicated linear and nonlinear PDEs.

## Acknowledgment

The authors would like to express their sincere thanks and gratitude to the reviewers for their valuable comments and suggestions for the improvement of this paper. The first author acknowledges the support by the Research Center, College of Science, King Saud University. The second author also gratefully acknowledges the partial support by the University Putra Malaysia under the Research University Grant Scheme (RUGS) 05-01-09-0720RU and FRGS 01-11-09-723FR.

## References

- [1] E. Alizadeh, M. Farhadi, K. Sedighi, H. R. Ebrahimi-Kebria, and A. Ghafourian, "Solution of the Falkner-Skan equation for wedge by Adomian Decomposition method," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 3, pp. 724–733, 2009.
- [2] A.-M. Wazwaz, "The modified decomposition method and Padé approximants for a boundary layer equation in unbounded domain," *Applied Mathematics and Computation*, vol. 177, no. 2, pp. 737–744, 2006.
- [3] N. Bellomo and R. A. Monaco, "A comparison between Adomian's decomposition methods and perturbation techniques for nonlinear random differential equations," *Journal of Mathematical Analysis and Applications*, vol. 110, no. 2, pp. 495–502, 1985.
- [4] R. Race, "On the Adomian decomposition method and comparison with Picard's method," *Journal of Mathematical Analysis and Applications*, vol. 128, pp. 480–483, 1987.
- [5] G. Adomian and R. Rach, "Noise terms in decomposition solution series," *Computers & Mathematics with Applications*, vol. 24, no. 11, pp. 61–64, 1992.
- [6] A. M. Wazwaz, "Necessary conditions for the appearance of noise terms in decomposition solution series," *Applied Mathematics and Computation*, vol. 81, pp. 1718–1740, 1987.
- [7] A. Kılıçman and H. Eltayeb, "A note on integral transforms and partial differential equations," *Applied Mathematical Sciences*, vol. 4, no. 3, pp. 109–118, 2010.
- [8] M. A. Asiru, "Sumudu transform and the solution of integral equations of convolution type," *International Journal of Mathematical Education in Science and Technology*, vol. 32, no. 6, pp. 906–910, 2001.
- [9] M. A. Aşiru, "Further properties of the Sumudu transform and its applications," *International Journal of Mathematical Education in Science and Technology*, vol. 33, no. 3, pp. 441–449, 2002.
- [10] A. Kadem and A. Kılıçman, "Note on transport equation and fractional Sumudu transform," *Computers & Mathematics with Applications*, vol. 62, no. 8, pp. 2995–3003, 2011.
- [11] H. Eltayeb, A. Kılıçman, and B. Fisher, "A new integral transform and associated distributions," *Integral Transforms and Special Functions*, vol. 21, no. 5, pp. 367–379, 2010.
- [12] A. Kılıçman and H. Eltayeb, "On the applications of Laplace and Sumudu transforms," *Journal of the Franklin Institute*, vol. 347, no. 5, pp. 848–862, 2010.
- [13] A. Kılıçman, H. Eltayeb, and R. P. Agarwal, "On Sumudu transform and system of differential equations," *Abstract and Applied Analysis*, vol. 2010, Article ID 598702, 11 pages, 2010.
- [14] A. Kılıçman, H. Eltayeb, and K. A. M. Atan, "A note on the comparison between Laplace and Sumudu transforms," *Bulletin of the Iranian Mathematical Society*, vol. 37, no. 1, pp. 131–141, 2011.
- [15] H. Agwa, F. Ali, and A. Kılıçman, "A new integral transform on time scales and its applications," *Advances in Difference Equations*, vol. 2012, article 60, 2012.
- [16] F. B. M. Belgacem, A. A. Karaballi, and S. L. Kalla, "Analytical investigations of the Sumudu transform and applications to integral production equations," *Mathematical Problems in Engineering*, vol. 2003, no. 3, pp. 103–118, 2003.

