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Research Article

Asymptotic Energy Estimates for Nonlinear Petrovsky Plate Model Subject to Viscoelastic Damping

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We consider the nonlinear Petrovsky plate model under the presence of long-time memory. Under suitable conditions, we show that the energy functional associated with the equation decays exponentially or polynomially to zero as time goes to infinity.

1. Introduction

Let Ω be a bounded domain of $\mathbb{R}^2$ with a sufficiently smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$. We will assume that $\Gamma_0 \cap \Gamma_1 = \emptyset$ and Γ_0, Γ_1 have positive measures. The vector $\boldsymbol{\nu} = (\nu_1, \nu_2)$ is the unit exterior normal and $\boldsymbol{\tau} = (-\nu_2, \nu_1)$ represents the tangential direction to Γ. Here, the variable w represents displacement of a plate occupying the domain Ω. The governing equation is given by

$$\begin{aligned} w''(t) - \gamma \Delta w''(t) + \Delta^2 w(t) + \Delta^2 \int_0^\infty h'(s) w^t(s) ds \\ = \operatorname{div}[\mathcal{C}(\sigma(t)) \nabla w(t)] \quad \text{in} \quad \Omega \times (0, \infty), \end{aligned} \tag{1.1}$$

where the notations $w^t(s) = w(t-s)$, $\ ' = \partial_t$. The positive constant γ is the proportional to the thickness of the plate, $h(t)$ is relaxation function, div stands for scalar divergence of a vector field, the stress resultant $\sigma(t)$ is given by

$$\sigma(t) := f(\nabla w(t)), \tag{1.2}$$

and the nonlinear function $f : \mathbb{R}^2 \to M$ is defined as $f(s) = (1/2)s \otimes s$ for all $s \in \mathbb{R}^2$. $\mathcal{C}$ is a linear operator defined on M with value on M, the space of 2×2 symmetric matrices, and defined as

$$\mathcal{C}(\sigma) = \frac{E}{d(1-\mu^2)}\left[(\mu \operatorname{tr}\sigma)I + (1-\mu)\sigma\right] \tag{1.3}$$

for any $\sigma \in M$, where I is the identity matrix and $\operatorname{tr}\sigma$ denotes the trace of σ. Moreover, $d > 0$ is the density of the shell, $E > 0$ denotes the Young modulus, and μ $(0 < \mu < 1/2)$ is the Poisson's ratio.

With (1.1), we associate the boundary conditions on the portion of the boundary Γ_0,

$$w = \partial_\nu w = 0 \quad \text{on } \Gamma_0 \times (0,\infty), \tag{1.4}$$

and the boundary conditions on the remaining portion of the boundary Γ_1,

$$\begin{gathered} \mathcal{B}_1 w(t) + \mathcal{B}_1 \int_0^\infty h'(s) w^t(s)ds = 0 \quad \text{on } \Gamma_1 \times (0,\infty), \\ \mathcal{B}_2 w(t) - \gamma \partial_\nu w''(t) + \mathcal{B}_2 \int_0^\infty h'(s) w^t(s)ds - \mathcal{C}(\sigma(t))\nu \cdot \nabla w(t) = 0 \quad \text{on } \Gamma_1 \times (0,\infty), \end{gathered} \tag{1.5}$$

where

$$\mathcal{B}_1 w = \Delta w + (1-\mu)B_1 w, \qquad \mathcal{B}_2 w = \partial_\nu \Delta w + (1-\mu)\partial_\tau B_2 w \tag{1.6}$$

and the boundary operators B_1 and B_2 are defined by

$$B_1 w = 2\nu_1\nu_2 w_{xy} - \nu_1^2 w_{yy} - \nu_2^2 w_{xx}, \qquad B_2 w = \left(\nu_1^2 - \nu_2^2\right) w_{xy} + \nu_1\nu_2\left(w_{yy} - w_{xx}\right). \tag{1.7}$$

With (1.1), we also associate the initial conditions

$$w(0) = w^0, \quad w'(0) = w^1 \quad \text{in } \Omega, \; w(s) = \chi(s), \; -\infty < s < 0, \tag{1.8}$$

where

$$\left(\chi(s), \chi'(s)\right) \in L_c^\infty\left(-\infty, 0; H^3 \times H^2\right), \tag{1.9}$$

the symbol L_c^∞ denotes the subspace of L^∞ such that there exists a constant T such that the functions vanish as $s < -T$.

This problem has its origin in the mathematical description of viscoelastic materials. It is well known that viscoelastic materials exhibit natural damping, which is due to the special property of these materials, to retain a memory of their past history.From the mathematical point of view, these damping effects are modeled by integro-differential operators. Therefore, the dynamics of viscoelastic materials are of great importance and interest as they have wide applications in natural sciences. From the physical point of view, the problem (1.1) describes the position $w(x, y, t)$ of the material particle (x, y) at time t, which is clamped in the portion Γ_0 of its boundary.

Models of Petrovsky type are of interest in applications in various areas in mathematical physics, as well as in geophysics and ocean acoustics [1, 2]. The Petrovsky type models without memory were discussed in [3, 4]. Messaoudi [3] considered the initial-boundary value problem

$$\begin{gathered} u_{tt} + \Delta^2 u + |u_t|^{m-2} u_t = |u|^{p-2} u, \quad x \in \Omega,\ t > 0, \\ u(x,t) = \partial_\nu u(x,t) = 0, \quad x \in \partial\Omega,\ t \geq 0, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega, \end{gathered} \tag{1.10}$$

established an existence result for (1.10), and showed that the solution continues to exist globally if $m \geq p$, however if $m < p$ and the initial energy is negative, the solution blows up in finite time. Chen and Zhou [4] proved that the solution of (1.10) blows up with positive initial energy. Moreover, she claimed that the solution blows up in finite time for vanishing initial energy under the condition $m = 2$ by different method.

The Petrovsky type equations with memory arouse the attention of mathematicians to study them. Alabau-Boussouira et al. [5] discussed the initial-boundary value problem of linear Petrovsky equation related to a plate model with memory,

$$\begin{gathered} u_{tt} + \Delta^2 u - \int_0^t g(t-s)\Delta^2 u(t,s)\,ds = 0 \quad \text{in } \Omega \times (0,\infty), \\ u = \partial_\nu u = 0 \quad \text{on } \partial\Omega \times (0,\infty), \\ u|_{t=0} = u_0, \quad u_t|_{t=0} = u_1 \quad \text{in } \Omega, \end{gathered} \tag{1.11}$$

and showed that the solution decays exponentially or polynomially as $t \to +\infty$ if the initial data is sufficient small. Yang [6] considered the problem in N-dimensional space,

$$\begin{gathered} u_{tt} + \Delta^2 u + \lambda u_t = \sum_{i=1}^{N} \frac{\partial}{\partial x_i} \sigma_i(u_{x_i}) \quad \text{in } \Omega \times (0,\infty), \\ u|_{\partial\Omega} = 0, \quad \left.\frac{\partial u}{\partial n}\right|_{\partial\Omega} = 0 \quad \text{on } [0,\infty), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad x \in \Omega, \end{gathered} \tag{1.12}$$

and proved that under rather mild conditions on nonlinear terms and initial data the above-mentioned problem admits a global weak solution and the solution decays exponentially to zero as $t \to \infty$ in the states of large initial data and small initial energy. In particular, in the case of space dimension $N = 1$, the weak solution is regularized to be a unique generalized solution. And if the conditions guaranteeing the global existence of weak solutions are not valid, then under the opposite conditions, the solutions of above-mentioned problem blow up in finite time. Muñoz Rivera et al. [7] considered the initial-boundary problem for viscoelastic plate equation,

$$\begin{gathered}
u_{tt} - \gamma\Delta u_{tt} + \Delta^2 u(t) - \int_0^t g(t-\tau)\Delta^2 u(\tau)d\tau = 0 \quad \text{in } \Omega \times (0,\infty), \\
u(x,y,0) = u_0(x,y), \quad u_t(x,y,0) = u_1(x,y) \quad \text{in } \Omega, \\
u = \partial_\nu u = 0 \quad \text{on } \Gamma_0 \times (0,\infty), \\
\mathcal{B}_1 u(t) + \mathcal{B}_1 \int_0^\infty g(s)u^t(s)ds = 0 \quad \text{on } \Gamma_1 \times (0,\infty), \\
\mathcal{B}_2 u(t) - \gamma\partial_\nu u''(t) + \mathcal{B}_2 \int_0^\infty g(s)u^t(s)ds = 0 \quad \text{on } \Gamma_1 \times (0,\infty),
\end{gathered} \tag{1.13}$$

and proved that the first and second order energies associated with its solution decay exponentially provided the kernel of the convolution also decays exponentially. When the kernel decays polynomially then the energy also decays polynomially. More precisely if the kernel g satisfies

$$g(t) \le -c_0 g^{1+(1/p)}(t), \quad g, g^{1+(1/p)} \in L^1(R) \quad \text{with } p > 2, \tag{1.14}$$

then the energy decays as $1/(1+t)^p$. On the recently related papers concerning the Petrovsky type models, the readers can see references [8–12].

In [13–15], Li et al. proved the existence uniqueness, uniform rates of decay, and limit behavior of the solution to nonlinear viscoelastic Marguerre-von Karman shallow shells system, respectively. To our best knowledge, we do not find the research report on the problem (1.1) which is considered in this paper.

Motivated by the above work, we obtain the energy functional associated with the equation decays exponentially or polynomially to zero as time goes to infinity. The main contribution of this paper are as follows. (a) The problem considered in this paper is nonlinear equation with integral dissipation, to our knowledge this model has not been considered; (b) the hypothesis on h and initial data are weaker; (c) we naturally define the energy by simple computation and only define simple auxiliary functionals to prove our result by precise priori estimates.

The outline of this paper is the following. In Section 2, we present some material needed to be proved. Section 3 contains the statement and the proof of our results.

2. Notations and Preliminaries

In this section, we will prepare some material needed in the proof of our main results. We use standard Lebesgue space $L^p(\Omega)$ and Sobolev space $H^m(\Omega)$ and adopt the following notations

$$(u,v) := (u,v)_{L^2(\Omega)}, \qquad \langle u,v\rangle := (u,v)_{L^2(\Gamma_1)}, \qquad \|u\|_p := \|u\|_{L^p(\Omega)}, \qquad \|u\| := \|u\|_{L^2(\Omega)}. \tag{2.1}$$

We denote

$$|\mathbf{A}(t)|^2 := \sum_{i,j} a_{ij}^2(t), \qquad \frac{d}{dt}\mathbf{A}(t) := \mathbf{A}'(t) = \left(a'_{ij}(t)\right), \qquad \mathbf{A}(t)\cdot \mathbf{B}(t) := \sum_{i,j} a_{ij}(t) b_{ij}(t),$$
$$(\mathbf{A}(t),\mathbf{B}(t)) := \sum_{i,j}\left(a_{ij}(t), b_{ij}(t)\right) \tag{2.2}$$

for any pair of tensors $\mathbf{A}(t) = (a_{ij}(t))$ and $\mathbf{B}(t) = (b_{ij}(t))$. We introduce the following space

$$W(\Omega) = \left\{w \in H^2(\Omega),\ w = \partial_\nu w = 0 \ \text{ on } \ \Gamma_0\right\}. \tag{2.3}$$

Define the bilinear form $a(\cdot,\cdot)$ as follows

$$a(w,v) = \int_\Omega \left[w_{xx}v_{xx} + w_{yy}v_{yy} + \mu\left(w_{xx}v_{yy} + w_{yy}v_{xx}\right) + 2(1-\mu)w_{xy}v_{xy}\right]dx\,dy. \tag{2.4}$$

For simplicity, we denote $a(w,w)$ by $a(w)$. For the relaxation function $h(t)$, we assume that

(A_1) $h : [0,+\infty) \rightarrow (0,+\infty)$ is a C^2 function satisfying

$$h(t) > 0, \qquad h'(t) < 0, \qquad h''(t) \geq 0. \tag{2.5}$$

(A_2) Both $h_\infty := h(\infty)$ and $h'_\infty := h'(\infty)$ exist, and

$$h_\infty > 0, \qquad h'_\infty = 0, \qquad h(0) = 1. \tag{2.6}$$

Hypotheses (A_1) assure that the viscoelastic energy (defined below) is nonincreasing and the assumption (A_2) means that the material behaves like an viscoelastic solid at $t = +\infty$ (cf. [16]).

Our results are based on the following existence theorem.

Theorem 2.1. *One assumes that $h(t)$ satisfies conditions (A_1)-(A_2). For any initial data*

$$\left(w^0, w^1\right) \in H^3(\Omega) \times H^2(\Omega) \tag{2.7}$$

subject to the compatibility conditions satisfied on the boundary Γ,

$$\begin{aligned}
&\nabla w^0 = \nabla w^1 = \mathbf{0}, \quad w^1 = w^0 = 0 \quad \text{on } \Gamma_0,\\
&\mathcal{B}_1\left(w^0 + \int_0^\infty h'(s)\chi(-s)ds\right) = 0 \quad \text{on } \Gamma_1.
\end{aligned} \tag{2.8}$$

Then for any $T > 0$*, there exists a unique global solution to* (1.1)–(1.10) *satisfying*

$$(w, w', w'') \in L^\infty\left(0, T; H^3(\Omega) \times H^2(\Omega) \times H^1(\Omega)\right). \tag{2.9}$$

Proof. We can use the method used in [13, 17] to show the existence and uniqueness as well as the regularity of the global solution. □

At First, the energy of the system must be properly defined. The total energy can be expected to consist of two parts. One part involves the current kinetic and strain energies, and the other will involve the past history of strains. To obtain the appropriate expression of energy functional, using assumption (A_2), we rewrite system (1.1) and (1.5) in the form

$$\begin{aligned}
w''(t) - \gamma\Delta w''(t) + h_\infty \Delta^2 w(t) + \Delta^2 \int_0^\infty h'(s)\left[w^t(s) - w(t)\right]ds&\\
= \operatorname{div}[\mathcal{C}(\sigma(t))\nabla w(t)] \quad \text{in } \Omega \times (0, \infty),&
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
&h_\infty \mathcal{B}_1 w(t) + \mathcal{B}_1 \int_0^\infty h'(s)\left[w^t(s) - w(t)\right]ds = 0 \quad \text{on } \Gamma_1 \times (0, \infty),\\
&h_\infty \mathcal{B}_2 w(t) - \gamma\partial_\nu w''(t) + \mathcal{B}_2 \int_0^\infty h'(s)\left[w^t(s) - w(t)\right]ds\\
&\qquad + \mathcal{C}(\sigma(t))\nu \cdot \nabla w(t) = 0 \quad \text{on } \Gamma_1 \times (0, \infty).
\end{aligned} \tag{2.11}$$

In order to define the energy functional, we give the following lemma.

Lemma 2.2. *Let* w *and* v *be the functions in* $H^4(\Omega) \cap W$*. Then, one has*

$$\int_\Omega \left(\Delta^2 w\right) v\,dx\,dy = a(w, v) + \int_{\Gamma_1} [(\mathcal{B}_2 w)v - (\mathcal{B}_1 w)\partial_\nu v]d\Gamma. \tag{2.12}$$

Proof. The definition of $a(w, v)$ gives

$$\int_\Omega \Delta w \Delta v\,dx\,dy = a(w, v) + \int_\Omega \left[(1-\mu)\left(w_{xx}v_{yy} + w_{yy}v_{xx}\right) - 2(1-\mu)w_{xy}v_{xy}\right]dx\,dy. \tag{2.13}$$

Using Green's formula, we see

$$\begin{aligned}\int_{\Omega}\left(\Delta^2 w\right)v\,dx\,dy &= \int_{\Gamma_1}(\partial_\nu \Delta w)v\,d\Gamma - \int_{\Gamma_1}\Delta w\,\partial_\nu v\,d\Gamma + \int_{\Omega}\Delta w\,\Delta v\,dx\,dy \\ &= \int_{\Gamma_1}(\partial_\nu \Delta w)v\,d\Gamma - \int_{\Gamma_1}\Delta w\,\partial_\nu v\,d\Gamma + a(w,v) \\ &\quad + \int_{\Omega}\left[(1-\mu)\left(w_{xx}v_{yy}+w_{yy}v_{xx}\right) - 2(1-\mu)w_{xy}v_{xy}\right]dx\,dy,\end{aligned} \tag{2.14}$$

$$\begin{aligned}\int_{\Omega}\left[\left(w_{xx}v_{yy}+w_{yy}v_{xx}\right) - 2w_{xy}v_{xy}\right]dx\,dy &= \int_{\Gamma_1}\left(w_{xx}v_y\nu_2 + w_{yy}v_x\nu_1\right)d\Gamma \\ &\quad - \int_{\Omega}\left(w_{xxy}v_y + w_{xyy}v_x\right)dx\,dy \\ &\quad - \int_{\Gamma_1}\left(w_{xy}v_y\nu_1 + w_{xy}v_x\nu_2\right)d\Gamma \\ &\quad + \int_{\Omega}\left(w_{xxy}v_y + w_{xyy}v_x\right)dx\,dy \\ &= \int_{\Gamma_1}\left(w_{xx}v_y\nu_2 + w_{yy}v_x\nu_1\right)d\Gamma \\ &\quad - \int_{\Gamma_1}\left(w_{xy}v_y\nu_1 + w_{xy}v_x\nu_2\right)d\Gamma.\end{aligned} \tag{2.15}$$

Using

$$\partial_\nu v = v_x\nu_1 + v_y\nu_2, \qquad \partial_\tau v = -v_x\nu_2 + v_y\nu_1, \tag{2.16}$$

we get

$$v_x = \partial_\nu v\nu_1 - \partial_\tau v\nu_2, \qquad v_y = \partial_\nu v\nu_2 + \partial_\tau v\nu_1. \tag{2.17}$$

Inserting (2.14) and (2.15) into (2.17) and noting

$$\int_{\Gamma_1}\partial_\tau (wv)d\Gamma = \int_{\Gamma}\partial_\tau (wv)d\Gamma = \int_{\Gamma}(wv)_x\,dx + (wv)_y\,dy = \int_{\Omega}\left[(wv)_{yx} - (wv)_{xy}\right]dxdy = 0, \tag{2.18}$$

we have

$$\int_{\Gamma_1} w\partial_\tau v\,d\Gamma = -\int_{\Gamma_1} v\partial_\tau w\,d\Gamma, \tag{2.19}$$

we obtain the conclusion. □

In order to define the energy function $E(t)$ of the problem (1.1)–(1.8), we give the following computations. Multiplying equation (2.10) by w', integrating the result over Ω and adding Green's formula, we get from Lemma 2.2 and (1.4) that

$$\begin{aligned}
&\int_\Omega w''(t)w'(t)dxdy + \gamma\int_\Omega \nabla w''(t)\cdot\nabla w'(t)dxdy + h_\infty a(w(t),w'(t))\\
&\quad+\int_{\Gamma_1}\left[w'(t)\left(h_\infty \mathcal{B}_2 w(t) - \gamma\partial_\nu w''(t)\right) - h_\infty \partial_\nu w'(t)\mathcal{B}_1 w(t)\right]d\Gamma\\
&\quad+\int_0^\infty h'(s)\left(\Delta^2 w^t(s) - \Delta^2 w(t),\, w'(t)\right)ds\\
&=\int_{\Gamma_1} \mathcal{C}(\sigma(t))\nabla w(t)\cdot\nu d\Gamma - \int_\Omega \mathcal{C}(\sigma(t))\nabla w(t)\cdot\nabla w'(t)dxdy\\
&=\int_{\Gamma_1} \mathcal{C}(\sigma(t))\nabla w(t)\cdot\nu d\Gamma - \int_\Omega \mathcal{C}(\sigma(t))\cdot\nabla w'(t)\otimes\nabla w(t)dxdy.
\end{aligned}\tag{2.20}$$

Clearly

$$\begin{aligned}
\left(\Delta^2 w^t(s) - \Delta^2 w(t),\, w'(t)\right) = &-\left(\Delta^2 w^t(s) - \Delta^2 w(t),\, w'^t(s) - w'(t)\right)\\
&+\left(\Delta^2 w'^t(s) - \Delta^2 w(t),\, w'^t(s)\right).
\end{aligned}\tag{2.21}$$

By applying Lemma 2.2, the first term on the right-hand side of (2.21) equals

$$\begin{aligned}
-\left(\Delta^2 w^t(s) - \Delta^2 w(t),\, w'^t(s) - w'(t)\right) = &-\frac{1}{2}\partial_t a\left(w^t(s) - w(t)\right)\\
&-\int_{\Gamma_1}\left[\left(w'^t(s) - w'(t)\right)\mathcal{B}_2\left(w^t(s) - w(t)\right)\right.\\
&\left.-\partial_\nu\left(w'^t(s) - w'(t)\right)\mathcal{B}_1\left(w^t(s) - w(t)\right)\right]d\Gamma.
\end{aligned}\tag{2.22}$$

In the same way, the second term on the right-hand side of (2.21) equals

$$\begin{aligned}
\left(\Delta^2 w'^t(s) - \Delta^2 w(t),\, w'^t(s)\right) = &-\frac{1}{2}\partial_s a\left(w^t(s) - w(t)\right)\\
&+\int_{\Gamma_1}\left[w'^t(s)\mathcal{B}_2\left(w^t(s) - w(t)\right)\right.\\
&\left.-\partial_\nu\left(w'^t(s)\right)\mathcal{B}_1\left(w^t(s) - w(t)\right)\right]d\Gamma.
\end{aligned}\tag{2.23}$$

Therefore, from (2.21)–(2.23), we have

$$\begin{aligned}\left(\Delta^2 w^t(s)-\Delta^2 w(t),\, w'(t)\right) = &-\frac{1}{2}\partial_t a\left(w^t(s)-w(t)\right)-\frac{1}{2}\partial_s a\left(w^t(s)-w(t)\right)\\ &+\int_{\Gamma_1}\left[w'(t)\mathcal{B}_2\left(w^t(s)-w(t)\right)-\partial_\nu w'(t)\mathcal{B}_1\left(w^t(s)-w(t)\right)\right]d\Gamma.\end{aligned} \tag{2.24}$$

Note

$$-\frac{1}{2}\int_0^\infty h'(s)\partial_t a\left(w^t(s)-w(t)\right)ds=-\frac{1}{2}\frac{d}{dt}\int_0^\infty h'(s)a\left(w^t(s)-w(t)\right)ds. \tag{2.25}$$

and (by $h'_\infty=0$)

$$\begin{aligned}-\frac{1}{2}\int_0^\infty h'(s)\partial_s a\left(w^t(s)-w(t)\right)ds &= -h'(s)a\left(w^t(s)-w(t)\right)\Big|_0^\infty+\frac{1}{2}\int_0^\infty h''(s)a\left(w^t(s)-w(t)\right)ds\\ &=\frac{1}{2}\int_0^\infty h''(s)a\left(w^t(s)-w(t)\right)ds.\end{aligned} \tag{2.26}$$

Consequently, we conclude form (2.24)–(2.26) that

$$\begin{aligned}\int_0^\infty h'(s)\left(\Delta^2 w^t(s)-\Delta^2 w(t), w'(t)\right)ds = &-\frac{1}{2}\frac{d}{dt}\int_0^\infty h'(s)a\left(w^t(s)-w(t)\right)ds\\ &+\frac{1}{2}\int_0^\infty h''(s)a\left(w^t(s)-w(t)\right)ds\\ &+\int_0^\infty h'(s)\int_{\Gamma_1}\left[w'(t)\mathcal{B}_2\left(w^t(s)-w(t)\right)\right.\\ &\left.-\partial_\nu w'(t)\mathcal{B}_1\left(w^t(s)-w(t)\right)\right]d\Gamma ds.\end{aligned} \tag{2.27}$$

Summing (2.20) and (2.27), using the boundary conditions (2.11) together with the symmetric of $\sigma(t)$ yields

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2+h_\infty a(w(t))-\int_0^\infty h'(s)a\left(w^t(s)-w(t)\right)ds\right)\\ &\quad+\int_\Omega \mathcal{C}(\sigma(t))\cdot\nabla w'(t)\otimes\nabla w(t)dxdy+\frac{1}{2}\int_0^\infty h''(s)a\left(w^t(s)-w(t)\right)ds=0.\end{aligned} \tag{2.28}$$

Note the relation

$$\int_{\Omega} \mathcal{C}(\sigma(t)) \cdot \nabla w'(t) \otimes \nabla w(t) dxdy = \frac{1}{4}\frac{d}{dt}\int_{\Omega} \mathcal{C}(\sigma(t)) \cdot \sigma(t) dxdy. \tag{2.29}$$

In fact

$$\begin{aligned}
&\int_{\Omega} \mathcal{C}(\sigma(t)) \cdot \nabla w'(t) \otimes \nabla w(t) dxdy \\
&\quad = \alpha \int_{\Omega} \begin{pmatrix} w_x w_x + \mu w_y w_y & (1-\mu) w_x w_y \\ (1-\mu) w_y w_x & w_y w_y + \mu w_x w_x \end{pmatrix} \cdot \begin{pmatrix} w'_x w_x & w'_x w_y \\ w'_y w_x & w'_y w_y \end{pmatrix} dxdy \\
&\quad = \alpha \int_{\Omega} \left[(w_x)^3 w'_x + (w_y)^3 w'_y + (w_y)^2 w_x w'_x + (w_x)^2 w_y w'_y \right] dxdy
\end{aligned} \tag{2.30}$$

with $\alpha = E/d(1-\mu^2)$. Denote $\sigma := \left(\begin{smallmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{smallmatrix}\right)$. Hence

$$\begin{aligned}
\mathcal{C}(\sigma) &= \alpha \begin{pmatrix} \sigma_{11} + \mu\sigma_{22} & (1-\mu)\sigma_{12} \\ (1-\mu)\sigma_{21} & \sigma_{22} + \mu\sigma_{11} \end{pmatrix}, \\
[\mathcal{C}(\sigma(t))]' \cdot \sigma(t) &= \alpha \left[\sigma'_{11}\sigma_{11} + \sigma'_{22}\sigma_{22} + \mu\left(\sigma'_{11}\sigma_{22} + \sigma'_{22}\sigma_{11}\right) \right. \\
&\qquad \left. + (1-\mu)\left(\sigma'_{12}\sigma_{12} + \sigma'_{21}\sigma_{21}\right) \right] \\
&= \mathcal{C}(\sigma(t)) \cdot [\sigma(t)]',
\end{aligned} \tag{2.31}$$

which implies that

$$\begin{aligned}
&\frac{d}{dt}\int_{\Omega} \mathcal{C}(\sigma(t)) \cdot \sigma(t) dxdy \\
&\quad = \int_{\Omega} [\mathcal{C}(\sigma(t))]' \cdot \sigma(t) dxdy + \int_{\Omega} \mathcal{C}(\sigma(t)) \cdot \sigma'(t) dxdy \\
&\quad = 2\int_{\Omega} [\mathcal{C}(\sigma(t))]' \cdot \sigma(t) dxdy \\
&\quad = 4\alpha \int_{\Omega} \left[(w_x)^3 w'_x + (w_y)^3 w'_y + (w_y)^2 w_x w'_x + (w_x)^2 w_y w'_y \right] dxdy.
\end{aligned} \tag{2.32}$$

From (2.30) and (2.32), we conclude that (2.29) holds. Combining (2.28) and (2.29), we conclude that

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\left[\left\| w'(t) \right\|^2 + \gamma \left\| \nabla w'(t) \right\|^2 + h_\infty a(w(t)) \right. \\
&\qquad \left. - \int_0^\infty h'(s) a\left(w^t(s) - w(t)\right) ds + \frac{1}{2}\int_{\Omega} \mathcal{C}(\sigma(t)) \cdot \sigma(t) dxdy \right] \\
&\quad = -\frac{1}{2}\int_0^\infty h''(s) a\left(w^t(s) - w(t)\right) ds.
\end{aligned} \tag{2.33}$$

The relation (2.33) inspires us to define energy functional as the following

$$E(t) = \frac{1}{2}\left(\|w'(t)\|^2 + \gamma\|\nabla w'(t)\|^2\right) + \frac{1}{2}\left(h_\infty a\left(w(t)\right) - \int_0^\infty h'(s)a\left(w^t(s) - w(t)\right)ds + \frac{1}{2}\int_\Omega C\left(\sigma(t)\right)\cdot\sigma(t)dxdy\right). \tag{2.34}$$

From (2.33) and (2.34), we obtain the following lemma.

Lemma 2.3. *Under the above notations and assumptions (A_1) and (A_2), one has that*

$$\frac{d}{dt}E(t) = -\frac{1}{2}\int_0^\infty h''(s)a\left(w^t(s) - w(t)\right)ds \le 0. \tag{2.35}$$

3. Main Results

In this section, $c, \overline{c}, \widetilde{c}$, and c_i denote some positive constants. Here, we need to point out that δ denotes the small enough different positive constant and C_δ denotes the different positive constant depending on δ in a different place, respectively. The main goal of this paper is to show, respectively, that the solution decays exponentially or polynomially to zero as the time goes to infinity under suitable conditions. Our main results are formulated below.

Theorem 3.1. *Let w be the global solution of the problem* (1.1)–(1.8) *with the conditions*

$$h'(t) \le ch'''(t), \qquad -c_3h'(t) \le h''(t) \le -c_4h'(t). \tag{3.1}$$

Then the energy functional $E(t)$ decays exponentially to zero as the time goes to infinity.

To prove our main result, we will give some important preliminaries.

Lemma 3.2. *Assume $w \in W(\Omega)$, functional $a(w)$ and tensor $\sigma(t)$ are defined as above. Then*

(1) $c_1\|\nabla^2 w\|^2 \le a(w) \le c_2\|\nabla^2 w\|^2$

(2) $\|\nabla w\|_{L^q}^2 \le c_3 a(w),\ q \ge 2$

(3) $\|w\|^2 \le c_4 a(w)$

(4) $c_5|\sigma(t)|^2 \le C(\sigma(t))\cdot\sigma(t) \le c_6|\sigma(t)|^2$.

Proof. (1) In fact

$$\begin{aligned}a(w) &= \int_\Omega \left\{ (w_{xx})^2 + \left(w_{yy}\right)^2 + 2\mu w_{xx} w_{yy} + 2(1-\mu)\left(w_{xy}\right)^2 \right\} dx\, dy \\ &\le \int_\Omega \left\{ (w_{xx})^2 + \left(w_{yy}\right)^2 + \mu\left[(w_{xx})^2 + \left(w_{yy}\right)^2\right] + 2(1-\mu)\left(w_{xy}\right)^2 \right\} dx\, dy \\ &= \int_\Omega \left\{ (1+\mu)\left[(w_{xx})^2 + \left(w_{yy}\right)^2\right] + 2(1-\mu)\left(w_{xy}\right)^2 \right\} dx\, dy,\end{aligned} \tag{3.2}$$

$$\begin{aligned}a(w) &= \int_\Omega \left\{ (w_{xx})^2 + \left(w_{yy}\right)^2 + 2\mu w_{xx} w_{yy} + 2(1-\mu)\left(w_{xy}\right)^2 \right\} dxdy \\ &\ge \int_\Omega \left\{ (w_{xx})^2 + \left(w_{yy}\right)^2 - \mu\left[(w_{xx})^2 + \left(w_{yy}\right)^2\right] + 2(1-\mu)\left(w_{xy}\right)^2 \right\} dx\, dy \\ &= \int_\Omega \left\{ (1-\mu)\left[(w_{xx})^2 + \left(w_{yy}\right)^2\right] + 2(1-\mu)\left(w_{xy}\right)^2 \right\} dx\, dy,\end{aligned} \tag{3.3}$$

$$\left\|\nabla^2 w\right\|^2 = \left(\nabla^2 w, \nabla^2 w\right) = \int_\Omega \left[(w_{xx})^2 + \left(w_{yy}\right)^2 + 2\left(w_{xy}\right)^2\right] dx\, dy. \tag{3.4}$$

Combining (3.2)–(3.4) and noting $0 < \mu < 1/2$, we finish the desire conclusion (1).

(2) By $H^1(\Omega) \hookrightarrow L^q(\Omega)$ $(q \ge 2)$, the definition of $W(\Omega)$, and Poincaré inquality, we know

$$\|\nabla w(t)\|^2_{L^q(\Omega)} \le c\|w(t)\|^2_{H^1(\Omega)} \le c\|w(t)\|^2_{W(\Omega)} \le c_3 a(w). \tag{3.5}$$

(3) Using Poincaré inequality, we get

$$\|w(t)\| \ \le C_* \|\nabla w(t)\|, \tag{3.6}$$

which with conclusion (2) shows the desire result.

(4) Denote $\sigma := \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$. Hence

$$\mathcal{C}(\sigma) = \alpha \begin{pmatrix} \sigma_{11} + \mu\sigma_{22} & (1-\mu)\sigma_{12} \\ (1-\mu)\sigma_{21} & \sigma_{22} + \mu\sigma_{11} \end{pmatrix}, \tag{3.7}$$

with $\alpha = E/d(1-\mu^2) > 0$, and

$$\mathcal{C}(\sigma) \cdot \sigma = \alpha\left[\sigma_{11}^2 + \sigma_{22}^2 + 2\mu\sigma_{11}\sigma_{22} + (1-\mu)\left(\sigma_{12}^2 + \sigma_{21}^2\right)\right], \tag{3.8}$$

which imply

$$\alpha\Big[(1-\mu)\Big(\sigma_{11}^2+\sigma_{22}^2\Big)+(1-\mu)\Big(\sigma_{12}^2+\sigma_{21}^2\Big)\Big] \\ \leq \mathcal{C}(\sigma)\cdot\sigma\leq\alpha\Big[(1+\mu)\Big(\sigma_{11}^2+\sigma_{22}^2\Big)+(1-\mu)\Big(\sigma_{12}^2+\sigma_{21}^2\Big)\Big]. \tag{3.9}$$

From (3.9) and noting $0<\mu<1/2$, we obtain the desire conclusion (4).

The proof is completed. □

Lemma 3.3. *Suppose $w, v \in W$ and $a(w,v)$ is defined as above, then*

$$a(w,v)\leq\delta a(w,w)+\frac{9}{4\delta}a(v,v). \tag{3.10}$$

Proof. In fact, for $\eta>0$ small enough,

$$\begin{aligned}|a(w,v)| &\leq \int_\Omega\Big|w_{xx}v_{xx}+w_{yy}v_{yy}+\mu(w_{xx}v_{yy}+w_{yy}v_{xx})+2(1-\mu)w_{xy}v_{xy}\Big|dx\,dy\\ &\leq \eta\int_\Omega\Big\{(1+\mu)\Big[(w_{xx})^2+(w_{yy})^2\Big]+2(1-\mu)(w_{xy})^2\Big\}dx\,dy\\ &\quad+\frac{1}{4\eta}\int_\Omega\Big\{(1+\mu)\Big[(v_{xx})^2+(v_{yy})^2\Big]+2(1-\mu)(v_{xy})^2\Big\}dx\,dy\\ &\leq \frac{1+\mu}{1-\mu}\eta\int_\Omega\Big\{(1-\mu)\Big[(w_{xx})^2+(w_{yy})^2\Big]+2(1-\mu)(w_{xy})^2\Big\}dx\,dy\\ &\quad+\frac{1+\mu}{1-\mu}\frac{1}{4\eta}\int_\Omega\Big\{(1-\mu)\Big[(v_{xx})^2+(v_{yy})^2\Big]+2(1-\mu)(v_{xy})^2\Big\}dx\,dy.\end{aligned} \tag{3.11}$$

Denote $\delta=((1+\mu)/(1-\mu))\eta$, then $((1+\mu)/(1-\mu))(1/4\eta)=((1+\mu)/(1-\mu))^2(1/4\delta)$. Owing to $1<(1+\mu)/(1-\mu)<3$, from (3.3) and (3.11), we obtain the conclusion. □

Lemma 3.4. *Define the functional*

$$\varphi(t):=\frac{1}{2}\int_\Omega\big[w(t)w'(t)+\gamma\nabla w'(t)\cdot\nabla w(t)\big]dx\,dy, \tag{3.12}$$

then

$$\frac{d}{dt}\varphi(t)\leq\frac{1}{2}\Big(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\Big)-\frac{1}{2}\Big(1-\delta+\int_0^\infty h'(s)ds\Big)a(w) \\ -\int_\Omega\mathcal{C}(\sigma(t))\cdot\sigma(t)dx\,dy+\frac{9}{8\delta}\int_0^\infty\big|h'(s)\big|a\big(w^t(s)-w(t)\big)ds. \tag{3.13}$$

Proof. Multiplying (1.1) with w and integrating the result over Ω, we obtain

$$\int_{\Omega}\left(w''(t)-\gamma\Delta w''(t)+\Delta^2 w+\Delta^2\int_0^{\infty}h'(s)w^t(s)ds-\ \text{div}\ [\mathcal{C}(\sigma(t))\nabla w(t)]\right)w dx\, dy=0, \tag{3.14}$$

which with Lemma 2.2 and (1.5) implies

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int_{\Omega}\left[w(t)w'(t)+\gamma\nabla w'(t)\cdot\nabla w(t)\right]dxdy\\
&\quad=\frac{1}{2}\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\right)+\frac{1}{2}\int_{\Omega}\left[w(t)w''(t)+\gamma\nabla w(t)\cdot\nabla w''(t)\right]dx\, dy\\
&\quad=\frac{1}{2}\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\right)+\frac{1}{2}\int_{\Omega}w(t)\left(w''(t)-\gamma\Delta w''(t)\right)dx\, dy+\frac{1}{2}\int_{\Gamma_1}w\partial_\nu w''(t)d\Gamma\\
&\quad=\frac{1}{2}\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\right)\\
&\qquad-\frac{1}{2}\int_{\Omega}w\left(\Delta^2 w+\Delta^2\int_0^{\infty}h'(s)w^t(s)ds\right.\\
&\qquad\qquad\left.-\ \text{div}[\mathcal{C}(\sigma(t))\nabla w(t)]\right)dxdy+\frac{1}{2}\int_{\Gamma_1}w\partial_\nu w''(t)d\Gamma\\
&\quad=\frac{1}{2}\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\right)-\frac{1}{2}\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\nabla w(t)\otimes\nabla w(t)dx\, dy-\frac{1}{2}a(w)\\
&\qquad-\frac{1}{2}a\left(\int_0^{\infty}h'(s)w^t(s)ds,\, w\right)-\frac{1}{2}\int_{\Gamma_1}w(t)\mathcal{B}_2\left(w+\int_0^{\infty}h'(s)w^t(s)\right)d\Gamma\\
&\qquad+\frac{1}{2}\int_{\Gamma_1}\partial_\nu w(t)\mathcal{B}_1\left(w+\int_0^{\infty}h'(s)w^t(s)\right)d\Gamma+\frac{1}{2}\int_{\Gamma_1}\left[\mathcal{C}(\sigma(t))\nu\cdot\nabla w\right]wd\Gamma\\
&\qquad+\frac{1}{2}\int_{\Gamma_1}w\partial_\nu w''(t)d\Gamma=\frac{1}{2}\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\right)\\
&\qquad-\frac{1}{2}\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\nabla w(t)\otimes\nabla w(t)dx\, dy-\frac{1}{2}a(w)-\frac{1}{2}a\left(\int_0^{\infty}h'(s)w^t(s)ds,\, w\right)\\
&\quad=\frac{1}{2}\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\right)-\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\sigma(t)dx\, dy\\
&\qquad-\frac{1}{2}\left(1+\int_0^{\infty}h'(s)ds\right)a(w)-\frac{1}{2}a\left(w,\int_0^{\infty}h'(s)\left(w^t(s)-w(t)\right)ds\right).
\end{aligned} \tag{3.15}$$

By Lemma 3.3, we know

$$\left|\frac{1}{2}a\left(w(t),\int_0^{\infty}h'(s)\left(w^t(s)-w(t)\right)ds\right)\right|\le\frac{1}{2}\delta a(w(t))+\frac{9}{8\delta}\int_0^{\infty}\left|h'(s)\right|a\left(w^t(s)-w(t)\right)ds. \tag{3.16}$$

Using (3.15)-(3.16), we get the conclusion. □

Lemma 3.5. *Suppose w is the solution of* (1.1)–(1.8) *and (A_1), (A_2), and* (3.1) *hold, then*

(1) $(\int_0^\infty h'(s)w^t(s)ds)' = h'(0)w(t) + \int_0^\infty h''(s)w^t(s)ds = \int_0^\infty h''(s)[w^t(s) - w(t)]ds$;

(2) $h''_\infty = 0$;

(3) $(\int_0^\infty h''(s)w^t(s)ds)' = h''(0)w(t) + \int_0^\infty h'''(s)w^t(s)ds = \int_0^\infty h'''(s)[w^t(s) - w(t)]ds$.

Proof. (1) By the formula of integral by part and (A_2), we deduce

$$\begin{aligned}\left(\int_0^\infty h'(s)w^t(s)ds\right)' &= \int_0^\infty h'(s)\partial_t w^t(s)ds = -\int_0^\infty h'(s)\partial_s w^t(s)ds \\ &= -h'(s)w^t(s)\big|_0^\infty + \int_0^\infty h''(s)w^t(s)ds = h'(0)w(t) + \int_0^\infty h''(s)w^t(s)ds \\ &= \int_0^\infty h''(s)\left[w^t(s) - w(t)\right]ds.\end{aligned} \tag{3.17}$$

(2) Using (A_2) and (3.1), we infer the conclusion.

(3) By $h''_\infty = 0$ and the formula of integral by part, we get the conclusion (3).

The proof is completed. □

Lemma 3.6. *Define the functional*

$$\begin{aligned}\psi(t) &= \gamma\int_{\Gamma_1}\partial_\nu w'(t)\left(h'(0)w(t) + \int_0^\infty h''(s)w^t(s)ds\right)d\Gamma \\ &\quad + \int_\Omega \left(w'(t) - \gamma\Delta w'(t)\right)\left(h'(0)w(t) + \int_0^\infty h''(s)w^t(s)ds\right)dx\,dy \\ &\quad + \frac{1}{2}a\left(\int_0^\infty h'(s)w^t(s)ds\right).\end{aligned} \tag{3.18}$$

then for some δ $(\delta > 0)$ small enough, there exist $C_\delta > 0$ and T $(T > 0)$ as $t > T$ the following inequality holds

$$\begin{aligned}\frac{d}{dt}\psi(t) &\le \frac{h'(0)}{2}\left(\|w'(t)\|^2 + \gamma\|\nabla w'(t)\|^2\right) + \delta\int_\Omega \mathcal{C}(\sigma(t))\cdot\sigma(t)dx\,dy \\ &\quad + C_\delta\int_0^\infty |h'(s)|a\left(w^t(s) - w(t)\right)ds.\end{aligned} \tag{3.19}$$

Proof. Multiplying (1.1) with $h'(0)w(t) + \int_0^\infty h''(s)w^t(s)ds$ and integrating the result over Ω, we have

$$\begin{aligned}&\int_\Omega\left(w''(t) - \gamma\Delta w''(t) + \Delta^2 w + \Delta^2\int_0^\infty h'(s)w^t(s)ds - \operatorname{div}[\mathcal{C}(\sigma(t))\nabla w(t)]\right) \\ &\quad\cdot\left(h'(0)w(t) + \int_0^\infty h''(s)w^t(s)ds\right)dxdy = 0,\end{aligned} \tag{3.20}$$

which with Lemma 3.5 gives

$$\begin{aligned}
&\frac{d}{dt}\int_{\Omega}(w'(t)-\gamma\Delta w'(t))\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)dxdy\\
&\quad=\int_{\Omega}(w''(t)-\gamma\Delta w''(t))\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)dxdy\\
&\qquad+\int_{\Omega}(w'(t)-\gamma\Delta w'(t))\left[h'(0)w'(t)+\left(\int_0^{\infty}h''(s)w^t(s)ds\right)'\right]dx\,dy\\
&\quad=-\int_{\Omega}\left(\Delta^2 w+\int_0^{\infty}h'(s)\Delta^2 w^t(s)ds-\operatorname{div}[\mathcal{C}(\sigma(t))\nabla w(t)]\right)\\
&\qquad\times\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)dxdy\\
&\qquad+\int_{\Omega}(w'(t)-\gamma\Delta w'(t))\left(h'(0)w'(t)+h''(0)w(t)+\int_0^{\infty}h'''(s)w^t(s)ds\right)dx\,dy.
\end{aligned}\tag{3.21}$$

By Green formula, we have

$$\begin{aligned}
&\int_{\Omega}(w'(t)-\gamma\Delta w'(t))\left(h'(0)w'(t)+h''(0)w(t)+\int_0^{\infty}h'''(s)w^t(s)ds\right)dx\,dy\\
&\quad=h'(0)\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\right)-\gamma\int_{\Gamma_1}\partial_{\nu}w'(t)\\
&\qquad\times\left(h'(0)w'(t)+h''(0)w(t)+\int_0^{\infty}h'''(s)w^t(s)ds\right)d\Gamma\\
&\qquad+\gamma\int_{\Omega}\nabla w'(t)\cdot\left(h''(0)\nabla w(t)+\int_0^{\infty}h'''(s)\nabla w^t(s)ds\right)dx\,dy\\
&\qquad+\int_{\Omega}w'(t)\left(h''(0)w+\int_0^{\infty}h'''(s)w^t(s)ds\right)dx\,dy.
\end{aligned}\tag{3.22}$$

Clearly

$$\begin{aligned}
&\gamma\int_{\Gamma_1}\partial_{\nu}w'(t)\left(h'(0)w'(t)+h''(0)w(t)+\int_0^{\infty}h'''(s)w^t(s)ds\right)d\Gamma\\
&\quad=\gamma\frac{d}{dt}\int_{\Gamma_1}\partial_{\nu}w'(t)\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)d\Gamma\\
&\qquad-\gamma\int_{\Gamma_1}\partial_{\nu}w''(t)\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)d\Gamma.
\end{aligned}\tag{3.23}$$

Inserting (3.22) and (3.23) into (3.21) yields

$$\begin{aligned}
&\frac{d}{dt}\int_{\Omega}\left(w'(t)-\gamma\Delta w'(t)\right)\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)dx\,dy\\
&\quad+\gamma\frac{d}{dt}\int_{\Gamma_1}\partial_\nu w'(t)\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)d\Gamma\\
&\quad=-\int_{\Omega}\left(\Delta^2 w+\int_0^{\infty}h'(s)\Delta^2 w^t(s)ds-\operatorname{div}[\mathcal{C}(\sigma(t))\nabla w(t)]\right)\\
&\quad\times\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)dx\,dy+h'(0)\left(\left\|w'(t)\right\|^2+\gamma\left\|\nabla w'(t)\right\|^2\right)\\
&\quad+\gamma\int_{\Gamma_1}\partial_\nu w''(t)\left(\int_0^{\infty}h''(s)\left[w^t(s)-w(t)\right]\right)d\Gamma\\
&\quad+\gamma\int_{\Omega}\nabla w'(t)\cdot\left(h''(0)\nabla w(t)+\int_0^{\infty}h'''(s)\nabla w^t(s)ds\right)dx\,dy\\
&\quad+\int_{\Omega}w'(t)\left(h''(0)w+\int_0^{\infty}h'''(s)w^t(s)ds\right)dx\,dy.
\end{aligned}\tag{3.24}$$

Applying Lemma 2.2, Lemma 3.5 and noting $a(w,v)=a(v,w)$, we have

$$\begin{aligned}
&-\int_{\Omega}\left(\Delta^2 w+\Delta^2\int_0^{\infty}h'(s)w^t(s)ds-\operatorname{div}[\mathcal{C}(\sigma(t))\nabla w(t)]\right)\\
&\quad\times\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)dx\,dy=-a\left(w,\int_0^{\infty}h''(s)\left[w^t(s)-w(t)\right]ds\right)\\
&\quad-\frac{1}{2}\frac{d}{dt}a\left(\int_0^{\infty}h'(s)w^t(s)ds,\int_0^{\infty}h'(s)w^t(s)ds\right)\\
&\quad-\int_{\Omega}\mathcal{C}(\sigma(t))\nabla w(t)\cdot\left(\int_0^{\infty}h''(s)\left[\nabla w^t(s)-\nabla w(t)\right]ds\right)dx\,dy\\
&\quad-\int_{\Gamma_1}\left[(\mathcal{B}_2 w)\left(\int_0^{\infty}h''(s)\left[w^t(s)-w(t)\right]ds\right)\right.\\
&\qquad\qquad\left.-(\mathcal{B}_1 w)\partial_\nu\left(\int_0^{\infty}h''(s)\left[w^t(s)-w(t)\right]ds\right)\right]d\Gamma\\
&\quad-\int_{\Gamma_1}\left(\mathcal{B}_2\int_0^{\infty}h'(s)w^t(s)ds-\left(C(\sigma(t))\nu\cdot\nabla w(t)\right)\right)\left(\int_0^{\infty}h''(s)\left[w^t(s)-w(t)\right]ds\right)d\Gamma\\
&\quad-\int_{\Gamma_1}\left(\mathcal{B}_1\int_0^{\infty}h'(s)w^t(s)ds\right)\partial_\nu\left(\int_0^{\infty}h''(s)\left[w^t(s)-w(t)\right]ds\right)d\Gamma.
\end{aligned}\tag{3.25}$$

Inserting (3.25) into (3.24) and owing to (1.5) yield

$$\begin{aligned}
\frac{d}{dt}\varphi(t) = &\frac{d}{dt}\int_{\Omega}(w'(t)-\gamma\Delta w'(t))\left(h'(0)w(t)+\int_0^{\infty}h''(s)w^t(s)ds\right)dxdy \\
&+\frac{1}{2}\frac{d}{dt}a\left(\int_0^{\infty}h'(s)w^t(s)ds\right)+\gamma\frac{d}{dt}\int_{\Gamma_1}\partial_\nu w'(t)\left(h'(0)w(t)+\int_0^{\infty}h''w^t(s)ds\right)d\Gamma \\
= &\, h'(0)\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\right) \\
&-\int_{\Omega}\mathcal{C}(\sigma(t))\nabla w(t)\cdot\left(\int_0^{\infty}h''(s)[\nabla w^t(s)-\nabla w(t)]ds\right)dxdy \\
&-a\left(w,\int_0^{\infty}h''(s)[w^t(s)-w(t)]ds\right) \\
&+\gamma\int_{\Omega}\nabla w'(t)\cdot\left(\int_0^{\infty}h'''(s)[\nabla w^t(s)-\nabla w(t)]ds\right)dxdy \\
&+\int_{\Omega}w'(t)\left(\int_0^{\infty}h'''(s)[\nabla w^t(s)-\nabla w(t)]ds\right)dxdy.
\end{aligned} \tag{3.26}$$

Now, we estimate some terms on the right-hand side of (3.26). The Lemma 3.3 and (3.1) imply

$$\left|a\left(w,\int_0^{\infty}h''(s)[w^t(s)-w(t)]ds\right)\right| \le \delta a(w,w)+C_\delta\int_0^{\infty}|h'(s)|a(w^t(s)-w(t))ds. \tag{3.27}$$

Using Cauchy inequality, Hölder inequality, and Lemma 3.2 and noting (3.1), we obtain

$$\begin{aligned}
&\left|\gamma\int_{\Omega}\nabla w'(t)\cdot\left(\int_0^{\infty}h'''(s)[\nabla w^t(s)-\nabla w(t)]ds\right)dx\,dy\right| \\
&\qquad\le \delta\gamma\|\nabla w'(t)\|^2+C_\delta\int_0^{\infty}|h'(s)|a(w^t(s)-w(t))ds, \\
&\left|\int_{\Omega}w'(t)\left(\int_0^{\infty}h'''(s)[\nabla w^t(s)-\nabla w(t)]ds\right)dx\,dy\right| \\
&\qquad\le \delta\|w'(t)\|^2+C_\delta\int_0^{\infty}|h'(s)|a(w^t(s)-w(t))ds.
\end{aligned} \tag{3.28}$$

Applying Hölder inequality, Lemma 3.2, and Lemma 2.3, we conclude

$$
\begin{aligned}
&\left|\int_{\Omega} \mathcal{C}(\sigma(t))\nabla w(t)\cdot\left(\int_0^{\infty} h''(s)\left[\nabla w^t(s)-\nabla w(t)\right]ds\right)dxdy\right| \\
&\quad = \left|\int_{\Omega} \mathcal{C}(\sigma(t))\cdot\left(\int_0^{\infty} h''(s)\nabla w(t)\otimes\left[\nabla w^t(s)-\nabla w(t)\right]ds\right)dxdy\right| \\
&\quad \le \delta\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\sigma(t)dxdy + C_\delta\int_0^{\infty}\left|h'(s)\right|\|\nabla w(t)\|^2\left\|\nabla w^t(s)-\nabla w(t)\right\|^2 ds \\
&\quad \le \delta\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\sigma(t)dxdy + a(w(t))C_\delta\int_0^{\infty}\left|h'(s)\right|a\left(w^t(s)-w(t)\right)ds \\
&\quad \le \delta\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\sigma(t)dxdy + \frac{1}{h_\infty}E(0)\overline{C}_\delta\int_0^{\infty}\left|h'(s)\right|a\left(w^t(s)-w(t)\right)ds \\
&\quad = \delta\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\sigma(t)dxdy + C_\delta\int_0^{\infty}\left|h'(s)\right|a\left(w^t(s)-w(t)\right)ds.
\end{aligned}
\tag{3.29}
$$

Combining (3.26)–(3.29), we conclude that

$$
\begin{aligned}
\frac{d}{dt}\psi(t) \le{}& \frac{h'(0)}{2}\left(\left\|w'(t)\right\|^2+\gamma\left\|\nabla w'(t)\right\|^2\right) \\
&+\delta\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\sigma(t)dxdy + C_\delta\int_0^{\infty}\left|h'(s)\right|a\left(w^t(s)-w(t)\right)ds.
\end{aligned}
\tag{3.30}
$$

The proof is completed. □

Proof of Theorem 3.1. Let

$$
F(t) = PE(t) + Q\varphi(t) + R\psi(t).
\tag{3.31}
$$

By Lemma 2.3, Lemma 3.4, and Lemma 3.6 and noting (3.1), we have

$$
\begin{aligned}
F'(t) \le{}& -P\frac{1}{2}\int_0^{\infty} h''(s)a\left(w^t(s)-w(t)\right)ds + Q\frac{1}{2}\left(\|w'(t)\|^2+\gamma\|\nabla w'(t)\|^2\right) \\
&- Q\frac{1}{2}\left(1-\delta+\int_0^{\infty} h'(s)ds\right)a(w(t)) - Q\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\sigma(t)dx\,dy \\
&+ Q\frac{9}{8\delta}\int_0^{\infty}\left|h'(s)\right|a\left(w^t(s)-w(t)\right)ds + R\frac{h'(0)}{2}\left(\left\|w'(t)\right\|^2+\gamma\left\|\nabla w'(t)\right\|^2\right) \\
&+ R\delta\int_{\Omega}\mathcal{C}(\sigma(t))\cdot\sigma(t)dxdy + RC_\delta\int_0^{\infty}\left|h'(s)\right|a\left(w^t(s)-w(t)\right)ds \\
\le{}& K_1\left(\left\|w'(t)\right\|^2+\gamma\left\|\nabla w'(t)\right\|^2\right) + K_2 a(w(t)) \\
&+ K_3\int_{\Omega}\mathcal{C}(\sigma(t))\times\sigma(t)dx\,dy + K_4\int_0^{\infty}\left|h'(s)\right|a\left(w^t(s)-w(t)\right)ds,
\end{aligned}
\tag{3.32}
$$

with

$$K_1 = \frac{1}{2}Rh'(0) + \frac{1}{2}Q, \qquad K_2 = -\frac{1}{2}Q\left(1-\delta+\int_0^\infty h'(s)ds\right) = -\frac{1}{2}Q(h_\infty-\delta), \qquad K_3 = -Q + R\delta,$$
$$K_4 = -\frac{1}{2}Pc_3 + \frac{9Q}{8\delta} + RC_\delta. \tag{3.33}$$

Fixing $R = 1$, $\delta = (1/2)\min\{h_\infty, -h'(0)\} > 0$, $Q = (1/2)(\delta - h'(0)) > 0$, $P > 9Q/4\delta c_3 + 2(C_\delta/c_3) > 0$, we obtain $K_i < 0$, $i = 1,2,3,4$. Under the conditions, we obtain from (2.34) and (3.32) that there exists a positive $\alpha > 0$ such that

$$\frac{d}{dt}F(t) \le -\alpha E(t). \tag{3.34}$$

Fixing a large enough positive constant P, we prove $F(t)$ and $E(t)$ are equivalent, that is,

$$M_1E(t) \le F(t) \le M_2E(t) \tag{3.35}$$

with M_1, $M_2 > 0$. Indeed, using Cauchy inequality, Lemma 3.2, and (2.34), we have

$$\begin{aligned}|R\varphi(t)| &= \left|\frac{Q}{2}\int_\Omega (w(t)w'(t) + \gamma\nabla w'(t)\cdot\nabla w(t))dxdy\right| \\ &\le \frac{Q}{4}\left[\|w(t)\|^2 + \gamma\|\nabla w(t)\|^2 + \|w'(t)\|^2 + \gamma\|\nabla w'(t)\|^2\right] \le \bar{c}a(w(t)) + \tilde{c}E(t) \le cE(t).\end{aligned} \tag{3.36}$$

Using the definition of $\psi(t)$ and Green's formula, we infer

$$\begin{aligned}\psi(t) = &\int_\Omega w'(t)\left(\int_0^\infty h''(s)\left[w^t(s) - w(t)\right]ds\right)dxdy \\ &+ \int_\Omega \gamma\nabla w'(t)\cdot\left(\int_0^\infty h''(s)\left[\nabla w^t(s) - \nabla w(t)\right]ds\right)dxdy + \frac{1}{2}a\left(\int_0^\infty h'(s)w^t(s)ds\right).\end{aligned} \tag{3.37}$$

Applying Cauchy inequality, Lemma 3.2 and noting (3.1), we conclude that

$$\begin{aligned}
&\int_\Omega w'(t)\left(\int_0^\infty h''(s)\left[w^t(s)-w(t)\right]ds\right)dxdy \\
&\quad+\int_\Omega \gamma\nabla w'(t)\cdot\left(\int_0^\infty h''(s)\left[\nabla w^t(s)-\nabla w(t)\right]ds\right)dx\,dy \\
&\le \delta\left\|w'(t)\right\|^2+\gamma\delta\ \left\|\nabla w'(t)\right\|^2 \\
&\quad+C_\delta\int_0^\infty \left|h'(s)\right|a\left(w^t(s)-w(t)\right)ds,
\end{aligned} \tag{3.38}$$

$$\begin{aligned}
\frac{1}{2}a\left(\int_0^\infty h'(s)w^t(s)ds\right) &\le C\left\|\nabla^2\left((h(0)-h_\infty)w(t)ds\right.\right. \\
&\quad\left.\left.+\int_0^\infty h'(s)\left[w^t(s)-w(t)\right]\right)\right\|^2 \\
&\le ca(w(t))+c\int_0^\infty \left|h'(s)\right|\left(w^t(s)-w(t)\right)ds.
\end{aligned} \tag{3.39}$$

Relations (3.37)-(3.39) imply that

$$\left|\varphi(t)\right|\le cE(t). \tag{3.40}$$

Fixing a large enough positive constant P, inequalities (3.36) and (3.40) show (3.35) hold. Relations (3.34) and (3.35) imply that there exists a positive constant $\beta>0$ such that

$$\frac{d}{dt}F(t)\le -\beta F(t), \tag{3.41}$$

which with Gronwall's inequality gives

$$F(t)\le F(0)e^{-\beta t}. \tag{3.42}$$

Combining (3.35) and (3.42), we finish the proof. □

Theorem 3.7. *Assume that w is the global solution of* (1.1)–(1.8) *with condition* $C|h'(t)|^{1+(1/p)}\le h''t), p>2$. *Then for some $T>0$ there exists a positive constant C, such that*

$$E(t)\le \frac{C}{(1+t)^{p/2}},\quad \forall t\ge T. \tag{3.43}$$

In order to prove Theorem 3.7, we quote the following technical lemma.

Lemma 3.8. *Suppose that* $w \in L^{\infty}(0, \infty; H^2(\Omega))$ *and* g *is a continuous function and there exists* $0 < \theta < 1$, *such that*

$$\int_0^{\infty} g^{1-\theta}(s)ds < \infty. \tag{3.44}$$

Then, there exists $C > 0$ *such that*

$$\begin{aligned}
&\int_0^{\infty} g(s)a\left(w^t(s) - w(t)\right)ds \\
&\quad \leq C\left\{\left(\int_0^{\infty} g^{1-\theta}(s)ds\right)\|v\|^2_{L^{\infty}(0,T;H^2(\Omega))}\right\}^{1/(\theta p+1)} \\
&\qquad \times \left\{\int_0^{\infty} g^{1+(1/p)}(s)a\left(w^t(s) - w(t)\right)dxdy\right\}^{\theta p/(\theta p+1)}.
\end{aligned} \tag{3.45}$$

Proof. Applying Hölder inequality, we obtain

$$\begin{aligned}
&\int_0^{\infty} g(s)a\left(w^t(s) - w(t)\right)ds \\
&\quad \leq \left\{\int_0^{\infty} |g(s)|^{1-\theta} a\left(w^t(s) - w(t)\right)ds\right\}^{1/(\theta p+1)} \\
&\qquad \times \left\{\int_0^{t} |g(s)|^{1+(1/p)} a\left(w^t(s) - w(t)\right)ds\right\}^{\theta p/(\theta p+1)} \\
&\quad \leq C\left\{\left(\int_0^{\infty} g^{1-\theta}(s)ds\right)\|w\|^2_{L^{\infty}(0,T;H^2(\Omega))}\right\}^{1/(\theta p+1)} \\
&\qquad \times \left\{\int_0^{\infty} g^{1+(1/p)}(s)a\left(w^t(s) - w(t)\right)ds\right\}^{\theta p/(\theta p+1)}.
\end{aligned} \tag{3.46}$$

This completes the proof of Lemma 3.8. □

Proof of Theorem 3.7. From the hypothesis $c|h'(t)|^{1+(1/p)} \leq h''(t)$, we have $[|h'|^{-1/p}(t)]' \geq c/p$. Integrating the above inequality over $[0, t]$, we get

$$|h'|(t) \leq c_7(1+t)^{-p}, \qquad c_7 > 0. \tag{3.47}$$

Fixing $\theta = 1/2$ in Lemma 3.8, then $(1-\theta)p = p/2 > 1$, hence we obtain

$$\int_0^{\infty} |h'|^{1-\theta}(s)ds \leq c_8 \int_0^{\infty} \frac{1}{(1+s)^{p/2}} ds < \infty. \tag{3.48}$$

Using this estimate into (3.46), noting (2.34) and Lemma 3.2, we get

$$\int_0^\infty |h'|(s)a(w^t(s)-w(t))ds \le c_9 E^{2/(p+2)}(0)\left(\int_0^\infty |h'|^{1+(1/p)}a(w^t(s)-w(t))ds\right)^{p/(p+2)}. \tag{3.49}$$

Denote $G(t)=Q\varphi(t)+\psi(t)$. Applying (3.32) and (3.49), we arrive at

$$\begin{aligned}\frac{d}{dt}G(t) &\le -\alpha_1 E(t)+\alpha_2\int_0^\infty |h'|(s)a(w^t(s)-w(t))ds\\ &\le -\alpha_1 E(t)+\alpha_2 c_9 E^{2/(p+2)}(0)\left(\int_0^\infty |h'|^{1+(1/p)}a(w^t(s)-w(t))ds\right)^{p/(p+2)}.\end{aligned} \tag{3.50}$$

Since $|G(t)|\le cE(t)$, applying Young inequality and from Lemma 3.2, (3.50), we deduce

$$\begin{aligned}\frac{d}{dt}\left[E^{2/p}(t)G(t)\right] &= \frac{2}{p}G(t)E^{(2/p)-1}(t)\frac{d}{dt}E(t)+E^{2/p}(t)\frac{d}{dt}G(t)\\ &\le -\frac{2}{p}cE^{2/p}(t)\frac{d}{dt}E(t)+E^{2/p}(t)\frac{d}{dt}G(t)\\ &\le -c_{10}\frac{d}{dt}E^{1+(2/p)}(t)-\alpha_1 E^{1+(2/p)}(t)\\ &\quad+\alpha_2 C_2 E^{2/(p+2)}(0)E^{2/p}(t)\left(\int_0^\infty |h'|^{1+(1/p)}a(w^t(s)-w(t))ds\right)^{p/(p+2)}\\ &\le -c_{10}\frac{d}{dt}E^{1+(2/p)}(t)-\alpha_1 E^{1+(2/p)}(t)+\alpha_2 C_2 E^{2/(p+2)}(0)\varepsilon E^{1+(2/p)}(t)\\ &\quad+\alpha_2 C_2 E^{2/(p+2)}(0)C_\varepsilon\int_0^\infty |h'|^{1+(1/p)}a(w^t(s)-w(t))ds.\end{aligned} \tag{3.51}$$

Since $c|h'|^{1+(1/p)}(t)\le h''(t)$, then

$$\int_0^\infty |h'|^{1+(1/p)}a(w^t(s)-w(t))ds \le \frac{1}{c}\int_0^\infty h''(s)a(w^t(s)-w(t))ds. \tag{3.52}$$

By Lemma 3.2, we have

$$\int_0^\infty h''(s)a(w^t(s)-w(t))ds = -2\frac{d}{dt}E(t). \tag{3.53}$$

Hence, we get

$$\int_0^\infty |h'|^{1+(1/p)}a(w^t(s)-w(t))ds \le -c_{11}\frac{d}{dt}E(t). \tag{3.54}$$

Taking ε small enough, from (3.51) and (3.54), we get

$$\frac{d}{dt}\left[E^{2/p}(t)G(t)\right] \leq -c_{10}\frac{d}{dt}E^{1+(2/p)}(t) - \frac{\alpha_1}{2}E^{1+(2/p)}(t) - c_{12}\frac{d}{dt}E(t). \tag{3.55}$$

Thus, we have

$$\frac{d}{dt}\left[E^{2/p}(t)(G(t) + c_{10}E(t))\right] \leq -c_{12}\frac{d}{dt}E(t) - \frac{\alpha_1}{2}E^{1+(2/p)}(t). \tag{3.56}$$

Let $\lambda > 0$ be a positive constant and define

$$H(t) = \lambda E(t) + E^{2/p}(t)[G(t) + c_{10}E(t)]. \tag{3.57}$$

Since $|G(t)| \leq cE(t)$, $(d/dt)E(t) \leq 0$, for λ large enough, we get

$$\frac{\lambda}{2}E(t) \leq H(t) \leq 2\lambda E(t), \quad \forall t \geq 0. \tag{3.58}$$

From Lemma 3.2 and (3.56), taking λ large enough, we obtain

$$\begin{aligned}\frac{d}{dt}H(t) &= \lambda\frac{d}{dt}E(t) + \frac{d}{dt}\left[E^{2/p}(t)(G(t) + C_0E(t))\right]\\ &\leq \lambda\frac{d}{dt}E(t) - c_{12}\frac{d}{dt}E(t) - \frac{\alpha_1}{2}E^{1+(2/p)}(t)\\ &= (\lambda - c_{12})\frac{d}{dt}E(t) - \frac{\alpha_1}{2}E^{1+(2/p)}(t) \leq -\frac{\alpha_1}{2}E^{1+(2/p)}(t).\end{aligned} \tag{3.59}$$

From (3.58) and (3.59), we have

$$\frac{d}{dt}H(t) \leq -c_{13}H^{1+(2/p)}(t). \tag{3.60}$$

Applying Gronwall inequality, we get

$$H(t) \leq \frac{c_{14}}{(1+t)^{p/2}}. \tag{3.61}$$

Hence, we obtain

$$E(t) \leq \frac{c_{15}}{(1+t)^{p/2}}. \tag{3.62}$$

This completes the proof of Theorem 3.7. □

4. Comment

In this paper, the key point is to construct suitable energy functional $E(t)$, find the appropriate auxiliary functionals $\varphi(t)$ and $\psi(t)$ by multiplier method, and prove that $F(t)$ and $E(t)$ are equivalent and $F(t)$ satisfies $F'(t) \leq -cF(t)$ by precise apriori estimates. Thanks to Lemma 3.8 and auxiliary functional $H(t)$, we show the polynomial decay result under suitable conditions on relaxation h.

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