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Research Article

Modified Function Projective Synchronization between Different Dimension Fractional-Order Chaotic Systems

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A modified function projective synchronization (MFPS) scheme for different dimension fractional-order chaotic systems is presented via fractional order derivative. The synchronization scheme, based on stability theory of nonlinear fractional-order systems, is theoretically rigorous. The numerical simulations demonstrate the validity and feasibility of the proposed method.

1. Introduction

Fractional-order calculus, which can be dated back to the 17th century [1, 2]. However, only in the last few decades, its application to physics and engineering has been addressed. So, the fractional-order calculus has attracted increasing attention only recently. On the other hand, complex bifurcation and chaotic phenomena have been found in many fractional-order dynamical systems. For example, the fractional-order Lorenz chaotic system [3], the fractional-order unified chaotic system [4], the fractional-order Chua chaotic circuit [5], the fractional-order modified Duffing chaotic system [6], and the fractional-order Rössler chaotic system [7, 8], and so on.

Nowadays, synchronization of chaotic systems and fractional-order chaotic systems has attracted much attention because of its applications in secure communication and control processing [9–21]. Many approaches have been reported for the synchronization of chaotic systems and fractional-order chaotic systems [12–19]. In 1999, Mainieri and Rehacek proposed projective synchronization (PS) [12] for chaotic systems, which has been extensively investigated in recent years because of its proportional feature in secure communications. Recently, a modified projective synchronization, which is called function projective synchronization (FPS) [13–15] has been reported. In FPS, the master and slave systems could be synchronized up to a scaling function, but not a constant. So, the unpredictability of the scaling function in FPS can additionally enhance the security of communication.

To the best of our knowledge, most of the existing FPS scheme for the fractional-order chaotic systems only discuss the same dimension. However, in many real physics systems, the synchronization is carried out through the oscillators with different dimension, especially the systems in biological science and social science [16–21]. Moreover, in some previous works [16, 17], all the nonlinear terms of response system or error system was absorbed. Referring to chaotic synchronization via fractional-order controller, there are a few results reported until now. Inspired by the above discussion, in this paper, we present a modified function projective synchronization (MFPS) scheme between different dimension fractional-order chaotic systems via fractional-order controller. The fractional-order controller is easily designed. The synchronization technique, based on tracking control and stability theory of nonlinear fractional-order systems, is theoretically rigorous. Our modified function projective synchronization (MFPS) scheme need not absorb all the nonlinear terms of response system. This is different from some previous works. Two examples are presented to demonstrate the effectiveness of the proposed MFPS scheme.

This paper is organized as follows. In Section 2, a modified function projective synchronization (MFPS) scheme is presented. In Section 3, two groups of examples are used to verify the effectiveness of the proposed scheme. The conclusion is finally drawn in Section 4.

2. The MFPS Scheme for Different Dimension Fractional-Order Chaotic Systems

The fractional-order chaotic drive and response systems with different dimension are defined as follows, respectively:

$$\frac{d^{q_d}x}{dt^{q_d}} = F_d(x), \tag{2.1}$$

$$\frac{d^{q_r}y}{dt^{q_r}} = F_r(y) + C(x,y), \tag{2.2}$$

where q_d $(0 < q_d < 1)$ and q_r $(0 < q_r < 1)$ are fractional order, and q_d may be different with q_r. $x \in R^n$, $y \in R^m$ $(n \neq m)$ are state vectors of the drive system (2.1) and response system (2.2), respectively. $F_d : R^n \to R^n$, $F_r : R^m \to R^m$ are two continuous nonlinear vector functions, and $C(x,y) \in R^m$ is a controller which will be designed later.

Definition 2.1. For the drive system (2.1) and response system (2.2), it is said to be modified function projective synchronization (MFPS) if there exist a controller $C(x,y)$ such that:

$$\lim_{t\to+\infty}\|e\| = \lim_{t\to+\infty}\|y - M(x)x\| = 0, \tag{2.3}$$

where $\|\cdot\|$ is the Euclidean norm, $M(x)$ is a $m \times n$ real matrix, and matrix element $M_{ij}(x)$ $(i = 1,2,\ldots m,\ j = 1,2,\ldots,n)$ are continuous bounded functions. $e_i = y_i - \sum_{j=1}^{n} M_{ij}x_j$ $(i = 1,2,\ldots m)$ are called MFPS error.

Remark 2.2. According to the view of tracking control, $M(x)x$ can be chosen as a reference signal. The MFPS in our paper is transformed into the problem of tracking control, that is the output signal y in system (2.2) follows the reference signal $M(x)x$.

In order to achieve the output signal y follows the reference signal $M(x)x$. Now, we define a compensation controller $C_1(x) \in R^m$ for response system (2.2) via fractional-order derivative $d^{q_r}(M(x)x)/dt^{q_r}$. The compensation controller is shown as follows:

$$C_1(x) = \frac{d^{q_r}(M(x)x)}{dt^{q_r}} - F_r(M(x)x), \tag{2.4}$$

and let controller $C(x,y)$ as follows:

$$C(x,y) = C_1(x) + C_2(x,y), \tag{2.5}$$

where $C_2(x,y) \in R^m$ is a vector function which will be designed later.

By controller (2.5) and compensation controller (2.4), the response system (2.2) can be changed as follows:

$$\frac{d^{q_r}e}{dt^{q_r}} = D_1(x,y)e + C_2(x,y), \tag{2.6}$$

where $D_1(x,y)e = F_r(y) - F_r(M(x)x)$, and $D_1(x,y) \in R^{m\times m}$. So, the MFPS between drive system (2.1) and response system (2.2) is transformed into the following problem: choose a suitable vector function $C_2(x,y)$ such that system (2.6) is asymptotically converged to zero.

In what follows we present the stability theorem for nonlinear fractional-order systems of commensurate order [22–25]. Consider the following nonlinear commensurate fractional-order autonomous system

$$D^q x = f(x), \tag{2.7}$$

the fixed points of system (2.7) is asymptotically stable if all eigenvalues (λ) of the Jacobian matrix $A = \partial f/\partial x$ evaluated at the fixed points satisfy $|\arg\lambda| > 0.5\pi q$. Where $0 < q < 1$, $x \in R^n$, $f : R^n \to R^n$ are continuous nonlinear functions, and the fixed points of this nonlinear commensurate fractional-order system are calculated by solving equation $f(x) = 0$.

Now, the following theorem is given based on the above discussion in order to achieve the MFPS between the drive system (2.1) and the response system (2.2).

Theorem 2.3. *Choose the control vector $C_2(x,y) = D_2(x,y)e$, and if $D_1(x,y) + D_2(x,y)$ satisfy the following conditions:*

(1) $d_{ij} = -d_{ji}$ $(i \neq j)$,

(2) $d_{ii} \leq 0$ *(all d_{ii} are not equal to zero),*

then the modified function projective synchronization (MFPS) between (2.1) *and* (2.2) *can be achieved. Where* $D_2(x,y) \in R^{m\times m}$*, and* d_{ij} $(i,j = 1,2,\dots m,$ *for all* $d_{ij} \in R)$ *are the matrix element of matrix* $D_1(x,y) + D_2(x,y)$*.*

Proof. Using $C_2(x,y) = D_2(x,\mathrm{y})e$, so fractional-order system (2.6) can be rewritten as follows:

$$\frac{d^{q_r}e}{dt^{q_r}} = \left[D_1(x,y) + D_2(x,y)\right]e. \tag{2.8}$$

Suppose λ is one of the eigenvalues of matrix $D_1(x,y)+D_2(x,y)$ and the corresponding non-zero eigenvector is φ, that is,

$$\left[D_1(x,y) + D_2(x,y)\right]\varphi = \lambda\varphi. \tag{2.9}$$

Take conjugate transpose (H) on both sides of (2.9), we yield

$$\overline{\left\{\left[D_1(x,y) + D_2(x,y)\right]\varphi\right\}^T} = \overline{\lambda}\varphi^H. \tag{2.10}$$

Equation (2.9) multiplied left by φ^H plus (2.10) multiplied right by φ, we derive that

$$\varphi^H\left\{\left[D_1(x,y) + D_2(x,y)\right] + \left[D_1(x,y) + D_2(x,y)\right]^H\right\}\varphi = \varphi^H\varphi\left(\lambda + \overline{\lambda}\right). \tag{2.11}$$

So,

$$\lambda + \overline{\lambda} = \frac{\varphi^H\left\{\left[D_1(x,y) + D_2(x,y)\right] + \left[D_1(x,y) + D_2(x,y)\right]^H\right\}\varphi}{\varphi^H\varphi}. \tag{2.12}$$

Because $d_{ij} = -d_{ji}$ $(i \neq j,$ for all $d_{ij} \in R)$ in matrix $D_1(x,y) + D_2(x,y)$, so

$$\lambda + \overline{\lambda} = \frac{\varphi^H\begin{pmatrix} 2d_{11} & 0 & \cdots & 0 \\ 0 & 2d_{22} & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 2d_{mm} \end{pmatrix}\varphi}{\varphi^H\varphi}. \tag{2.13}$$

Because $d_{ii} \leq 0$ (for all $d_{ii} \in R$), and all d_{ii} are not equal to zero. So,

$$\lambda + \overline{\lambda} \leq 0. \tag{2.14}$$

From (2.14), we have

$$\left|\arg\lambda\left[D_1(x,y) + D_2(x,y)\right]\right| \geq 0.5\pi > 0.5q_r\pi. \tag{2.15}$$

According to the stability theorem for nonlinear fractional-order systems of commensurate order [22–25], system (2.8) is asymptotically stable. That is

$$\lim_{t \to +\infty} \|e\| = 0. \tag{2.16}$$

Therefore,

$$\lim_{t \to +\infty} \|e\| = \lim_{t \to +\infty} \left\| y - M(x)x \right\| = 0. \tag{2.17}$$

This indicates that the modified function projective synchronization between drive system (2.1) and response system (2.2) will be obtained. The proof is completed. □

Remark 2.4. Theorem 2.3 indicates that the condition of the MFPS between drive system (2.1) and response system (2.2) are $|\arg \lambda[D_1(x, y) + D_2(x, y)]| > 0.5 q_r \pi$. So, in practical applications, we can easily choose the matrix $D_2(x, y)$ according to the matrix $D_1(x, y)$. Moreover, in order to reserve all the nonlinear terms in response system or error system, the controller in our work may be complex than the controller reported by [16, 17]. But, all the nonlinear terms in response system or error system are absorbed in [16, 17].

Remark 2.5. Perhaps our result can be extended to the modified function projective synchronization of complex networks of fractional order chaotic systems [26–28] and the complex fractional-order multi scroll chaotic systems [29–31]. But, the modified function projective synchronization for complex networks and complex fractional-order multi-scroll chaotic systems would be much more complex. Further work on this issue is an ongoing research topic in our group.

3. Applications

In this section, to illustrate the effectiveness of the proposed MFPS scheme for different dimension fractional-order chaotic systems. Two groups of examples are considered and their numerical simulations are performed.

3.1. The MFPS between 3-Dimensional Fractional-Order Lorenz System and 4-Dimensional Fractional-Order Hyperchaotic System

The fractional-order Lorenz [3] system is described as follows:

$$\begin{aligned} D^{q_r} y_1 &= 10\left(y_2 - y_1\right) \\ D^{q_r} y_2 &= 28 y_1 - y_2 - y_1 y_3 \\ D^{q_r} y_3 &= y_1 y_2 - \frac{8 y_3}{3}. \end{aligned} \tag{3.1}$$

The fractional-order Lorenz system exhibits chaotic behavior [3] for $q_r \geq 0.993$. The chaotic attractor for $q_r = 0.995$ is shown in Figure 1.

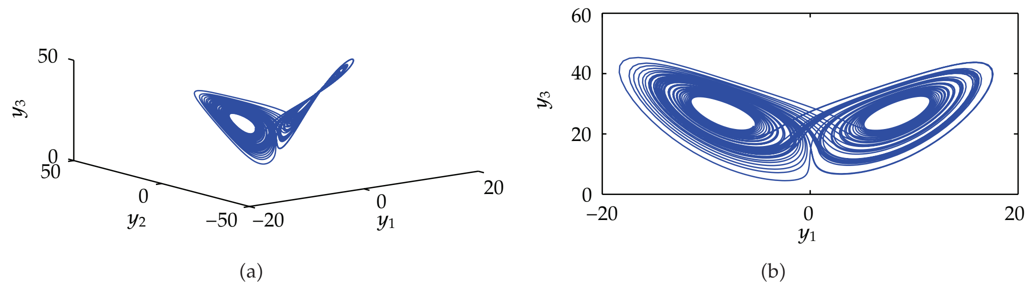


Figure 1: Chaotic attractors of the fractional-order Lorenz system (3.1) for $q_r = 0.995$.

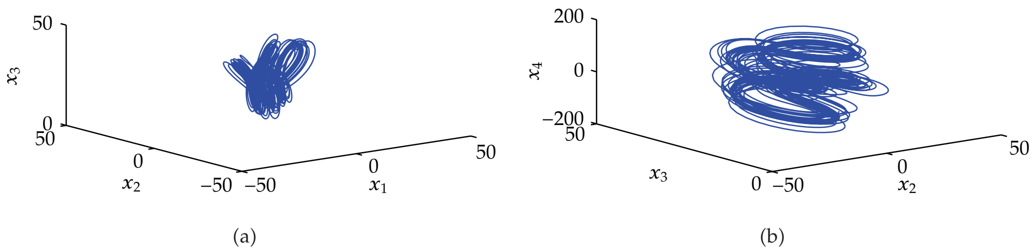


Figure 2: Hyperchaotic attractors of the fractional-order system (3.2) for $q_d = 0.95$.

Recently, Pan et al. constructed a hyperchaotic system [17]. Its corresponded fractional-order system is described as follows:

$$\begin{aligned} D^{q_d} x_1 &= 10(x_2 - x_1) + x_4 \\ D^{q_d} x_2 &= 28x_1 - x_1 x_3 \\ D^{q_d} x_3 &= x_1 x_2 - \frac{8x_3}{3} \\ D^{q_d} x_4 &= -x_1 x_3 + 1.3x_4. \end{aligned} \tag{3.2}$$

The hyperchaotic attractor of system (3.2) for $q_d = 0.95$ is shown in Figure 2.

Consider the fractional-order hyperchaotic system (3.2) with fractional-order $q_d = 0.95$ as drive system, and the fractional-order Loren system with fractional-order $q_r = 0.995$ as response system. According to the above mentioned, we can obtain

$$F_r(y) - F_r(M(x)x) = D_1(x, y)e = \begin{pmatrix} -10 & 10 & 0 \\ 28 - y_3 & -1 & -\sum_{j=1}^{4} M_{1j} x_j \\ y_2 & \sum_{j=1}^{4} M_{1j} x_j & -\dfrac{8}{3} \end{pmatrix} e. \tag{3.3}$$

Now, we can choose

$$D_2(x,y) = \begin{pmatrix} 0 & 0 & -y_2 \\ -38 + y_3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.4}$$

So,

$$D_1(x,y) + D_2(x,y) = \begin{pmatrix} -10 & 10 & -y_2 \\ -10 & -1 & -\sum_{j=1}^{4} M_{1j}x_j \\ y_2 & \sum_{j=1}^{4} M_{1j}x_j & -\dfrac{8}{3} \end{pmatrix}. \tag{3.5}$$

According to the above theorem, the MFPS between the 3-dimensional fractional-order Lorenz system (3.1) and the 4-dimensional fractional-order hyperchaotic system (3.2) can be achieved. For example, choose $M(x) = \begin{pmatrix} 1+x_2 & 2 & 1+x_2 & 3 \\ 2 & x_1 & 1 & x_2 \\ 2 & 1 & x_4 & 0.5 \end{pmatrix}$. The corresponding numerical result is shown in Figure 3, in which the initial conditions are $x(0) = (2,1,2,1)^T$, and $y(0) = (18,13,13.5)^T$, respectively.

3.2. *The MFPS between 4-Dimensional Fractional-Order Hyperchaotic Lǔ System and 3-Dimensional Fractional-Order Arneodo Chaotic System*

In 2002, Lü and Chen reported a new chaotic system [32], which be called Lü chaotic system. The Lü chaotic system is different from the Lorenz and Chen system. Based on Lü chaotic system, the hyperchaotic Lü chaotic system and the fractional-order hyperchaotic Lü system have been constructed recently. The fractional-order hyperchaotic Lü system [16] is described by the following

$$\begin{gathered} D^{q_r} y_1 = 36(y_2 - y_1) + y_4 \\ D^{q_r} y_2 = 20y_2 - y_1 y_3 \\ D^{q_r} y_3 = y_1 y_2 - 3y_3 \\ D^{q_r} y_4 = y_1 y_3 - y_4. \end{gathered} \tag{3.6}$$

The hyperchaotic attractor of system (3.6) for $q_r = 0.96$ is shown in Figure 4.

The fractional order Arneodo chaotic system [16] is defined as follows:

$$\begin{gathered} D^{q_d} x_1 = x_2 \\ D^{q_d} x_2 = x_3 \\ D^{q_d} x_3 = 5.5x_1 - 3.5x_2 - x_3 - x_1^3. \end{gathered} \tag{3.7}$$

The chaotic attractor of system (3.7) for $q_d = 0.998$ is shown in Figure 5.

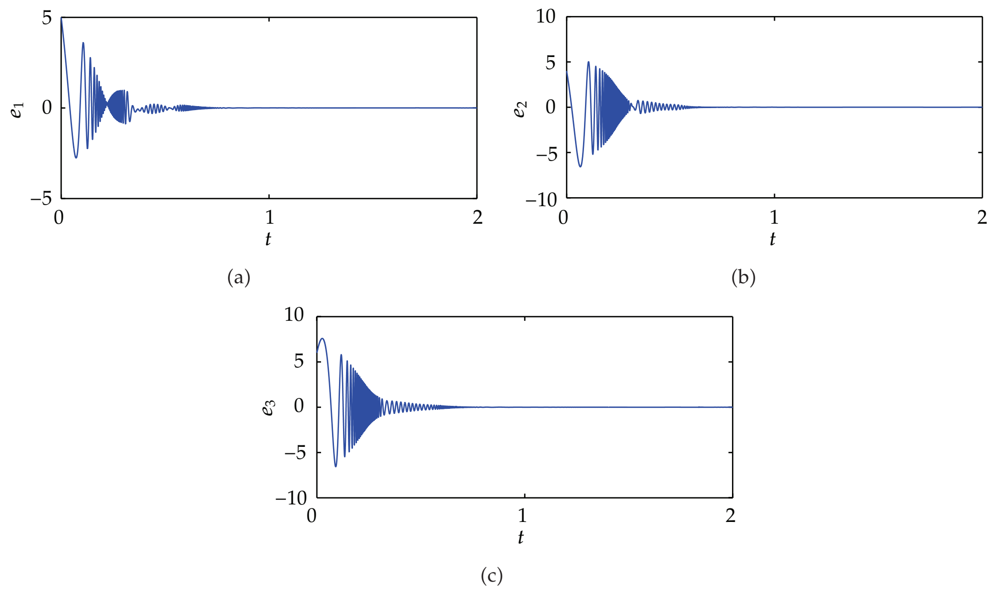


Figure 3: The modified function projective synchronization errors between the fractional-order Lorenz system (3.1) and the fractional-order hyperchaotic system (3.2).

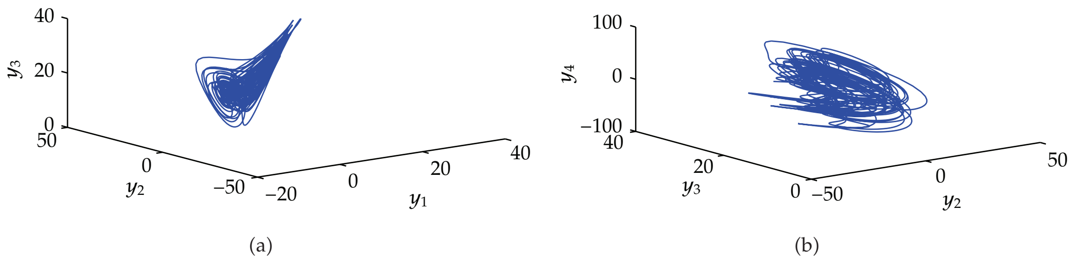
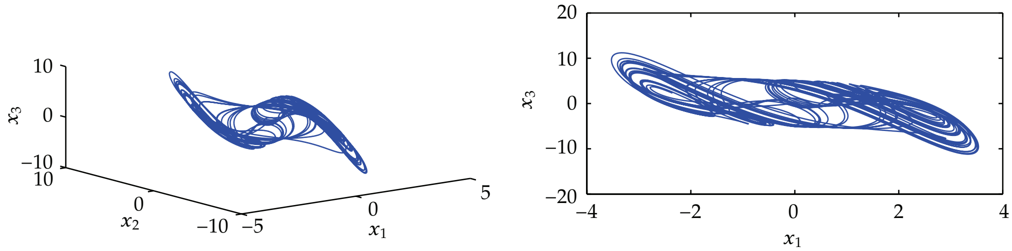


Figure 4: Hyperchaotic attractors of the fractional-order hyperchaotic Lŭ system (3.6) for $q_r = 0.96$.

Consider the fractional-order Arneodo chaotic system (3.7) with fractional-order $q_d = 0.998$ as drive system, and the fractional-order hyperchaotic Lŭ system (3.6) with fractional-order $q_r = 0.96$ as response system. According to the above mentioned, we can yield

$$F_r(y) - F_r(M(x)x) = D_1(x,y)e = \begin{pmatrix} -36 & 36 & 0 & 1 \\ -y_3 & 20 & -\sum_{j=1}^{3} M_{1j}x_j & 0 \\ y_2 & \sum_{j=1}^{3} M_{1j}x_j & -3 & 0 \\ y_3 & 0 & \sum_{j=1}^{3} M_{1j}x_j & -1 \end{pmatrix} e. \tag{3.8}$$



(a) (b)

Figure 5: Chaotic attractors of the fractional-order Arneodo chaotic system (3.7) for $q_d = 0.998$.

Now, we can choose

$$D_2(x,y) = \begin{pmatrix} 0 & 0 & -y_2 & 0 \\ -36 + y_3 & -21 & 0 & 0 \\ 0 & 0 & 0 & -\sum_{j=1}^{3} M_{1j} x_j \\ -1 - y_3 & 0 & 0 & 0 \end{pmatrix}. \tag{3.9}$$

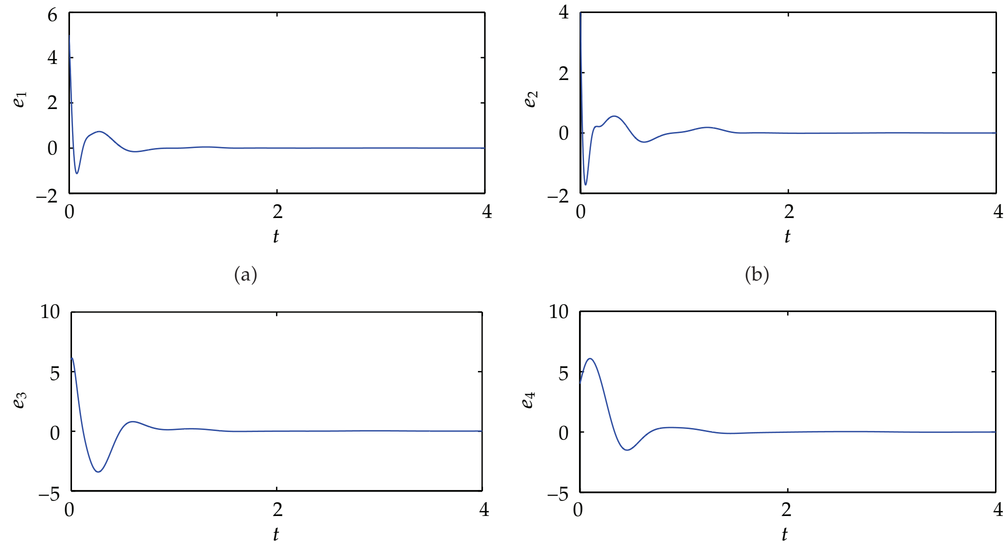
So,

$$D_1(x,y) + D_2(x,y) = \begin{pmatrix} -36 & 36 & -y_2 & 1 \\ -36 & -1 & -\sum_{j=1}^{3} M_{1j} x_j & 0 \\ y_2 & \sum_{j=1}^{3} M_{1j} x_j & -3 & -\sum_{j=1}^{3} M_{1j} x_j \\ -1 & 0 & \sum_{j=1}^{3} M_{1j} x_j & -1 \end{pmatrix}. \tag{3.10}$$

According to above theorem, the MFPS between the 4-dimensional fractional-order hyperchaotic Lŭ system (3.6) and the 3-dimensional fractional-order Arneodo chaotic system (3.7) can be achieved. For example, choose $M(x) = \begin{pmatrix} 1+x_2 & 0 & 0 \\ 0 & 1+x_3 & 0 \\ 0 & 0 & 0.5+x_1 \\ 1 & 1-x_1 & 1 \end{pmatrix}$. The corresponding numerical result is shown in Figure 6, in which the initial conditions are $x(0) = (2,2,2)^T$, and $y(0) = (11,10,11,2)^T$, respectively.

4. Conclusions

In this paper, based on the stability theory of the fractional-order system and the tracking control, a modified function projective synchronization scheme for different dimension fractional-order chaotic systems is addressed. The derived method in the present paper shows that the modified function projective synchronization between drive system and response system with different dimensions can be achieved. The modified function projective synchronization between 3-dimensional fractional-order Lorenz system and 4-dimensional



(a) (b) (c) (d)

Figure 6: The modified function projective synchronization errors between the fractional-order system (3.6) and the following fractional-order system:

fractional-order hyperchaotic system, and the modified function projective synchronization between the 4-dimensional fractional-order hyperchaotic Lŭ system, and the 3-dimensional fractional-order Arneodo chaotic system, are chosen to illustrate the proposed method. Numerical experiments shows that the present method works very well, which can be used for other chaotic systems.

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