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Research Article

Solvability of Nonlinear Integral Equations of Volterra Type

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This paper deals with the existence of continuous bounded solutions for a rather general nonlinear integral equation of Volterra type and discusses also the existence and asymptotic stability of continuous bounded solutions for another nonlinear integral equation of Volterra type. The main tools used in the proofs are some techniques in analysis and the Darbo fixed point theorem via measures of noncompactness. The results obtained in this paper extend and improve essentially some known results in the recent literature. Two nontrivial examples that explain the generalizations and applications of our results are also included.

1. Introduction

It is well known that the theory of nonlinear integral equations and inclusions has become important in some mathematical models of real processes and phenomena studied in mathematical physics, elasticity, engineering, biology, queuing theory economics, and so on (see, [1–3]). In the last decade, the existence, asymptotical stability, and global asymptotical stability of solutions for various Volterra integral equations have received much attention, see, for instance, [1, 4–22] and the references therein.

In this paper, we are interested in the following nonlinear integral equations of Volterra type:

$$x(t) = f\left(t, x(a(t)), (Hx)(b(t)), \int_0^{\alpha(t)} u(t, s, x(c(s)))ds\right), \quad \forall t \in \mathbb{R}_+, \tag{1.1}$$

$$x(t) = h\left(t, x(t), \int_0^{\alpha(t)} u(t, s, x(c(s)))ds\right), \quad \forall t \in \mathbb{R}_+, \tag{1.2}$$

where the functions f, h, u, a, b, c, α and the operator H appearing in (1.1) are given while $x = x(t)$ is an unknown function.

To the best of our knowledge, the papers dealing with (1.1) and (1.2) are few. But some special cases of (1.1) and (1.2) have been investigated by a lot of authors. For example, Arias et al. [4] studied the existence, uniqueness, and attractive behaviour of solutions for the nonlinear Volterra integral equation with nonconvolution kernels

$$x(t) = \int_0^t k(t,s)g(x(s))ds, \quad \forall t \in \mathbb{R}_+. \tag{1.3}$$

Using the monotone iterative technique, Constantin [13] got a sufficient condition which ensures the existence of positive solutions of the nonlinear integral equation

$$x(t) = L(t) + \int_0^t \left[M(t,s)x(s) + K(t,s)g(x(s))\right]ds, \quad \forall t \in \mathbb{R}_+. \tag{1.4}$$

Roberts [21] examined the below nonlinear Volterra integral equation

$$x(t) = \int_0^t k(t-s)G((x(s),s))ds, \quad \forall t \in \mathbb{R}_+, \tag{1.5}$$

which arose from certain models of a diffusive medium that can experience explosive behavior; utilizing the Darbo fixed point theorem and the measure of noncompactness in [7], Banaś and Dhage [6], Banaś et al. [8], Banaś and Rzepka [9, 10], Hu and Yan [16] and Liu and Kang [19] investigated the existence and/or asymptotic stability and/or global asymptotic stability of solutions for the below class of integral equations of Volterra type:

$$x(t) = (Tx)(t)\int_0^t u(t,s,x(s))ds, \quad \forall t \in \mathbb{R}_+, \tag{1.6}$$

$$x(t) = f(t,x(t)) + \int_0^t u(t,s,x(s))ds, \quad \forall t \in \mathbb{R}_+, \tag{1.7}$$

$$x(t) = f(t,x(t))\int_0^t u(t,s,x(s))ds, \quad \forall t \in \mathbb{R}_+, \tag{1.8}$$

$$x(t) = g(t,x(t)) + x(t)\int_0^t u(t,s,x(s))ds, \quad \forall t \in \mathbb{R}_+, \tag{1.9}$$

$$x(t) = f(t,x(t)) + g(t,x(t))\int_0^t u(t,s,x(s))ds, \quad \forall t \in \mathbb{R}_+, \tag{1.10}$$

$$x(t) = f(t,x(\alpha(t))) + \int_0^{\beta(t)} g\left(t,s,x(\gamma(s))\right)ds, \quad \forall t \in \mathbb{R}_+, \tag{1.11}$$

respectively. By means of the Schauder fixed point theorem and the measure of noncompactness in [7], Banaś and Rzepka [11] studied the existence of solutions for the below nonlinear quadratic Volterra integral equation:

$$x(t) = p(t) + f(t,x(t))\int_0^t v(t,s,x(s))ds, \quad \forall t \in \mathbb{R}_+. \tag{1.12}$$

Banaś and Chlebowicz [5] got the solvability of the following functional integral equation

$$x(t) = f_1\left(t, \int_0^t k(t,s) f_2(s, x(s)) ds\right), \quad \forall t \in \mathbb{R}_+ \tag{1.13}$$

in the space of Lebesgue integrable functions on $\mathbb{R}_+$. El-Sayed [15] studied a differential equation of neutral type with deviated argument, which is equivalent to the functional-integral equation

$$x(t) = f\left(t, \int_0^{H(t)} x(s) ds, x(h(t))\right), \quad \forall t \in \mathbb{R}_+ \tag{1.14}$$

by the technique linking measures of noncompactness with the classical Schauder fixed point principle. Using an improvement of the Krasnosel'skii type fixed point theorem, Taoudi [22] discussed the existence of integrable solutions of a generalized functional-integral equation

$$x(t) = g(t, x(t)) + f_1\left(t, \int_0^t k(t,s) f_2(s, x(s)) ds\right), \quad \forall t \in \mathbb{R}_+. \tag{1.15}$$

Dhage [14] used the classical hybrid fixed point theorem to establish the uniform local asymptotic stability of solutions for the nonlinear quadratic functional integral equation of mixed type

$$x(t) = f(t, x(\alpha(t)))\left(q(t) + \int_0^{\beta(t)} u(t, s, x(\gamma(s))) ds\right), \quad \forall t \in \mathbb{R}_+. \tag{1.16}$$

The purpose of this paper is to prove the existence of continuous bounded solutions for (1.1) and to discuss the existence and asymptotic stability of continuous bounded solutions for (1.2). The main tool used in our considerations is the technique of measures of noncompactness [7] and the famous fixed point theorem of Darbo [23]. The results presented in this paper extend proper the corresponding results in [6, 9, 10, 15, 16, 19]. Two nontrivial examples which show the importance and the applicability of our results are also included.

This paper is organized as follows. In the second section, we recall some definitions and preliminary results and prove a few lemmas, which will be used in our investigations. In the third section, we state and prove our main results involving the existence and asymptotic stability of solutions for (1.1) and (1.2). In the final section, we construct two nontrivial examples for explaining our results, from which one can see that the results obtained in this paper extend proper several ones obtained earlier in a lot of papers.

2. Preliminaries

In this section, we give a collection of auxiliary facts which will be needed further on. Let $\mathbb{R} = (-\infty, \infty)$ and $\mathbb{R}_+ = [0, \infty)$. Assume that $(E, \|\cdot\|)$ is an infinite dimensional Banach space with zero element θ and B_r stands for the closed ball centered at θ and with radius r. Let $B(E)$ denote the family of all nonempty bounded subsets of E.

Definition 2.1. Let D be a nonempty bounded closed convex subset of the space E. A operator $f : D \to E$ is said to be a *Darbo operator* if it is continuous and satisfies that $\mu(fA) \le k\mu(A)$ for each nonempty subset A of D, where $k \in [0,1)$ is a constant and μ is a measure of noncompactness on $B(E)$.

The Darbo fixed point theorem is as follows.

Lemma 2.2 (see [23]). *Let D be a nonempty bounded closed convex subset of the space E and let $f : D \to D$ be a Darbo operator. Then f has at least one fixed point in D.*

Let $BC(\mathbb{R}_+)$ denote the Banach space of all bounded and continuous functions $x : \mathbb{R}_+ \to \mathbb{R}$ equipped with the standard norm

$$\|x\| = \sup\{|x(t)| : t \in \mathbb{R}_+\}. \tag{2.1}$$

For any nonempty bounded subset X of $BC(\mathbb{R}_+)$, $x \in X, t \in \mathbb{R}_+, T > 0$ and $\varepsilon \ge 0$, define

$$\begin{aligned}
&\omega^T(x,\varepsilon) = \sup\{|x(p) - x(q)| : p,q \in [0,T] \text{ with } |p-q| \le \varepsilon\},\\
&\omega^T(X,\varepsilon) = \sup\left\{\omega^T(x,\varepsilon) : x \in X\right\}, \qquad \omega_0^T(X) = \lim_{\varepsilon\to 0}\omega^T(X,\varepsilon),\\
&\omega_0(X) = \lim_{T\to+\infty}\omega_0^T(X), \qquad X(t) = \{x(t) : x \in X\},\\
&\operatorname{diam} X(t) = \sup\{|x(t) - y(t)| : x,y \in X\},\\
&\mu(X) = \omega_0(X) + \limsup_{t\to+\infty} \operatorname{diam} X(t).
\end{aligned} \tag{2.2}$$

It can be shown that the mapping μ is a measure of noncompactness in the space $BC(\mathbb{R}_+)$ [4].

Definition 2.3. Solutions of an integral equation are said to be *asymptotically stable* if there exists a ball B_r in the space $BC(\mathbb{R}_+)$ such that for any $\varepsilon > 0$, there exists $T > 0$ with

$$|x(t) - y(t)| \le \varepsilon \tag{2.3}$$

for all solutions $x(t), y(t) \in B_r$ of the integral equation and any $t \ge T$.

It is clear that the concept of asymptotic stability of solutions is equivalent to the concept of uniform local attractivity [9].

Lemma 2.4. *Let φ and $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ be functions with*

$$\lim_{t\to+\infty}\psi(t) = +\infty, \qquad \limsup_{t\to+\infty}\varphi(t) < +\infty. \tag{2.4}$$

Then

$$\limsup_{t\to+\infty}\varphi(\psi(t)) = \limsup_{t\to+\infty}\varphi(t). \tag{2.5}$$

Proof. Let $\limsup_{t\to+\infty}\varphi(t) = A$. It follows that for each $\varepsilon > 0$, there exists $T > 0$ such that

$$\varphi(t) < A + \varepsilon, \quad \forall t \geq T. \tag{2.6}$$

Equation (2.4) means that there exists $C > 0$ satisfying

$$\psi(t) \geq T, \quad \forall t \geq C. \tag{2.7}$$

Using (2.6) and (2.7), we infer that

$$\varphi(\psi(t)) < A + \varepsilon, \quad \forall t \geq C, \tag{2.8}$$

that is, (2.5) holds. This completes the proof. □

Lemma 2.5. *Let $a : \mathbb{R}_+ \to \mathbb{R}_+$ be a differential function. If for each $T > 0$, there exists a positive number $\overline{a}_T$ satisfying*

$$0 \leq a'(t) \leq \overline{a}_T, \quad \forall t \in [0,T], \tag{2.9}$$

then

$$\omega^T(x \circ a, \varepsilon) \leq \omega^{a(T)}(x, \overline{a}_T\varepsilon), \quad \forall (x,\varepsilon) \in BC(R_+) \times (0,+\infty). \tag{2.10}$$

Proof. Let $T > 0$. It is clear that (2.9) yields that the function a is nondecreasing in $[0,T]$ and for any $t, s \in [0,T]$, there exists $\xi \in (0,T)$ satisfying

$$|a(t) - a(s)| = a'(\xi)|t-s| \leq \overline{a}_T|t-s| \tag{2.11}$$

by the mean value theorem. Notice that (2.9) means that $a(t) \in [a(0), a(T)] \subseteq [0, a(T)]$ for each $t \in [0,T]$, which together with (2.11) gives that

$$\begin{aligned} \omega^T(x \circ a, \varepsilon) &= \sup\{|x(a(t)) - x(a(s))| : t, s \in [0,T], |t-s| \leq \varepsilon\} \\ &\leq \sup\{\left|x(p) - x(q)\right| : p, q \in [a(0), a(T)], \left|p-q\right| \leq \overline{a}_T\varepsilon\} \\ &\leq \omega^{a(T)}(x, \overline{a}_T\varepsilon), \quad \forall (x,\varepsilon) \in BC(R_+) \times (0,+\infty), \end{aligned} \tag{2.12}$$

which yields that (2.10) holds. This completes the proof. □

Lemma 2.6. *Let $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ be a function with $\lim_{t\to+\infty}\varphi(t) = +\infty$ and X be a nonempty bounded subset of $BC(\mathbb{R}_+)$. Then*

$$\omega_0(X) = \lim_{T\to+\infty}\omega_0^{\varphi(T)}(X). \tag{2.13}$$

Proof. Since X is a nonempty bounded subset of $BC(\mathbb{R}_+)$, it follows that $\omega_0(X) = \lim_{T\to+\infty}\omega_0^T(X)$. That is, for given $\varepsilon > 0$, there exists $M > 0$ satisfying

$$\left|\omega_0^T(X) - \omega_0(X)\right| < \varepsilon, \quad \forall T > M. \tag{2.14}$$

It follows from $\lim_{t\to+\infty}\varphi(t) = +\infty$ that there exists $L > 0$ satisfying

$$\varphi(T) > M, \quad \forall T > L. \tag{2.15}$$

By means of (2.14) and (2.15), we get that

$$\left|\omega_0^{\varphi(T)}(X) - \omega_0(X)\right| < \varepsilon, \quad \forall T > L, \tag{2.16}$$

which yields (2.13). This completes the proof. □

3. Main Results

Now we formulate the assumptions under which (1.1) will be investigated.

(H1) $f : \mathbb{R}_+ \times \mathbb{R}^3 \to \mathbb{R}$ is continuous with $f(t,0,0,0) \in BC(\mathbb{R}_+)$ and $\overline{f} = \sup\{|f(t,0,0,0)| : t \in \mathbb{R}_+\}$;

(H2) $a, b, c, \alpha : \mathbb{R}_+ \to \mathbb{R}_+$ satisfy that a and b have nonnegative and bounded derivative in the interval $[0,T]$ for each $T > 0$, c and α are continuous and α is nondecreasing and

$$\lim_{t\to+\infty} a(t) = \lim_{t\to+\infty} b(\mathrm{t}) = +\infty; \tag{3.1}$$

(H3) $u : \mathbb{R}_+^2 \times \mathbb{R} \to \mathbb{R}$ is continuous;

(H4) there exist five positive constants r, M, M_0, M_1, and M_2 and four continuous functions $m_1, m_2, m_3, g : \mathbb{R}_+ \to \mathbb{R}_+$ such that g is nondecreasing and

$$\begin{gathered}\left|f(t,v,w,z) - f(t,p,q,y)\right| \le m_1(t)\left|v-p\right| + m_2(t)\left|w-q\right| + m_3(t)\left|z-y\right|, \\ \forall t \in \mathbb{R}_+,\ v,p \in [-r,r],\ w,q \in \left[-g(r), g(r)\right],\ z,y \in [-M,M],\end{gathered} \tag{3.2}$$

$$\lim_{t\to+\infty} \sup\left\{ m_3(t)\int_0^{\alpha(t)} |u(t,s,v(c(s))) - u(t,s,w(c(s)))|ds : v,w \in B_r \right\} = 0, \tag{3.3}$$

$$\sup\left\{ m_3(t)\int_0^{\alpha(t)} |u(t,s,v(c(s)))|ds : t \in \mathbb{R}_+, v \in B_r \right\} \le M_0, \tag{3.4}$$

$$\sup\left\{ \int_0^{\alpha(t)} |u(t,s,v(c(s)))|ds : t \in \mathbb{R}_+, v \in B_r \right\} \le M, \tag{3.5}$$

$$\sup\{m_i(t) : t \in \mathbb{R}_+\} \leq M_i, \quad i \in \{1,2\}, \tag{3.6}$$

$$M_1 r + M_2 g(r) + M_0 + \overline{f} \leq r, \qquad M_1 + QM_2 < 1; \tag{3.7}$$

(H5) $H : BC(\mathbb{R}_+) \to BC(R_+)$ satisfies that $H : B_r \to BC(\mathbb{R}_+)$ is a Darbo operator with respect to the measure of noncompactness of μ with a constant Q and

$$|(Hx)(b(t))| \leq g(|x(b(t))|), \quad \forall (x,t) \in B_r \times \mathbb{R}_+. \tag{3.8}$$

Theorem 3.1. *Under Assumptions* (H1)–(H5), (1.1) *has at least one solution* $x = x(t) \in B_r$.

Proof. Let $x \in B_r$ and define

$$(Fx)(t) = f\left(t, x(a(t)), (Hx)(b(t)), \int_0^{\alpha(t)} u(t,s,x(c(s)))ds\right), \quad \forall t \in \mathbb{R}_+. \tag{3.9}$$

It follows from (3.9) and Assumptions (H1)–(H5) that Fx is continuous on $\mathbb{R}_+$ and that

$$\begin{aligned}
|(Fx)(t)| &\leq \left| f\left(t, x(a(t)), (Hx)(b(t)), \int_0^{\alpha(t)} u(t,s,x(c(s)))ds\right) - f(t,0,0,0)\right| + |f(t,0,0,0)| \\
&\leq m_1(t)|x(a(t))| + m_2(t)|(Hx)(b(t))| + m_3(t)\left|\int_0^{\alpha(t)} u(t,s,x(c(s)))ds\right| + \overline{f} \\
&\leq M_1 r + M_2 g(r) + M_0 + \overline{f} \\
&\leq r, \quad \forall t \in \mathbb{R}_+,
\end{aligned} \tag{3.10}$$

which means that Fx is bounded on $\mathbb{R}_+$ and $F(B_r) \subseteq B_r$.

We now prove that

$$\mu(FX) \leq (M_1 + M_2 Q)\mu(X), \quad \forall X \subseteq B_r. \tag{3.11}$$

Let X be a nonempty subset of B_r. Using (3.2), (3.6), and (3.9), we conclude that

$$\begin{aligned}
&|(Fx)(t) - (Fy)(t)| \\
&\quad = \left| f\left(t, x(a(t)), (Hx)(b(t)), \int_0^{\alpha(t)} u(t,s,x(c(s)))ds\right)\right. \\
&\qquad \left. - f\left(t, y(a(t)), (Hy)(b(t)), \int_0^{\alpha(t)} u(t,s,y(c(s)))ds\right)\right|
\end{aligned}$$

$$\begin{aligned}
&\le m_1(t)\big|x(a(t))-y(a(t))\big| + m_2(t)\big|(Hx)(b(t))-(Hy)(b(t))\big| \\
&\quad + m_3(t)\int_0^{\alpha(t)}\big|u(t,s,x(c(s)))-u(t,s,y(c(s)))\big|ds \\
&\le M_1\big|x(a(t))-y(a(t))\big| + M_2\big|(Hx)(b(t))-(Hy)(b(t))\big| \\
&\quad + \sup\left\{m_3(t)\int_0^{\alpha(t)}|u(t,s,w(c(s)))-u(t,s,z(c(s)))|ds : w,z\in B_r\right\}, \quad \forall x,y\in X,\ t\in\mathbb{R}_+,
\end{aligned} \tag{3.12}$$

which yields that

$$\begin{aligned}
\operatorname{diam}(FX)(t) &\le M_1 \operatorname{diam} X(a(t)) + M_2 \operatorname{diam}(HX)(b(t)) \\
&\quad + \sup\left\{m_3(t)\int_0^{\alpha(t)}|u(t,s,w(c(s)))-u(t,s,z(c(s)))|ds : w,z\in B_r\right\}, \\
&\quad \forall t\in\mathbb{R}_+,
\end{aligned} \tag{3.13}$$

which together with (3.3), Assumption (H2) and Lemma 2.4 ensures that

$$\begin{aligned}
&\limsup_{t\to+\infty} \operatorname{diam}(FX)(t) \\
&\quad\le M_1 \limsup_{t\to+\infty} \operatorname{diam} X(a(t)) + M_2 \limsup_{t\to+\infty} \operatorname{diam}(HX)(b(t)) \\
&\qquad + \limsup_{t\to+\infty} \sup\left\{m_3(t)\int_0^{\alpha(t)}|u(t,s,w(c(s)))-u(t,s,z(c(s)))|ds : w,z\in B_r\right\} \\
&\quad= M_1 \limsup_{t\to+\infty} \operatorname{diam} X(t) + M_2 \limsup_{t\to+\infty} \operatorname{diam}(HX)(t),
\end{aligned} \tag{3.14}$$

that is,

$$\limsup_{t\to+\infty} \operatorname{diam}(FX)(t) \le M_1 \limsup_{t\to+\infty} \operatorname{diam} X(t) + M_2 \limsup_{t\to+\infty} \operatorname{diam}(HX)(t). \tag{3.15}$$

For each $T>0$ and $\varepsilon>0$, put

$$\begin{aligned}
&M_{3T} = \sup\{m_3(p) : p\in[0,T]\}, \\
&u_r^T = \sup\{|u(p,q,v)| : p\in[0,T],\ q\in[0,\alpha(T)],\ v\in[-r,r]\}, \\
&\omega_r^T(u,\varepsilon) = \sup\{|u(p,\tau,v)-u(q,\tau,v)| : p,q\in[0,T],\ |p-q|\le\varepsilon,\ \tau\in[0,\alpha(T)],\ v\in[-r,r]\}, \\
&\omega^T(u,\varepsilon,r) = \sup\{|u(p,q,v)-u(p,q,w)| : p\in[0,T],\ q\in[0,\alpha(T)],\ v,w\in[-r,r],\ |v-w|\le\varepsilon\}, \\
&\omega_r^T(f,\varepsilon,g(r)) = \sup\{|f(p,v,w,z)-f(q,v,w,z)| : p,\ q\in[0,T],\ |p-q|\le\varepsilon, \\
&\qquad v\in[-r,r],\ w\in[-g(r),g(r)],\ z\in[-M,M]\}.
\end{aligned} \tag{3.16}$$

Let $T > 0$, $\varepsilon > 0$, $x \in X$ and $t, s \in [0,T]$ with $|t-s| \le \varepsilon$. It follows from (H2) that there exist $\overline{a}_T$ and $\overline{b}_T$ satisfying

$$0 \le a'(t) \le \overline{a}_T, \quad 0 \le b'(t) \le \overline{b}_T, \quad \forall t \in [0,T]. \tag{3.17}$$

In light of (3.2), (3.6), (3.9), (3.16), (3.17), and Lemma 2.5, we get that

$$\begin{aligned}
&|(Fx)(t)-(Fx)(s)| \\
&\quad \le \left| f\left(t, x(a(t)), (Hx)(b(t)), \int_0^{\alpha(t)} u(t,\tau,x(c(\tau)))d\tau\right)\right. \\
&\qquad \left. - f\left(t, x(a(s)), (Hx)(b(s)), \int_0^{\alpha(s)} u(s,\tau,x(c(\tau)))d\tau\right)\right| \\
&\qquad + \left| f\left(t, x(a(s)), (Hx)(b(s)), \int_0^{\alpha(s)} u(s,\tau,x(c(\tau)))d\tau\right)\right. \\
&\qquad\quad \left. - f\left(s, x(a(s)), (Hx)(b(s)), \int_0^{\alpha(s)} u(s,\tau,x(c(\tau)))d\tau\right)\right| \\
&\quad \le m_1(t)|x(a(t))-x(a(s))| + m_2(t)|(Hx)(b(t))-(Hx)(b(s))| \\
&\qquad + m_3(t)\left[\left|\int_{\alpha(s)}^{\alpha(t)} |u(t,\tau,x(c(\tau)))|d\tau\right| + \int_0^{\alpha(s)} |u(t,\tau,x(c(\tau)))-u(s,\tau,x(c(\tau)))|d\tau\right] \\
&\qquad + \sup\{|f(p,v,w,z)-f(q,v,w,z)| : p,q\in[0,T],\ |p-q|\le\varepsilon, \\
&\qquad\qquad v\in[-r,r],\ w\in[-g(r),g(r)],\ z\in[-M,M]\} \\
&\quad \le M_1\omega^T(x\circ a,\varepsilon) + M_2\omega^T((Hx)\circ b,\varepsilon) \\
&\qquad + M_{3T}|\alpha(t)-\alpha(s)|\sup\{|u(p,\tau,v)| : p\in[0,T],\ \tau\in[0,\alpha(T)],\ v\in[-r,r]\} \\
&\qquad + M_{3T}\alpha(T)\sup\{|u(p,\tau,v)-u(q,\tau,v)| : p,q\in[0,T],\ |p-q|\le\varepsilon,\ \tau\in[0,\alpha(T)],\ v\in[-r,r]\} \\
&\qquad + \omega_r^T(f,\varepsilon,g(r)) \\
&\quad \le M_1\omega^{a(T)}(x,\overline{a}_T\varepsilon) + M_2\omega^{b(T)}\left(Hx,\overline{b}_T\varepsilon\right) + M_{3T}\omega^T(\alpha,\varepsilon)u_r^T \\
&\qquad + M_{3T}\alpha(T)\omega_r^T(u,\varepsilon) + \omega_r^T(f,\varepsilon,g(r)),
\end{aligned} \tag{3.18}$$

which implies that

$$\begin{aligned}
\omega^T(Fx,\varepsilon) &\le M_1\omega^{a(T)}(x,\overline{a}_T\varepsilon) + M_2\omega^{b(T)}\left(Hx,\overline{b}_T\varepsilon\right) + M_{3T}\omega^T(\alpha,\varepsilon)u_r^T \\
&\quad + M_{3T}\alpha(T)\omega_r^T(u,\varepsilon) + \omega_r^T(f,\varepsilon,g(r)), \quad \forall T>0,\ \varepsilon>0,\ x\in X.
\end{aligned} \tag{3.19}$$

Notice that Assumptions (H1)–(H3) imply that the functions $\alpha = \alpha(t)$, $f = f(t,p,q,v)$ and $u = u(t,y,z)$ are uniformly continuous on the sets $[0,T]$,$[0,T] \times [-r,r] \times [-g(r),g(r)] \times [-M,M]$ and $[0,T] \times [0,\alpha(T)] \times [-r,r]$, respectively. It follows that

$$\lim_{\varepsilon \to 0} \omega^T(\alpha,\varepsilon) = \lim_{\varepsilon \to 0} \omega_r^T(u,\varepsilon) = \lim_{\varepsilon \to 0} \omega_r^T(f,\varepsilon,g(r)) = 0. \tag{3.20}$$

In terms of (3.19) and (3.20), we have

$$\omega_0^T(FX) \leq M_1 \omega_0^{a(T)}(X) + M_2 \omega_0^{b(T)}(HX), \tag{3.21}$$

letting $T \to +\infty$ in the above inequality, by Assumption (H2) and Lemma 2.6, we infer that

$$\omega_0(FX) \leq M_1 \omega_0(X) + M_2 \omega_0(HX). \tag{3.22}$$

By means of (3.15), (3.22), and Assumption (H5), we conclude immediately that

$$\begin{aligned}
\mu(FX) &= \omega_0(FX) + \limsup_{t \to +\infty} \operatorname{diam}(FX)(t) \\
&\leq M_1 \omega_0(X) + M_2 \omega_0(HX) + M_1 \limsup_{t \to +\infty} \operatorname{diam} X(t) + M_2 \limsup_{t \to +\infty} \operatorname{diam}(HX)(t) \\
&= M_1 \mu(X) + M_2 \Big(\omega_0(HX) + \limsup_{t \to +\infty} \operatorname{diam}(HX)(t) \Big) \\
&\leq (M_1 + M_2 Q) \mu(X),
\end{aligned} \tag{3.23}$$

that is, (3.11) holds.

Next we prove that F is continuous on the ball B_r. Let $x \in B_r$ and $\{x_n\}_{n \geq 1} \subset B_r$ with $\lim_{n \to \infty} x_n = x$. It follows from (3.3) that for given $\varepsilon > 0$, there exists a positive constant T such that

$$\sup\left\{ m_3(t) \int_0^{\alpha(t)} |u(t,s,v(c(s))) - u(t,s,w(c(s)))| ds : v,w \in B_r \right\} < \frac{\varepsilon}{3}, \quad \forall t > T. \tag{3.24}$$

Since $u = u(t,s,v)$ is uniformly continuous in $[0,T] \times [0,\alpha(T)] \times [-r,r]$, it follows from (3.16) that there exists $\delta_0 > 0$ satisfying

$$\omega^T(u,\delta,r) < \frac{\varepsilon}{1 + 3M_{3T}\alpha(T)}, \quad \forall \delta \in (0,\delta_0). \tag{3.25}$$

By Assumption (H5) and $\lim_{n \to \infty} x_n = x$, we know that there exists a positive integer N such that

$$(1 + M_1)\|x_n - x\| + M_2 \|Hx_n - Hx\| < \frac{1}{2} \min\{\varepsilon, \delta_0\}, \quad \forall n > N. \tag{3.26}$$

In view of (3.2), (3.6), (3.9), (3.24)–(3.26), and Assumption (H2), we gain that for any $n > N$ and $t \in \mathbb{R}_+$

$$
\begin{aligned}
&|(Fx_n)(t) - (Fx)(t)| \\
&= \left| f\left(t, x_n(a(t)), (Hx_n)(b(t)), \int_0^{\alpha(t)} u(t,s,x_n(c(s)))ds\right) \right. \\
&\qquad \left. - f\left(t, x(a(t)), (Hx)(b(t)), \int_0^{\alpha(t)} u(t,s,x(c(s)))ds\right) \right| \\
&\le m_1(t)|x_n(a(t)) - x(a(t))| + m_2(t)|(Hx_n)(b(t)) - (Hx)(b(t))| \\
&\quad + m_3(t)\int_0^{\alpha(t)} |u(t,s,x_n(c(s))) - u(t,s,x(c(s)))|ds \\
&\le M_1\|x_n - x\| + M_2\|Hx_n - Hx\| \\
&\quad + \max\left\{ \sup_{\tau > T} \sup\left\{ m_3(\tau)\int_0^{\alpha(\tau)} \left|u(\tau,s,z(c(s))) - u(\tau,s,y(c(s)))\right|ds : z, y \in B_r \right\}, \right. \\
&\qquad\qquad \sup_{\tau \in [0,T]} \sup\left\{ m_3(\tau)\int_0^{\alpha(\tau)} \left|u(\tau,s,z(c(s))) - u(\tau,s,y(c(s)))\right|ds : z, \right. \\
&\qquad\qquad\qquad \left.\left. y \in B_r, \|z - y\| \le \frac{\delta_0}{2} \right\} \right\} \\
&< \frac{\varepsilon}{2} + \max\left\{ \frac{\varepsilon}{3}, \sup_{\tau \in [0,T]} \left\{ m_3(\tau)\int_0^{\alpha(\tau)} \omega^T\left(u, \frac{\delta_0}{2}, r\right)ds \right\} \right\} \\
&< \frac{\varepsilon}{2} + \max\left\{ \frac{\varepsilon}{3}, \frac{M_{3T}\alpha(T)\varepsilon}{1 + 3M_{3T}\alpha(T)} \right\} \\
&= \frac{5\varepsilon}{6},
\end{aligned}
\tag{3.27}
$$

which yields that

$$\|Fx_n - Fx\| < \varepsilon, \quad \forall n > N, \tag{3.28}$$

that is, F is continuous at each point $x \in B_r$.

Thus Lemma 2.2 ensures that F has at least one fixed point $x = x(t) \in B_r$. Hence (1.1) has at least one solution $x = x(t) \in B_r$. This completes the proof. □

Now we discuss (1.2) under below hypotheses:

(H6) $h : \mathbb{R}_+ \times \mathbb{R}^2 \to \mathbb{R}$ is continuous with $h(t,0,0) \in BC(\mathbb{R}_+)$ and $\overline{h} = \sup\{|h(t,0,0)| : t \in \mathbb{R}_+\}$;

(H7) $c, \alpha : \mathbb{R}_+ \to \mathbb{R}_+$ are continuous and α is nondecreasing;

(H8) there exist two continuous functions $m_1, m_3 : \mathbb{R}_+ \to \mathbb{R}_+$ and four positive constants r, M, M_0 and M_1 satisfying (3.3)–(3.5),

$$\left|h(t,v,w) - h(t,p,q)\right| \le m_1(t)\left|v-p\right| + m_3(t)\left|w-q\right|, \quad \forall t \in \mathbb{R}_+,\ v,\ p \in [-r,r],\ w,q \in [-M,M], \tag{3.29}$$

$$\sup\{m_1(t) : t \in \mathbb{R}_+\} \le M_1, \tag{3.30}$$

$$M_0 + \overline{h} \le r(1-M_1). \tag{3.31}$$

Theorem 3.2. *Under Assumptions* (H3) *and* (H6)–(H8), (1.2) *has at least one solution* $x = x(t) \in B_r$. *Moreover, solutions of* (1.2) *are asymptotically stable.*

Proof. As in the proof of Theorem 3.1, we conclude that (1.2) possesses at least one solution in B_r.

Now we claim that solutions of (1.2) are asymptotically stable. Note that r, M_0, and M_1 are positive numbers and $\overline{h} \ge 0$, it follows from (3.31) that $M_1 < 1$. In terms of (3.3), we infer that for given $\varepsilon > 0$, there exists $T > 0$ such that

$$\sup\left\{m_3(t)\int_0^{\alpha(t)} |u(t,s,v(c(s))) - u(t,s,w(c(s)))|ds : v,w \in B_r\right\} < \varepsilon(1-M_1), \quad \forall t \ge T. \tag{3.32}$$

Let $z = z(t), y = y(t)$ be two arbitrarily solutions of (1.2) in B_r. According to (3.29)–(3.32), we deduce that

$$\begin{aligned}
&\left|z(t)-y(t)\right| \\
&\quad = \left|h\left(t,z(t),\int_0^{\alpha(t)} u(t,\tau,z(c(\tau)))d\tau\right) - h\left(t,y(t),\int_0^{\alpha(t)} u(t,\tau,y(c(\tau)))d\tau\right)\right| \\
&\quad \le m_1(t)\left|z(t)-y(t)\right| + m_3(t)\int_0^{\alpha(t)}\left|u(t,\tau,z(c(\tau))) - u(t,\tau,y(c(\tau)))\right|d\tau \\
&\quad \le M_1\left|z(t)-y(t)\right| + \sup\left\{m_3(t)\int_0^{\alpha(t)}|u(t,\tau,v(c(\tau))) - u(t,\tau,w(c(\tau)))|d\tau : v,w \in B_r\right\} \\
&\quad < M_1\left|z(t)-y(t)\right| + \varepsilon(1-M_1), \quad \forall t \ge T,
\end{aligned} \tag{3.33}$$

which means that

$$\left|z(t)-y(t)\right| < \varepsilon \tag{3.34}$$

whenever z, y are solutions of (1.2) in B_r and $t \ge T$. Hence solutions of (1.2) are asymptotically stable. This completes the proof. □

Remark 3.3. Theorems 3.1 and 3.2 generalize Theorem 3.1 in [6], Theorem 2 in [9], Theorem 3 in [10], Theorem 1 in [15], Theorem 2 in [16], and Theorem 3.1 in [19]. Examples 4.1 and 4.2 in the fourth section show that Theorems 3.1 and 3.2 substantially extend the corresponding results in [6, 9, 10, 15, 16, 19].

4. Examples

In this section, we construct two nontrivial examples to support our results.

Example 4.1. Consider the following nonlinear integral equation of Volterra type:

$$\begin{aligned} x(t) = {} & \frac{1}{10+\ln^3(1+t^3)} + \frac{tx^2(1+3t^2)}{200(1+t)} + \frac{1}{10\sqrt{3}+20t+3|x(t^3)|} \\ & + \frac{1}{9+\sqrt{1+t}}\left(\int_0^{t^2} \frac{s^2x(4s)\sin\left(\sqrt{1+t^3s^5}-(t-s)^4x^3(4s)\right)}{1+t^{12}+\left|stx^3(4s)-3s^3+7(t-2s)^2\right| x^2(4s)}ds\right)^3, \quad \forall t\in\mathbb{R}_+. \end{aligned} \tag{4.1}$$

Put

$$\begin{aligned} f(t,v,w,z) &= \frac{1}{10+\ln^3(1+t^3)} + \frac{tv^2}{200(1+t)} + \frac{1}{10\sqrt{3}+20t+|w|} + \frac{z^3}{9+\sqrt{1+t}}, \\ u(t,s,p) &= \frac{s^2p\sin\left(\sqrt{1+t^3s^5}-(t-s)^4p^3\right)}{1+t^{12}+\left|stp^3-3s^3+7(t-2s)^2\right|p^2}, \\ a(t) &= 1+3t^2, \quad b(t)=t^3, \quad c(t)=4t, \quad \alpha(t)=t^2, \quad \forall t,s\in\mathbb{R}_+,\ v,w,z,p\in\mathbb{R}. \end{aligned} \tag{4.2}$$

Let

$$\begin{aligned} & r\in\left[\frac{9-\sqrt{51}}{2},\frac{9+\sqrt{51}}{2}\right], \quad M=3, \quad M_0=\frac{9r}{10}, \quad M_1=\frac{r}{100}, \quad M_2=\frac{1}{300}, \quad \overline{f}=\frac{1}{10}, \\ & Q=3, \quad m_1(t)=\frac{rt}{100(1+t)}, \quad m_2(t)=\frac{1}{\left(10\sqrt{3}+20t\right)^2}, \quad m_3(t)=\frac{3M^2}{9+\sqrt{1+t}}, \\ & (Hy)(t)=3y(t), \quad g(t)=3t, \quad \forall t\in\mathbb{R}_+, y\in BC(\mathbb{R}_+). \end{aligned} \tag{4.3}$$

It is easy to verify that (3.6) and Assumptions (H1)–(H3) and (H5) are satisfied. Notice that

$$M_1r+M_2g(r)+M_0+\overline{f}\le r \iff r\in\left[\frac{9-\sqrt{51}}{2},\frac{9+\sqrt{51}}{2}\right],$$

$$\left|f(t,v,w,z)-f(t,p,q,y)\right|$$

$$
\begin{aligned}
&\le \frac{t}{200(1+t)}\left|v^2-p^2\right| + \left|\frac{1}{10\sqrt{3}+20t+|w|} - \frac{1}{10\sqrt{3}+20t+|q|}\right| \\
&\quad + \frac{1}{9+\sqrt{1+t}}\left|z^3-y^3\right| \le m_1(t)|v-p| + m_2(t)|w-q| + m_3(t)|z-y|, \\
&\qquad \forall t\in\mathbb{R}_+,\ v,p\in[-r,r],\ w,q\in[-g(r),g(r)],\ z,y\in[-M,M], \\
&\sup\left\{m_3(t)\int_0^{\alpha(t)}|u(t,s,v(c(s)))-u(t,s,w(c(s)))|ds : v,w\in B_r\right\} \\
&= \sup\left\{\frac{3M^2}{9+\sqrt{1+t}}\int_0^{t^2}\left|\frac{s^2v(4s)\sin\left(\sqrt{1+t^3s^5}-(t-s)^4v^3(4s)\right)}{1+t^{12}+\left|stv^3(4s)-3s^3+7(t-2s)^2\right|v^2(4s)}\right.\right. \\
&\qquad\qquad \left.\left. -\frac{s^2w(4s)\sin\left(\sqrt{1+t^3s^5}-(t-s)^4w^3(4s)\right)}{1+t^{12}+\left|stw^3(4s)-3s^3+7(t-2s)^2\right|w^2(4s)}\right| ds : v,w\in B_r\right\} \\
&\le \frac{3M^2}{9+\sqrt{1+t}}\int_0^{t^2}\frac{2s^2}{(1+t^{12})^2}\left[r\left(1+t^{12}\right)+r^3\left(r^3t^3+3r^6+7\left(t+2t^2\right)^2\right)\right]ds \\
&\le \frac{2M^2t^6}{\left(9+\sqrt{1+t}\right)(1+t^{12})^2}\left\{r\left(1+t^{12}\right)+r^3\left[r^3t^3+3r^6+7\left(t+2t^2\right)^2\right]\right\} \longrightarrow 0 \quad \text{as } t\longrightarrow+\infty, \\
&\sup\left\{m_3(t)\int_0^{\alpha(t)}|u(t,s,v(c(s)))|ds : t\in\mathbb{R}_+, v\in B_r\right\} \\
&= \sup\left\{\frac{3M^2}{9+\sqrt{1+t}}\int_0^{t^2}\left|\frac{s^2v(4s)\sin\left(\sqrt{1+t^3s^5}-(t-s)^4v^3(4s)\right)}{1+t^{12}+\left|stv^3(4s)-3s^3+7(t-2s)^2\right|v^2(4s)}\right| ds : t\in\mathbb{R}_+, v\in B_r\right\} \\
&\le \sup\left\{\frac{3M^2}{10}\int_0^t\frac{s^2ds}{1+t^{12}} : t\in\mathbb{R}^+\right\} \\
&= \frac{rM^2}{20} < M_0, \\
&\sup\left\{\int_0^{\alpha(t)}|u(t,s,v(c(s)))|ds : t\in\mathbb{R}_+, v\in B_r\right\} \\
&= \sup\left\{\frac{3M^2}{9+\sqrt{1+t}}\int_0^{t^2}\left|\frac{s^2v(4s)\sin\left(\sqrt{1+t^3s^5}-(t-s)^4v^3(4s)\right)}{1+t^{12}+\left|stv^3(4s)-3s^3+7(t-2s)^2\right|v^2(4s)}\right| ds : t\in\mathbb{R}_+, v\in B_r\right\} \\
&\le \sup\left\{\frac{3M^2}{10}\int_0^t\frac{s^2ds}{1+t^{12}} : t\in\mathbb{R}^+\right\} \\
&= \frac{r}{6} \le M,
\end{aligned}
\tag{4.4}
$$

that is, (3.2)–(3.5) and (3.7) hold. Hence all Assumptions of Theorem 3.1 are fulfilled. Consequently, Theorem 3.1 ensures that (4.1) has at least one solution $x = x(t) \in B_r$. However Theorem 3.1 in [6], Theorem 2 in [9],Theorem 3 in [10], Theorem 1 in [15], Theorem 2 in [16], and Theorem 3.1 in [19] are unapplicable for (4.1).

Example 4.2. Consider the following nonlinear integral equation of Volterra type:

$$\begin{aligned} x(t) = {} & \frac{3+\sin^4\left(\sqrt{1+t^2}\right)}{16} + \frac{x^2(t)}{8+t^2} \\ & + \frac{1}{1+t}\left(\int_0^{\sqrt{t}} \frac{sx^3(1+s^2)}{1+t^2+s\cos^2(1+t^3s^7x^2(1+s^2))}ds\right)^2, \quad \forall t \in \mathbb{R}_+. \end{aligned} \tag{4.5}$$

Put

$$\begin{aligned} & h(t,v,w) = \frac{3+\sin^4\left(\sqrt{1+t^2}\right)}{16} + \frac{v^2}{8+t^2} + \frac{w^2}{1+t}, \\ & u(t,s,p) = \frac{sp^3}{1+t^2+s\cos^2(1+t^3s^7p^2)}, \\ & \alpha(t) = \sqrt{t}, \quad c(t) = 1+t^2, \quad m_1(t) = \frac{1}{8+t^2}, \quad m_3(t) = \frac{4}{1+t}, \quad \forall t,s \in \mathbb{R}_+, v,w,p \in \mathbb{R}, \\ & r = \frac{1}{2}, \qquad M = 2, \qquad M_0 = M_1 = \frac{1}{8}, \qquad \overline{h} = \frac{1}{4}. \end{aligned} \tag{4.6}$$

It is easy to verify that (3.30), (3.31) and Assumptions (H6) and (H7) are satisfied. Notice that

$$\begin{aligned} & \left|h(t,v,w) - h(t,p,q)\right| \\ & \quad \le \frac{1}{8+t^2}\left|v^2-p^2\right| + \frac{1}{1+t}\left|w^2-q^2\right| \\ & \quad \le m_1(t)\left|v-p\right| + m_3(t)\left|z-y\right|, \quad \forall t \in \mathbb{R}_+,\ v,\ p \in [-r,r],\ w,\ q \in [-M,M], \\ & \sup\left\{ m_3(t)\int_0^{\alpha(t)} |u(t,s,v(c(s))) - u(t,s,w(c(s)))| ds : x,\ y \in B_r \right\} \\ & \quad = \sup\left\{ \frac{4}{1+t}\int_0^{\sqrt{t}} \left| \frac{sv^3(1+s^2)}{1+t^2+s\cos^2(1+t^3s^7v^2(1+s^2))} \right.\right. \\ & \qquad\qquad \left.\left. - \frac{sw^3(1+s^2)}{1+t^2+s\cos^2(1+t^3s^7w^2(1+s^2))}ds \right| ds : v,\ w \in B_r \right\} \\ & \quad \le \frac{4r^3t\left(1+\sqrt{t}+t^2\right)}{(1+t)(1+t^2)^2} \longrightarrow 0 \quad \text{as } t \longrightarrow +\infty, \end{aligned}$$

$$\sup\left\{m_3(t)\int_0^{\alpha(t)}|u(t,s,v(c(s)))|ds : t\in\mathbb{R}_+,\ v\in B_r\right\}$$
$$=\sup\left\{\frac{4}{1+t}\int_0^{\sqrt{t}}\left|\frac{sv^3(1+s^2)}{1+t^2+s\cos^2(1+t^3s^7v^2(1+s^2))}ds\right| : t\in\mathbb{R}_+,\ v\in B_r\right\}$$
$$\leq \sup\left\{\frac{4r^3}{1+t}\int_0^{\sqrt{t}}\frac{s}{1+t^2}ds : t\in\mathbb{R}^+\right\}$$
$$\leq \sup\left\{\frac{t}{4(1+t)(1+t^2)} : t\in\mathbb{R}^+\right\}$$
$$\leq \frac{1}{8}=M_1,$$
$$\sup\left\{\int_0^{\alpha(t)}|u(t,s,v(c(s)))|ds : t\in\mathbb{R}_+,\ v\in B_r\right\}$$
$$=\sup\left\{\int_0^{\sqrt{t}}\left|\frac{sv^3(1+s^2)}{1+t^2+s\cos^2(1+t^3s^7v^2(1+s^2))}ds\right| : t\in\mathbb{R}_+,\ v\in B_r\right\}$$
$$\leq \sup\left\{r^3\int_0^{\sqrt{t}}\frac{s}{1+t^2}ds : t\in\mathbb{R}^+\right\}$$
$$=\frac{1}{32}<M, \tag{4.7}$$

which yield (3.3)–(3.5). That is, all Assumptions of Theorem 3.2 are fulfilled. Therefore, Theorem 3.2 guarantees that (4.5) has at least one solution $x = x(t) \in B_r$. Moreover, solutions of (4.5) are asymptotically stable. But Theorem 2 in [9], Theorem 3 in [10], Theorem 1 in [15], Theorem 2 in [16], and Theorem 3.1 in [19] are invalid for (4.5).

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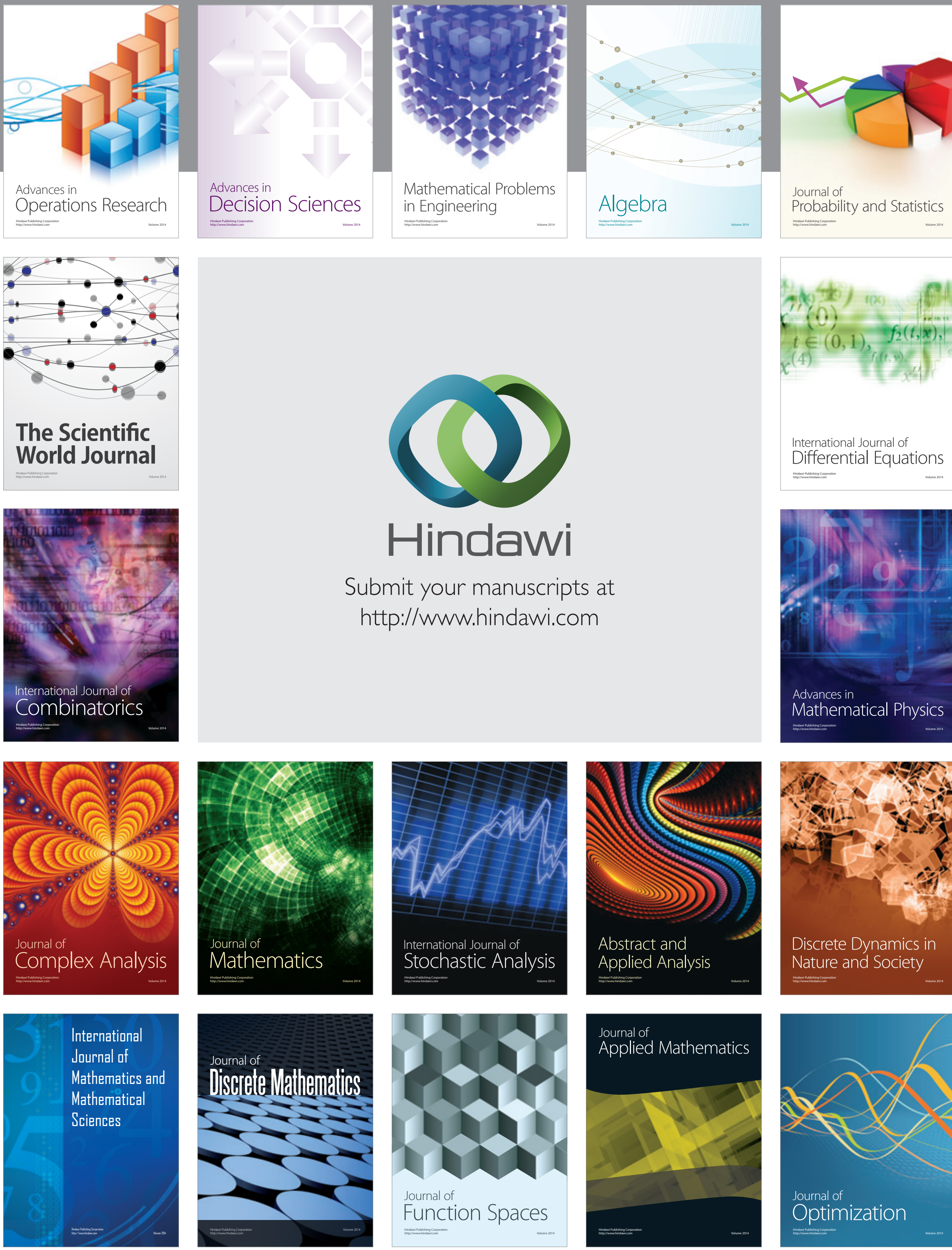
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