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Letter to the Editor

Periodic Solution of the Hematopoiesis Equation

Ji-Huan He

National Engineering Laboratory for Modern Silk, College of Textile and Clothing Engineering, Soochow University, 199 Ren-Ai Road, Suzhou 215123, China

Correspondence should be addressed to Ji-Huan He; hejihuan@suda.edu.cn

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Wu and Liu (2012) presented some results for the existence and uniqueness of the periodic solutions for the hematopoiesis model. This paper gives a simple approach to find an approximate period of the model.

Wu and Liu studied the following hematopoiesis model [1]:

$$x'(t) = -ax(t) + \frac{\beta\theta^n}{\theta^n + x^n(t-\tau)}, \quad (1)$$

where x denotes the density of mature cells in blood circulation. The physical meaning of other parameters is referred to [1].

Equation (1) admits periodic solutions as revealed in [1]. Hereby we suggest a simple approach to the search for an approximate period of (1) using a simple amplitude-frequency formulation [2–5]. To this end, we rewrite (1) in the form

$$\begin{aligned} &x'(t)\theta^n + x'(t)x^n(t-\tau) + ax(t)\theta^n \\ &\quad + ax(t)x^n(t-\tau) - \beta\theta^n = 0. \end{aligned} \quad (2)$$

Assume that the periodic solution can be expressed in the form

$$x(t) = A\cos\omega t. \quad (3)$$

Submitting (3) into (2) results in the following residual:

$$\begin{aligned} R(\omega,t) = &-A\omega\theta^n\sin\omega t \\ &- A^{1+n}\omega\sin\omega t\cos^n\omega(t-\tau) + a\theta^n A\cos\omega t \\ &+ aA^{1+n}\cos\omega t\cos^n\omega(t-\tau) - \beta\theta^n. \end{aligned} \quad (4)$$

In order to use the amplitude-frequency formulation [2–5], we choose two trial frequencies and locate them at $t = \pi/(4\omega)$.

Setting $\omega_1 = 1$, $\omega_1 t = \pi/4$, and $\omega_1 = 2$, $\omega_2 t = \pi/4$, respectively, we have

$$\begin{aligned} R_1 = &-\frac{\sqrt{2}}{2}A\theta^n - \frac{\sqrt{2}}{2}A^{1+n}\cos^n\left(\frac{\pi}{4}-\tau\right) \\ &+ \frac{\sqrt{2}}{2}a\theta^n A + \frac{\sqrt{2}}{2}aA^{1+n}\cos^n\left(\frac{\pi}{4}-\tau\right) - \beta\theta^n, \\ R_2 = &-\sqrt{2}A\theta^n - \sqrt{2}A^{1+n}\cos^n\left(\frac{\pi}{4}-2\tau\right) \\ &+ \frac{\sqrt{2}}{2}a\theta^n A + \frac{\sqrt{2}}{2}aA^{1+n}\cos^n\left(\frac{\pi}{4}-2\tau\right) - \beta\theta^n. \end{aligned} \quad (5)$$

The frequency can be then obtained approximately in the form [2–5]

$$\begin{aligned} \omega^2 &= \frac{R_1\omega_1^2 - R_2\omega_2^2}{R_1 - R_2} = \frac{R_1 - 4R_2}{R_1 - R_2} \\ &= \left(\frac{7\sqrt{2}}{2}A\theta^n + \frac{7\sqrt{2}}{2}A^{1+n}\cos^n\left(\frac{\pi}{4}-2\tau\right) - \frac{3\sqrt{2}}{2}a\theta^n A \right. \\ &\quad \left. -\frac{3\sqrt{2}}{2}aA^{1+n}\cos^n\left(\frac{\pi}{4}-2\tau\right) + 3\beta\theta^n\right) \end{aligned}$$

$$\times\left(\left(\sqrt{2}-\frac{\sqrt{2}}{2}\right)A\theta^{n} -\left(\sqrt{2}-\frac{\sqrt{2}}{2}\right)A^{1+n}\cos^{n}\left(\frac{\pi}{4}-\tau\right)\right)^{-1}. \quad (6)$$

This formulation has been widely used to solve periodic solutions of various nonlinear oscillators [6–13], and it is often called as He's frequency formulation, He's amplitude-frequency formulation, or He's frequency-amplitude formulation. In case $\omega^2 < 0$, no period solution is admitted. A similar criterion is given for a nonlinear equation arising in electrospinning process [14].

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References

[1] J. Wu and Y. Liu, "Fixed point theorems and uniqueness of the periodic solution for the hematopoiesis models," *Abstract and Applied Analysis*, vol. 2012, Article ID 387193, 7 pages, 2012.

[2] J.-H. He, "Some asymptotic methods for strongly nonlinear equations," *International Journal of Modern Physics B*, vol. 20, no. 10, pp. 1141–1199, 2006.

[3] J. H. He, "Asymptotic methods for Solitary Solutions and Compactons," *Abstract and Applied Analysis*, vol. 2012, Article ID 916793, 130 pages, 2012.

[4] J. H. He, "Comment on 'He's frequency formulation for nonlinear oscillators'," *European Journal of Physics*, vol. 29, no. 4, pp. L19–L22, 2008.

[5] J. H. He, "An improved amplitude-frequency formulation for nonlinear oscillators," *The International Journal of Nonlinear Sciences and Numerical Simulation*, vol. 9, no. 2, pp. 211–212, 2008.

[6] Z. F. Ren, "He's frequency-amplitude formulation for nonlinear oscillators," *International Journal of Modern Physics B*, vol. 25, no. 17, pp. 2379–2382, 2011.

[7] D. Younesian, H. Askari, Z. Saadatnia, and M. KalamiYazdi, "Frequency analysis of strongly nonlinear generalized Duffing oscillators using He's frequency-amplitude formulation and He's energy balance method," *Computers & Mathematics with Applications*, vol. 59, no. 9, pp. 3222–3228, 2010.

[8] A. E. Ebaid, "Analytical periodic solution to a generalized nonlinear oscillator: application of He's frequency-amplitude formulation," *Mechanics Research Communications*, vol. 37, no. 1, pp. 111–112, 2010.

[9] H.-L. Zhang, "Application of He's amplitude-frequency formulation to a nonlinear oscillator with discontinuity," *Computers & Mathematics with Applications*, vol. 58, no. 11-12, pp. 2197–2198, 2009.

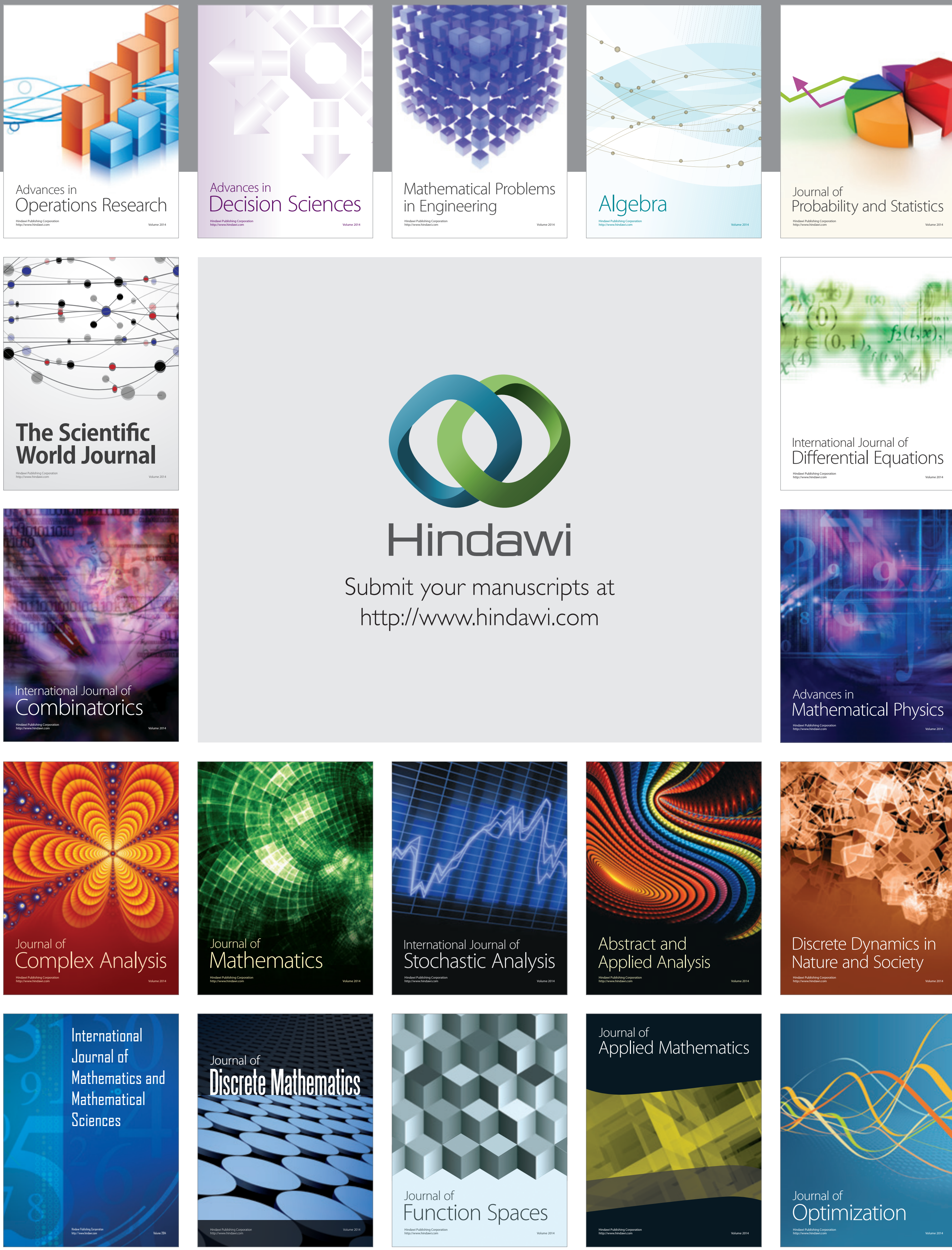
[10] X.-C. Cai and W.-Y. Wu, "He's frequency formulation for the relativistic harmonic oscillator," *Computers & Mathematics with Applications*, vol. 58, no. 11-12, pp. 2358–2359, 2009.

[11] Y.-N. Zhang, F. Xu, and L.-l. Deng, "Exact solution for nonlinear Schrödinger equation by He's frequency formulation," *Computers & Mathematics with Applications*, vol. 58, no. 11-12, pp. 2449–2451, 2009.

[12] A. E. Ebaid, "Oscillations in an $x^{(2n+2)/(2n+1)}$ potential via he's frequency-amplitude formulation," *Zeitschrift fur Naturforschung A*, vol. 64, no. 12, pp. 877–878, 2009.

[13] Z. F. Ren, G. Q. Liu, Y. X. Kang et al., "Application of He's amplitude-frequency formulation to nonlinear oscillators with discontinuities," *Physica Scripta*, vol. 80, no. 4, Article ID 045003, 2009.

[14] J. H. He, H. Y. Kong, R. R. Yang et al., "Review on fiber morphology obtained by the bubble electrospinning and Blown bubble spinning," *Thermal Science*, vol. 16, no. 5, pp. 1263–1279, 2012.



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