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Research Article

Structure of Intuitionistic Fuzzy Sets in Γ-Semihyperrings

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As we know, intuitionistic fuzzy sets are extensions of the standard fuzzy sets. Now, in this paper, the basic definitions and properties of intuitionistic fuzzy Γ-hyperideals of a Γ-semihyperring are introduced. A few examples are presented. In particular, some characterizations of Artinian and Noetherian Γ-semihyperring are given.

1. Introduction

The theory of fuzzy sets proposed by Zadeh [1] has achieved a great success in various fields. In 1971, Rosenfeld [2] introduced fuzzy sets in the context of group theory and formulated the concept of a fuzzy subgroup of a group. Since then, many researchers are engaged in extending the concepts of abstract algebra to the framework of the fuzzy setting.

The concept of a fuzzy ideal of a ring was introduced by Liu [3]. The concept of intuitionistic fuzzy set was introduced and studied by Atanassov [4–6] as a generalization of the notion of fuzzy set. In [7], Biswas studied the notion of an intuitionistic fuzzy subgroup of a group. In [8], Kim and Jun introduced the concept of intuitionistic fuzzy ideals of semirings. Also, in [9], Gunduz and Davvaz studied the universal coefficient theorem in the category of intuitionistic fuzzy modules. Also see [10, 11].

In 1964, Nobusawa introduced Γ-rings as a generalization of ternary rings. Barnes [12] weakened slightly the conditions in the definition of Γ-ring in the sense of Nobusawa. Barnes [12], Luh [13], and Kyuno [14] studied the structure of Γ-rings and obtained various generalization analogous to corresponding parts in ring theory. The concept of Γ-semigroups was introduced by Sen and Saha [15, 16] as a generalization of semigroups and ternary semigroups. Then the notion of Γ-semirings introduced by Rao [17].

Algebraic hyperstructures represent a natural extension of classical algebraic structures, and they were introduced by the French mathematician Marty [18]. Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Since then, hundreds of papers and several books have been written on this topic, see [19–22].

In [23–25], Davvaz et al. introduced the notion of a Γ-semihypergroup as a generalization of a semihypergroup. Many classical notions of semigroups and semihypergroups have been extended to Γ-semihypergroups and a lot of results on Γ-semihypergroups are obtained. In [26–29], Davvaz et al. studied the notion of a Γ-semihyperring as a generalization of semiring, a generalization of a semihyperring, and a generalization of a Γ-semiring.

The study of fuzzy hyperstructures is an interesting research topic of fuzzy sets. There is a considerable amount of work on the connections between fuzzy sets and hyperstructures, see [20]. In [30], Davvaz introduced the notion of fuzzy subhypergroups as a generalization of fuzzy subgroups, and this topic was continued by himself and others. In [31], Leoreanu-Fotea and Davvaz studied fuzzy hyperrings. Recently, Davvaz et al. [32–35] considered the intuitionistic fuzzification of the concept of algebraic hyperstructurs and investigated some properties of such hyperstructures.

Now, in this work we introduce the notion of an Atanassov's intuitionistic fuzzy hyperideals of a semihyperrings and investigate some basic properties about it.

2. Basic Definitions

Let S be a nonempty set, and let $\wp^\bullet(S)$ be the set of all non-empty subsets of S. A hyperoperation on S is a map $\circ : S \times S \to \wp^\bullet(S)$, and the couple $(S, \circ)$ is called a *hypergrupoid*. If A and B are non-empty subsets of S, then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \qquad x \circ A = \{x\} \circ A, \qquad A \circ x = A \circ \{x\}. \tag{1}$$

A hypergrupoid $(S, \circ)$ is called a *semihypergroup* if for all $x, y,$ and z of S we have $(x \circ y) \circ z = x \circ (y \circ z)$. That is,

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v. \tag{2}$$

A semihypergroup $(S, \circ)$ is called a *hypergroup* if for all $x \in S$, $x \circ S = S \circ x = S$.

A *semihyperring* is an algebraic structure $(S, +, \cdot)$ which satisfies the following properties:

(1) $(S, +)$ is a *commutative semihypergroup*; that is,

(i) $(x + y) + z = x + (y + z)$,

(ii) $x + y = y + x$, for all $x, y, z \in S$.

(2) $(S, \cdot)$ is a semihypergroup; that is, $(x \cdot y) \cdot z = x \cdot (y \cdot z)$, for all $x, y, z \in S$.

(3) The multiplication is distributive with respect to hyperoperation $+$; that is,

$$x \cdot (y + z) = x \cdot y + x \cdot z, \qquad (x + y) \cdot z = x \cdot z + y \cdot z, \tag{3}$$

for all $x, y, z \in S$.

(4) The element $0 \in S$ is an *absorbing element;* that is, $0 \cdot x = x \cdot 0 = 0$, for all $x \in S$. A semihyperring $(S, +, \cdot)$ is called *commutative* if $x \cdot y = y \cdot x$ for all $x, y \in S$. Vougiouklis in [22] studied the notion of semihyperrings in a general form. That is, both sum and multiplication are hyperoperations.

A semihyperring S has identity element if there exists $1_S \in S$, such that $1_S \cdot x = x \cdot 1_S = x$, for all $x \in S$. An element $x \in S$ is called *unit* if there exists $y \in S$, such that $y \cdot x = x \cdot y = 1_S$. A non-empty subset A of a semihyperring $(S, +, \cdot)$ is called *subsemihyperring* if $x + y \subseteq A$ and $x \cdot y \subseteq A$, for all $x, y \in A$. A left hyperideal of a semihyperring S is a non-empty subset I of S satisfying the following:

(i) $x + y \subseteq I$, for all $x, y \in I$,

(ii) $x \cdot a \subseteq I$, for all $a \in I$ and $x \in S$.

Let S and Γ be two non-empty sets. Then, S is called a Γ-*semihypergroup;* if for every hyperoperation $\gamma \in \Gamma$, $\alpha, \beta \in \Gamma$, and $x, y, z \in S$, we have

$$(x\beta y)\gamma z = x\beta(y\gamma z). \tag{4}$$

For example, $S = [0, 1]$ and $\Gamma \subseteq S$. For all $\alpha \in \Gamma$ and for all $x, y \in S$, we define $x\alpha y = [0, \max\{x, \alpha, y\}]$. Then, S is a Γ-semihypergroup.

The concept of Γ-*semihyperring* was introduced and studied by Dehkordi and Davvaz [26–28]. We recall the following definition from [26].

Definition 1. Let S be a commutative semihypergroup, and Γ be a commutative group. Then, S is called a Γ-*semihyperring* if there exists a map $S \times \Gamma \times S \to \wp^*(S)$ (image to denoted by $x\gamma y$) satisfying the following conditions:

(i) $x\alpha(y + z) = x\alpha y + x\alpha z$,

(ii) $(x + y)\alpha z = x\alpha z + y\alpha z$,

(iii) $x(\alpha + \beta)z = x\alpha z + x\beta z$,

(iv) $x\alpha(y\beta z) = (x\alpha y)\beta z$,

for all $x, y, z \in S$ and $\alpha, \beta \in \Gamma$.

In the above definition, if S is a semigroup, then S is called a multiplicative Γ-*semihyperring*. A Γ-*semihyperring* S is called *commutative* if $x\alpha y = y\alpha x$, for all $x, y \in S$ and $\alpha \in \Gamma$. We say that a Γ-semihyperring S is with zero, if there exists $0 \in S$, such that $x \in x + 0$ and $0 \in 0\alpha x$, for all $x \in S$ and $\alpha \in \Gamma$. Let A and B be two non-empty subsets of a Γ-*semihyperring* and $x \in S$. We define

$$A + B = \{x \mid x \in a + b,\ a \in A,\ b \in B\}, \qquad A\Gamma B = \{x \mid x \in a\alpha b,\ a \in A,\ b \in B,\ \alpha \in \Gamma\}. \tag{5}$$

A non-empty subset of S_1 of Γ-semihyperring S is called a *sub*-Γ-*semihyperring* if it is closed with respect to the multiplication and addition. In other words, a non-empty subset S_1 of Γ-semihyperring S is a sub Γ-semihyperring if

$$S_1 + S_1 \subseteq S_1, \qquad S_1 \Gamma S_1 \subseteq S_1. \tag{6}$$

Example 2. Let $(S, +, \cdot)$ be a semiring, and let Γ be a subsemiring of S. We define

$$x\gamma y = \langle x, \gamma, y \rangle, \tag{7}$$

the ideal generated by $x,\ \gamma,$ and y, for all $x, y \in S$ and $\gamma \in \Gamma$. Then, it is not difficult to see that S is a multiplicative Γ-semihyperring.

Example 3. Let $S = \mathbb{Q}^+$, $\Gamma = \{\gamma_i \mid i \in \mathbb{N}\}$ and $A_i = i\mathbb{Z}^+$. We define $x\gamma_i y \to xA_i y$ for every $\gamma \in \Gamma$ and $x, y \in S$. Then, S is a Γ-semihyperring under ordinary addition and multiplication.

Example 4. Let $(S, +, \cdot)$ be a semihyperring, and let $M_{m,n}(S)$ the set of all $m \times n$ matrices with entries in S. We define $\circ : M_{m,n}(S) \times M_{n,m}(S) \times M_{m,n}(S) \to P^*(M_{m,n}(S))$ by

$$A \circ B \circ C = \{Z \in M_{m,n}(S) \mid Z \in ABC\}, \tag{8}$$

for all $A, C \in M_{m,n}(S)$ and $B \in M_{n,m}(S)$.

Then, $M_{m,n}(S)$ is $M_{n,m}(S)$-semihyperring [29]. Now, suppose that

$$X = \left\{ \left(a_{ij}\right)_{m,n} \mid a_{11} \neq 0,\ a_{21} \neq 0,\ a_{ij} = 0 \text{ otherwise} \right\}. \quad (9)$$

Now, it is easy to see that X is a sub Γ-semihyperring of S.

A non-empty subset I of a Γ-semihyperring S is a *left* (*right*) Γ-*hyperideal* of S if for any $I_1, I_2 \subseteq I$ implies $I_1 + I_2 \subseteq I$ and $I\Gamma S \subseteq I$ $(S\Gamma I \subseteq I)$ and is a Γ-*hyperideal* of S if it is both left and right Γ-*hyperideal*.

Example 5. Consider the following:

$$S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{Z}^{+} \cup \{0\} \right\},$$
$$A_i = \left\{ \begin{bmatrix} ie & 0 \\ 0 & if \end{bmatrix} \mid e, f \in 2\mathbb{Z}^{+} \right\}, \quad \forall i \in \mathbb{N}, \quad (10)$$
$$\Gamma = \{\gamma_i \mid i \in \mathbb{N}\}.$$

Then, S is a Γ-semihyperring under the matrix addition and the hyperoperation as follows:

$$x\gamma_i y = xA_i y, \quad (11)$$

for all $x, y \in S$ and $\gamma_i \in \Gamma$. Let

$$I_1 = \left\{ \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \mid a, b \in \mathbb{Z}^{+} \cup \{0\} \right\},$$
$$I_2 = \left\{ \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} \mid a, c \in \mathbb{Z}^{+} \cup \{0\} \right\}, \quad (12)$$
$$I_3 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in 2\mathbb{Z}^{+} \cup \{0\} \right\}.$$

Then, I_1 is a right Γ-hyperideal of S, but not left. I_2 is a left Γ-hyperideal of S, but not right. I_3 is both right and left Γ-hyperideal of S.

A fuzzy subset μ of a non-empty set S is a function from S to $[0,1]$. For all $x \in S$ μ^c, the complement of μ is the fuzzy subset defined by $\mu^c(x) = 1 - \mu(x)$. The intersection and the union of two fuzzy subsets μ and λ of S, denoted by $\mu \cap \lambda$ and $\mu \cup \lambda$, are defined by

$$(\mu \cap \lambda)(x) = \min\{\mu(x), \lambda(x)\},$$
$$(\mu \cup \lambda)(x) = \max\{\mu(x), \lambda(x)\}. \quad (13)$$

Definition 6. Let S be a Γ-semihyperring, and let μ be a fuzzy subset of S. Then,

(1) μ is called a *fuzzy left* Γ-*hyperideal* of S if

$$\min\{\mu(x), \mu(y)\} \leq \inf_{z \in x+y} \{\mu(z)\},$$
$$\mu(y) \leq \inf_{z \in x\gamma y} \{\mu(z)\} \quad \forall x, y \in S,\ \forall \gamma \in \Gamma. \quad (14)$$

(2) μ is called a *fuzzy right* Γ-*hyperideal* of S if

$$\min\{\mu(x), \mu(y)\} \leq \inf_{z \in x+y} \{\mu(z)\},$$
$$\mu(x) \leq \inf_{z \in x\gamma y} \{\mu(z)\} \quad \forall x, y \in S,\ \forall \gamma \in \Gamma. \quad (15)$$

(3) μ is called a *fuzzy* Γ-*hyperideal* of S if μ is both a fuzzy left Γ-hyperideal and fuzzy right Γ-hyperideal of S.

Example 7. Let $S_1 = \mathbb{Z}_6$, $\Gamma = \{\gamma_2, \gamma_3\}$, $S_2 = \{\overline{0}, \overline{3}\}$, and $S_3 = \{\overline{0}\}$ be non-empty subsets of S_1. We define $x\gamma_i y = xS_i y$, for every $\gamma_i \in \Gamma$ and $x, y \in S_1$. Then, S_1 is a Γ-semihypering. Now, we define the fuzzy subset μ of S_1 as follows:

$$\mu(x) = \begin{cases} 0 & \text{if } x = \overline{1}, \overline{2}, \overline{4}, \overline{5}, \\ \dfrac{1}{3} & \text{if } x = \overline{3}, \\ 1 & \text{if } x = \overline{0}. \end{cases} \quad (16)$$

It is easy to see that μ is a fuzzy Γ-hyperideal of S_1.

3. Atanassov's Intuitionistic Fuzzy Γ-Hyperideals

The concept of intuitionistic fuzzy set was introduced and studied by Atanassov [4–6]. Intuitionistic fuzzy sets are extensions of the standard fuzzy sets. An intuitionistic fuzzy set A in a non-empty set S has the form $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in S\}$. Here, $\mu_A : S \to [0,1]$ is the degree of membership of the element $x \in S$ to the set A, and $\lambda_A : S \to [0,1]$ is the degree of nonmembership of the element $x \in S$ to the set A. We have also $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$, for all $x \in S$.

Example 8 (see [35]). Consider the universe $X = \{10, 100, 500, 1000, 1200\}$. An intuitionistic fuzzy set "Large" of X denoted by A and may be defined by

$$A = \{\langle 10, 0.01, 0.9\rangle, \langle 100, 0.1, 0.88\rangle, \langle 500, 0.4, 0.05\rangle,$$
$$\langle 1000, 0.8, 0.1\rangle, \langle 1200, 1, 0\rangle\}. \quad (17)$$

One may define an intuitionistic fuzzy set "Very Large" denoted by B, as follows:

$$\mu_B(x) = (\mu_A(x))^2,$$
$$\lambda_B(x) = 1 - (1 - \lambda_A(x))^2, \quad (18)$$

for all $x \in X$. Then,

$$B = \{\langle 10, 0.0001, 0.99\rangle, \langle 100, 0.01, 0.9856\rangle,$$
$$\langle 500, 0.16, 0.75\rangle, \langle 1000, 0.64, 0.19\rangle, \langle 1200, 1, 0\rangle\}. \quad (19)$$

For the sake of simplicity, we shall use the symbol $A = (\mu_A, \lambda_A)$ instead of $A = \{(x, \mu_A(x), \lambda_A(x)) \mid x \in S\}$. Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be two intuitionistic fuzzy sets of S. Then, the following expressions are defined in [4] as follows:

(1) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\lambda_A(x) \geq \lambda_B(x)$, for all $x \in S$,

(2) $A^C = \{(x, \lambda_A(x), \mu_A(x)) \mid x \in S\}$,

(3) $A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\}) \mid x \in S\}$,

(4) $A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\}) \mid x \in S\}$,

(5) $\Box A = \{(x, \mu_A(x), \mu_A^c(x)) \mid x \in S\}$,

(6) $\Diamond A = \{(x, \lambda_A^c(x), \lambda_A(x)) \mid x \in S\}$.

Now, we introduce the notion of intuitionistic fuzzy Γ-hyperideals of Γ-semihyperrings.

Definition 9. Let S be a Γ-semihyperring.

(1) An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in S is called a *left intuitionistic fuzzy* Γ*-hyperideal* of Γ-semihyperring S if

(i) $\min\{\mu_A(x), \mu_A(y)\} \le \inf_{z \in x+y}\{\mu_A(z)\}$ and $\max\{\lambda_A(x), \lambda_A(y)\} \ge \sup_{z \in x+y}\{\lambda_A(z)\}$,

(ii) $\mu_A(y) \le \inf_{z \in x\gamma y}\{\mu_A(z)\}$ and $\sup_{z \in x\gamma y}\{\lambda_A(z)\} \le \lambda_A(y)$,

for all $x, y \in S$, $\gamma \in \Gamma$.

(2) An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in S is called a *right intuitionistic fuzzy* Γ*-hyperideal* of Γ-semihyperring S if

(i) $\min\{\mu_A(x), \mu_A(y)\} \le \inf_{z \in x+y}\{\mu_A(z)\}$and $\max\{\lambda_A(x), \lambda_A(y)\} \ge \sup_{z \in x+y}\{\lambda_A(z)\}$,

(ii) $\mu_A(x) \le \inf_{z \in x\gamma y}\{\mu_A(z)\}$ and $\sup_{z \in x\gamma y}\{\lambda_A(z)\} \le \lambda_A(x)$,

for all $x, y \in S$, $\gamma \in \Gamma$.

(3) An intuitionistic fuzzy set $A = (\mu_A, \lambda_A)$ in S is called an *intuitionistic fuzzy* Γ*-hyperideal* of Γ-semihyperring S if it is both left intuitionistic fuzzy Γ-hyperideal and right intuitionistic fuzzy Γ-hyperideal of S.

Example 10. Let S_1 be a Γ-semihyperring in Example 7. We define the fuzzy subsets μ_A and λ_A of S_1 as follows:

$$\mu(x) = \begin{cases} 0 & \text{if } x = \overline{1}, \overline{2}, \overline{4}, \overline{5}, \\ \dfrac{1}{4} & \text{if } x = \overline{3}, \\ \dfrac{2}{5} & \text{if } x = \overline{0}, \end{cases}$$
$$\lambda(x) = \begin{cases} \dfrac{1}{2} & \text{if } x = \overline{1}, \overline{2}, \overline{4}, \overline{5}, \\ \dfrac{1}{3} & \text{if } x = \overline{3}, \\ \dfrac{3}{10} & \text{if } x = \overline{0}. \end{cases} \tag{20}$$

Then, A is an intuitionistic fuzzy Γ-hyperideal of S_1.

Example 11. Let S be a Γ-semihyperring defined in Example 5. Suppose that

$$\mu_{A_i}(x) = \begin{cases} \dfrac{5}{8} & \text{if } a, b, c, d \in 2\mathbb{Z}^+ \cup \{0\}, \\ \dfrac{2}{5} & \text{otherwise}, \end{cases}$$
$$\lambda_{A_i}(x) = \begin{cases} \dfrac{1}{9} & \text{if } a, b, c, d \in 2\mathbb{Z}^+ \cup \{0\}, \\ \dfrac{4}{7} & \text{otherwise}, \end{cases} \tag{21}$$

for all $x \in S$. Then, $A_i = (\mu_{A_i}, \lambda_{A_i})$ is an intuitionistic fuzzy Γ-hyperideal of S.

Theorem 12. *Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy Γ-hyperideal of Γ-semihyperring S and $0 \le t \le 1$. We define an intuitionistic fuzzy set $B = (\mu_B, \lambda_B)$ in S by $\mu_B(x) = t\mu_A(x)$ and $\lambda_B(x) = (1-t)\lambda_B(x)$, for all $x \in S$. Then, $B = (\mu_B, \lambda_B)$ is an intuitionistic fuzzy Γ-hyperideal of S.*

Proof. Note that $0 \le \mu_B(x) + \lambda_B(x) = t\mu_A(x) + (1-t)\lambda_A(x) \le t + 1 - t = 1$. □

Theorem 13. *Let $A = (\mu_A, \lambda_A)$ be an intuitionistic fuzzy Γ-hyperideal of Γ-semihyperring S. We define an intuitionistic fuzzy set $B = (\mu_B, \lambda_B)$ in S by $\mu_B(x) = (\mu_A(x))^2$ and $\lambda_B(x) = 1 - (1 - \lambda_B(x))^2$, for all $x \in S$. Then, $B = (\mu_B, \lambda_B)$ is an intuitionistic fuzzy Γ-hyperideals of S.*

Proof. Since $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy Γ-hyperideal, we have $\inf_{z \in x+y}\{\mu_A(z)\} \ge \min\{\mu_A(x), \mu_A(y)\}$ and so

$$\left(\inf_{z \in x+y}\{\mu_A(z)\}\right)^2 \ge (\min\{\mu_A(x), \mu_A(y)\})^2. \tag{22}$$

Hence, $\inf_{z \in x+y}\{(\mu_A(z))^2\} \ge \min\{(\mu_A(x))^2, (\mu_A(y))^2\}$; that is,

$$\inf_{z \in x+y}\{\mu_B(z)\} \ge \min\{\mu_B(x), \mu_B(y)\}. \tag{23}$$

Since $\inf_{z \in x\gamma y}\{\mu_A(z)\} \ge \mu_A(y)$, so $(\inf_{z \in x\gamma y}\{\mu_A(z)\})^2 \ge (\mu_A(y))^2$ which implies that $\inf_{z \in x\gamma y}\{(\mu_A(z))^2\} \ge (\mu_A(y))^2$. Hence, $\inf_{z \in x\gamma y}\{\mu_B(z)\} \ge \mu_B(y)$. Also, we have

$$\sup_{z \in x+y}\{\lambda_A(z)\} \le \max\{\lambda_A(x), \lambda_A(y)\}$$
$$\Longrightarrow \inf_{z \in x+y}\{-\lambda_A(z)\} \ge \min\{-\lambda_A(x), -\lambda_A(y)\}$$
$$\Longrightarrow 1 - \inf_{z \in x+y}\{-\lambda_A(z)\} \ge 1 + \min\{-\lambda_A(x), -\lambda_A(y)\}$$

$$\begin{aligned}
&\Longrightarrow \inf_{z\in x+y}\{1-\lambda_A(z)\} \geq \min\{1-\lambda_A(x), 1-\lambda_A(y)\}\\
&\Longrightarrow \left(\inf_{z\in x+y}\{1-\lambda_A(z)\}\right)^2\\
&\quad \geq (\min\{1-\lambda_A(x), 1-\lambda_A(y)\})^2\\
&\Longrightarrow \inf_{z\in x+y}\left\{(1-\lambda_A(z))^2\right\}\\
&\quad \geq \min\left\{(1-\lambda_A(x))^2, (1-\lambda_A(y))^2\right\}\\
&\Longrightarrow \sup_{z\in x+y}\left\{-(1-\lambda_A(z))^2\right\}\\
&\quad \leq \max\left\{-(1-\lambda_A(x))^2, -(1-\lambda_A(y))^2\right\}\\
&\Longrightarrow 1+\sup_{z\in x+y}\left\{-(1-\lambda_A(z))^2\right\}\\
&\quad \leq 1+\max\left\{-(1-\lambda_A(x))^2, -(1-\lambda_A(y))^2\right\}\\
&\Longrightarrow \sup_{z\in x+y}\left\{1-(1-\lambda_A(z))^2\right\}\\
&\quad \leq \max\left\{1-(1-\lambda_A(x))^2, 1-(1-\lambda_A(y))^2\right\}\\
&\Longrightarrow \sup_{z\in x+y}\{\lambda_B(z)\} \leq \max\{\lambda_B(x), \lambda_B(y)\}.
\end{aligned} \tag{24}$$

Moreover, we have

$$\begin{aligned}
&\sup_{z\in x\gamma y}\{\lambda_A(z)\} \leq \lambda_A(y)\\
&\quad\Longrightarrow \inf_{z\in x\gamma y}\{-\lambda_A(z)\} \geq -\lambda_A(y)\\
&\quad\Longrightarrow 1-\inf_{z\in x\gamma y}\{-\lambda_A(z)\} \geq 1-\lambda_A(y)\\
&\quad\Longrightarrow \inf_{z\in x\gamma y}\{1-\lambda_A(z)\} \geq 1-\lambda_A(y)\\
&\quad\Longrightarrow \left(\inf_{z\in x\gamma y}\{1-\lambda_A(z)\}\right)^2 \geq (1-\lambda_A(y))^2\\
&\quad\Longrightarrow \inf_{z\in x\gamma y}\left\{(1-\lambda_A(z))^2\right\} \geq (1-\lambda_A(y))^2\\
&\quad\Longrightarrow \sup_{z\in x+y}\left\{-(1-\lambda_A(z))^2\right\} \leq -(1-\lambda_A(y))^2\\
&\quad\Longrightarrow 1+\sup_{z\in x+y}\left\{-(1-\lambda_A(z))^2\right\} \leq 1-(1-\lambda_A(y))^2\\
&\quad\Longrightarrow \sup_{z\in x+y}\left\{1-(1-\lambda_A(z))^2\right\} \leq 1-(1-\lambda_A(y))^2\\
&\quad\Longrightarrow \sup_{z\in x+y}\{\lambda_B(z)\} \leq \lambda_B(y).
\end{aligned} \tag{25}$$

We proved the theorem for left intuitionistic fuzzy Γ-hyperideals. For the proof of right intuitionistic fuzzy Γ-hyperideals similar proof is used. □

Lemma 14. *If $\{A_i = (\mu_{A_i}, \lambda_{A_i})\}$ is a collection of intuitionistic fuzzy Γ-hyperideals of S, then $\cap_{i\in\Lambda}A_i$ and $\cup_{i\in\Lambda}A_i$ are intuitionistic fuzzy Γ-hyperideals of S, too.*

Proof. The proof is straightforward. □

Theorem 15. *An intuitionistic fuzzy subset $A = (\mu_A, \lambda_A)$ of a Γ-semihyperring S is a left (res. right) intuitionistic fuzzy Γ-hyperideal of S if and only if for every $t \in [0,1]$, the fuzzy sets μ_A and λ_A^c are left (res. right) fuzzy Γ-hyperideals of S.*

Proof. Assume that $A = (\mu_A, \lambda_A)$ is a left intuitionistic fuzzy Γ-hyperideal of S. Clearly, by using the definition, we have that μ_A is a left (res. right) fuzzy Γ-hyperideal.

Also,

$$\begin{aligned}
\inf_{z\in x+y}\{\lambda_A^c(z)\} &= \inf_{z\in x+y}\{1-\lambda_A(z)\}\\
&= 1-\sup_{z\in x+y}\{\lambda_A(z)\}\\
&\geq \min\{1-\lambda_A(x), 1-\lambda_A(y)\}\\
&= \min\{\lambda_A^c(x), \lambda_A^c(y)\},\\
\inf_{z\in x\gamma y}\{\lambda_A^c(z)\} &= \inf_{z\in x\gamma y}\{1-\lambda_A(z)\}\\
&= 1-\sup_{z\in x\gamma y}\{\lambda_A(z)\}\\
&\geq 1-\lambda_A(y)\\
&= \lambda_A^c(y),
\end{aligned} \tag{26}$$

for all $x, y \in S$ and $\gamma \in \Gamma$.

Conversely, suppose that μ_A and λ_A^c are left (res. right) fuzzy Γ-hyperideals of S. We have $\min\{\mu_A(x), \mu_A(y)\} \leq \inf_{z\in x+y}\{\mu_A(z)\}$ and $\mu_A(y) \leq \inf_{z\in x\gamma y}\{\mu_A(z)\}$, for all $x, y \in S$ and $\gamma \in \Gamma$. We obtain

$$\begin{aligned}
\sup_{z\in x+y}\{\lambda_A(z)\} &= \sup_{z\in x+y}\{1-\lambda_A^c(z)\}\\
&= 1-\inf_{z\in x+y}\{\lambda_A^c(z)\}\\
&\leq \max\{1-\lambda_A^c(x), 1-\lambda_A^c(y)\}\\
&= \max\{\lambda_A(x), \lambda_A(y)\},\\
\sup_{z\in x\gamma y}\{\lambda_A(z)\} &= \sup_{z\in x\gamma y}\{1-\lambda_A^c(z)\}\\
&= 1-\inf_{z\in x\gamma y}\{\lambda_A^c(z)\}\\
&\leq 1-\lambda_A^c(y)\\
&= \lambda_A(y).
\end{aligned} \tag{27}$$

Therefore, $A = (\mu_A, \lambda_A)$ is a left intuitionistic fuzzy Γ-hyperideal of S.

For any fuzzy set μ of S and any $t \in [0,1]$, $U(\mu;t) = \{x \in S \mid \mu(x) \geq t\}$ is called an *upper bound t-level cut* of μ, and $L(\mu;t) = \{x \in S \mid \mu(x) \leq t\}$ is called a *lower bound t-level cut* of μ. □

Theorem 16. *An intuitionistic fuzzy subset A of a Γ-semihyperring S is a left (right) intuitionistic fuzzy Γ-hyperideal of S if and only if for every $t, s \in [0,1]$, the subsets $U(\mu_A;t)$ and $L(\lambda_A;s)$ of S are left (res. right) Γ-hyperideals, when they are non-empty.*

Proof. Assume that $A = (\mu_A, \lambda_A)$ is a left intuitionistic fuzzy Γ-hyperideal of S. Let $x, y \in U(\mu_A;t)$. Since $\inf_{z\in x+y}\mu_A(z) \geq \min\{\mu_A(x), \mu_A(y)\} \geq t$, then we have $x + y \subseteq U(\mu_A;t)$. Let $x \in S$ and $y \in U(\mu_A;t)$. We have $\inf_{z\in x\gamma y}\mu_A(z) \geq \mu_A(y) \geq t$. Therefore, $z \in U(\mu_A;t)$ and so $x\gamma y \subseteq U(\mu_A;t)$.

Now, let $x, y \in L(\lambda_A;s)$. Since $\lambda_A(x) \leq s$ and $\lambda_A(y) \leq s$, we have $\sup_{z\in x+y}\{\lambda_A(z)\} \leq \max\{\lambda_A(x), \lambda_A(y)\} \leq s$. Then, $z \in L(\lambda_A;s)$, and so $x + y \subseteq L(\lambda_A;s)$. Now, for all $x \in S$, $y \in L(\lambda_A;s)$ and $\gamma \in \Gamma$ we have $\sup_{z\in x\gamma y}\{\lambda_A(z)\} \leq \lambda_A(y) \leq s$. Therefore, $z \in L(\lambda_A;s)$ and so $x\gamma y \subseteq L(\lambda_A;s)$.

Conversely, suppose that all non-empty level sets $U(\mu_A;t)$ and $L(\lambda_A;s)$ are left Γ-hyperideals. Let $x, y \in S$ and $\gamma \in \Gamma$. Let $t_0 = \mu_A(x)$, $t_1 = \mu_A(y)$ and $s_0 = \lambda_A(x)$, $s_1 = \lambda_A(y)$ with $t_0 \leq t_1$, $s_0 \leq s_1$, then $x, y \in U(\mu_A;t_0)$ and $x, y \in L(\lambda_A;s_1)$. Since

$$x + y \subseteq U(\mu_A;t_0), \qquad x + y \subseteq L(\lambda_A;s_1), \tag{28}$$

we have

$$\begin{gathered} t_0 = \min\{\mu_A(x), \mu_A(y)\} \leq \inf_{z\in x+y}\{\mu_A(z)\}, \\ s_1 = \max\{\lambda_A(x), \lambda_A(y)\} \geq \sup_{z\in x+y}\{\lambda_A(z)\}. \end{gathered} \tag{29}$$

Since $U(\mu_A;t)$ and $L(\lambda_A;t)$ are left Γ-hyperideals, and we have $z \in x\gamma y \subseteq U(\mu_A;t)$ and $z \in x\gamma y \subseteq U(\lambda_A;s)$, we have $\mu_A(z) \geq t_0$ and $\lambda_A(z) \leq s_1$. And so $\inf_{z\in x\gamma y}\mu_A(z) \geq \mu_A(y)$ and $\sup_{z\in x\gamma y}\lambda_A(z) \leq \lambda_A(y)$. Hence, $A = (\mu_A, \lambda_A)$ is a left intuitionistic fuzzy Γ-hyperideal of S. □

Corollary 17. *Let I be a left (res. right) Γ-hyperideal of a Γ-semihyperring S. We define fuzzy sets μ_A and λ_A as follows:*

$$\begin{gathered} \mu_A = \begin{cases} a_0 & \text{if } x \in I, \\ a_1 & \text{if } x \in S - I, \end{cases} \\ \lambda_A = \begin{cases} b_0 & \text{if } x \in I, \\ b_1 & \text{if } x \in S - I, \end{cases} \end{gathered} \tag{30}$$

where $0 \leq a_1 < a_0$, $0 \leq b_0 < b_1$ and $a_i + b_i \leq 1$ for $i = 0, 1$. Then, $A = (\mu_A, \lambda_A)$ is a left intuitionistic fuzzy Γ-hyperideal of S and $U(\mu;a_0) = I = L(\lambda;b_0)$.

Now, we give the following theorem about the intuitionistic fuzzy sets $\Box A$ and $\Diamond A$.

Theorem 18. *If $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy Γ-hyperideal of S, then so are $\Box A$ and $\Diamond A$.*

Proof. Since $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy Γ-hyperideal of S, we have

$$\begin{gathered} \min\{\mu_A(x), \mu_A(y)\} \leq \inf_{z\in x+y}\{\mu_A(z)\}, \\ \mu_A(x) \leq \inf_{z\in x\gamma y}\{\mu_A(z)\}, \end{gathered} \tag{31}$$

for all $x, y, z \in S$ and $\gamma \in \Gamma$. Moreover,

$$\begin{aligned} \sup_{z\in x+y}\{\mu_A^c(z)\} &= \sup_{z\in x+y}\{1 - \mu_A(z)\} \\ &= 1 - \inf_{z\in x+y}\mu_A(z) \\ &\leq \max\{1 - \mu_A(x), 1 - \mu_A(y)\} \\ &= \max\{\mu_A^c(x), \mu_A^c(y)\}, \\ \sup_{z\in x\gamma y}\{\mu_A^c(z)\} &= \sup_{z\in x\gamma y}\{1 - \mu_A(z)\} \\ &= 1 - \inf_{z\in x\gamma y}\mu_A(z) \\ &\leq \max\{1 - \mu_A(x), 1 - \mu_A(y)\} \\ &= \max\{\mu_A^c(x), \mu_A^c(y)\}. \end{aligned} \tag{32}$$

This shows that $\Box A$ is an intuitionistic fuzzy Γ-hyperideal of S. For the other part one can use the similar way.

Note that if we have a fuzzy Γ-hyperideal A, then $\Box A = A = \Diamond A$. While for a proper intuitionistic fuzzy Γ-hyperideal A, we have $\Box A \subset A \subset \Diamond A$ and $\Box A \neq A \neq \Diamond A$. □

Theorem 19. *If $A = (\mu_A, \lambda_A)$ is a left intuitionistic fuzzy Γ-hyperideal of S, then we have,*

$$\begin{gathered} \mu_A(x) = \sup\{t \in [0,1] \mid x \in U(\mu_A;t)\}, \\ \lambda_A(x) = \inf\{s \in [0,1] \mid x \in L(\lambda_A;s)\}, \end{gathered} \tag{33}$$

for all $x \in S$.

Proof. The proof is straightforward. □

Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be two intuitionistic fuzzy Γ-hyperideals of S. Then, the product of A and B denoted by $A\Gamma B$ is defined as follows:

$$(\mu_A, \lambda_A)\,\Gamma\,(\mu_B, \lambda_B)(z) = (t_1, t_2), \tag{34}$$

where

$$\begin{gathered} t_1 = \begin{cases} \sup\limits_{z\in x\gamma y}\{\min(\theta_1(x), \sigma_1(y)) \mid x, y \in S, \gamma \in \Gamma\}, \\ 0 \qquad \text{otherwise}, \end{cases} \\ t_2 = \begin{cases} \inf\limits_{z\in x\gamma y}\{\max(\theta_2(x), \sigma_2(y)) \mid x, y \in S, \gamma \in \Gamma\}, \\ 0 \qquad \text{otherwise}. \end{cases} \end{gathered} \tag{35}$$

Let χ_I be the characteristic function of a left (res. right) Γ-hyperideal I of S. Then, $I = (\chi_I, {\chi_I}^C)$ is a left (res. right) intuitionistic fuzzy Γ-hyperideal of S.

Theorem 20. *If S is a Γ-semihyperring and A is an intuitionistic fuzzy Γ-hyperideal of S, then $X\Gamma A \subseteq A$, where $X = (\chi_S, \chi_S^c)$ and χ_S is the characteristic function of S.*

Proof. The proof is straightforward. □

Proposition 21. *Let S be a Γ-semihyperring, and let A be an intuitionistic fuzzy Γ-hyperideal of S. Then,*

(1) *if w is a fixed element of S, then the set ${\mu_A}^w = \{x \in S \mid \mu(x) \geq \mu(w)\}$ and ${\lambda_A}^w = \{x \in S \mid \lambda(x) \leq \lambda(w)\}$ are Γ-hyperideals of S,*

(2) *the sets $U = \{x \in S \mid \mu_A(x) = 0\}$ and $L = \{x \in S \mid \lambda_A(x) = 0\}$ are Γ-hyperideals of S.*

Proof. (1) Suppose that $x, y \in {\mu_A}^w$. Then, we have

$$\inf_{z\in x+y} \mu_A(z) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \mu(w), \tag{36}$$

which implies that $z \in U(\mu_A; \mu(w))$ and so $x + y \subseteq U(\mu_A; \mu(w))$. Similarly, suppose that $x, y \in {\lambda_A}^w$. We have $\sup_{z\in x+y} \lambda_A(z) \leq \max\{\lambda_A(x), \lambda_A(y)\} \leq \mu(w)$ which implies that $z \in U(\lambda_A; \lambda(w))$ and so $x + y \subseteq U(\lambda_A; \lambda(w))$. Assume that $x \in S$, $y \in {\mu_A}^w$ and $\gamma \in \Gamma$. Then, we have $\inf_{z\in x\gamma y} \mu_A(z) \geq \mu_A(y) \geq \mu(w)$ which implies that $z \in U(\mu_A; \mu(w))$ and so $x\gamma y \subseteq U(\mu_A; \mu(w))$. Similarly, suppose that $x \in {\lambda_A}^w$, $y \in S$ and $\gamma \in \Gamma$. We have $\sup_{z\in x\gamma y} \lambda_A(z) \leq \lambda_A(x) \leq \mu(w)$ which implies that $z \in U(\lambda_A; \lambda(w))$ and so $x\gamma y \subseteq U(\lambda_A; \lambda(w))$.

(2) One can easily prove the second part. □

Definition 22. Let S be a Γ-semihypering, and let $\overline{S}$ be a $\overline{\Gamma}$-semihyperring. If there exists a map $\varphi : S \to \overline{S}$ and a bijection $f : \Gamma \to \overline{\Gamma}$, such that

$$\begin{gathered} \varphi(x + y) = \varphi(x) + \varphi(y), \\ \varphi(x\gamma y) = \varphi(x) f(\gamma) \varphi(y), \end{gathered} \tag{37}$$

for all $x, y \in S$ and $\gamma \in \Gamma$, then we say (φ, f) is a *homomorphism* from S to $\overline{S}$. Also, if φ is a bijection, then (φ, f) is called an *isomorphism*, and S and $\overline{S}$ are *isomorphic*.

Definition 23. Let $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ be two nonempty intuitionistic fuzzy subsets of S and $\bar{S}$, respectively, and let $\varphi : S \to \overline{S}$ be a map. Then,

(1) the *inverse image* of B under φ is defined by

$$\varphi^{-1}(B) = \left(\varphi^{-1}(\mu_B), \varphi^{-1}(\lambda_B)\right), \tag{38}$$

where

$$\varphi^{-1}(\mu_B) = \mu_B \circ \varphi, \qquad \varphi^{-1}(\lambda_B) = \lambda_B \circ \varphi, \tag{39}$$

(2) the *image* of A under φ, denoted by $\varphi(A)$, is the intuitionistic fuzzy set in $\overline{S}$ defined by $\varphi(A) = (\varphi(\mu_A), \varphi^*(\lambda_A))$, where for each $y \in \overline{S}$

$$\begin{aligned} &\varphi(\mu_A)(y) \\ &\quad = \begin{cases} \sup\limits_{x\in\varphi^{-1}(y)} \mu_A(x) & \text{if } \varphi^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases} \\ &\varphi^*(\lambda_A)(y) \\ &\quad = \begin{cases} \inf\limits_{x\in\varphi^{-1}(y)} \lambda_A(x) & \text{if } \varphi^{-1}(y) \neq \emptyset, \\ 1 & \text{otherwise.} \end{cases} \end{aligned} \tag{40}$$

Now, the next theorem is about image and preimage of intuitionistic fuzzy sets.

Theorem 24. *Let S be a Γ-semihyperring, let $\overline{S}$ be a $\overline{\Gamma}$-semihyperring, and let (φ, f) be a homomorphism from S to $\overline{S}$. Then,*

(1) *if $A = (\mu_A, \lambda_A)$ is an intuitionistic fuzzy Γ-hyperideal of S, then $\varphi(A)$ is an intuitionistic fuzzy $\overline{\Gamma}$-hyperideal of $\overline{S}$,*

(2) *if $B = (\mu_B, \lambda_B)$ is an intuitionistic fuzzy $\overline{\Gamma}$-hyperideal of $\overline{S}$, then $\varphi^{-1}(B)$ is an intuitionistic fuzzy Γ-hyperideal of S.*

Proof. The proof is straightforward. □

4. Artinian and Noetherian Γ-Semihypergroups

In this section, we give some characterizations of Artinian and Noetherian Γ-semihyperrings.

Definition 25. Let S be a Γ-semihypering. Then, S is called *Noetherian* (*Artinian* resp.) if S satisfies the ascending (descending) chain condition on Γ-hyperideals. That is, for any Γ-hyperideals of S, such that

$$\begin{gathered} I_1 \subseteq I_2 \subseteq I_3 \subseteq \cdots \subseteq I_i \cdots \\ \left(I_1 \supseteq I_2 \supseteq I_3 \supseteq \cdots \supseteq I_i \cdots\right), \end{gathered} \tag{41}$$

there exists $n \in \mathbb{N}$, such that $I_i = I_{i+1}$, for all $i \geq n$.

Proposition 26 (see [28]). *Let S be a Γ-semihypering. Then, the following conditions are equivalent:*

(1) *S is Noetherian.*

(2) *S satisfies the maximum condition for Γ-hyperideals.*

(3) *Every Γ-hyperideal of S is finitely generated.*

Theorem 27. *If every intuitionistic fuzzy Γ-hyperideal of Γ-semihypering has finite number of values, then S is Artinian.*

Proof. Suppose that every intuitionistic fuzzy Γ-hyperideal of Γ-semihyperings has finite number of values, and S is not Artinian. So, there exists a strictly descending chain

$$S = I_0 \supseteq I_1 \supseteq I_2 \supseteq \cdots \tag{42}$$

of Γ-hyperideals of S. We define the intuitionistic fuzzy set A by

$$\mu_A(x) = \begin{cases} \dfrac{n}{n+1} & \text{if } x \in I_n - I_{n+1}, \\ 1 & \text{if } x \in \bigcap_{n=0}^{\infty} I_n, \end{cases}$$
$$\lambda_A(x) = \begin{cases} \dfrac{1}{n+1} & \text{if } x \in I_n - I_{n+1}, \\ 0 & \text{if } x \in \bigcap_{n=0}^{\infty} I_n. \end{cases} \tag{43}$$

It is easy to see that A is an intuitionistic fuzzy Γ-hyperideal of S. We have contradiction because of the definition of A, which depends on $I_0 \supseteq I_1 \supseteq I_2 \supseteq \cdots$ infinitely descending chain of Γ-hyperideals of S. □

Theorem 28. *Let S be a Γ-semihypering. Then the following statements are equivalent:*

(1) *S is Noetherian.*

(2) *The set of values of any intuitionistic fuzzy Γ-hyperideal of S is a well-ordered subset of* $[0, 1]$.

Proof. (1) ⇒ (2) Let A be an intuitionistic fuzzy Γ-hyperideal of Γ-semihypering. Assume that the set of values of A is not a well-ordered subset of $[0, 1]$. Then, there exits a strictly infinite decreasing sequence $\{t_n\}$, such that $\mu_A(x) = t_n$ and $\lambda_A(x) = 1 - t_n$. Let $I_n = \{x \in S \mid \mu(x) \geq t_n\}$ and $J_n = \{x \in S \mid \lambda(x) \leq 1 - t_n\}$. Then, $I_1 \subseteq I_2 \subseteq \cdots$ and $J_1 \subseteq J_2 \subseteq \cdots$ are strictly infinite decreasing chains of Γ-hyperideals of S. This contradicts with our hypothesis.

(2) ⇒ (1) Suppose that $I_1 \subseteq I_2 \subseteq \cdots$ is a strictly infinite ascending chain Γ-hyperideals of S. Let $I = \bigcup_{n \in \mathbb{N}} I_n$. It is easy to see that I is a Γ-hyperideal of S. Now, we define

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \notin I, \\ \dfrac{1}{k} & \text{where } k = \min\{n \in \mathbb{N} \mid x \in I_n\}, \end{cases}$$
$$\lambda_A(x) = \begin{cases} 1 & \text{if } x \notin I, \\ \dfrac{k-1}{k+1} & \text{where } k = \max\{n \in \mathbb{N} \mid x \in I_n\}. \end{cases} \tag{44}$$

Clearly, A is an intuitionistic fuzzy Γ-hyperideal of S. Since the chain is not finite, A has strictly infinite ascending sequence of values. This contradicts that the set of the values of fuzzy Γ-hyperideal is not well ordered. □

Theorem 29. *If a Γ-semihyperring S both Artinian and Noetherian, then every intuitionistic fuzzy Γ-hyperideal of S is finite valued.*

Proof. Suppose that A is an intuitionistic fuzzy Γ-hyperideal of S, $\mathrm{Im}(\mu_A)$ and $\mathrm{Im}(\lambda_A)$ are not finite. According the previous theorem, we consider the following two cases.

Case 1. Suppose that $t_1 < t_2 < t_3 < \cdots$ is strictly increasing sequence in $\mathrm{Im}(\mu_A)$ and $s_1 > s_2 > s_3 \cdots$ is strictly decreasing sequence in $\mathrm{Im}(\lambda_A)$. Now, we obtain

$$U(\mu_A; t_1) \supset U(\mu_A; t_2) \supset U(\mu_A; t_3) \supset \cdots,$$
$$L(\lambda_A; s_1) \supset L(\lambda_A; s_2) \supset L(\lambda_A; s_3) \supset \cdots \tag{45}$$

are strictly descending and ascending chains of Γ-hyperideals, respectively. Since S is both Artinian and Noetherian there exists a natural number i, such that $U(\mu_A; t_i) = U(\mu_A; t_{i+n})$ and $L(\lambda_A; s_i) = L(\lambda_A; s_{i+n})$ for $n \geq 1$. This means that $t_i = t_{i+n}$ and $s_i = s_{i+n}$. This is a contradiction.

Case 2. Assume that $t_1 > t_2 > t_3 > \cdots$, $s_1 < s_2 < s_3 \cdots$ are strictly decreasing and increasing sequence in $\mathrm{Im}(\mu_A)$ and $\mathrm{Im}(\lambda_A)$, respectively. From these equalities we conclude that

$$U(\mu_A; t_1) \subset U(\mu_A; t_2) \subset U(\mu_A; t_3) \subset \cdots,$$
$$L(\lambda_A; s_1) \subset L(\lambda_A; s_2) \subset L(\lambda_A; s_3) \subset \cdots \tag{46}$$

are strictly ascending and descending chains of Γ-hyperideals, respectively. Because of the hypothesis there exists a natural number j, such that $U(\mu_A; t_i) = U(\mu_A; t_{i+n})$ and $L(\lambda_A; s_i) = L(\lambda_A; s_{i+n})$ for $n \geq 1$. This implies that $t_i = t_{i+n}$ and $s_i = s_{i+n}$ which is a contradiction. □

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