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Research Article

General Solutions of Fully Fuzzy Linear Systems

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We propose a method to approximate the solutions of fully fuzzy linear system (FFLS), the so-called *general solutions*. So, we firstly solve the 1-cut position of a system, then some unknown spreads are allocated to each row of an FFLS. Using this methodology, we obtain some general solutions which are placed in the well-known solution sets like Tolerable solution set (TSS) and Controllable solution set (CSS). Finally, we solved two examples in order to demonstrate the ability of the proposed method.

1. Introduction

Systems of simulations linear equations play major role in various areas such as mathematics, statistics, and social sciences. Since in many applications, at least some of thesystem's parameters and measurements are represented by fuzzy rather than crisp numbers, therefore, it is important to develop mathematical models and numerical procedures that would appropriately treat general fuzzy linear systems and solve them.

The system of linear equations $A\widetilde{X} = \widetilde{b}$ where the coefficient matrix A is crisp, while $\widetilde{b}$ is a fuzzy number vector, is called a fuzzy system of linear equations (FSLE). Fuzzy linear systems have been studied by many authors. The first person who suggested the solution for solving FSLE was Fridman. Friedman et al. [1] proposed a general model to solve FSLE by using an embedding approach. Following Friedman et al. [1], Ma et al. [2] analyzed the solution of the duality of fuzzy systems. Allahviranloo et al. suggested some famous numerical methods for solving an FSLE [3–8]. Also, in [9, 10], Abbasbandy et al. proposed the LU-decomposition method and the Steepest descent method to solve system, respectively. For more research see [11–22].

The linear system $\widetilde{A}\widetilde{X} = \widetilde{b}$, where the elements, $\widetilde{a}_{ij}$, of the matrix $\widetilde{A}$ and the elements, $\widetilde{b}_i$, of the vector $\widetilde{b}$ are fuzzy numbers, is called a fully fuzzy linear system (FFLS).

Buckley and Qu in their consecutive works [23–25] proposed different solutions for FFLSs. Also, they found relation between these solutions. Based on their works, Muzzioli and Reynaerts in [26] studied FFLS of the form $A_1x + b_1 = A_2x + b_2$, while for implementing their method $2^{n(n+1)}$ crisp systems should be solved.

Consequently, Dehghan et al. have studied some methods for solving FFLS. They have represented Cramer's rule, Gaussian elimination, fuzzy LU decomposition (Doolittle algorithm), and its simplification; they also have showed the applicability of linear programming approach for overdetermined FFLS in [27–29]. Also, in [30], Allahviranloo and Mikaeilvand proposed an analytical method to obtain solution of FFLS by an embedding method. Their method is constructed based on obtaining a nonzero solution of the FFLS.

Vroman et al. in their continuous works [31–33] suggested two methods for solving system. In [33], they have proposed Cramer's rule to solve FFLS approximately, then they proved that their solution is better than Buckley and

Qu's approximate solution vector. Furthermore, they have proposed an algorithm to improve their method to solve FFLS by parametric functions [32].

Recently, Allahviranloo et al. [34] have proposed a new practical method to solve an FFLS based on the 1-cut expansion. In their method, some spreads and then some new solutions have been derived that belong to TSS or CSS. Note that they have obtained some spreads which are symmetric.

We show that, using the proposed method in the present paper, we can obtain better solutions. On the other hand, the created errors in some certain cases with respect to the proposed distance are less than the errors that are obtained via Allahviranloo et al.'s method [34].

The structure of this paper is organized as follows.

In Section 2, we discuss concisely some important basic concepts and definitions which will be used later. In Section 3, we present our new method and concentrate on how we could derive the linear general spreads of fuzzy vector solution corresponding to TSS or CSS. The proposed method is illustrated by solving some examples in Section 4, and conclusion is drawn in Section 5.

2. Preliminaries

Let $P_k(\mathbb{R})$ denote the family of all nonempty compact convex subset of $\mathbb{R}$.

A nonempty bounded subset A of $\mathbb{R}$ is called convex if and only if

$$(1-k)x+ky \in A \quad \text{for every } x, y \in A, k \in [0,1]. \tag{1}$$

The basic definition of fuzzy numbers is given in [35–38].

Definition 1. A fuzzy number is a function such as $u : \mathbb{R} \to [0,1]$ satisfying the following properties:

(1) u is normal, that is, there exists an $x_0 \in \mathbb{R}$ such that $u(x_0) = 1$;

(2) u is fuzzy convex, that is, $u(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}$ for any $x, y \in \mathbb{R}, \lambda \in [0,1]$;

(3) u is upper semicontinuous;

(4) $\overline{\{x \in \mathbb{R} : u(x) > 0\}}$ is compact.

The set of all fuzzy real numbers is denoted by E.

An alternative definition of fuzzy number is as follows.

Definition 2. A fuzzy number u in parametric form is a pair $(\underline{u}, \overline{u})$ of functions $\underline{u}(r), \overline{u}(r), 0 \leq r \leq 1$, which satisfies the following requirements:

(1) $\underline{u}(r)$ is a bounded monotonic increasing left continuous function;

(2) $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function;

(3) $\underline{u}(r) \leq \overline{u}(r), 0 \leq r \leq 1$.

A popular fuzzy number is the symmetric triangular fuzzy number $S[x_0, \sigma]$ centered at x_0 with basic 2σ:

$$u(x) = \begin{cases} \dfrac{1}{\sigma}(x - x_0 + \sigma), & x_0 - \sigma \leq x \leq x_0, \\ \dfrac{1}{\sigma}(x_0 + \sigma - x), & x_0 \leq x \leq x_0 + \sigma, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

We define arbitrary $u = (\underline{u}(r), \overline{u}(r))$, $v = (\underline{v}(r), \overline{v}(r))$, addition, subtraction, and multiplication:

(1) $\underline{u+v}(r) = \underline{u}(r) + \underline{v}(r)$, $\overline{u+v}(r) = \overline{u}(r) + \overline{v}(r)$,

(2) $\underline{u-v}(r) = \underline{u}(r) - \overline{v}(r)$, $\overline{u-v}(r) = \overline{u}(r) - \underline{v}(r)$,

(3) $\underline{uv}(r) = \min\{\underline{u}(r)\underline{v}(r), \underline{u}(r)\overline{v}(r), \overline{u}(r)\underline{v}(r), \overline{u}(r)\overline{v}(r)\}$,
$\overline{uv}(r) = \max\{\underline{u}(r)\underline{v}(r), \underline{u}(r)\overline{v}(r), \overline{u}(r)\underline{v}(r), \overline{u}(r)\overline{v}(r)\}$.

Definition 3. The Hausdorff distance between fuzzy numbers given by $d : E \times E \to R_+ \cup \{0\}$,

$$d(u,v) = \sup_{r\in[0,1]} \max\left\{\left|\underline{u}(r), \underline{v}(r)\right|, \left|\overline{u}(r), \overline{v}(r)\right|\right\}, \tag{3}$$

where $u = (\underline{u}(r), \overline{u}(r))$, $v = (\underline{v}(r), \overline{v}(r)) \subset R$, is utilized in [11]. Then it is easy to see that d is a metric in E and has the following properties (see [36]):

(1) $d(u+w, v+w) = d(u,v)$, for all $u, v, w \in E$,

(2) $d(k \cdot u, k \cdot v) = |k| d(u,v)$, for all $k \in R, u, v \in E$,

(3) $d(u+v, w+e) \leq d(u,w) + d(v,e)$, for all $u, v, w, e \in E$,

(4) (d, E) is a complete metric space.

Also, we define the distance between two fuzzy vectors (each vector with fuzzy components) AX and $\tilde{b}$ as follows:

$$D\left(AX, \tilde{b}\right) = \sum_{i=1}^{n} d\left(AX_i, \tilde{b}_i\right), \quad i = 1, 2, \ldots, n, \tag{4}$$

where AX_i is the ith row of FFLS and $\tilde{b}_i$ is the ith compnent of fuzzy vector $\tilde{b}$.

Definition 4. The $n \times n$ linear systems of equations

$$\begin{aligned} &\tilde{a}_{11}\tilde{x}_1 + \tilde{a}_{12}\tilde{x}_2 + \cdots + \tilde{a}_{1n}\tilde{x}_n = \tilde{b}_1 \\ &\tilde{a}_{21}\tilde{x}_1 + \tilde{a}_{22}\tilde{x}_2 + \cdots + \tilde{a}_{2n}\tilde{x}_n = \tilde{b}_2 \\ &\vdots \\ &\tilde{a}_{n1}\tilde{x}_1 + \tilde{a}_{n2}\tilde{x}_2 + \cdots + \tilde{a}_{nn}\tilde{x}_n = \tilde{b}_n, \end{aligned} \tag{5}$$

where the elements, $\tilde{a}_{ij}$, of the coefficient matrix $\tilde{A}$, $1 \leq i, j \leq n$ and the elements, $\tilde{b}_i$, of the vector $\tilde{b}$ are fuzzy numbers, are called fully fuzzy linear systems (FFLSs).

Definition 5. A fuzzy vector $\tilde{X} = (\tilde{x}_1, \ldots, \tilde{x}_n)^t$, given by $\tilde{x}_i(r) = [\overline{x}_i(r), \underline{x}_i(r)]$, is called the solution of (10) if

$$\underline{\sum_{j=1}^{n} a_{ij}x_j} = \sum_{j=1}^{n} \underline{a_{ij}x_j} = \underline{b}_i, \qquad \overline{\sum_{j=1}^{n} a_{ij}x_j} = \sum_{j=1}^{n} \overline{a_{ij}x_j} = \overline{b}_i. \tag{6}$$

Definition 6 (see [34, 39]). The united solution set (USS), the tolerable solution set (TSS), and controllable solution set (CSS) for the system (10) are, respectively, as follows:

$$\begin{aligned} X_{\exists\exists} &= \left\{x' \in R^n : \left(\exists A' \in A\right)\left(\exists b' \in b\right) \text{ s.t. } A'x' = b'\right\} \\ &= \left\{x' \in R^n : Ax' \cap b' \neq \emptyset\right\}, \\ X_{\forall\exists} &= \left\{x' \in R^n : \left(\forall A' \in A\right)\left(\exists b' \in b\right) \text{ s.t. } A'x' = b'\right\} \\ &= \left\{x' \in R^n : Ax' \subseteq b\right\}, \\ X_{\exists\forall} &= \left\{x' \in R^n : \left(\forall b' \in b\right)\left(\exists A' \in A\right) \text{ s.t. } A'x' = b'\right\} \\ &= \left\{x' \in R^n : Ax' \supseteq b\right\}. \end{aligned} \tag{7}$$

Definition 7. A fuzzy vector $\widetilde{X} = (\widetilde{x}_1, \ldots, \widetilde{x}_n)^t$, given by $\widetilde{x}_i(r) = [\underline{x}_i(r), \overline{x}_i(r)]$, $0 \leq r \leq 1$, is called the minimal symmetric solution of (5) which is placed in CSS, if for any arbitrary symmetric solution $\widetilde{Y} = (\widetilde{y}_1, \ldots, \widetilde{y}_n)^t$ which is placed in CSS and $\widetilde{Y}(1) = \widetilde{X}(1)$, we have

$$\left(\widetilde{Y} \supseteq \widetilde{X}\right), \quad \text{that is, } \left(\widetilde{y}_i \supseteq \widetilde{x}_i\right), \quad \text{that is, } \left(\sigma_{\widetilde{y}_i} \geq \sigma_{\widetilde{x}_i}\right), \quad \forall i = 1, \ldots, n, \tag{8}$$

where $\sigma_{\widetilde{y}_i}$ and $\sigma_{\widetilde{x}_i}$ are symmetric spreads of $\widetilde{y}_i$ and $\widetilde{x}_i$, respectively. See [34, 39].

Definition 8. A fuzzy vector $\widetilde{X} = (\widetilde{x}_1, \ldots, \widetilde{x}_n)^t$, given by $\widetilde{x}_i(r) = [\underline{x}_i(r), \overline{x}_i(r)]$, $0 \leq r \leq 1$, is called the maximal symmetric solution of (5) which is placed in TSS, if for any arbitrary symmetric solution $\widetilde{Z} = (\widetilde{z}_1, \ldots, \widetilde{z}_n)^t$ which is placed in TSS and $\widetilde{Z}(1) = \widetilde{X}(1)$, we have

$$\left(\widetilde{X} \supseteq \widetilde{Z}\right), \quad \text{that is, } \left(\widetilde{x}_i \supseteq \widetilde{z}_i\right), \quad \text{that is, } \left(\sigma_{\widetilde{x}_i} \geq \sigma_{\widetilde{z}_i}\right), \quad \forall i = 1, \ldots, n, \tag{9}$$

where $\sigma_{\widetilde{x}_i}$ and $\sigma_{\widetilde{z}_i}$ are symmetric spreads of $\widetilde{x}_i$ and $\widetilde{z}_i$, respectively. See [34, 39].

3. General Solutions

In this section, we suggest a novel and practical method to obtain general solutions of FFLS. To this end, we solve the 1-cut system (5), which is a crisp system. So, we solve the following crisp system:

$$\sum_{j=1}^{n} \widetilde{a}_{ij}(1)\, x_j = \widetilde{b}_i(1), \quad i = 1, \ldots, n, \tag{10}$$

where $\widetilde{b}_i(1), \widetilde{a}_{ij}(1) \in \mathbb{R}$ and x_j, $j = 1, \ldots, n$ are unknown crisp variables which determine by solving system (10). Therefore, we fuzzify, the obtained solution from the crisp system (10), by allocating some unknown general spreads (asymmetric spreads) to each row of the system (10).

Then, crisp system (10) is converted to the following system $2n$ linear equations:

$$\begin{aligned} &\left[\underline{a}_{11}(r), \overline{a}_{11}(r)\right]\left[x_1 - \alpha_1(r), x_1 + \beta_1(r)\right] + \cdots \\ &\quad + \left[\underline{a}_{1n}(r), \overline{a}_{1n}(r)\right]\left[x_n - \alpha_1(r), x_n + \beta_1(r)\right] = \left[\underline{b}_1(r), \overline{b}_1(r)\right] \\ &\left[\underline{a}_{21}(r), \overline{a}_{21}(r)\right]\left[x_1 - \alpha_2(r), x_1 + \beta_2(r)\right] + \cdots \\ &\quad + \left[\underline{a}_{2n}(r), \overline{a}_{2n}(r)\right]\left[x_n - \alpha_2(r), x_n + \beta_2(r)\right] = \left[\underline{b}_2(r), \overline{b}_2(r)\right] \\ &\quad \vdots \\ &\left[\underline{a}_{n1}(r), \overline{a}_{n1}(r)\right]\left[x_1 - \alpha_n(r), x_1 + \beta_n(r)\right] + \cdots \\ &\quad + \left[\underline{a}_{nn}(r), \overline{a}_{nn}(r)\right]\left[x_n - \alpha_n(r), x_n + \beta_n(r)\right] = \left[\underline{b}_n(r), \overline{b}_n(r)\right]. \end{aligned} \tag{11}$$

In the above system, x_j, $j = 1, \ldots, n$ are of the obtained of crisp system (11) and $\alpha_i(r) > 0$, $\beta_i(r) > 0$, $i = 1, \ldots, n$ are unknown spreads. However, for obtaining general solutions of the FFLS, first we have to solve the above equation system, which requires finding the general spreads solution. Now, to solve system (11), we suppose the following one type for the components of fuzzy matrix:

$$I = \left\{(i, j) \in N_n \times N_n \mid \widetilde{a}_{ij} > 0\right\}, \quad N_n = 1, 2, \ldots, n. \tag{12}$$

Remark 9. Without loss of generality, we explain our method with the assumption that, in interval $[x_i - \alpha_i(r), x_i + \beta_i(r)]$, function $x_i - \alpha_i(r)$ is positive. We just remove some types which elements of fuzzy matrixes are negative and positive-negative, and also zero does exists in the support of elements of fuzzy matrixes and fuzzy solution. Moreover, the ordering $>$ means that $a_{ij} > 0$ if and only if $\underline{a}_{ij}(0) > 0$ and $a_{ij} < 0$ if and only if $\overline{a}_{ij}(0) < 0$.

$$\text{Type (1)}: I = \left\{(i, j) \in N_n \times N_n \mid \widetilde{a}_{ij} > 0\right\}, \quad \left|I_1\right| = n^2. \tag{13}$$

Because of positivity of elements of fuzzy matrix $\widetilde{A}$, the ith row of system (11) is the supposed like the following:

$$\begin{aligned} &\left[\underline{a}_{i1}(r), \overline{a}_{i1}(r)\right]\left[x_1 - \alpha_i(r), x_1 + \beta_i(r)\right] + \cdots \\ &\quad + \left[\underline{a}_{in}(r), \overline{a}_{in}(r)\right]\left[x_n - \alpha_i(r), x_n + \beta_i(r)\right] = \left[\underline{b}_i(r), \overline{b}_i(r)\right]. \end{aligned} \tag{14}$$

Since, we considered $\widetilde{A}$ is positive, the compact form of the above equations are calculated as follows:

$$\sum_{j=1}^{n} \underline{a}_{ij}(r)\left(x_j - \alpha_i(r)\right) = \underline{b}_i(r), \quad i = 1, \ldots, n, \tag{15}$$

$$\sum_{j=1}^{n} \overline{a}_{ij}(r)\left(x_j + \beta_i(r)\right) = \overline{b}_i(r), \quad i = 1, \ldots, n, \tag{16}$$

in which (15) and (16) are rendered, respectively, to

$$\alpha_i(r) = f_1\left(x_1, \ldots, x_n, \underline{a}_{i1}(r), \ldots, \underline{a}_{in}(r), \underline{b}_i(r)\right), \quad i = 1, \ldots, n, \tag{17}$$

$$\beta_i(r) = f_2\left(x_1, \ldots, x_n, \overline{a}_{i1}(r), \ldots, \overline{a}_{in}(r), \overline{b}_i(r)\right), \quad i = 1, \ldots, n. \tag{18}$$

We offer 4 ways, to determine the spreads of solutions of the FFLS, which are gained as follows:

$$\begin{aligned} \alpha_G^-(r) &= \min_{0\le r\le 1}\{|\alpha_i(r)|\}, \quad i = 1, \ldots, n, \\ \beta_G^-(r) &= \min_{0\le r\le 1}\{|\beta_i(r)|\}, \quad i = 1, \ldots, n, \end{aligned} \tag{19}$$

$$\begin{aligned} \alpha_G^+(r) &= \max_{0\le r\le 1}\{|\alpha_i(r)|\}, \quad i = 1, \ldots, n, \\ \beta_G^+(r) &= \max_{0\le r\le 1}\{|\beta_i(r)|\}, \quad i = 1, \ldots, n, \end{aligned} \tag{20}$$

in which the obtained $\widetilde{X}(r) = (\widetilde{x}_1(r), \ldots, \widetilde{x}_n(r))^t$, by using (19) or (20), are as follows:

$$\begin{aligned} \widetilde{x}_i(r) &= \left[x_i - \alpha_G^-(r), x_i + \beta_G^-(r)\right], \\ \widetilde{x}_i(r) &= \left[x_i - \alpha_G^+(r), x_i + \beta_G^+(r)\right]. \end{aligned} \tag{21}$$

Since, spreads $\{\alpha_G^-(r), \alpha_G^+(r), \beta_G^-(r), \beta_G^+(r)\}$ are not linear functions, they may be piece-wise linear functions. Thus, for obtaining linear spreads, we have to make some changes in the structure of the obtained spreads. So, linear forms of spreads are as follows:

$$\begin{aligned} \alpha_i(r) &= \frac{\sum_{j=1}^n \underline{a}_{ij}(r)x_j - \underline{b}_i(r) + \delta_i - \delta_i r}{\sum_{j=1}^n \underline{a}_{ij}(r)}, \\ \beta_i(r) &= \frac{\overline{b}_i(r) + \gamma_i - \gamma_i r - \sum_{j=1}^n \overline{a}_{ij}(r)x_j}{\sum_{j=1}^n \overline{a}_{ij}(r)}. \end{aligned} \tag{22}$$

Let $\underline{a}_{ij}(r) \in [\underline{a}_{ij}(0), a_{ij}(1)]$, where supp $\widetilde{a}_{ij} = [\underline{a}_{ij}(0), \overline{a}_{ij}(0)]$, then

$$\begin{aligned} \min_{0\le r\le 1}\left\{\sum_{j=1}^n \underline{a}_{ij}(r)\right\} &= \sum_{j=1}^n \min_{0\le r\le 1}\left\{\underline{a}_{ij}(r)\right\} \\ &= \sum_{j=1}^n \underline{a}_{ij}(0), \quad i = 1, \ldots, n, \\ \max_{0\le r\le 1}\left\{\sum_{j=1}^n \underline{a}_{ij}(r)\right\} &= \sum_{j=1}^n \max_{0\le r\le 1}\left\{\underline{a}_{ij}(r)\right\} \\ &= \sum_{j=1}^n \underline{a}_{ij}(1), \quad i = 1, \ldots, n. \end{aligned} \tag{23}$$

Similarly, consider $\overline{a}_{ij}(r) \in [a_{ij}(1), \overline{a}_{ij}(0)]$, then

$$\begin{aligned} \min_{0\le r\le 1}\left\{\sum_{j=1}^n \overline{a}_{ij}(r)\right\} &= \sum_{j=1}^n \min_{0\le r\le 1}\left\{\overline{a}_{ij}(r)\right\} \\ &= \sum_{j=1}^n \overline{a}_{ij}(1), \quad i = 1, \ldots, n, \\ \max_{0\le r\le 1}\left\{\sum_{j=1}^n \overline{a}_{ij}(r)\right\} &= \sum_{j=1}^n \max_{0\le r\le 1}\left\{\overline{a}_{ij}(r)\right\} \\ &= \sum_{j=1}^n \overline{a}_{ij}(0), \quad i = 1, \ldots, n. \end{aligned} \tag{24}$$

Therefore, we get

$$\begin{aligned} \alpha_i^l(r) &= \frac{\sum_{j=1}^n \underline{a}_{ij}(r)x_j - \underline{b}_i(1) + \delta_i - \delta_i r}{\sum_{j=1}^n \underline{a}_{ij}(1)}, \quad i = 1, \ldots, n, \\ \alpha_i^u(r) &= \frac{\sum_{j=1}^n \underline{a}_{ij}(r)x_j - \underline{b}_i(1) + \delta_i - \delta_i r}{\sum_{j=1}^n \underline{a}_{ij}(0)}, \quad i = 1, \ldots, n, \\ \beta_i^l(r) &= \frac{\overline{b}_i(r) + \gamma_i - \gamma_i r - \sum_{j=1}^n \overline{a}_{ij}(r)x_j}{\sum_{j=1}^n \overline{a}_{ij}(0)}, \quad i = 1, \ldots, n, \\ \beta_i^u(r) &= \frac{\overline{b}_i(r) + \gamma_i - \gamma_i r - \sum_{j=1}^n \overline{a}_{ij}(r)x_j}{\sum_{j=1}^n \overline{a}_{ij}(1)}, \quad i = 1, \ldots, n. \end{aligned} \tag{25}$$

So, we consider some situations on linear asymmetric spreads of solutions as follows:

$$\begin{aligned} \alpha_G^{-,l,u}(r) &= \min_{0\le r\le 1}\left\{\left|\alpha_i^l(r)\right|, \left|\alpha_i^u(r)\right|\right\}, \\ \beta_G^{-,l,u}(r) &= \min_{0\le r\le 1}\left\{\left|\beta_i^l(r)\right|, \left|\beta_i^u(r)\right|\right\}, \\ \alpha_G^{+,l,u}(r) &= \max_{0\le r\le 1}\left\{\left|\alpha_i^l(r)\right|, \left|\alpha_i^u(r)\right|\right\}, \\ \beta_G^{+,l,u}(r) &= \max_{0\le r\le 1}\left\{\left|\beta_i^l(r)\right|, \left|\beta_i^u(r)\right|\right\}. \end{aligned} \tag{26}$$

Hence, the fuzzy vector solution of FFLS, by using general spreads, will be obtained like the following:

$$\begin{aligned} \widetilde{X}_G^{-,l,u}(r) &= \left(\widetilde{x}_1^{-,l,u}(r), \ldots, \widetilde{x}_n^{-,l,u}(r)\right)^t, \ \text{s.t. } \widetilde{x}_i^{-,l,u}(r) \\ &= \left[x_i - \alpha_G^{-,l,u}(r), x_i + \beta_G^{-,l,u}(r)\right], \\ \widetilde{X}_G^{+,l,u}(r) &= \left(\widetilde{x}_1^{+,l,u}(r), \ldots, \widetilde{x}_n^{+,l,u}(r)\right)^t, \ \text{s.t. } \widetilde{x}_i^{+,l,u}(r) \\ \widetilde{X}_G^{+,l,u}(r) &= \left[x_i - \alpha_G^{+,l,u}(r), x_i + \beta_G^{+,l,u}(r)\right]. \end{aligned} \tag{27}$$

However, the obtained spreads of the FFLS in crisp manner should be zero, which we will talk about in the following part. Have in mind this feature is applicable for the mentioned type.

Proposition 10. *Consider the linear asymmetric spreads* (26) *and the solutions of FFLS derived by* (27). *Then,*

(1) $\alpha_G^{-,l,u}(1) = \alpha_G^{+,l,u}(1) = \beta_G^{-,l,u}(1) = \beta_G^{+,l,u}(1) = 0$,

(2) $\widetilde{X}_G^{-,l,u}(1) = \widetilde{X}_G^{+,l,u}(1) = X^c$.

Proof. Based on the proposed method, we have assumed that 1-cut position is crisp (since the fuzzy values are triangular). So, all spreads are zero, that is,

$$\alpha_G^{-,l,u}(1) = \alpha_G^{+,l,u}(1) = \beta_G^{-,l,u}(1) = \beta_G^{+,l,u}(1) = 0, \tag{28}$$

and then we deduce that the solutions in this case coincide with the 1-cut solution, that is,

$$\widetilde{X}_G^{-,l,u}(1) = \widetilde{X}_G^{+,l,u}(1) = X^c, \tag{29}$$

which completes the proof. □

Theorem 11. *The solution of fully fuzzy linear system* (10) *is a fuzzy vector.*

Proof. We state the proof for the solution, $\widetilde{X}_G^{-,l,u}$, and the proof for the other solution, $\widetilde{X}_G^{+,l,u}$, is similar. So, we omit it. It is easy to verify that $\widetilde{X}_G^{-,l,u}$ satisfies

$$x_i - \alpha_G^{-,l,u}(r) \le x_i + \beta_G^{-,l,u}(r), \tag{30}$$

for each $0 \le r \le 1$. Also, let us consider $0 < r_1 \le r_2 \le 1$, then we have

$$\widetilde{X}_G^{-,l,u}(r_1) \ge \widetilde{X}_G^{-,l,u}(r_2). \tag{31}$$

So, we deduce that $\widetilde{X}_G^{-,l,u}$ is a fuzzy vector which completes the proof. □

Theorem 12. *For given spreads from* (26) *and the solutions of FFLS given by* (27), *one has the following properties:*

(1) $\widetilde{X}_G^{-,l,u} \in TSS$,

(2) $\widetilde{X}_G^{+,l,u} \in CSS$.

Proof. Let us consider $\widetilde{x}_i^{-,l,u}(r) = [x_i - \alpha_G^{-,l,u}(r), x_i + \beta_G^{-,l,u}(r)]$, then based on the proposed approach, we have

$$A\widetilde{X}_G^{-,l,u} \subseteq \widetilde{b}, \tag{32}$$

which shows that $\widetilde{X}_G^{-,l,u} \in$ TSS. Similarly, using the proposed method and definition of $\widetilde{X}_G^{+,l,u}$, one has

$$A\widetilde{X}_G^{+,l,u} \supseteq \widetilde{b}, \tag{33}$$

which indicates that $\widetilde{X}_G^{+,l,u} \in$ CSS. □

Theorem 13. *Consider $\widetilde{X}_G^{-,l,u}$ and $\widetilde{X}_G^{+,l,u}$ as defined in Theorem 12, then one has the following:*

(1) *$\widetilde{X}_G^{-,l,u}$ is maximal general solution in TSS,*

(2) *$\widetilde{X}_G^{+,l,u}$ is minimal general solution in CSS.*

Proof. Based on the definition of maximal and minimal general solutions, the proof is straightforward. □

Now, we provide some useful result to show the difference between proposed method and the symmetric solutions [34, 39].

Theorem 14. *Assuming that $\alpha_G^{-,l,u} \le \beta_G^{-,l,u}$, then one has:*

$$X_{sym}^{-,l,u} \subseteq X_G^{-,l,u}. \tag{34}$$

Proof. By comparing obtained results for the symmetric solution $X_{\mathrm{sym}}^{-,l,u}$, proposed in [34, 39], the proof is straightforward. □

The following results show that, under certain conditions, the approach suggested in the present paper has less errors than the Allahviranloo et al.'s method [34].

Theorem 15. *Assuming that $\alpha_G^{-,l,u} = \beta_G^{-,l,u}$, then one has:*

$$X_{sym}^{-,l,u} = X_G^{-,l,u}. \tag{35}$$

Also, let $\alpha_G^{+,l,u} = \beta_G^{+,l,u}$, then we have:

$$X_{sym}^{+,l,u} = X_G^{+,l,u}. \tag{36}$$

Proof. By comparing obtained results in [34], the proof is straightforward. □

Theorem 16. *If $\alpha_G^{-,l,u}(r) \le \beta_G^{-,l,u}(r)$, then the following property holds:*

$$d\left(AX_G^{-,l,u}, b\right) \le d\left(AX_{sym}^{-,l,u}, b\right). \tag{37}$$

Proof. Based on definitions of $X_{\mathrm{sym}}^{-,l,u}$ and $X_G^{-,l,u}$, we have

$$\begin{aligned} X_{\mathrm{sym}}^{-,l,u} &= \left[\left[x_1 - \alpha_G^{-,l,u}, x_1 + \alpha_G^{-,l,u}\right], \dots, \right. \\ &\qquad \left.\left[x_n - \alpha_G^{-,l,u}, x_n + \alpha_G^{-,l,u}\right]\right]^t, \\ X_G^{-,l,u} &= \left[\left[x_1 - \alpha_G^{-,l,u}, x_1 + \beta_G^{-,l,u}\right], \dots, \right. \\ &\qquad \left.\left[x_n - \alpha_G^{-,l,u}, x_n + \beta_G^{-,l,u}\right]\right]^t. \end{aligned} \tag{38}$$

Clearly, in the mentioned fuzzy vector solutions, the lower functions of each component, $x_i - \alpha_G^{-,l,u}$, $i = 1, \dots, n$, are equivalent. So, we can state that $\underline{X}_{\mathrm{sym}}^{-,l,u} = \underline{X}_G^{-,l,u}$.

Now, we discuss the upper functions of the fuzzy vector solutions $\overline{X}_{\mathrm{sym}}^{-,l,u}$ and $\overline{X}_G^{-,l,u}$. Since $\alpha_G^{-,l,u} \le \beta_G^{-,l,u}$, we define the following positive function:

$$\eta(r) = \beta_G^{-,l,u}(r) - \alpha_G^{-,l,u}(r), \quad 0 \le r \le 1. \tag{39}$$

Then, after simple calculations we obtain:

$$\left(A\overline{X}_{\text{sym}}^{-,l,u}\right)_i(r) + \left[0, \sum_{j=1}^{n} \overline{a}_{ij}(r)\,\eta(r)\right] = \left(A\overline{X}_{G}^{-,l,u}\right)_i(r), \tag{40}$$

for all $r \in [0,1]$ and $i = 1,2,\ldots,n$. It is easy to verify that $\sum_{j=1}^{n} \overline{a}_{ij}(r)\eta(r)$ is a positive function:

$$\left(A\overline{X}_{G}^{-,l,u}\right)_i(r) \le \left(A\overline{X}_{\text{sym}}^{-,l,u}\right)_i(r). \tag{41}$$

Consequently, we have

$$\left|\left(A\overline{X}_{G}^{-,l,u}\right)_i(r) - \overline{b}_i\right| \le \left|\left(A\overline{X}_{\text{sym}}^{-,l,u}\right)_i(r) - \overline{b}_i\right|, \quad 1 \le i \le n. \tag{42}$$

Thus,

$$\begin{aligned} d\left(AX_{G}^{-,l,u}, b\right) &= \sum_{i=1}^{n} d\left(\left(AX_{G}^{-,l,u}\right)_i(r), b_i(r)\right) \\ &\le \sum_{i=1}^{n} d\left(\left(AX_{\text{sym}}^{-,l,u}\right)_i(r), b_i(r)\right) \\ &= d\left(AX_{\text{sym}}^{-,l,u}, b\right), \end{aligned} \tag{43}$$

which completes the proof. □

4. Examples

In this section, we take an example which has been solved in [28, 30, 34] with their proposed methods. We obtain the maximal-minimal general solutions, which are placed in a TSS and CSS, respectively.

Example 17. Consider the following FFLS:

$$\widetilde{A} = \begin{pmatrix} (4+r, 6-r) & (5+r, 8-2r) \\ (6+r, 7) & (4, 5-r) \end{pmatrix}, \qquad \widetilde{b} = \begin{pmatrix} (40+10r, 67-17) \\ (43+5r, 55-7r) \end{pmatrix}. \tag{44}$$

Then, the crisp solution of 1-cut of the FFLS is $\widetilde{X} = (x_1, x_2)^t = (4,5)^t$. So, we fuzzify the crisp system as follows:

$$\begin{aligned} &[4+r, 6-r]\,[4-\alpha_1(r), 4+\beta_1(r)] \\ &\quad + [5+r, 8-2r]\,[5-\alpha_1(r), 5+\beta_1(r)] = [40+10r, 67-17r] \\ &[6+r, 7]\,[4-\alpha_2(r), 4+\beta_2(r)] \\ &\quad + [4, 5-r]\,[5-\alpha_2(r), 5+\beta_2(r)] = [43+5r, 55-7r]. \end{aligned} \tag{45}$$

Now, we ought to solve the equations to find the spreads:

$$(4+r)\left(4-\alpha_1(r)\right) + (5+r)\left(5-\alpha_1(r)\right) = 40+10r, \tag{46}$$

$$(6-r)\left(4+\beta_1(r)\right) + (8-2r)\left(5+\beta_1(r)\right) = 67-17r, \tag{47}$$

$$(6+r)\left(4-\alpha_2(r)\right) + 4\left(5-\alpha_2(r)\right) = 43+5r, \tag{48}$$

$$7\left(4+\beta_2(r)\right) + (5-r)\left(5+\beta_2(r)\right) = 55-7r, \tag{49}$$

which, the obtained spreads from (46)–(49), are as follows, respectively:

$$\begin{aligned} \alpha_1(r) &= \frac{1-r}{9+2r}, \\ \beta_1(r) &= \frac{3-3r}{14-3r}, \\ \alpha_2(r) &= \frac{1-r}{10+r}, \\ \beta_2(r) &= \frac{2-2r}{12-r}. \end{aligned} \tag{50}$$

After, the obtain spreads, by using (26), we determine linear unsymmetric spreads of solutions of the FFLS:

$$\begin{aligned} \alpha_G^{-,l,u}(r) &= \frac{1-r}{11}, \\ \beta_G^{-,l,u}(r) &= \frac{2-2r}{12}, \\ \alpha_G^{+,l,u}(r) &= \frac{1-r}{9}, \\ \beta_G^{+,l,u}(r) &= \frac{3-3r}{11}, \end{aligned} \tag{51}$$

In this way, unsymmetric solutions, in which the use of the above mention spreads, will be obtained as follows:

$$\begin{aligned} &\widetilde{X}_G^{-,l,u}(r) \\ &\quad = \left(\left[4 - \frac{1-r}{11}, 4 + \frac{2-2r}{12}\right], \left[5 - \frac{1-r}{11}, 5 + \frac{2-2r}{12}\right]\right)^t, \\ &\widetilde{X}_G^{+,l,u}(r) \\ &\quad = \left(\left[4 - \frac{1-r}{9}, 4 + \frac{3-3r}{11}\right], \left[5 - \frac{1-r}{9}, 5 + \frac{3-3r}{11}\right]\right)^t. \end{aligned} \tag{52}$$

We insert the obtained solutions (52), into FFLS, in order to compare the differences between the values of row 1 with $\widetilde{b}_1$ and the values of row 2 with $\widetilde{b}_2$ (see Figures 1 and 2).

Furthermore, the solutions of the suggested method are plotted to compare with Dehghan's method (D) which is offered in [28] and Allahviranloo's methods (A) which are proposed in [30, 34] (see Figures 3 and 4).

Note that Dehghan's solution [28] is given by

$$\widetilde{X}_D(r) = \left[\left[\frac{43}{11} + \frac{r}{11}, 4\right], \left[\frac{54}{11} + \frac{r}{11}, \frac{11}{2}, \frac{r}{2}\right]\right]^t, \tag{53}$$

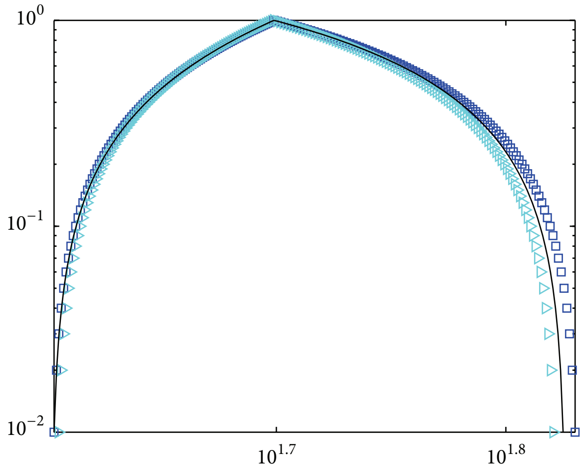

FIGURE 1: Compare $\widetilde{b}_1(-)$ and the value of the first row for $\widetilde{x}_G^{-,l,u}$ $(\triangleright)$ and $\widetilde{x}_G^{+,l,u}(\square)$.

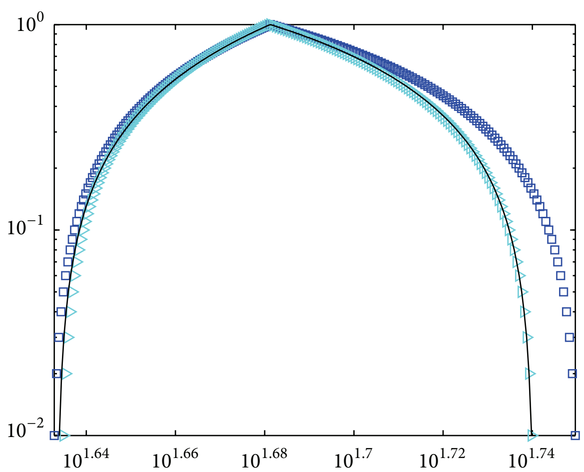

FIGURE 2: Compare $\widetilde{b}_2(-)$ and the value of the first row for $\widetilde{x}_G^{-,l,u}$ $(\triangleright)$ and $\widetilde{x}_G^{+,l,u}$ $(\square)$.

and Allahviranloo's solutions [34] are given by

$$X_{\text{sym}}^{-,l}(r)=\left[\left[4-\frac{1-r}{11},4+\frac{1-r}{11}\right],\left[5-\frac{1-r}{11},5+\frac{1-r}{11}\right]\right]^t,$$
$$X_{\text{sym}}^{-,u}(r)=\left[\left[4-\frac{1-r}{10},4+\frac{1-r}{10}\right],\left[5-\frac{1-r}{10},5+\frac{1-r}{10}\right]\right]^t,$$
$$X_{\text{sym}}^{+,l}(r)=\left[\left[4-\frac{3-3r}{14},4+\frac{3-3r}{14}\right],\left[5-\frac{3-3r}{14},5+\frac{3-3r}{14}\right]\right]^t,$$
$$X_{\text{sym}}^{+,u}(r)=\left[\left[4-\frac{3-3r}{11},4+\frac{3-3r}{11}\right],\left[5-\frac{3-3r}{11},5+\frac{3-3r}{11}\right]\right]^t, \tag{54}$$

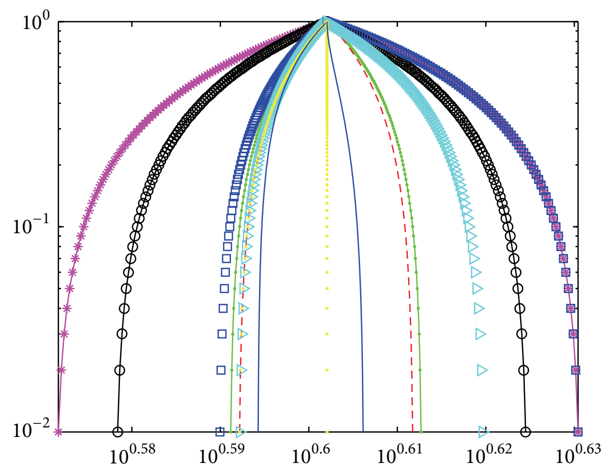

FIGURE 3: Compare the proposed solution $\widetilde{x}_1^{-,l,u}(\triangleright)$, $\widetilde{x}_1^{+,l,u}(\square)$ with $(\widetilde{x}_1)_D(\bullet\bullet)$, $(\widetilde{x}_1^{-,l})_A(—)$, $(\widetilde{x}_1^{-,u})_A(\cdot-)$, $(\widetilde{x}_1^{+,l})_A(-\text{o})$, $(\widetilde{x}_1^{+,u})_A(-*)$ and $(\widetilde{x}_1)_A(-)$.

and also the proposed solution in [30] has been obtained as follows:

$$\widetilde{X}_A(r)=\left[\left[\frac{-5r^2-28r-55}{-r^2-7r-14},\frac{-3r^3+4r^2+189r-630}{-r^3+3r^2+44r-156}\right],\right.$$
$$\left.\left[\frac{-5r^2-37r-68}{-r^2-7r-14},\frac{7r^2+22r-139}{r^2+3r-26}\right]\right]^t. \tag{55}$$

Example 18. Consider the following fully fuzzy linear system:

$$\widetilde{A}=\begin{pmatrix}(1+2r,5-2r) & (1+r,2)\\ (1,3-2r) & (3+r,5-r)\end{pmatrix},$$
$$\widetilde{b}=\begin{pmatrix}(5+3r,10-2r)\\ (2+4r,12-6r)\end{pmatrix}. \tag{56}$$

The 1-cut solution of system is $\widetilde{X}=(x_1,\ x_2)^t=(2,1)^t$. So, we have

$$\alpha_1(r)=\frac{2-2r}{-2-3r},$$
$$\beta_1(r)=\frac{2-2r}{-7+2r},$$
$$\alpha_2(r)=\frac{3-3r}{4+r},$$
$$\beta_2(r)=\frac{1-r}{8-3r}. \tag{57}$$

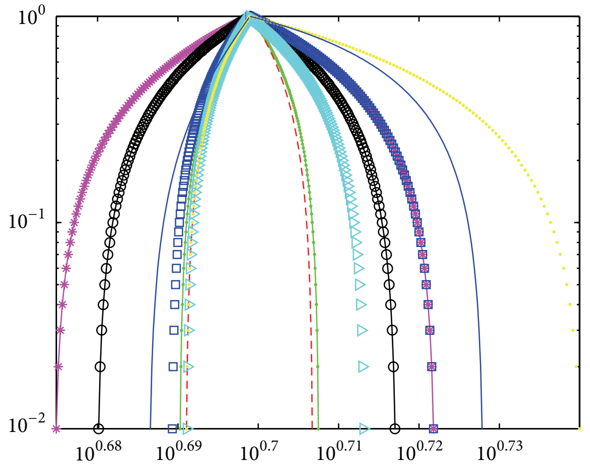

FIGURE 4: Compare the proposed solution $\tilde{x}_2^{-,l,u}(\triangleright)$, $\tilde{x}_2^{+,l,u}(\square)$ with $(\tilde{x}_2)_D(\bullet\bullet)$, $(\tilde{x}_2^{-,l})_A(—)$, $(\tilde{x}_2^{-,u})_A(\cdot-)$, $(\tilde{x}_2^{+,l})_A(-\mathrm{o})$, $(\tilde{x}_2^{+,u})_A(-*)$ and $(\tilde{x}_2)_A(-)$.

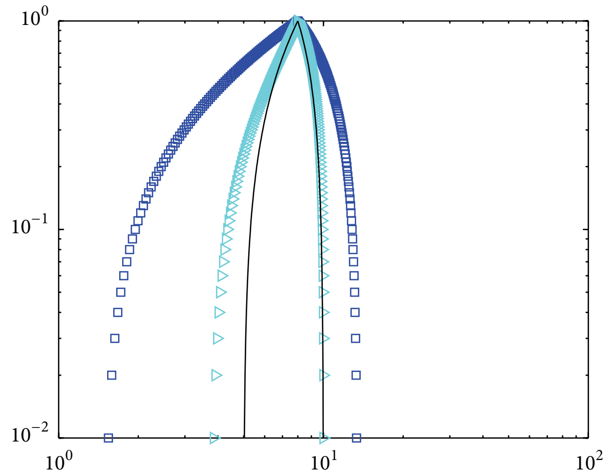

FIGURE 5: Compare $\tilde{b}_1(-)$ and the value of the first row for $\tilde{x}_G^{-,l,u}(\triangleright)$ and $\tilde{x}_G^{+,l,u}(\square)$.

Then, asymmetric linear spreads are derived as the following:

$$\alpha_G^{-,l,u}(r) = \frac{2-2r}{5},$$
$$\beta_G^{-,l,u}(r) = \frac{1-r}{8},$$
$$\alpha_G^{+,l,u}(r) = \frac{2-2r}{2}, \tag{58}$$
$$\beta_G^{+,l,u}(r) = \frac{2-2r}{5}.$$

Using asymmetric spreads, we obtain

$$\widetilde{X}_G^{-,l,u}(r) = \left(\left[2-\frac{2-2r}{5}, 2+\frac{1-r}{8}\right], \left[1-\frac{2-2r}{5}, 1+\frac{1-r}{8}\right]\right)^t, \tag{59}$$

$$\widetilde{X}_G^{+,l,u}(r) = \left(\left[2-\frac{2-2r}{2}, 2+\frac{2-2r}{5}\right], \left[1-\frac{2-2r}{2}, 1+\frac{3-3r}{11}\right]\right)^t. \tag{60}$$

We depict all the solutions via Figures 5 and 6.

5. Conclusion

In this paper, we presented a practicl method for determining the general solutions of a fully fuzzy linear system. To do so, we firstly solved the system in 1-cut form, then we fuzzify 1-cut solution of the FFLS by devoting general spreads. Therefore, the crisp system was changed into a new system that we should have obtained its spreads.

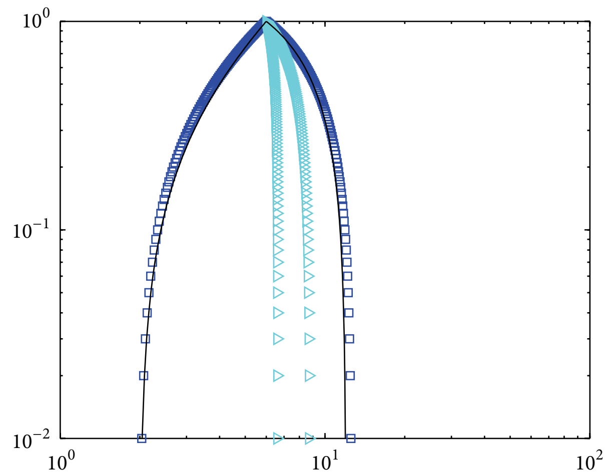

FIGURE 6: Compare $\tilde{b}_2(-)$ and the value of the first row for $\tilde{x}_G^{-,l,u}(\triangleright)$ and $\tilde{x}_G^{+,l,u}(\square)$.

Moreover, we have discussed the obtained result which was placed in the TSS and CSS. Furthermore, we have established that, under certain conditions, proposed method has less errors than the previously reported symmetric solutions. This method is a new approach to find the general solutions of the fully fuzzy linear systems. Also, the presented method always gives a fuzzy vector solution.

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References

[1] M. Friedman, M. Ming, and A. Kandel, "Fuzzy linear systems," *Fuzzy Sets and Systems*, vol. 96, no. 2, pp. 201–209, 1998.

[2] M. Ma, M. Friedman, and A. Kandel, "Duality in fuzzy linear systems," *Fuzzy Sets and Systems*, vol. 109, no. 1, pp. 55–58, 2000.

[3] T. Allahviranloo, "Numerical methods for fuzzy system of linear equations," *Applied Mathematics and Computation*, vol. 155, no. 2, pp. 493–502, 2004.

[4] T. Allahviranloo, "Successive over relaxation iterative method for fuzzy system of linear equations," *Applied Mathematics and Computation*, vol. 162, no. 1, pp. 189–196, 2005.

[5] T. Allahviranloo, "The Adomian decomposition method for fuzzy system of linear equations," *Applied Mathematics and Computation*, vol. 163, no. 2, pp. 553–563, 2005.

[6] T. Allahviranloo and M. Afshar Kermani, "Solution of a fuzzy system of linear equation," *Applied Mathematics and Computation*, vol. 175, no. 1, pp. 519–531, 2006.

[7] T. Allahviranloo, E. Ahmady, N. Ahmady, and Kh. S. Alketaby, "Block Jacobi two-stage method with Gauss-Seidel inner iterations for fuzzy system of linear equations," *Applied Mathematics and Computation*, vol. 175, no. 2, pp. 1217–1228, 2006.

[8] T. Allahviranloo, "A comment on fuzzy linear systems," *Fuzzy Sets and Systems*, vol. 140, no. 3, p. 559, 2003.

[9] S. Abbasbandy, R. Ezzati, and A. Jafarian, "LU decomposition method for solving fuzzy system of linear equations," *Applied Mathematics and Computation*, vol. 172, no. 1, pp. 633–643, 2006.

[10] S. Abbasbandy and A. Jafarian, "Steepest descent method for system of fuzzy linear equations," *Applied Mathematics and Computation*, vol. 175, no. 1, pp. 823–833, 2006.

[11] T. Allahviranloo and M. Ghanbari, "On the algebraic solution of fuzzy linear systems based on interval theory," *Applied Mathematical Modelling*, vol. 36, no. 11, pp. 5360–5379, 2012.

[12] E. Amrahov and I. N. Askerzade, "Strong solutions of the fuzzy linear systems," *Computer Modeling in Engineering & Sciences*, vol. 76, no. 3-4, pp. 207–216, 2011.

[13] B. Asady, S. Abbasbandy, and M. Alavi, "Fuzzy general linear systems," *Applied Mathematics and Computation*, vol. 169, no. 1, pp. 34–40, 2005.

[14] R. B. Kearfott, *Rigorous Global Search: Continuous Problems*, vol. 13 of *Nonconvex Optimization and Its Applications*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1996.

[15] R. Ezzati, "Solving fuzzy linear systems," *Soft Comput*, vol. 15, pp. 193–197, 2011.

[16] N. Gasilov, E. Amrahov, A. G. Fatullayev, H. Karakaş, and O. Akın, "A geometric approach to solve fuzzy linear systems," *Computer Modeling in Engineering & Sciences*, vol. 75, no. 3-4, pp. 189–203, 2011.

[17] A. Ghomashi, S. Salahshour, and H. Hakimzadeh, "Approximating solutions of fully fuzzy linear systems: a financial case study," *Journal of Intelligent and Fuzzy Systems*. In press.

[18] A. Kumar, N. Babbar, and A. Bansal, "A new computational method to solve fully fuzzy linear systems for negative coefficient matrix," *International Journal of Manufacturing Technology and Management*, vol. 25, pp. 19–32, 2012.

[19] N. Kumar and A. Bansal, "A new approach for solving fully fuzzy linear systems," *Advances in Fuzzy Systems*, vol. 2011, Article ID 943161, 8 pages, 2011.

[20] K. Wang and B. Zheng, "Inconsistent fuzzy linear systems," *Applied Mathematics and Computation*, vol. 181, no. 2, pp. 973–981, 2006.

[21] X. Wang, Z. Zhong, and M. Ha, "Iteration algorithms for solving a system of fuzzy linear equations," *Fuzzy Sets and Systems*, vol. 119, no. 1, pp. 121–128, 2001.

[22] B. Zheng and K. Wang, "General fuzzy linear systems," *Applied Mathematics and Computation*, vol. 181, no. 2, pp. 1276–1286, 2006.

[23] J. J. Buckley and Y. Qu, "Solving systems of linear fuzzy equations," *Fuzzy Sets and Systems*, vol. 43, no. 1, pp. 33–43, 1991.

[24] J. J. Buckley and Y. Qu, "Solving linear and quadratic fuzzy equations," *Fuzzy Sets and Systems*, vol. 38, no. 1, pp. 43–59, 1990.

[25] J. J. Buckley and Y. Qu, "Solving fuzzy equations: a new solution concept," *Fuzzy Sets and Systems*, vol. 39, no. 3, pp. 291–301, 1991.

[26] S. Muzzioli and H. Reynaerts, "Fuzzy linear systems of the form $A_1x + b_1 = A_2x + b_2$," *Fuzzy Sets and Systems*, vol. 157, no. 7, pp. 939–951, 2006.

[27] M. Dehghan and B. Hashemi, "Iterative solution of fuzzy linear systems," *Applied Mathematics and Computation*, vol. 175, no. 1, pp. 645–674, 2006.

[28] M. Dehghan, B. Hashemi, and M. Ghatee, "Computational methods for solving fully fuzzy linear systems," *Applied Mathematics and Computation*, vol. 179, no. 1, pp. 328–343, 2006.

[29] M. Dehghan, B. Hashemi, and M. Ghatee, "Solution of the fully fuzzy linear systems using iterative techniques," *Chaos, Solitons & Fractals*, vol. 34, no. 2, pp. 316–336, 2007.

[30] T. Allahviranloo and N. Mikaeilvand, "Non zero solutions of the fully fuzzy linear systems," *Applied and Computational Mathematics*, vol. 10, no. 2, pp. 271–282, 2011.

[31] A. Vroman, G. Deschrijver, and E. E. Kerre, "A solution for systems of linear fuzzy equations in spite of the non-existence of a field of fuzzy numbers," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 13, no. 3, pp. 321–335, 2005.

[32] A. Vroman, G. Deschrijver, and E. E. Kerre, "Solving systems of linear fuzzy equations by parametric functions—an improved algorithm," *Fuzzy Sets and Systems*, vol. 158, no. 14, pp. 1515–1534, 2007.

[33] A. Vroman, G. Deschrijver, and E. E. Kerre, "Solving systems of linear fuzzy equations by parametric functions," *IEEE Transactions on Fuzzy Systems*, vol. 15, pp. 370–384, 2007.

[34] T. Allahviranloo, S. Salahshour, and M. Khezerloo, "Maximal- and minimal symmetric solutions of fully fuzzy linear systems," *Journal of Computational and Applied Mathematics*, vol. 235, no. 16, pp. 4652–4662, 2011.

[35] D. Dubois and H. Prade, *Fuzzy Sets and Systems: Theory and Applications*, vol. 144 of *Mathematics in Science and Engineering*, Academic Press, New York, NY, USA, 1980.

[36] R. Goetschel, Jr. and W. Voxman, "Elementary fuzzy calculus," *Fuzzy Sets and Systems*, vol. 18, no. 1, pp. 31–43, 1986.

[37] M. L. Puri and D. A. Ralescu, "Fuzzy random variables," *Journal of Mathematical Analysis and Applications*, vol. 114, no. 2, pp. 409–422, 1986.

[38] H. J. Zimmermann, *Fuzzy Sets Theory and Applications*, Kluwer, Dorrecht, The Netherlands, 1985.

[39] T. Allahviranloo and S. Salahshour, "Fuzzy symmetric solutions of fuzzy linear systems," *Journal of Computational and Applied Mathematics*, vol. 235, no. 16, pp. 4545–4553, 2011.


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