Hindawi Publishing Corporation
Advances in Difference Equations
Volume 2010, Article ID 954684, 15 pages
doi:10.1155/2010/954684

Research Article

Dynamical Analysis of a Delayed Predator-Prey System with Birth Pulse and Impulsive Harvesting at Different Moments

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Received 21 August 2010; Accepted 22 September 2010

Academic Editor: Kanishka Perera

We consider a delayed Holling type II predator-prey system with birth pulse and impulsive harvesting on predator population at different moments. Firstly, we prove that all solutions of the investigated system are uniformly ultimately bounded. Secondly, the conditions of the globally attractive prey-extinction boundary periodic solution of the investigated system are obtained. Finally, the permanence of the investigated system is also obtained. Our results provide reliable tactic basis for the practical biological economics management.

1. Introduction

Theories of impulsive differential equations have been introduced into population dynamics lately [1, 2]. Impulsive equations are found in almost every domain of applied science and have been studied in many investigation [3–11], they generally describe phenomena which are subject to steep or instantaneous changes. In [11], Jiao et al. suggested releasing pesticides is combined with transmitting infective pests into an SI model. This may be accomplished using selecting pesticides and timing the application to avoid periods when the infective pesticides would be exposed or placing the pesticides in a location where the transmitting infective pests would not contact it. So an impulsive differential equation to model the process of releasing infective pests and spraying pesticides at different fixed moment was represented as

$$\left.\begin{aligned}\frac{dS(t)}{dt}&=rS(t)\left(1-\frac{S(t)+\theta I(t)}{K}\right)-\beta S(t)I(t),\\ \frac{dI(t)}{dt}&=\beta S(t)I(t)-I(t),\end{aligned}\right\}\quad t\neq(n-1+l)\tau,\ t\neq n\tau,$$

$$
\begin{aligned}
&\left.\begin{aligned}\Delta S(t) &= -\mu_1 S(t),\\ \Delta I(t) &= -\mu_2 I(t),\end{aligned}\right\} \quad t = (n-1+l)\tau,\ n = 1,2,\ldots,\\
&\left.\begin{aligned}\Delta S(t) &= 0,\\ \Delta I(t) &= \mu,\end{aligned}\right\} \quad t = n\tau,\ n = 1,2,\ldots.
\end{aligned}
\tag{1.1}
$$

The biological meaning of the parameters in System (1.1) can refer to Literature [11].

Clack [12] has studied the optimal harvesting of the logistic equation, a logistic equation without exploitation as follows:

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right), \tag{1.2}$$

where $x(t)$ represents the density of the resource population at time t, r is the intrinsic growth rate. the positive constant K is usually referred as the environmental carrying capacity or saturation level. Suppose that the population described by logistic equation (1.1) is subject to harvesting at rate $h(t)$ = constant or under the catch-per-unit effort hypothesis $h(t) = Ex(t)$. Then the equations of the harvested population revise, respectively, as following

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - h, \tag{1.3}$$

or

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - Ex(t), \tag{1.4}$$

where E denotes the harvesting effort.

Moreover, in most models of population dynamics, increase in population due to birth are assumed to be time dependent, but many species reproduce only during a period of the year. In between these pulses of growth, mortality takes its toll, and the population decreases. In this paper, we suggest impulsive differential equations to model the process of periodic birth pulse and impulsive harvesting. Combining (1.2) and (1.4), we can obtain a single population model with birth pulse and impulsive harvesting at different moments

$$
\begin{aligned}
\frac{dx(t)}{dt} &= -dx(t), \quad t \neq (n+l)\tau,\ t \neq (n+1)\tau,\\
\Delta x(t) &= x(t)(a - bx(t)), \quad t = (n+l)\tau,\\
\Delta x(t) &= -\mu x(t), \quad t = (n+1)\tau,\ n \in Z^+,
\end{aligned}
\tag{1.5}
$$

where $x(t)$ is the density of the population. d is the death rate. The population is birth pulse as intrinsic rate of natural increase and density dependence rate of predator population are denoted by a, b, respectively. The pulse birth and impulsive harvesting occurs every τ period

(τ is a positive constant). $\Delta x(t) = x(t^+) - x(t)$. $x(t)(a - bx(t))$ represents the birth effort of predator population at $t = (n + l)\tau$, $0 < l < 1$, $n \in Z_+$. $0 \le \mu \le 1$ represents the harvesting effort of predator population at $t = (n + 1)\tau$, $n \in Z_+$.

But in the natural world, there are many species (especially insects) whose individual members have a life history that takes them through two stages, immature and mature. In [13], a stage-structured model of population growth consisting of immature and mature individuals was analyzed, where the stage-structured was modeled by introduction of a constant time delay. Other models of population growth with time delays were considered in [3, 5–7, 13]. The following single- species stage-structured model was introduced by Aiello and Freedman [14] as follows:

$$\begin{aligned} x'(t) &= \beta y(t) - r x(t) - \beta e^{-r\tau} y(t-\tau), \\ y'(t) &= \beta e^{-r\tau} y(t-\tau) - \eta_2 y^2(t), \end{aligned} \tag{1.6}$$

where $x(t), y(t)$ represent the immature and mature populations densities, respectively, τ represents a constant time to maturity, and β, r and η_2 are positive constants. This model is derived as follows. We assume that at any time $t > 0$, birth into the immature population is proportional to the existing mature population with proportionality constant β. We then assume that the death rate of immature population is proportional to the existing immature population with proportionality constant r. We also assume that the death rate of mature population is of a logistic nature, that is, proportional to the square of the population with proportionality constant η_2. In this paper, we consider a delayed Holling type II predator-prey system with birth pulse and impulsive harvesting on predator population at different moments.

The organization of this paper is as follows. In the next section, we introduce the model. In Section 3, some important lemmas are presented. In Section 4, we give the globally asymptotically stable conditions of prey-extinction periodic solution of System (2.1), and the permanent condition of System (2.1). In Section 5, a brief discussion is given in the last section to conclude this paper.

2. The Model

In this paper, we consider a delayed Holling type II predator-prey model with birth pulse and impulsive harvesting on predator population at different moments

$$\begin{aligned}
&\frac{dx_1(t)}{dt} = r x_2(t) - r e^{-w\tau_1} x_2(t-\tau_1) - w x_1(t), \\
&\frac{dx_2(t)}{dt} = r e^{-w\tau_1} x_2(t-\tau_1) - \frac{\beta x_2(t)}{m + x_2(t)} y(t) - d_1 x_2(t), \qquad t \ne (n+l)\tau,\ t \ne (n+1)\tau, \\
&\frac{dy(t)}{dt} = \frac{k\beta x_2(t)}{m + x_2(t)} y(t) - d_2 y(t), \\
&\Delta x_1(t) = 0, \\
&\Delta x_2(t) = 0, \qquad\qquad t = (n+l)\tau,\ n = 1, 2 \ldots, \\
&\Delta y(t) = y(t)\big(a - b y(t)\big),
\end{aligned}$$

$$\begin{aligned}
&\Delta x_1(t) = 0,\\
&\Delta x_2(t) = 0, \qquad t = (n+1)\tau,\ n = 1,2\ldots,\\
&\Delta y(t) = -\mu y(t),
\end{aligned} \tag{2.1}$$

the initial conditions for (2.1) are

$$(\varphi_1(\zeta), \varphi_2(\zeta), \varphi_3(\zeta)) \in C_+ = C\left([-\tau_1, 0], R_+^3\right), \quad \varphi_i(0) > 0, \quad i = 1,2,3, \tag{2.2}$$

where $x_1(t), x_2(t)$ represent the densities of the immature and mature prey populations, respectively. $y(t)$ represents the density of predator population. $r > 0$ is the intrinsic growth rate of prey population. τ_1 represents a constant time to maturity. w is the natural death rate of the immature prey population. d_1 is the natural death rate of the mature prey population. d_2 is the natural death rate of the predator population. The predator population consumes prey population following a Holling type-II functional response with predation coefficients β, and half-saturation constant m. k is the rate of conversion of nutrients into the reproduction rate of the predators. The predator population is birth pulse as intrinsic rate of natural increase and density dependence rate of predator population are denoted by a, b, respectively. The pulse birth and impulsive harvesting occurs every τ period (τ is a positive constant). $\Delta y(t) = y(t^+) - y(t)$. $y(t)(a - by(t))$ represents the birth effort of predator population at $t = (n+l)\tau, 0 < l < 1, n \in Z_+$. $0 \leq \mu \leq 1$ represents the harvesting effort of predator population at $t = (n+1)\tau, n \in Z_+$. In this paper, we always assume that $\tau < (1/d)\ln(1+a)$.

Before going into any details, we simplify model (2.1) and restrict our attention to the following model:

$$\begin{aligned}
&\left.\begin{aligned}
&\frac{dx_2(t)}{dt} = re^{-w\tau_1}x_2(t-\tau_1) - \frac{\beta x_2(t)}{m + x_2(t)}y(t) - d_1x_2(t),\\
&\frac{dy(t)}{dt} = \frac{k\beta x_2(t)}{m + x_2(t)}y(t) - d_2y(t),
\end{aligned}\right\} \quad t \neq (n+l)\tau,\ t \neq (n+1)\tau,\\
&\left.\begin{aligned}
&\Delta x_2(t) = 0,\\
&\Delta y(t) = y(t)(a - by(t)),
\end{aligned}\right\} \quad t = (n+l)\tau,\ n = 1,2,\ldots,\\
&\left.\begin{aligned}
&\Delta x_2(t) = 0,\\
&\Delta y(t) = -\mu y(t),
\end{aligned}\right\} \quad t = (n+1)\tau,\ n = 1,2,\ldots,
\end{aligned} \tag{2.3}$$

the initial conditions for (2.3) are

$$(\varphi_2(\zeta), \varphi_3(\zeta)) \in C_+ = C\left([-\tau_1, 0], R_+^2\right), \quad \varphi_i(0) > 0, \quad i = 2,3. \tag{2.4}$$

3. The Lemma

Before discussing main results, we will give some definitions, notations and lemmas. Let $R_+ = [0, \infty)$, $R_+^3 = \{x \in R^3 : x > 0\}$. Denote $f = (f_1, f_2, f_3)$ the map defined by the right hand of system (2.1). Let $V : R_+ \times R_+^3 \to R_+$, then V is said to belong to class V_0, if

(i) V is continuous in $(n\tau, (n+l)\tau] \times R_+^3$ and $((n+l)\tau, (n+1)\tau] \times R_+^3$, for each $x \in R_+^3$, $n \in Z_+$, $\lim_{(t,y)\to((n+l)\tau^+,x)} V(t,y) = V((n+l)\tau^+, x)$ and $\lim_{(t,y)\to((n+1)\tau^+,x)} V(t,y) = V((n+1)\tau^+, x)$ exist.

(ii) V is locally *Lipschitzian* in x.

Definition 3.1. $V \in V_0$, then for $(t,z) \in (n\tau, (n+l)\tau] \times R_+^3$ and $((n+l)\tau, (n+1)\tau] \times R_+^3$, the upper right derivative of $V(t,z)$ with respect to the impulsive differential system (2.1) is defined as

$$D^+V(t,z) = \lim_{h\to 0} \sup \frac{1}{h}\left[V(t+h, z+hf(t,z)) - V(t,z)\right]. \tag{3.1}$$

The solution of (2.1), denote by $z(t) = (x(t), y(t))^T$, is a piecewise continuous function $x{:}R_+ \to R_+^3$, $z(t)$ is continuous on $(n\tau, (n+l)\tau] \times R_+^3$ and $((n+l)\tau,\ (n+1)\tau] \times R_+^3 (n \in Z_+,\ 0 \le l \le 1)$. Obviously, the global existence and uniqueness of solutions of (2.1) is guaranteed by the smoothness properties of f, which denotes the mapping defined by right-side of system (2.1) Lakshmikantham et al. [1]. Before we have the the main results. we need give some lemmas which will be used as follows.

Now, we show that all solutions of (2.1) are uniformly ultimately bounded.

Lemma 3.2. *There exists a constant* $M > 0$ *such that* $x_1(t) \le M/k$, $x_2(t) \le M/k$, $y(t) \le M$ *for each solution* $(x_1(t), x_2(t), y(t))$ *of* (2.1) *with all t large enough.*

Proof. Define $V(t) = kx_1(t) + kx_2(t) + y(t)$.

(i) If $d_1 > r$, then $d = \min\{d_1, d_2, d_1 - r\}$, when $t \ne n\tau$, we have

$$D^+V(t) + dV(t) = -k(d_1 - r - d)x_1(t) - k(d_2 - d)x_2(t) - (d_2 - d)y(t) \triangleq \xi \le 0. \tag{3.2}$$

When $t = (n+l-1)\tau$,

$$\begin{aligned} V((n+l)\tau^+) &= kx((n+l)\tau) + y((n+l)\tau) + y((n+l)\tau)(a - by((n+l)\tau)) \\ &= V((n+l)\tau) - b\Big(y((n+l)\tau) - \frac{a}{2b}\Big)^2 + \frac{a^2}{4b} \\ &\le V((n+l)\tau) + \frac{a^2}{4b}. \end{aligned} \tag{3.3}$$

For convenience, we make a notation as $\xi_1 \triangleq a^2/4b$. When $t = n\tau$,

$$V((n+1)\tau^+) = kx((n+1)\tau) + (1-\mu)y((n+1)\tau) \le V((n+1)\tau). \tag{3.4}$$

From [17, Lemma 2.2, Page 23] for $t \in ((n-1)\tau, (n+l-1)\tau]$ and $((n+l-1)\tau, n\tau]$, we have

$$V(t) \le V(0^+)e^{-dt} + \frac{\xi}{d}\left(1-e^{-dt}\right) + \xi_1 \frac{e^{-d(t-\tau)}}{1-e^{-d\tau}} + \xi_1 \frac{e^{d\tau}}{e^{d\tau}-1} \longrightarrow \frac{\xi}{d} + \xi_1 \frac{e^{d\tau}}{e^{d\tau}-1}, \quad \text{as } t \longrightarrow \infty. \tag{3.5}$$

(ii) If $d_1 < r$, then $d = 0$, we can easily obtain

$$V(t) \le V(0^+), \quad \text{as } t \longrightarrow \infty. \tag{3.6}$$

So $V(t)$ is uniformly ultimately bounded. Hence, by the definition of $V(t)$, there exists a constant $M > 0$ such that $x(t) \le M/k$, $y(t) \le M$ for t large enough. The proof is complete. □

If $x(t) = 0$, we have the following subsystem of System (2.1):

$$\begin{aligned} \frac{dy(t)}{dt} &= -d_2 y(t), \quad t \ne (n+l)\tau, \ t \ne (n+1)\tau, \\ \Delta y(t) &= y(t)\left(a - by(t)\right), \quad t = (n+l)\tau, \\ \Delta y(t) &= -\mu y(t), \quad t = (n+1)\tau, \ n \in Z^+. \end{aligned} \tag{3.7}$$

We can easily obtain the analytic solution of System (3.7) between pulses, that is,

$$y(t) = \begin{cases} y(n\tau^+)e^{-d_2(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ \left[(1+a)e^{-d_2 l\tau} y(n\tau^+) + be^{-2d_2 l\tau} y^2(n\tau^+)\right] e^{-d_2(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau). \end{cases} \tag{3.8}$$

Considering the last two equations of system (3.7), we have the stroboscopic map of System (3.7) as follows:

$$y((n+1)\tau^+) = (1-\mu)(1+a)e^{-d_2\tau} y(n\tau^+) - (1-\mu)be^{-d_2(1+l)\tau} y^2(n\tau^+). \tag{3.9}$$

The are two fixed points of (3.9) are obtained as $G_1(0)$ and $G_2(y^*)$, where

$$y^* = \frac{1+a}{b}e^{d_2 l\tau} - \frac{1}{(1-\mu)b}e^{d_2(1+l)\tau} \quad \text{with } \mu < 1 - \frac{1}{1+a}e^{d_2\tau}. \tag{3.10}$$

Lemma 3.3. (i) *If $\mu > 1 - (1/(1+a))e^{d_2\tau}$, the fixed point $G_1(0)$ is globally asymptotically stable;* (ii) *if $\mu < 1 - (1/(1+a))e^{d_2\tau}$, the fixed point $G_2(y^*)$ is globally asymptotically stable.*

Proof. For convenience, make notation $y_n = y(n\tau^+)$, then Difference equation (3.9) can be rewritten as

$$y_{n+1} = F(y_n). \tag{3.11}$$

(i) If $\mu > 1-(1/(1+a))e^{d_2\tau}$, $G_1(0)$ is the unique fixed point, we have

$$\left.\frac{dF(y)}{dy}\right|_{y=0} = (1-\mu)(1+a)e^{-d_2\tau} < 1, \tag{3.12}$$

then $G_1(0)$ is globally asymptotically stable.

(ii) If $\mu < 1-(1/(1+a))e^{d_2\tau}$, $G_1(0)$ is unstable. For

$$\left.\frac{dF(y)}{dy}\right|_{y=y^*} = -(1-\mu)(1+a)e^{-d_2\tau} + 2 < 1, \tag{3.13}$$

then $G_1(y^*)$ is globally asymptotically stable. This complete the proof.

□

It is well known that the following lemma can easily be proved.

Lemma 3.4. (i) *If $\mu > 1-(1/(1+a))e^{d_2\tau}$, the triviality periodic solution of System* (3.7) *is globally asymptotically stable;*

(ii) *if $\mu < 1-(1/(1+a))e^{d_2\tau}$, the periodic solution of System* (3.7)

$$\widetilde{y(t)} = \begin{cases} y^* e^{-d_2(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ \left[(1+a)e^{-d_2 l\tau}y^* + be^{-2d_2 l\tau}(y^*)^2\right]e^{-d_2(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau) \end{cases} \tag{3.14}$$

is globally asymptotically stable. Here,

$$y^* = \frac{1+a}{b}e^{d_2 l\tau} - \frac{1}{(1-\mu)b}e^{d_2(1+l)\tau}. \tag{3.15}$$

Lemma 3.5 (see [22]). *Consider the following delay equation:*

$$x'(t) = a_1 x(t-\tau) - a_2 x(t) = 0, \tag{3.16}$$

one assumes that $a_1, a_2, \tau > 0$; $x(t) > 0$ for $-\tau \le t \le 0$. Assume that $a_1 < a_2$. Then

$$\lim_{t\to\infty} x(t) = 0. \tag{3.17}$$

4. The Dynamics

In this section, we will firstly obtain the sufficient condition of the global attractivity of prey-extinction periodic solution of System (2.1) with (2.2).

Theorem 4.1. *If*

$$\mu < 1 - \frac{1}{1+a} e^{d_2 \tau}, \tag{4.1}$$

$$re^{-w\tau_1} < \frac{k\beta}{km+M} \left\{ \left[e^{-d_2 l\tau} + (1+a) e^{-d_2 \tau} \right] y^* + b e^{-d_2 (1+l)\tau} \left(y^* \right)^2 \right\} + d_1 \tag{4.2}$$

hold, the prey-extinction solution $(0, 0, \widetilde{y(t)})$ *of System* (2.1) *with* (2.2) *is globally attractive*

$$y^* = \frac{1+a}{b} e^{d_2 l\tau} - \frac{1}{(1-\mu) b} e^{d_2 (1+l)\tau}. \tag{4.3}$$

Proof. It is clear that the global attraction of prey-extinction periodic solution $(0, 0, \widetilde{y(t)})$ of System (2.1) with (2.2) is equivalent to the global attraction of prey-extinction periodic solution $(0, \widetilde{y(t)})$ of System (2.3). So we only devote to System (2.3) with (2.4). Since

$$re^{-w\tau_1} < \frac{k\beta}{km+M} \left\{ \left[e^{-d_2 l\tau} + (1+a) e^{-d_2 \tau} \right] y^* + b e^{-d_2 (1+l)\tau} \left(y^* \right)^2 \right\} + d_1, \tag{4.4}$$

we can choose ε_0 sufficiently small such that

$$re^{-w\tau_1} < \frac{k\beta}{km+M} \left\{ \left[e^{-d_2 l\tau} + (1+a) e^{-d_2 \tau} \right] y^* + b e^{-d_2 (1+l)\tau} \left(y^* \right)^2 - \varepsilon_0 \right\} + d_1. \tag{4.5}$$

It follows from that the second equation of System (2.3) with (2.4) that $dy(t)/dt \geq -d_2 y(t)$. So we consider the following comparison impulsive differential system:

$$\begin{aligned} \frac{dx(t)}{dt} &= -d_2 x(t), \quad t \neq (n+l)\tau, \ t \neq (n+1)\tau, \\ \Delta x(t) &= x(t)(a - bx(t)), \quad t = (n+l)\tau, \\ \Delta x(t) &= -\mu x(t), \quad t = (n+1)\tau. \end{aligned} \tag{4.6}$$

In view of Condition (4.1) and Lemma 3.4, we obtain that the periodic solution of System (4.6)

$$\widetilde{x(t)} = \begin{cases} x^* e^{-d_2 (t - n\tau)}, & t \in [n\tau, (n+l)\tau), \\ \left[(1+a) e^{-d_2 l\tau} x^* + b e^{-2 d_2 l\tau} (x^*)^2 \right] e^{-d_2 (t - (n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \tag{4.7}$$

is globally asymptotically stable. Here,

$$x^* = \frac{1+a}{b} e^{d_2 l\tau} - \frac{1}{(1-\mu) b} e^{d_2 (1+l)\tau}. \tag{4.8}$$

By the comparison theorem of impulsive equation (see [13, Theorem 3.1.1]), we have $y(t) \geq x(t)$ and $x(t) \to \widetilde{x(t)} = \widetilde{y(t)}$ as $t \to \infty$. Then there exists an integer $k_2 > k_1$, $t > k_2$ such that

$$y(t) \geq x(t) \geq \widetilde{y(t)} - \varepsilon_0, \quad n\tau < t \leq (n+1)\tau, \quad n > k_2, \tag{4.9}$$

that is

$$\begin{aligned} y(t) > \widetilde{y(t)} - \varepsilon_0 \geq \left\{ \left[e^{-d_2 l\tau} + (1+a)e^{-d_2\tau} \right] y^* + be^{-d_2(1+l)\tau} \left(y^*\right)^2 \right\} - \varepsilon_0 \triangleq \varrho, \\ n\tau < t \leq (n+1)\tau, \quad n > k_2. \end{aligned} \tag{4.10}$$

From (2.3), we get

$$\frac{dx_2(t)}{dt} \leq re^{-w\tau_1} x_2(t-\tau_1) - \left(\frac{k\beta\varrho}{km+M} + d_1 \right) x_2(t), \quad t > n\tau + \tau_1, \quad n > k_2. \tag{4.11}$$

Consider the following comparison differential system:

$$\frac{dz(t)}{dt} = re^{-w\tau_1} z(t-\tau_1) - \left(\frac{k\beta\varrho}{km+M} + d_1 \right) z(t), \quad t > n\tau + \tau_1, \quad n > k_2, \tag{4.12}$$

from (4.5), we have $re^{-w\tau_1} < (k\beta\varrho/(km+M) + d_1)$. According to Lemma 3.5, we have $\lim_{t\to\infty} z(t) = 0$.

Let $(x_2(t), y(t))$ be the solution of system (2.3) with initial conditions (2.4) and $x_2(\zeta) = \varphi_2(\zeta)$ $(\zeta \in [-\tau_1, 0])$, $y(t)$ is the solution of system (4.12) with initial conditions $z(\zeta) = \varphi_2(\zeta)(\zeta \in [-\tau_1, 0])$. By the comparison theorem, we have $\lim_{t\to\infty} x_2(t) < \lim_{t\to\infty} z(t) = 0$. Incorporating into the positivity of $x_2(t)$, we know that

$$\lim_{t\to\infty} x_2(t) = 0. \tag{4.13}$$

Therefore, for any $\varepsilon_1 > 0$ (sufficiently small), there exists an integer $k_3(k_3\tau > k_2\tau + \tau_1)$ such that $x_2(t) < \varepsilon_1$ for all $t > k_3\tau$.

For System (2.3), we have

$$-d_2 y(t) \leq \frac{dy(t)}{dt} \leq \left(-d_2 + \frac{k\beta\varepsilon_1}{m+\varepsilon_1} \right) y(t), \tag{4.14}$$

then we have $z_1(t) \leq y(t) \leq z_2(t)$ and $z_1(t) \to \widetilde{y(t)}$, $z_2(t) \to \widetilde{y(t)}$ as $t \to \infty$, while $z_1(t)$ and $z_2(t)$ are the solutions of

$$\begin{gathered}
\frac{dz_1(t)}{dt} = -d_2 z_1(t), \quad t \neq (n+l)\tau,\ t \neq (n+1)\tau, \\
\Delta z_1(t) = z_1(t)(a + b z_1(t)), \quad t = (n+l)\tau, \\
\Delta z_1(t) = -\mu z_1(t), \quad t = (n+1)\tau, \\
\frac{dz_2(t)}{dt} = \left(-d_2 + \frac{k\beta\varepsilon_1}{m+\varepsilon_1}\right) z_2(t), \quad t \neq (n+l)\tau,\ t \neq (n+1)\tau, \\
\Delta z_2(t) = z_2(t)(a + b z_2(t)), \quad t = (n+l)\tau, \\
\Delta z_2(t) = -\mu z_2(t), \quad t = (n+1)\tau,
\end{gathered} \tag{4.15}$$

respectively,

$$\widetilde{z_2(t)} = \begin{cases} z_2^* e^{(-d_2 + k\beta\varepsilon_1/(m+\varepsilon_1))(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ \left[(1+a) e^{(-d_2+k\beta\varepsilon_1/(m+\varepsilon_1))l\tau} z_2^* + b e^{2(-d_2+k\beta\varepsilon_1/(m+\varepsilon_1))l\tau} \left(z_2^*\right)^2\right] & \\ \quad \times e^{(-d_2+k\beta\varepsilon_1/(m+\varepsilon_1))(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \tag{4.16}$$

Here,

$$z_2^* = \frac{1+a}{b} e^{d_2 - (k\beta\varepsilon_1/(m+\varepsilon_1))l\tau} - \frac{1}{(1-\mu)b} e^{(d_2 - k\beta\varepsilon_1/(m+\varepsilon_1))(1+l)\tau}. \tag{4.17}$$

Therefore, for any $\varepsilon_2 > 0$. there exists a integer k_4, $n > k_4$ such that

$$\widetilde{y(t)} - \varepsilon_2 < y(t) < \widetilde{y(t)} + \varepsilon_2, \tag{4.18}$$

Let $\varepsilon_1 \to 0$, so we have

$$\widetilde{y(t)} - \varepsilon_2 < y(t) < \widetilde{y(t)} + \varepsilon_2, \tag{4.19}$$

for t large enough, which implies $y(t) \to \widetilde{y(t)}$ as $t \to \infty$. This completes the proof. □

The next work is to investigate the permanence of the system (2.4). Before starting our theorem, we give the definition of permanence of system (2.4).

Definition 4.2. System (2.1) is said to be permanent if there are constants $m, M > 0$ (independent of initial value) and a finite time T_0 such that for all solutions $(x_1(t), x_2(t), y(t))$ with all initial values $x_1(t) > 0$, $x_2(0^+) > 0$, $y(0^+) > 0$, $m \leq x_1(t) < M/k$, $x_2(t) \leq M/k$, $m \leq x_3(t) \leq M$ holds for all $t \geq T_0$. Here T_0 may depend on the initial values $(x_1(0^+), x_2(0^+), y(0^+))$.

Theorem 4.3. *If*

$$re^{-w\tau_1} > \frac{\beta}{m}\left\{\left[1+(1+a)e^{(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}\right]v^* + be^{2(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}(v^*)^2\right\} + d_1, \tag{4.20}$$

then there is a positive constant q such that each positive solution $(x_2(t), y(t))$ of (2.3) *with* (2.4) *satisfies*

$$x_2(t) \geq q, \tag{4.21}$$

for t large enough, where x_2^* is determined as the following equation:

$$\begin{aligned}
&\left[1+(1+a)e^{(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}\right] \times \left[\frac{1+a}{b}e^{d_2-(k\beta x_2^*/(m+x_2^*))l\tau} - \frac{1}{(1-\mu)b}e^{(d_2-k\beta x_2^*/(m+x_2^*))(1+l)\tau}\right] \\
&\quad + be^{2(-d_2+k\beta x_2^*/(m+x_2^*))l\tau} \times \left(\frac{1+a}{b}e^{d_2-(k\beta x_2^*/(m+x_2^*))l\tau} - \frac{1}{(1-\mu)b}e^{(d_2-k\beta x_2^*/(m+x_2^*))(1+l)\tau}\right)^2 \\
&= \frac{m}{\beta}\left(re^{-w\tau_1} - d_1\right).
\end{aligned} \tag{4.22}$$

Proof. The first equation of System (2.3) can be rewritten as

$$\frac{dx_2(t)}{dt} = \left(re^{-w\tau_1} - \frac{\beta y(t)}{m+x_2(t)} - d_1\right)x_2(t) - re^{-w\tau_1}\frac{d}{dt}\int_{t-\tau_1}^{t} x_2(u)du. \tag{4.23}$$

Let us consider any positive solution $(x_2(t), y(t))$ of System (2.3). According to(4.23), $V(t)$ is defined as

$$V(t) = x_2(t) + re^{-w\tau_1}\int_{t-\tau_1}^{t} x_2(u)du. \tag{4.24}$$

We calculate the derivative of $V(t)$ along the solution of (2.3) as follows:

$$\frac{dV(t)}{dt} = \left[re^{-w\tau_1} - \frac{\beta y(t)}{m+x_2(t)} - d_1\right]x_2(t), \tag{4.25}$$

Equation (4.25) can also be written

$$\frac{dV(t)}{dt} > \left[re^{-w\tau_1} - \frac{\beta}{m}y(t) - d_1\right]x_2(t). \tag{4.26}$$

We claim that for any $t_0 > 0$, it is impossible that $x_2(t) < x_2^*$ for all $t > t_0$. Suppose that the claim is not valid. Then there is a $t_0 > 0$ such that $x_2(t) < x_2^*$ for all $t > t_0$. It follows from the second equation of System (2.3) that for all $t > t_0$,

$$\frac{dy(t)}{dt} < \left(\frac{k\beta x_2^*}{m + x_2^*} - d_2\right) y(t). \tag{4.27}$$

Consider the following comparison impulsive system for all $t > t_0$

$$\begin{aligned} \frac{dv(t)}{dt} &= \left(\frac{k\beta x_2^*}{m + x_2^*} - d_2\right) v(t), \quad t \neq (n+l)\tau, (n+1)\tau, \\ \Delta v(t) &= v(t)(a - bv(t)), \quad t = (n+l)\tau, \\ \Delta v(t) &= -\mu v(t), \quad t = (n+1)\tau. \end{aligned} \tag{4.28}$$

By Lemma 3.4, we obtain

$$\widetilde{v(t)} = \begin{cases} v^* e^{(-d_2 + k\beta x_2^*/(m+x_2^*))(t-n\tau)}, & t \in [n\tau, (n+l)\tau), \\ \left[(1+a)e^{(-d_2+k\beta x_2^*/(m+x_2^*))l\tau} v^* + b e^{2(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}(v^*)^2\right] & \\ \quad \times e^{(-d_2+k\beta x_2^*/(m+x_2^*))(t-(n+l)\tau)}, & t \in [(n+l)\tau, (n+1)\tau), \end{cases} \tag{4.29}$$

is the unique positive periodic solution of (4.28) which is globally asymptotically stable, where

$$v^* = \frac{1+a}{b} e^{d_2 - (k\beta x_2^*/(m+x_2^*))l\tau} - \frac{1}{(1-\mu)b} e^{(d_2 - k\beta x_2^*/(m+x_2^*))(1+l)\tau}. \tag{4.30}$$

By the comparison theorem for impulsive differential equation [1, 2], we know that there exists $t_1 (> t_0 + \tau_1)$ such that the following inequality holds for $t \geq t_1$:

$$y(t) \leq \widetilde{v(t)} + \varepsilon. \tag{4.31}$$

Thus,

$$y(t) \leq \left[1 + (1+a)e^{(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}\right] v^* + b e^{2(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}(v^*)^2 + \varepsilon, \tag{4.32}$$

for all $t \geq t_1$. For convenience, we make notation as $\sigma = [1 + (1+a)e^{(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}]v^* + b e^{2(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}(v^*)^2 + \varepsilon$. From (4.20), we can choose a ε such that have

$$r e^{-w\tau_1} > \frac{\beta}{m}\left[1 + (1+a)e^{(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}\right] v^* + b e^{2(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}(v^*)^2 + \varepsilon + d_1, \tag{4.33}$$

By (4.26), we have

$$V'(t) > x_2(t)\left(re^{-w\tau_1} - \frac{\beta}{m}\sigma - d_1\right), \tag{4.34}$$

for all $t > t_1$. Set

$$x_2^m = \min_{t\in[t_1,t_1+\tau_1]} x_2(t), \tag{4.35}$$

We will show that $x_2(t) \geq x_2^m$ for all $t \geq t_1$. Suppose the contrary. Then there is a $T_0 > 0$ such that $x_2(t) \geq x_2^m$ for $t_1 \leq t \leq t_1 + \tau_1 + T_0$, $x_2(t_1 + \tau_1 + T_0) = x_2^m$ and $x_2'(t_1 + \tau_1 + T_0) < 0$. Hence, the first equation of system (2.3) and (4.33) imply that

$$\begin{aligned}
x_2'(t_1 + \tau_1 + T_0) &= re^{-w\tau_1}x_2(t_1 + T_0) - \frac{\beta x_2(t_1 + \tau_1 + T_0)y(t_1 + \tau_1 + T_0)}{m + x_2(t_1 + \tau_1 + T_0)} \\
&\quad - d_1x_2(t_1 + \tau_1 + T_0), \\
&\geq \left(re^{-w\tau_1} - \frac{\beta}{m}\sigma - d_1\right)x_2^m \\
&> 0.
\end{aligned} \tag{4.36}$$

This is a contradiction. Thus, $x_2(t) \geq x_2^m$ for all $t > t_1$. As a consequence, (4.26) and (4.33) lead to

$$V'(t) > x_2^m\left(re^{-w\tau_1} - \frac{\beta}{m}\sigma - d_1\right) > 0, \tag{4.37}$$

for all $t > t_1$. This implies that as $t \to \infty$, $V(t) \to \infty$. It is a contradiction to $V(t) \leq M(1 + r\tau_1e^{-w\tau_1})$. Hence, the claim is complete.

By the claim, we are left to consider two case. First, $x_2(t) \geq x_2^*$ for all t large enough. Second, $x_2(t)$ oscillates about x_2^* for t large enough.

Define

$$q = \min\left\{\frac{x_2^*}{2}, q_1\right\}, \tag{4.38}$$

where $q_1 = x_2^* e^{-(\beta M/(m+M)+d_1)\tau_1}$. We hope to show that $x_2(t) \geq q$ for all t large enough. The conclusion is evident in first case. For the second case, let $t^* > 0$ and $\xi > 0$ satisfy $x_2(t^*) = x_2(t^* + \xi) = x_2^*$ and $x_2(t) < x_2^*$ for all $t^* < t < t^* + \xi$ where t^* is sufficiently large such that

$$y(t) < \sigma \quad \text{for } t^* < t < t^* + \xi, \tag{4.39}$$

$x_2(t)$ is uniformly continuous. The positive solutions of (2.3) are ultimately bounded and $x_2(t)$ is not affected by impulses. Hence, there is a T $(0 < t < \tau_1$ and T is dependent of the choice of t^*) such that $x_2(t^*) > x_2^*/2$ for $t^* < t < t^* + T$. If $\xi < T$, there is nothing to prove. Let us consider the case $T < \xi < \tau_1$. Since $x_2'(t) > -(\beta M/(m+M) + d_1)x_2(t)$ and $x_2(t^*) = x_2^*$, it is clear that $x_2(t) \geq q_1$ for $t \in [t^*, t^* + \tau_1]$. Then, proceeding exactly as the proof for the above claim. We see that $x_2(t) \geq q_1$ for $t \in [t^* + \tau_1, t^* + \xi]$. Because the kind of interval $t \in [t^*, t^* + \xi]$ is chosen in an arbitrary way (we only need t^* to be large). We concluded $x_2(t) \geq q$ for all large t. In the second case. In view of our above discussion, the choice of q is independent of the positive solution, and we proved that any positive solution of (2.3) satisfies $x_2(t) \geq q$ for all sufficiently large t. This completes the proof of the theorem. □

From Theorems 4.1 and 4.3, we can easily obtain the following theorem.

Theorem 4.4. *If*

$$re^{-w\tau_1} > \frac{\beta}{m}\left\{\left[1 + (1+a)e^{(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}\right]v^* + be^{2(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}(v^*)^2\right\} + d_1, \tag{4.40}$$

then System (2.1) *with* (2.2) *is permanent, where* x_2^* *is determined as the following equation:*

$$\begin{aligned}&\left[1 + (1+a)e^{(-d_2+k\beta x_2^*/(m+x_2^*))l\tau}\right] \times \left[\frac{1+a}{b}e^{d_2-k\beta x_2^*/(m+x_2^*)l\tau} - \frac{1}{(1-\mu)b}e^{(d_2-k\beta x_2^*/(m+x_2^*))(1+l)\tau}\right]\\ &\quad + be^{2(-d_2+k\beta x_2^*/(m+x_2^*))l\tau} \times \left(\frac{1+a}{b}e^{d_2-(k\beta x_2^*/(m+x_2^*))l\tau} - \frac{1}{(1-\mu)b}e^{(d_2-k\beta x_2^*/(m+x_2^*))(1+l)\tau}\right)^2\\ &= \frac{m}{\beta}\left(re^{-w\tau_1} - d_1\right).\end{aligned} \tag{4.41}$$

5. Discussion

In this paper, considering the fact of the biological source management, we consider a delayed Holling type II predator-prey system with birth pulse and impulsive harvesting on predator population at different moments. We prove that all solutions of System (2.1) with (2.2) are uniformly ultimately bounded. The conditions of the globally attractive prey-extinction boundary periodic solution of System (2.1) with (2.2) are obtained. The permanence of the System (2.1) with (2.2) is also obtained. The results show that the successful management of a renewable resource is based on the concept of a sustain yield, that is, an exploitation does not the threaten the long-term persistence of the species. Our results provide reliable tactic basis for the practical biological resource management.

Acknowledgments

This work was supported by National Natural Science Foundation of China (no. 10961008), the Nomarch Foundation of Guizhou Province (no. 2008035), the Science Technology Foundation of Guizhou Education Department (no. 2008038), and the Science Technology Foundation of Guizhou (no. 2010J2130).

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