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Review Article

Electrodynamics of Radiating Charges

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The theory of electrodynamics of radiating charges is reviewed with special emphasis on the role of the Schott energy for the conservation of energy for a charge and its electromagnetic field. It is made clear that the existence of radiation from a charge is not invariant against a transformation between two reference frames that has an accelerated motion relative to each other. The questions whether the existence of radiation from a uniformly accelerated charge with vanishing radiation reaction force is in conflict with the principle of equivalence and whether a freely falling charge radiates are reviewed. It is shown that the resolution of an electromagnetic "perpetuum mobile paradox" associated with a charge moving geodetically along a circular path in the Schwarzschild spacetime requires the so-called tail terms in the equation of motion of a charged particle.

1. Introduction

The nonrelativistic version of the equation of motion of a radiating charged particle was discussed already more than a hundred years ago by Lorentz [1],

$$m_0\mathbf{a} = \mathbf{f}_{\text{ext}} + \frac{m_0\tau_0 d\mathbf{a}}{dT}, \qquad \tau_0 = \frac{q^2}{6\pi\varepsilon_0 m_0 c^3}, \tag{1.1}$$

where $\mathbf{a}$ is the ordinary (Newtonian) acceleration of the particle. The time τ_0 is of the same order of magnitude as the time taken by light to move a distance equal to *the classical electron radius*, that is, $\tau_0 \approx 10^{-23}$ seconds. The general solution of the equation is

$$\mathbf{a}(T) = e^{T/\tau_0}\left[\mathbf{a}(0) - \frac{1}{m\tau_0}\int_0^T e^{-T'/\tau_0}\mathbf{f}_{\text{ext}}(T')dT'\right]. \tag{1.2}$$

Hence, the charge performs a *runaway motion*, that is, it accelerates away even when $\mathbf{f}_{\text{ext}} = 0$ unless one chooses the initial condition

$$m\tau_0 \mathbf{a}(0) = \int_0^\infty e^{-T'/\tau_0} \mathbf{f}_{\text{ext}}(T')dT'. \tag{1.3}$$

By combining (1.1) and (1.3), one obtains

$$m\mathbf{a}(T) = \int_0^\infty e^{-s} \mathbf{f}_{\text{ext}}(T + \tau_0 s)ds. \tag{1.4}$$

This equation shows that the acceleration of the charge at a point of time is determined by the future force. Hence, when runaway motion is removed *preacceleration* appears.

The relativistic generalization of the equation was originally found by Abraham in 1905 [2]. A new deduction of the Lorentz covariant equation of motion was given by Dirac in 1938 [3]. This equation is therefore called the Lorentz-Abraham-Dirac equation, or for short, *the LAD equation*. In the 4-vector notation invented by Minkowski in 1908, and *referring to an inertial frame in flat spacetime*, the equation of motion of a radiating charged particle takes the form

$$F^\mu_{\text{ext}} + F^\mu_A = m_0 \dot{U}^\mu, \tag{1.5}$$

where $U^\mu = dX^\mu/d\tau$ is the 4-velocity of the particle and the dot denotes the ordinary differentiation with respect to the proper time of the particle. Here,

$$F^\mu_A \equiv m_0\tau_0\left(\dot{A}^\mu - g^2 U^\mu\right), \quad g^2 = A_\alpha A^\alpha, \tag{1.6}$$

is called the *Abraham 4-force*, g is the proper acceleration of the charged particle with respect to an inertial frame, and

$$A^\mu \mathbf{e}_\mu = \frac{d}{d\tau}(U^\mu \mathbf{e}_\mu) \tag{1.7}$$

is the 4-acceleration of the particle.

Let the components of the 4-acceleration be $(A^0, \mathbf{A})$. Fritz Rohrlich called the spatial component of the Abraham 4-force, $\mathbf{f}_A$, for *the field reaction force*, and separated it in two forces, $\mathbf{f}_A = \mathbf{f}_S + \mathbf{f}_R$, where $\mathbf{f}_S$ is the *Schott force* (called the acceleration reaction force by Rohrlich) and $\mathbf{f}_R$ the *radiation reaction force.* The Schott force is given by

$$\mathbf{f}_S = m_0\tau_0 \frac{d\mathbf{A}}{dT}, \tag{1.8}$$

where T is the inertial laboratory time, and the radiation reaction force is

$$\mathbf{f}_R = -m_0\tau_0 g^2 \mathbf{v}, \tag{1.9}$$

where $\mathbf{v}$ is the ordinary velocity of the particle.

The field reaction force is

$$\mathbf{f}_A = m_0 \tau_0 \frac{d\mathbf{A}}{dT} - m_0 \tau_0 g^2 \mathbf{v}. \tag{1.10}$$

For a freely moving particle both the Schott force and the field reaction force vanish.

Gal'tsov and Spirin [4] have reviewed and compared two different approaches to radiation reaction in the classical electrodynamics of point charges: a local calculation of the self-force, using the equation of motion and a global calculation consisting in integration of the electromagnetic energy-momentum flux through a hypersurface encircling the worldline. With reference to Dirac [3] and Teitelboim [5], they interpreted the Schott force physically in the following way: the Schott force is the finite part of the derivative of the momentum of the electromagnetic field, which is bound to the charge.

In an inertial reference frame the Abraham 4-force may also be written as

$$F_A^{\mu} = m_0 \tau_0 \gamma (\mathbf{v} \cdot \dot{\mathbf{g}}, \dot{\mathbf{g}}), \tag{1.11}$$

where the vector $\mathbf{g}$ is the proper acceleration of the charged particle with respect to an inertial frame. The Abraham 4-force is written in terms of the field reaction force as

$$F_A^{\mu} = \gamma (\mathbf{v} \cdot \mathbf{f}_A, \mathbf{f}_A). \tag{1.12}$$

Hence, the field reaction force can be written as

$$\mathbf{f}_A = m_0 \tau_0 \dot{\mathbf{g}}. \tag{1.13}$$

According to Larmor's formula, the energy radiated by the particle per unit time is

$$P_L = m_0 \tau_0 g^2. \tag{1.14}$$

It should be noted that this version of Larmor's formula is only valid with respect to an inertial frame meaning that it is only valid in a frame where there is no acceleration of gravity.

In flat spacetime uniformly accelerated motion is defined by $\dot{\mathbf{g}} = 0$. Hence, no field reaction force acts upon a uniformly accelerated charge. However, if one investigates the electromagnetic field of such a charge, one finds that a uniformly accelerated charge emits radiation in accordance with Larmor's formula.

The 4-acceleration is defined as the covariant directional derivative of the 4-velocity along the four-velocity. Since free particles move along geodesic curves that are defined by the condition that their tangent vectors, that is, the 4-velocity, are connected by parallel transport, it follows that in curved as well as in flat spacetime a freely falling charge has vanishing 4-acceleration. Hence, in an inertial frame a freely moving charge will not radiate. However, it will be shown in Section 4 that this is not so in a non-inertial frame where the acceleration of gravity does not vanish. Then, a generalization of (1.14) will be needed.

2. Localization and Physical Interpretation of the Schott Energy

In a recent article on relativistic particle motion and radiation reaction in electrodynamics Hammond [6] has discussed the question whether energy is conserved for uniformly accelerated motion of a radiating charge. He wrote that the Schott energy (defined in (2.5) below) destroys our concept of what conservation of energy should be. Hammond pointed out an important problem that is rather disturbing as long as the Schott energy only appears as a term in the energy conservation equation without any physical interpretation telling us what sort of energy it is. In order to remedy this defect in classical electrodynamics Eriksen and Grøn [7–10] and also Rowland [11] worked out a physical interpretation of the Schott energy in terms of the energy of the electromagnetic field produced by an accelerating charge. The results of this research will be summarized in this section.

From the equation of motion (1.5) we get the energy equation

$$\mathbf{v} \cdot \mathbf{F}_{\text{ext}} = \gamma^{-1}\left(m_0 \dot{U}^0 - F_A^0\right) = m_0 \gamma^3 \mathbf{v} \cdot \mathbf{a} - \mathbf{v} \cdot \mathbf{f}_A = \frac{dE_k}{dT} - \mathbf{v} \cdot \mathbf{f}_A, \tag{2.1}$$

where $\mathbf{a}$ is the ordinary acceleration and $E_k = (\gamma - 1)m_0 c^2$ is the kinetic energy of the particle. In an instantaneous inertial rest frame of the charge the radiation reaction force $\mathbf{f}_R$ vanishes. The reason is that in this frame the charge radiates isotropically. Hence, in this frame the field reaction force $\mathbf{f}_A$ reduces to the Schott force $\mathbf{f}_S$, which here takes the form

$$\mathbf{f}_S = m_0 \tau_0 \frac{d\mathbf{g}}{dT}, \quad \mathbf{g} = \frac{d\mathbf{v}}{dT}. \tag{2.2}$$

For particles moving with a velocity $v \ll c$ we can use this expression for the Schott force, and the work done by this force is

$$\int_0^T \mathbf{f}_S \cdot \mathbf{v}\, dT = m_0 \tau_0 \int_0^T \frac{d\mathbf{g}}{dT} \cdot \mathbf{v} dT = m_0 \tau_0 [\mathbf{g} \cdot \mathbf{v}]_0^T - m_0 \tau_0 \int_0^T g^2 dT. \tag{2.3}$$

The first term vanishes if the acceleration or the velocity vanishes at the boundary of the integration region or for periodic motion, integrating over a whole number of periods. Then, the work performed by the Schott force accounts for the radiated energy as given by the last term.

The power due to the field reaction force is

$$\mathbf{v} \cdot \mathbf{f}_A = m_0 \tau_0 \frac{d}{dT}\left(\gamma^4 \mathbf{v} \cdot \mathbf{a}\right) - P_L = -\frac{dE_S}{dT} - \frac{dE_R}{dT}, \tag{2.4}$$

where E_R is the energy of the radiation field and E_S is the *Schott energy* defined by

$$E_S \equiv -m_0 \tau_0 \gamma^4 \mathbf{v} \cdot \mathbf{a} = -m_0 \tau_0 A^0. \tag{2.5}$$

The Schott energy was called "acceleration energy" by Schott [12]. It is negative if the acceleration is in the same direction as the velocity and positive if it is in the opposite direction. By means of (2.4) the energy equation (2.1) may now be written as

$$\frac{dW_{\text{ext}}}{dT} = \mathbf{v} \cdot \mathbf{F}_{\text{ext}} = \frac{d}{dT}(R_K + E_S + E_R). \tag{2.6}$$

In the case of uniformly accelerated motion, $\dot{\mathbf{g}} = 0$, the field reaction force, $\mathbf{f}_A$, vanishes. From (2.4) and (2.6) it follows that in this case

$$\mathbf{v} \cdot \mathbf{F}_{\text{ext}} = \frac{dE_K}{dT}. \tag{2.7}$$

Hence, in this case all of the work performed by the external force goes to increase the kinetic energy of the charged particle. From (2.6) it is seen that then all of the radiated energy comes from the Schott energy. But what is the Schott energy?

In the article "The equations of motion of classical charges" Rohrlich wrote [13]: "If the Schott energy is expressed by the electromagnetic field, it would describe an energy content of the near field of the charged particle which can be changed *reversibly*. In periodic motion energy is borrowed, returned, and stored in the near-field during each period. Since the time of energy measurement is usually large compared to such a period only the average energy is of interest and that average of the Schott energy rate vanishes. Uniformly accelerated motion permits one to borrow energy from the near-field for large *macroscopic* time-intervals, and no averaging can be done because at no two points during the motion is the acceleration four-vector the same. Nobody has so far shown in detail just how the Schott energy occurs in the near-field, how it is stored, borrowed, and so forth."

This challenge was taken up by Eriksen and Grøn, who gave an answer in a series of articles [7–10]. Pearle [14] has summarized works of Fulton and Rohrlich [15] and Teitelboim [5] on this problem and writes: "The physical meaning of the Schott term has been puzzled over for a long time. Its zero component represents a power which adds "Schott acceleration energy" to the electron and its associated electromagnetic field. The work done by an external force not only goes into electromagnetic radiation and into increasing the electron's kinetic energy, but it causes an increase in the "Schott acceleration energy" as well. This change can be ascribed to a change in the "bound" electromagnetic energy in the electron's induction field, just as the last term of (2.6) can be ascribed to a change in the "free" electromagnetic energy in the electron's radiation field. What meaning should be given to the Schott term? Teitelboim has argued convincingly that when an electron accelerates, its near-field is modified so that a correct integration of the electromagnetic four-momentum of the electron includes not only the Coulomb 4-momentum $(q^2/8\pi\varepsilon_0 r)U^\mu$, but an extra four-momentum of the bound electromagnetic field."

It remains to obtain a precise localization of the Schott field energy. Eriksen and Grøn [10] obtained the following result. The Schott energy is inside a spherical light front touching the front end of a moving Lorentz contracted charged particle. At the point of time T the

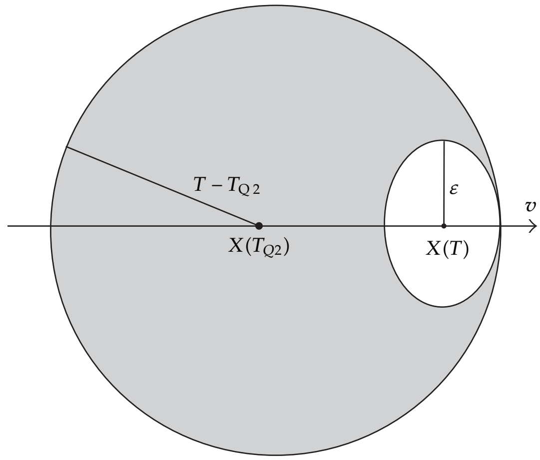

Figure 1: The figure shows a Lorentz contracted charged particle with proper radius ε moving to the right with velocity v. The field is observed at a point of time T, and at this moment the centre of the particle is at the position $X(T)$. The circle is a field front produced at the retarded point of time T_{Q2} when the centre of the particle was at the position $X(T_{Q2})$. The field front is chosen such that it just touches the front of the particle. The Schott energy is localized in the shaded region between the field front and the ellipsoid representing the surface of the particle. In the figure the velocity is chosen to be $v = 0.6$.

radius of the eikonal consisting of light emitted at the point of time T_{Q2} that represents the boundary of the distribution of the Schott energy is (Figure 1)

$$T - T_{Q2} = r_0 \sqrt{\frac{1 + v/c}{1 - v/c}}. \tag{2.8}$$

Here, $r_0 = \varepsilon$ represents the proper radius of the particle and v is the absolute value of its velocity. The horizontal extension of the hatched region is

$$L = 2c(T - T_{Q2}) - 2r_0 \sqrt{\frac{1 - v^2}{c^2}} = 2r_0 \left(\frac{v}{c}\right) \sqrt{\frac{(1 + v/c)}{(1 - v/c)}}. \tag{2.9}$$

Note that $L \to 0$ when $v \to 0$. Unless the velocity of the charge is close to that of light, the Schott energy is localized just outside the surface of the charged particle. This is illustrated in Figure 2.

We can now see, from a field point of view, why the Schott energy E_S depends on the velocity of the particle as well as its acceleration and vanishes in the rest frame of the charge. The reason is that the size of the region C containing the Schott energy depends on the velocity of the charge, with the size going to zero as the velocity goes to zero.

With Eriksen and Grøn's results as a point of departure Rowland [11] has worked out a new interpretation of the Schott energy. He considered a charged particle moving initially in the negative x-direction with a constant velocity $\beta = v/c = -0.745$. At time $t = t_1$ it starts to accelerate uniformly in the positive x-direction. The particle's location at $t = t_1$ is labeled $x(t_1)$ in Figure 3, its current position $x(t)$, and its virtual position at the current time $x_v(t)$ (the particle's virtual position is where it would have been if it had not started accelerating at

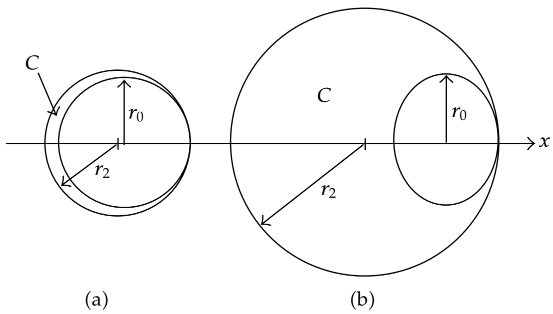

Figure 2: The figure shows how the size of the region C, as given by (2.9), depends on the velocity of the particle. In both parts of the figure the radius r_0 of the particle is the same. The difference is that in (a) $\beta = 0.1$ while in (b) $\beta = 0.6$.

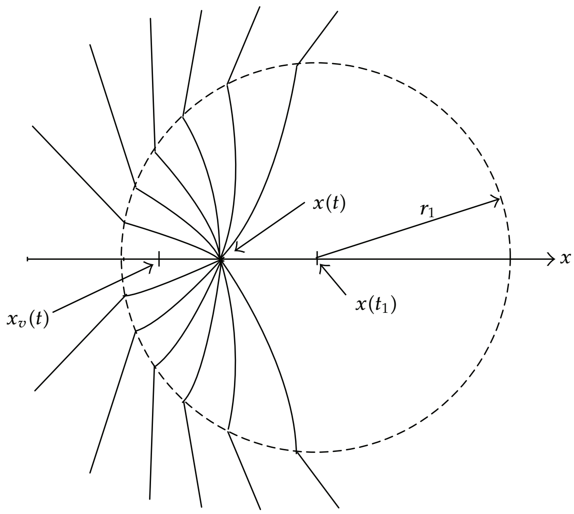

Figure 3: The figure shows the electrical field of a charged particle moving initially in the negative x-direction with a constant velocity $\beta = -0.745$. At time $t = t_1$ it starts to accelerate uniformly in the positive x-direction. The particle's location at $t = t_1$ is labeled $x(t_1)$, its current position $x(t)$, and its virtual position at the current time $x_v(t)$. The dashed circle in the diagram is the location of the light cone emanating from $x(t_1)$ at the current instant. Outside this light cone the particle's field is simply a Lorentz contracted Coulomb field focused on $x_v(t)$, while inside the light cone the field is that of a uniformly accelerating charge with circular field lines.

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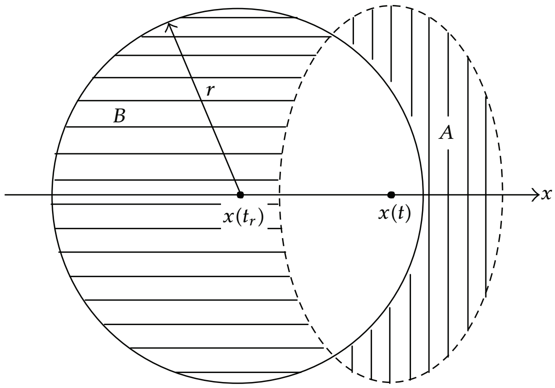
Consider for a moment a charged particle that moves with constant velocity in the positive x-direction. Figure 4 shows the difference between a Lorentz contracted sphere (dashed line) centered at the instantaneous current (C) position of the charge, $x(t)$, and the retarded (ret) sphere (solid line) with the same radius $r = c(t - t_r)$ centered at the position, $x(t_r)$, of the particle at time $t_r < t$. The bound (b) field energies $E_{b\text{cur}}$ and $E_{b\text{ret}}$ *out*side the current and retarded surfaces differ because the energy in region A is included in the



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calculation of $E_{b\mathrm{ret}}$ but not $E_{b\mathrm{cur}}$, while region B is included in the calculation of $E_{b\mathrm{cur}}$ but not $E_{b\mathrm{ret}}$. The example shown is for $\beta = 0.8$.

The energy in the electric field outside a spherical shell of radius r and with charge q moving with constant velocity in the laboratory, as measured in the rest frame of the shell, is

$$E_0(r) = \frac{q^2}{8\pi\varepsilon_0 r}. \tag{2.10}$$

The bound energy in the electric field outside a Lorentz contracted spherical shell, centered at the *current* position of the charge, is

$$E_{b\mathrm{cur}}(\beta, r) = \gamma\left(1 + \frac{\beta}{3}\right)E_0(r). \tag{2.11}$$

The bound energy in the electric field outside a spherical shell, centered at the *retarded* position of the charge, is

$$E_{b\mathrm{ret}}(\beta, r) = \gamma E_{b\mathrm{cur}}(\beta, r) = \left(1 + \frac{4}{3}\gamma^2\beta^2\right)E_0(r). \tag{2.12}$$

The space outside the current position of the charge is divided into three regions (Figure 5).

In region A the field is just that of a charge moving with the constant velocity v_1, and since this region is centered at a retarded position of the charge, the total energy in region A is given by (2.12)

$$E_b(A) = \left(1 + \frac{4}{3}\gamma^2\beta^2\right)\frac{q^2}{8\pi\varepsilon_0 r}. \tag{2.13}$$

Region B lies between two spheres with retarded radii $r_2 = c(t - t_2)$ and $r_1 = c(t - t_1)$, with $t_1 < t_2 < t$. Since the charge has been accelerating between t_1 and t_2, $E(B)$ has contributions

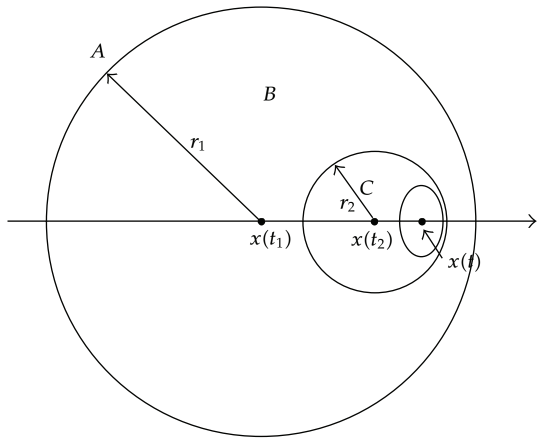

Figure 5: The regions surrounding an accelerating charge where field energies are calculated. Here $x(t_1)$ is the particle position when it started accelerating, and $r_1 = c(t - t_1)$. Thus there is just a Lorentz contracted Coulomb field in the region A, while there exist both generalized Coulomb and radiation fields in region B. The innermost ellipse is the cross-section through the Lorentz contracted sphere (i.e., it is an ellipsoid) centered at the current position $x(t)$ of the charge and withradius r_0 (not shown on this diagram) in the rest frame of the particle. Hence the charge has a finite radius that is necessary to provide a cut-off for the energy integrals to keep them finite. Region C where Eriksen and Grøn localized the Schott energy, lies between this ellipsoid and a retarded sphere tangential to the ellipsoid. Furthermore, $x(t_2)$ is the particle position at time $t = t_2$ and $r_2 = c(t - t_2)$. The relationship between t and t_2 is given in (2.8).

E_b and E from, respectively, bound and radiation fields. Using (2.12) to interpret Eriksen and Grøn's results, one finds that

$$E_b(B) = E_{b\mathrm{ret}}(\beta_2, r_2) - E_{b\mathrm{ret}}(\beta_1, r_1), \tag{2.14}$$

where $\beta_2 = \beta(t_2)$, and

$$E_r(B) = E_r(t_1, t_2) = \int_{t_1}^{t_2} P_L dt = m_0 \tau_0 g^2 (t_2 - t_1). \tag{2.15}$$

Here, P_L is the radiated effect as given by Larmor's formula (1.14). The bound energy in the region $A + B$ outside the sphere with retarded radius r_2 is

$$E_b(A + B) = E_{b\mathrm{ret}}(\beta_2, r_2). \tag{2.16}$$

If the charge had been moving with its current velocity, that is, its velocity at the point of time t, for its entire history, the energy in the region $A + B$ would have been $E_{b\mathrm{ret}}(\beta(t), r_2)$. Rowland has shown that to the first order in $\delta t = t - t_2$

$$E_b(A + B) - E_{b\mathrm{ret}}(\beta(t), r_2) \approx 2E_S. \tag{2.17}$$

In other words the bound field energy in $A+B$ differs from that of a charged particle moving with the current velocity by twice the Schott energy. The acceleration determines in part how much β_2 differs from $\beta(t)$. It is therefore not surprising that E_S depends upon the acceleration.

The region C is the volume between a Lorentz contracted sphere of radius r_0 with center at the current position of the charge and a spherical light front of radius r_2. Using (2.10), (2.11), and (2.6) to interpret Eriksen and Grøn's results, one finds that the energy of the fields in the region C is

$$E_b(C) \approx E_{b\text{cur}}(\beta(t), r_0) - E_{b\text{ret}}(\beta(t), r_2) - E_S(\beta(t), a(t)). \tag{2.18}$$

The first term at the right- hand side is the bound energy in the electric field outside a Lorentz contracted spherical shell, centered at the *current* position of the charge as given in (2.6). The second term is the bound energy in the electric field outside a spherical shell, centered at the *retarded* position of the charge. The difference, $\Delta E_{b\text{cur}} = E_{b\text{cur}}(\beta(t), r_0) - E_{b\text{ret}}(\beta(t), r_2)$, is the energy that would have been in the region C if the charged particle had moved with constant velocity for its entire history. For the accelerated particle the field lines are curved as shown in the Figure 6, and there is more energy in the region C than if the particle had moved with constant velocity. Equation (2.18) can be written as

$$E_S = \Delta E_{b\text{cur}} - E_b(C), \tag{2.19}$$

which leads to the following interpretation: *the Schott energy is the difference between the bound field energy of an accelerated charged particle if the particle had moved with its current velocity for its entire history and the actual bound field energy of the particle*. With this understanding one may say that *the Schott energy is that part of the bound field energy of an accelerated charged particle that is due to its acceleration, and it is localized close to the particle*.

We can now provide a description of how *the LAD equation implies energy-momentum conservation even during the strange runaway motion*. This understanding comes from the detailed investigations of energy-momentum relationships of an accelerated charge and its electromagnetic field that has been reviewed above. It is important to recognize that an accelerated charge is not an isolated system. *It is the charge and its electromagnetic field that are the isolated system.* During runaway motion there is an increasingly negative Schott energy and an increasing Schott momentum directed oppositely to the motion of the charge. It follows from (2.6) with $W_{\text{ext}} = 0$ that the energy and momentum of the charge and its field are conserved during runaway motion. The bound field close to the charge where the Schott energy is acts upon the charge, accelerates it, increases its kinetic energy and also is transformed into radiation energy, while the charge acts back on the field in accordance with Newton's third law. These are internal forces in the isolated charge-field system. The internal forces involving the Schott energy and its transformation to radiation energy then cause a redistribution of energy and momentum between the particle, its near field, and the radiation it emits.

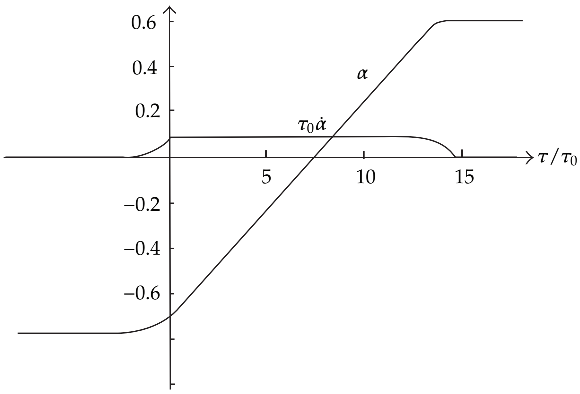

Figure 6: We have here plotted the rapidity $\alpha = \operatorname{arctanh}\beta$ and its rate of change as functions of proper time. Here $\alpha(-\infty) = -0.8$ and we have chosen $\tau_2 = 14\tau_0$.

3. Motion of a Charge Moving into a Constant Electric Field Which Stops the Charge and Accelerates It Back.

We will now consider a situation analyzed by Eriksen and Grøn [16–18] that makes clear the essential role played by the Schott energy for energy conservation. The particle enters a region H with a constant electric field at $\tau = \tau_1$ giving the particle a constant proper acceleration in the opposite direction of its velocity and leaves the electrical field at $\tau = \tau_2$. Assuming $\tau_0 \ll \tau_2 - \tau_1$, that is, that the time τ_0 is much less than the time the charge is inside the region H, the solution of the LAD equation is

$$\dot{\alpha} = g e^{(\tau-\tau_1)/\tau_0}, \quad \alpha = \alpha_0 + g\tau_0 e^{(\tau-\tau_1)/\tau_0}, \quad \tau \leq \tau_1, \tag{3.1}$$

where $\alpha_0 < 0$ is the limiting initial value of the rapidity for $\tau \to -\infty$. There is preacceleration both before the particle enters the electric field and before it leaves it (Figure 6).

The values of the kinetic energy of the particle and the Schott energy, respectively, for $\tau = -\infty$, are

$$E_K = m_0(\cosh\alpha_0 - 1), \qquad E_S = 0. \tag{3.2}$$

At the moment $\tau = \tau_1$ when the particle enters H, the values of these energies are

$$E_K = m_0(\cosh\alpha_1 - 1), \qquad E_S = -\frac{2}{3}Q^2 g \sinh\alpha_1. \tag{3.3}$$

To second order in $g\tau_0$, which we assume is much less than the velocity of light, $c = 1$, the changes in the kinetic energy and the Schott energy from $\tau = -\infty$ to $\tau = \tau_1$ are

$$\Delta E_K = \gamma_0 m_0 v_0 g\tau_0 + \frac{1}{2}\gamma_0 m_0 g^2\tau_0^2, \qquad \Delta E_S = -\gamma_0 m_0 v_0 g\tau_0 - \gamma_0 m_0 g^2\tau_0^2. \tag{3.4}$$

From the energy equation (2.6), which for $\tau < \tau_1$ takes the form $\Delta E_R + \Delta E_K + \Delta E_S = 0$, it follows that the radiated energy during this period is

$$\Delta E_R = \frac{1}{2}\gamma_0 m_0 g^2 \tau_0^2. \tag{3.5}$$

From (3.3)–(3.5) it follows that the particle during the preacceleration gets an increase of the Schott energy which is nearly equal to the loss of kinetic energy of the particle. Only a minor part of the particle loss in kinetic energy (second order in $g\tau_0$) is radiated away.

The change of velocity of the particle, $\tanh\alpha_1 - \tanh\alpha_0$, during the preacceleration may be expressed as

$$v_1 - v_0 = \frac{\sinh g\tau_0}{\gamma_1\gamma_0}, \tag{3.6}$$

which shows that $v_1 \to v_0$ when $\gamma_0 \to \infty$.

We shall now consider the energy budget during the time when the charge is within H, that is, for $\tau_1 < \tau < \tau_2$. From (2.6) and (3.1) we get an energy equation $\dot{W}_{\text{ext}} = \dot{E}_R + \dot{E}_K + \dot{E}_S$ with the following rates of change per unit proper time:

$$\begin{gathered} \dot{W}_{\text{ext}} = m_0 g \sinh\alpha, \qquad \dot{E}_K = m_0\dot{\alpha}\sinh\alpha, \\ \dot{E}_S = m_0(g - \dot{\alpha})\sinh\alpha - \frac{2}{3}Q^2\dot{\alpha}^2\cosh\alpha, \qquad \dot{E}_R = -\frac{2}{3}Q^2\dot{\alpha}^2\cosh\alpha. \end{gathered} \tag{3.7}$$

The equations shall now be interpreted for the case that $\dot{\alpha} = g$, that is, when the motion of the particle is approximately hyperbolic. According to (3.1) this is the case when $\tau \ll \tau_2 - \tau_0$, that is, when the particle is not too near its exit from H. In this region $\dot{W}_{\text{ext}} = \dot{E}_K$ so that there is no other effect of the external force than a change in the kinetic energy. Thus, the sum of the kinetic energy of the charge and its potential energy in the field of force accelerating it in H is approximately constant, and the radiated energy is taken from the Schott energy, which according to (3.7) decreases uniformly with time,

$$\frac{dE_S}{dT} = \frac{\dot{E}_S}{\gamma} = -\frac{2}{3}Q^2 g^2. \tag{3.8}$$

When the particle arrives at the point where it turns back, there is no Schott energy left, and at this moment the energy that has been radiated by the particle is equal to its loss of kinetic energy during the preacceleration. The energy has been radiated as a pulse with energy ΔE_R given in (3.5), during the preacceleration, and then with a constant effect $(2/3)Q^2 g^2$ during the hyperbolic motion. The situation changes when the particle approaches the position where it leaves H, that is, when $\tau \approx \tau_2 - \tau_0$. Then, the particle experiences a new nonnegligible preacceleration, which reduces the acceleration from $\approx g$to 0, and the emitted power is reduced from $\approx (2/3)Q^2 g^2$ to 0. The velocity still increases during this period, but less than in the case of hyperbolic motion. The Schott energy, which until now (in H) has decreased at a constant rate, increases from the negative value $-(2/3)Q^2 g\gamma v$ to zero. All the energies E_R, E_K, and E_S increase during this preacceleration. The energy is provided by the work of the external force $F_{\text{ext}} = m_0 g$, or in other words from the loss of potential energy of the particle in the field of this force. In the region where the motion can be considered as hyperbolic, $\dot{\alpha} = g =$ constant, and the reaction force $m_0\tau_0\ddot{\alpha}$ vanishes. Here, $F_{\text{ext}} = m_0 g$ is the only force acting upon the particle, and $E_K + E_P =$ constant. This is no longer the case when the particle approaches the exit of H, where the preacceleration makes $\ddot{\alpha} \neq 0$.

In order to make a complete energy budget in the region H, we must know the proper time τ_2 when the particle leaves H. The position $X(\tau)$ of the particle at a point of time τ is given by

$$X(\tau) - X_1 = \int_{\tau_1}^{\tau} \gamma v \, d\tau = \int_{\tau_2}^{\tau} \sinh\alpha \, d\tau, \tag{3.9}$$

where α is given by (3.1). The point of time τ_2 when the particle leaves H is found from the equation $X(\tau_2) = X_1$. Solving the integral (to the second order in $g\tau_0$) the equation reads

$$g\int_{\tau_1}^{\tau_2} \sinh\alpha \, d\tau = \left(1 - g^2\tau_0^2\right)\cosh\left(\alpha_1 + g(\tau_2 - \tau_1)\right) - \cosh\alpha_1, \tag{3.10}$$

where we have utilized $\tau_0 \ll \tau_2 - \tau_1$. We get the following solution to the second order in $g\tau_0$:

$$\tau_2 - \tau_1 = -\frac{1}{g}\left(2\alpha_1 + g^2\tau_0^2 \coth\alpha_1\right), \tag{3.11}$$

where $\alpha_1 = \alpha_0 + g\tau_0$ is the rapidity of the particle at the moment it enters H. The term $-2\alpha_1/g$, which is dominating, is the proper time that the particle would have spent inside H if the motion had been hyperbolic. Then $\dot{\alpha} = g$, so the travelling proper time would be $\Delta\tau = \Delta\alpha/g$, where $\Delta\alpha = -2\alpha_1$ is the increase of α during the motion in H. Equation (3.11) tells that $\tau_2 - \tau_1$ is a little larger than this value.

Inserting τ_2 from (3.11) into the the expression for α in equation (3.1), we get

$$\alpha(\tau_2) = -\alpha_1 - g\tau_0 - g^2\tau_0^2 \coth\alpha_1, \tag{3.12}$$

which gives

$$\cosh\alpha(\tau_2) = \left(1 + \frac{3}{2}g^2\tau_0^2\right)\cosh\alpha_1 + g\tau_0 \sinh\alpha_1. \tag{3.13}$$

From this we find the following (negative) changes of the kinetic energy of the charge and its Schott energy during the period, $\tau_1 < \tau < \tau_2$, when the charge moves in H:

$$\Delta E_K = m_0 g\tau_0\gamma_1 v_1 + \frac{3}{2}m_0 g^2\tau_0^2\gamma_1, \qquad \Delta E_S = m_0 g\tau_0\gamma_1 v_1. \tag{3.14}$$

Since the total work performed by the external force F_{ext} upon the charge during its motion in H vanishes, the energy equation (2.6) gives for the energy radiated by the charge during this motion

$$\Delta E_R = -\Delta E_K - \Delta E_S = -m_0 g \tau_0 \gamma_1 v_1 - \frac{3}{2} m_0 g^2 \tau_0^2 \gamma_1. \tag{3.15}$$

The dominating term, $-2m_0 g \tau_0 \gamma_1 v_1$, may be interpreted as the energy radiated by a particle with exact hyperbolic motion. This is seen as follows. Using

$$g = \dot{\alpha} = \gamma \frac{d\alpha}{dT} = \cosh\alpha \frac{d\alpha}{dT} = \frac{d(\sinh\alpha)}{dT}, \tag{3.16}$$

for hyperbolic motion, leads to

$$\Delta(\sinh\alpha) = g\Delta T. \tag{3.17}$$

Thus, the time that the charge stays inside H is

$$\Delta T = -\left(\frac{2}{g}\right)\sinh\alpha_1. \tag{3.18}$$

The dominating term in (3.15) may be written as

$$m_0 g^2 \tau_0 \frac{-2\sinh\alpha_1}{g} = m_0 g^2 \tau_0 \Delta T = \frac{2}{3} Q^2 g^2 \Delta T \tag{3.19}$$

in agreement with Larmor's formula.

According to (3.14) the Schott energy and the kinetic energy decrease by about the same amount, which means that the Schott energy and the kinetic energy give approximately the same contribution to the radiated energy.

To get the complete energy budget from $\tau = -\infty$ to $\tau = \infty$ we utilize $W_{\text{ext}} = 0$ and $E_S(-\infty) = E_S(\infty) = 0$. Then, according to (3.15), $\Delta E_R = \Delta E_K = 0$, where ΔE_R is the sum of the expressions in (3.4). Expressing the relationships in terms of the velocity v_0 (which is negative) at $\tau = -\infty$, we find that the radiated energy is

$$\Delta E_R = \Delta E_K = E_K(-\infty) - E_K(\infty) = -2m_0 g \tau_0 \gamma_0 v_0 - 3m_0 g^2 \tau_0^2 \gamma_0. \tag{3.20}$$

The kinetic energy, the mechanical energy (i.e., the sum of the kinetic and potential energy), the Schott energy, and the radiated energy as functions of proper time are shown in Figures 7 and 8.

Let us summarize what happens to the particle and its energy from $\tau = -\infty$ to $\tau = \infty$. The charge comes from an infinitely far region with constant velocity. It moves towards a region H with, say, a constant electrical field antiparallel to its direction of motion. Approaching H it gets an increasing preacceleration, which causes the kinetic energy of the particle to decrease. A Schott energy of about the same magnitude appears. Also a small

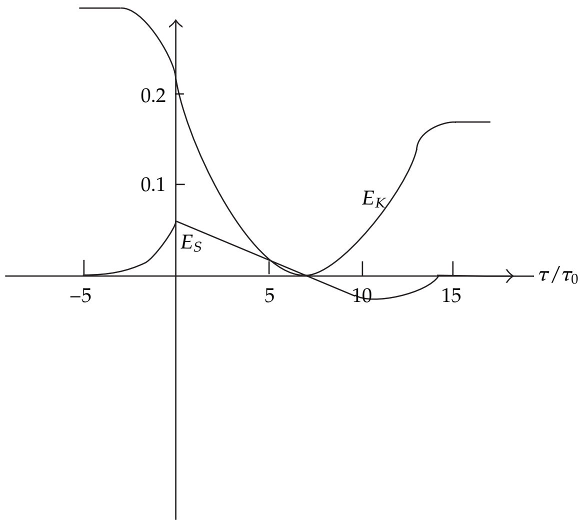

Figure 7: Kinetic energy and Schott energy in units of m_0 as functions of proper time for the motion given in (3.1).

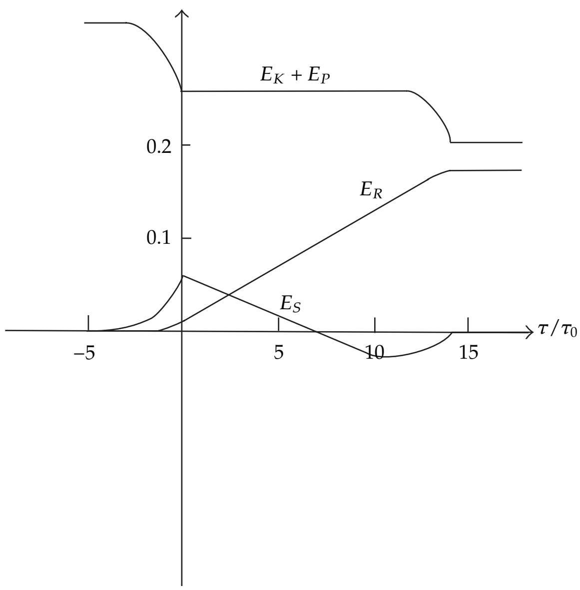

Figure 8: Mechanical energy, that is, $E_K + E_P$, Schott energy, and radiated energy as functions of proper time. Here E_P is the potential energy in the force field $m_0 g$ with $E_P = 0$ for $X > X_1$, and $E_P = m_0 g(X_1 - X)$. Note that $E_K + E_P + E_S + E_R =$ constant. All energies are in units of m_0.

amount of energy is radiated away by the particle. In the region H the particle moves approximately hyperbolically until it experiences a new preacceleration before it leaves H. During the hyperbolic part of the motion the external work performed by the field force upon the particle is used only to change the kinetic energy of the particle. The particle radiates at a constant rate, and the radiated energy comes from the Schott energy, which decreases steadily during this part of the motion. Before the particle leaves H the preacceleration decreases the acceleration towards zero. The particle still radiates although the Schott energy now increases. What happens all together while the particle is in H is that the kinetic energy and the Schott energy decrease by about the same amount, giving about the same contribution to the radiated energy. When the particle has left H and disappears towards an infinite remote region, the Schott energy has vanished again. The particle has lost kinetic energy, and this loss of energy is equal to the energy that the particle has radiated.

Pauri and Vallisneri [19] have commented this situation in the following way:

"We can imagine that the energy flux radiated by the charge during the uniformly accelerated motion is being borrowed from the *divergent* energy of the electromagnetic field near the charge, which effectively acts as an infinite *reservoir*. While draining energy, the field becomes more and more different from the pure velocity field of an inertial charge; when hyperbolic motion finally ends, the extended force must provide all the energy that is necessary to re-establish the original structure of the field." They did not mention the Schott energy, but its existence is implicit in what they wrote.

4. The Principle of Equivalence and Noninvariance of Electromagnetic Radiation

The foundation of the principle of equivalence is that at a certain point of spacetime every free particle instantaneously at rest falls with the same acceleration independent of its composition. A consequence of this is that if a proton and a neutron are falling from the same point in spacetime, they will fall together with the same acceleration. However, the proton will emit electromagnetic radiation and the neutron not. So where does the radiated energy come from?

Think of the following two situations.

(1) Consider a freely falling charge, for example, in a uniformly accelerated reference frame (UAF). It emits radiation in accordance with Larmor's formula. However, as observed by a comoving observer it is at rest in an inertial frame. Hence, this observer would say that the charge does not radiate.

(2) Consider now a charge with uniform acceleration in the inertial frame. This charge, too, emits radiation in accordance with Larmor's formula. But it is at rest in UAF.

If the detection of radiation depended only on the state of motion of the charge, an observer in UAF would detect radiation from a charge at rest in UAF. However, in this situation there is nothing that can provide the radiation energy since the situation is static, so the assumption that whether a charge radiates or not depends only upon its state of motion, cannot be correct.

Both situations (1) and (2) mean that the existence of radiation cannot be invariant against a transformation between an inertial and noninertial reference frame.

In order to have a discussion of the question whether the existence of radiation from a charged particle is invariant against a transformation between an inertial- and an accelerated reference frame we need a precise definition of *electromagnetic radiation*. This will be given below following Rohrlich [20].

4.1. The Definition of Electromagnetic Radiation

The rate of radiation energy emission will be defined as a Lorentz invariant, but not generally invariant, quantity. Hence, in this section all components of tensor quantities will refer to an inertial reference frame (and we use units so that $c = 1$). The component formulae will be generalized to expressions valid also with respect to a uniformly accelerated reference frame in Section 4.4.

Given a point charge q following a trajectory $X^{\mu}(\tau)$, the electromagnetic field produced by the charge, $F^{\mu\nu}$, is measured at a point P with coordinates X_P^{μ}. The point P is connected to an emission point Q by a null vector $R^{\mu} = X_P^{\mu} - X^{\mu}$, that is, $R_{\mu}R^{\mu} = 0$. The spatial distance between P and Q in the inertial system in which the charge is instantaneously at rest is

$$\rho = -U_{\mu}R^{\mu} > 0, \tag{4.1}$$

where U^{μ} represents the 4-velocity of the charge at the retarded point Q. We now define a spacelike vector

$$N^{\mu} = \left(\frac{1}{\rho}\right)R^{\mu} - U^{\mu}, \tag{4.2}$$

obeying

$$N_{\mu}N^{\mu} = 1, \quad N_{\mu}U^{\mu} = 0. \tag{4.3}$$

The vector R^{μ} can therefore be written as

$$R^{\mu} = \rho(N^{\mu} + U^{\mu}). \tag{4.4}$$

In this notation the Liénard-Wiechert potentials are

$$L^{\mu} = e\frac{U^{\mu}}{\rho}, \qquad e \equiv \frac{q}{4\pi\varepsilon_{0.}}. \tag{4.5}$$

The retarded electromagnetic field is given by

$$F^{\mu\nu} = -\left(\frac{2e}{\rho^2}\right)N^{[\mu}U^{\nu]} - \left(\frac{2e}{\rho}\right)\left[U^{[\mu}A^{\nu]} + N^{[\mu}\left(U^{\nu]}\ A_N + A^{\nu}\right)\right], \qquad A_N = A_{\lambda}N^{\lambda}. \tag{4.6}$$

Here, the antisymmetrization bracket around the upper indices is defined by $a^{[\mu}b^{\nu]} = (1/2)(a^{\mu}b^{\nu} - a^{\nu}b^{\mu})$. The first term is the generalized Coulomb field, and the second term, which vanishes if and only if the components of the four-acceleration with respect to an inertial frame vanish, is the *radiation field*, by definition.

The electromagnetic energy tensor is

$$T_{\mu\nu} = \varepsilon_0\left[F_{\mu}^{\lambda}F_{\lambda\nu} - \left(\frac{1}{4}\right)\eta_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right], \tag{4.7}$$

where the components of the Minkowski metric tensor have been used since all quantities are decomposed in an inertial frame in this section. Inserting (4.6) into this expression one finds the energy-momentum tensor at a point P due to the field emission at the retarded point Q on the world line of a point charge q,

$$T^{\mu\nu} = \frac{\varepsilon_0 e^2}{\rho^4}\left(N^\mu N^\nu - U^\mu U^\nu - \frac{1}{2}\eta^{\mu\nu}\right) + \frac{2\varepsilon_0 e^2}{\rho^3}\left[A_N V^\mu V^\nu - V^{(\mu}\left(U^{\nu)} A_N + A^\nu\right)\right] + \frac{\varepsilon_0 e^2}{\rho^2} V^\mu V^\nu \left(A_N^2 - g^2\right). \tag{4.8}$$

Here, $V^\mu = N^\mu + U^\mu$, and the symmetrization bracket around the upper indices is defined by $a^{(\mu} b^{\nu)} = (1/2)(a^\mu b^\nu + a^\nu b^\mu)$.

Let Σ represent a spherical surface in the instantaneous inertial rest frame of the charge at the point Q. We shall calculate the rate of change of electromagnetic field energy inside this surface in the limit of a very large radius so that the surface is in the wave zone of the charge. Energy-momentum conservation can be expressed in the following way: the rate of change of electromagnetic energy-momentum inside this surface is equal to the flux of electromagnetic energy-momentum through the surface,

$$\frac{dP^\mu}{d\tau} = -\lim_{\Sigma\to\infty}\frac{d}{d\tau}\int_\Sigma T^{\mu\nu} d^3\sigma_\nu = -\lim_{\Sigma\to\infty}\frac{d}{d\tau}\int_\Sigma T^{\mu\nu} N_\nu d\tau \rho^2 d\Omega. \tag{4.9}$$

Inserting expression (4.8) gives

$$\frac{dP^\mu}{d\tau} = \varepsilon_0 e^2 \lim_{\Sigma\to\infty}\int_\Sigma V^\mu\left(g^2 - A_N^2\right) d\Omega = \frac{8\pi\varepsilon_0}{3} e^2 g^2 U^\mu. \tag{4.10}$$

This 4-vector is the total rate of energy and momentum emission in form of radiation. The Lorentz invariant emitted power is

$$P_L = -U_\mu \frac{dP^\mu}{d\tau} = \frac{8\pi\varepsilon_0}{3} e^2 g^2 \tag{4.11}$$

in agreement with Larmor's formula (1.14).

Rohrlich [20] then showed that the radius of the spherical surface Σ can be chosen to be small. The sphere need not be in the wave zone of the charge. This was demonstrated in the following way. The rate of energy and momentum which crosses the surface Σ in the direction N^μ per unit solid angle is

$$\frac{dP^\mu}{d\tau d\Omega} = -\frac{\varepsilon_0 e^2}{2}\frac{N^\mu}{\rho^2} + \frac{\varepsilon_0 e^2}{\rho}\left(A^\mu - A_N N^\mu\right) + \varepsilon_0 e^2 V^\mu\left(g^2 - A_N^2\right). \tag{4.12}$$

This quantity is a 4-vector. The first two terms are both spacelike vectors and may be interpreted as the Coulomb 4-momentum and the cross-term between Coulomb and radiation fields. *The last term is a null vector and describes pure radiation. It is independent of* ρ. Since the two first terms are orthogonal to the velocity vector, we get

$$-U^{\mu}\frac{dP^{\mu}}{d\tau d\Omega} = \varepsilon_0 e^2\left(g^2 - A_N^2\right). \tag{4.13}$$

This is the invariant formulation of Poynting's formula for the radiation energy rate per unit solid angle. Because this expression is independent of ρ, it follows that the radiated power through the surface Σ is given by

$$P_L = \frac{d}{d\tau}\int_{\Sigma} T^{\mu\nu} d^3\sigma_{\nu} = \frac{8\pi\varepsilon_0}{3}e^2 g^2, \tag{4.14}$$

that is, we do not need to take the limit of an infinitely large radius of the surface Σ. This result permits one to establish a criterion for testing whether a charge is emitting radiation at a given instant, by measuring the fields only and without having to do so at a distance large compared to the emitted wave length.

The criterion for radiation is as follows: given the world line of a charge and an arbitrary instant τ_0 on it. Consider a sphere Σ of *arbitrary radius* r in the instantaneous inertial rest system S_0 of the charge at the proper time τ_0 with center at the charge at that instant. Measure the electromagnetic fields $F^{\mu\nu}$ on Σ at the time $\tau_0 + r$ and evaluate the integral

$$P_L(\tau_0) = \int T^{0k} N_k r^2 d\Omega = \int_{\Sigma} \mathbf{S}\cdot\mathbf{N} d^2\sigma, \tag{4.15}$$

where $\mathbf{N}$ is the Poynting vector. The value of this integral is the Lorentz invariant rate of radiation energy P_L at time τ_0 and vanishes if and only if the charge did not radiate at that instant.

Ginzburg [21] has pointed out that according to this definition a uniformly accelerated charge radiates although there is no wave zone in this case, and it is not suitable, then, to speak about the appearance of photons.

It should be noted that an accelerated observer may very well measure a different rate of radiation from that given in (4.15) or even no radiation at all. The concept of radiation emitted from a charged particle as measured by an accelerated observer will be described in Section 4.4.

4.2. On the Concept "Gravitational Field"

The principle of equivalence is usually stated as follows: the physical effects of a homogeneous gravitational field due to a mass distribution are equivalent to the physical effects of the artificial gravitational field in an accelerated reference frame. However, Hammond [6] writes that in the general theory of relativity the curvature of spacetime replaces the Newtonian concept of a gravitation field. For a uniformly accelerated frame (in fact for any accelerated frame in Minkowski spacetime) the curvature tensor vanishes: there is no gravitational field.

However, in applications of the principle of equivalence it is necessary to distinguish between the tidal and the nontidal components of a gravitational field. This distinction was made clear by Grøn and Vøyenli [22, 23], and a related discussion of the concept "gravitational field" has been given by Brown [24].

The distinction between a tidal and a nontidal gravitational field is based on the geodesic equation and the equation of geodesic deviation. Consider two nearby points P_0 and P in spacetime and two geodesics, one passing through P_0 and one through P. Let $\mathbf{n}$ be the distance vector between P_0 and P. The geodesics are assumed to be parallel at P_0 and P, so that $(d\mathbf{n}/d\tau)_{P_0} = 0$. Using (53) of [25] we find that a Taylor expansion about the point P_0 gives the following formula for the acceleration of a free particle at P:

$$\left(\frac{d^2x^i}{d\tau^2}\right)_P = -\left(\Gamma^i_{\alpha\beta}u^\alpha u^\beta\right)_{P_0} - \left(\Gamma^i_{\alpha\beta,\gamma}u^\alpha u^\beta\right)_{P_0} n^\gamma. \tag{4.16}$$

The first term at the right hand side represents the acceleration of a free particle at P_0 and contains, for example, the centrifugal acceleration and the Coriolis acceleration in a rotating reference frame.

We define the gravitational field strength at the point P, $\mathbf{g}$, as the acceleration of a free particle instantaneously at rest. Then, the spatial components of the four-velocity vanish. Using the proper time of the particle as time coordinate gives $u^0 = 1$, and (4.16) simplifies to

$$g^i = -\left(\Gamma^i_{00}\right)_{P_0} - \left(\Gamma^i_{00,\gamma}\right)_{P_0} n^\gamma. \tag{4.17}$$

Grøn and Vøyenli [22] have shown that in a stationary metric this equation can be written as

$$g^i = -\Gamma^i_{00} + \left(\Gamma^k_{00}\,\Gamma^i_{jk} - \Gamma^\sigma_{0j}\,\Gamma^i_{\sigma 0}\right)n^j - R^i_{0j0}\,n^j, \tag{4.18}$$

where the Christoffel symbols and the components of the Riemann curvature tensor are evaluated at the point P_0. The first term at the right hand side of this equation represents the acceleration of gravity at the point P_0, that is, it represents the uniform part of the gravitational field. The second term represents the nonuniform part of the gravitational field, which is also present in a noninertial reference frame in flat spacetime, for example, the nonuniformity of the centrifugal field in a rotating reference frame. The last term represents the tidal effects, which in the general theory are proportional to the spacetime curvature.

This suggests the following separation of a gravitational field into a nontidal part and a tidal part

$$g^i = g^i_{\mathrm{NT}} + g^i_{\mathrm{T}}, \tag{4.19}$$

where the non-tidal part is given by

$$g^i_{\mathrm{NT}} = -\Gamma^i_{00} + \left(\Gamma^k_{00}\,\Gamma^i_{jk} - \Gamma^\sigma_{0j}\,\Gamma^i_{\sigma 0}\right)n^j \tag{4.20}$$

and the tidal part by

$$g^i_{\mathrm{T}} = -R^i_{0j0}\,n^j. \tag{4.21}$$

The non-tidal gravitational field can be transformed away by going into a local inertial frame. The tidal gravitational field cannot be transformed away. The mathematical expression of these properties is that the non-tidal gravitational field is given by Christoffel symbols, and they are not tensor components. All of them can be transformed away. But the tidal gravitational field is given in terms of the Riemann curvature tensor of spacetime, which cannot be transformed away.

A gravitational field caused by a mass distribution may be called a *permanent gravitational field*. In general such a field has both a tidal and a non-tidal component. However, the gravitational fields experienced in rotating or accelerated reference frames in flat spacetime are purely non-tidal.

With this background, making it clear that there exists a gravitational field in a uniformly accelerated reference frame in flat spacetime, a brief review of earlier works on the compatibility of the equivalence principle with the invariance or noninvariance of radiation produced by an accelerated charge will now be presented.

4.3. Previous Discussions on Radiation by an Accelerated Charge and the Principle of Equivalence

The present discussion will be based on the following formulation of the principle of equivalence. According to the principle of equivalence the physical effects of the non-tidal gravitational field in a noninertial frame of reference are equivalent to the effects of the non-tidal component of a permanent gravitational field caused by a mass distribution. The adjective "non-tidal" means that the principle of equivalence does only have a local validity. It is valid in a region of spacetime sufficiently small that tidal effects of a permanent gravitational field cannot be measured with the equipment of the laboratory. Hence, according to the principle of equivalence in such a laboratory one cannot perform any experiment making it possible to decide, for example, whether one is in an accelerated laboratory in space or in a laboratory at rest on the surface of the Earth.

In 1955 Bondi and Gold [26] discussed radiation from a uniformly accelerated charge in relation to the principle of equivalence. They showed that a uniformly accelerating charge radiates and then wrote: "The principle of equivalence states that it is impossible to distinguish between the action on a particle of matter of a constant acceleration or of static support in a gravitational field. This might be thought to raise a paradox when a charged particle, statically supported in a gravitational field, is considered, for it might be thought that a radiation field is required to assure that no distinction can be made between the cases of gravitation and acceleration—But there can be no static radiation field, and as the whole system is static the electromagnetic field cannot depend upon time." They resolved this apparent contradiction by arguing that hyperbolic motion requires a homogeneous gravitational field of infinite extension and that such a field does not exist in nature.

In 1960 Fulton and Rohrlich [15] took up the discussion whether the existence of radiation from a uniformly accelerated charge is in conflict with the principle of equivalence. First they performed a calculation based on Maxwell's equations showing that a uniformly accelerated charge radiates. Then they used the LAD-equation to show that in spite of the vanishing radiation reaction upon a uniformly accelerating charge this is not in conflict with energy conservation. Finally they discussed the principle of equivalence and wrote: "A particle which is falling freely in a homogeneous gravitational field should appear to an observer who is falling with it, like a particle at rest in an inertial frame. When the particle is charged, the observer can establish the presence of a gravitational field by looking for radiation. If he observes radiation from the charge, he knows that he and the charge are falling in a gravitational field; if he observes no radiation, he knows that he and the particle are in a force free region of space."

Fulton and Rohrlich solved the problem in a similar way as Bondi and Gold, noting that radiation can only be determined far from the emitting charge and that the principle of equivalence has only a local validity, and hence the problem disappears.

The problem discussed by Bondi and Gold and by Fulton and Rohrlich and the way they solved it indicate that at this time it was an underlying assumption that the existence of radiation from an accelerated charge is invariant against a transformation between an accelerated and an inertial reference frame, and hence that an observer in the accelerated frame for whom the charge is permanently at rest will measure radiation from the charge.

Rohrlich [27] presented a discussion of the problem in 1963 containing a breakthrough in the understanding of the invariance properties of electromagnetic radiation. He calculated the electromagnetic field of a charge at rest in a uniformly accelerated reference frame (UAF) and found a purely electric static field. An observer at rest in AF experiences a static homogeneous gravitational field (SHGF). Rohrlich thus concluded that a *charge at rest in a SHGF does not radiate*. As observed from an inertial frame this charge is accelerated and radiates in agreement with Larmor's formula. Hence, the next conclusion is that, the existence of radiation from an accelerated charge is *not* invariant against a transformation from an inertial frame to the accelerated permanent rest frame of the charge.

Rohrlich then referred to Rosen's [28] calculation of the electromagnetic field of a freely falling charge in a SHGF, which showed that the freely falling charge emits radiation. As observed from a comoving observer this charge is permanently at rest in an inertial frame and obviously does not radiate. So the next conclusion is that the existence of radiation from a freely falling charge is not invariant against a transformation from the accelerated frame with a SHGF to the inertial rest frame of the charge. Rohrlich noted that since *radiation is not a generally invariant concept* the question whether the charge *really* radiates is meaningless unless it is referred to a particular coordinate system. He also emphasized that in this connection a definition of radiation that does not depend on taking the wave-zone limit is essential [20] and concluded: "These considerations are important in that they confirm the validity of the principle of equivalence also for charged test particles."

Similar results were obtained by Kovetz and Tauber [29] in 1969. But instead of defining a SHGF as that in a uniformly accelerated reference frame in flat spacetime, they defined it by taking a local limit of the Schwarzschild spacetime. Their results agree with those of Rohrlich: a charge falling freely in a SHGF radiates as measured by a supported observer but not as measured by a comoving freely falling observer, and a supported charge in a SHGF radiates as measured by a freely falling observer but not as measured by a supported one.

In the same year Ginzburg [21] presented a thorough discussion of the relationship of the non-invariance of radiation and the principle of equivalence. He arrived at the same results as Rohrlich and Kovetz and Tauber, and he also discussed the physical meaning of the non-invariance of radiation from the point of view of the photon picture of radiation. Similar results were later deduced by direct calculation, essentially using Rohrlich's definition of radiation, by Piazzese and Rizzi [30], by Ginzburg and Ya Eroshenko [31], and by Shariati and Khorrami [32]. Matsas [33] obtained related results in an analysis based upon the photon picture of radiation.

By studying the electromagnetic field associated with a uniformly accelerated charge Boulware [34] made in 1980 an important discovery that he summarized in the following way: "The equivalence principle paradox that the co-accelerating observer measures no radiation while a freely falling observer measures the standard radiation of an accelerated charge is resolved by noting that all the radiation goes into the region of space time inaccessible to the co-accelerating observer." The same result was later deduced in a slightly simpler way by de Almeida and Saa [35].

There have been some later works where these results have not been accepted [36–39]. They seem to be based upon a different conception of radiation associated with the wave zone and a possible existence of photons, leading to a generally invariant character of radiation. However, the main stream definition of radiation is that of Rohrlich [20], making radiated effect Lorentz invariant, but not generally invariant, which is the definition adopted in here. On the other hand Singal [36] argued that neither a freely falling charge nor a charge at rest in a SHGF is observed to radiate irrespective of the state of motion of the observer, and Fabbri [38] concluded similarly for a freely falling charge. These results were countered by Parrott [40]. Furthermore, Logunov et al. [37] claimed: "A charge at rest in an inertial system cannot radiate, and this assertion does not depend on the system of coordinates in which the charge is considered. It is for this reason that the charge emits no radiation in the accelerated system either." Concerning this Ginzburg et al. [41] wrote: "A charge may be considered to emit radiation if the flow of the Poynting vector across the surface surrounding the charge differs from zero. Such a definition, which is reasonable and justified, implies that a uniformly accelerated charge may radiate although there is no wave zone in this case, and one cannot speak about the appearance of photons. Logunov and co-workers understand by radiation only the field in the wave zone." Finally Lyle [39] has written a book on this problem, but he too seems implicitly to have defined radiation in the same way as Logunov and coworkers and hence is outside the main stream physics on these matters.

4.4. Mathematical Expression of the Noninvariance of Radiation

According to Larmor's formula (1.14), the effect radiated from an accelerated charge is proportional to the scalar product of the 4-acceleration of the charge with itself. This is a scalar and should be invariant against arbitrary coordinate transformations. However, the components of the 4-acceleration written with capital letters are assumed to refer to an inertial frame. Hence, when one is going to write down a formula for the effect radiated by an accelerated charge, which is to be valid also in an accelerated frame, Larmor's formula has to be generalized. Such generalizations have been given by Kretzschmar and Fugmann [42, 43] and by Hirayama [44, 45].

Following Hirayama we shall write down Larmor's formula in a uniformly accelerated reference frame. Using the Rindler coordinates the line element in the uniformly accelerated reference frame takes the form

$$ds^2 = -g^2x^2dt^2 + dx^2 + dy^2 + dz^2. \tag{4.22}$$

Here g is the acceleration of gravity in the uniformly accelerated reference frame defined in the same way as the surface gravity of the Kerr-Newman spacetime, namely as $\ddot{x}/u^t$, where $\ddot{x}$ is the acceleration of a free particle instantaneously at rest as given by the geodesic equation, $\ddot{x} = -\Gamma^x_{tt}\dot{t}^2$. Calculating the Christoffel symbol from the metric (4.22) gives $\Gamma^x_{tt} = g^2x$. Using that the four-velocity of a particle instantaneously at rest is $u^t = \dot{t} = (-g_{tt})^{-1/2} = 1/gx$, we obtain $\ddot{x}/u^t = (-g_{tt})^{-1/2}\Gamma^x_{tt} = g$.

The 4-velocity and 4-acceleration of a charged particle moving along the x-axis in the Rindler frame are

$$\begin{aligned} v^\mu &= \frac{dx^\mu}{d\tau} = \gamma(1, v, 0, 0), \quad \gamma = \left(g^2x^2 - v^2\right)^{-1/2}, \\ a^\mu &= \frac{dv^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta}v^\alpha v^\beta = \gamma^4\left(a + g^2x - \frac{2v^2}{x}\right)\left(v, g^2x, 0, 0\right), \end{aligned} \tag{4.23}$$

where $v = dx/dt$ and $a = dv/dt$. Inserting $v = a = 0$, we find that the 4-velocity and 4-acceleration of the reference particles in the Rindler frame are

$$u^\mu = \left(\frac{1}{gx}, 0, 0, 0\right), \qquad g^\mu = \left(0, \frac{1}{x}, 0, 0\right). \tag{4.24}$$

Calculating the acceleration scalar of the reference particles, we find that it is equal to the acceleration of gravity in the Rindler frame, $(g_\mu g^\mu)^{1/2} = g$. The projection tensor on the plane orthogonal to the world line of the charged particle has components

$$h^\mu_\nu = \delta^\mu_\nu + v_\nu v^\mu. \tag{4.25}$$

Hirayama [44] defines an acceleration 4-vector

$$\alpha^\mu = h^\mu_\nu(a^\nu - g^\nu - gu^\nu). \tag{4.26}$$

At a moment, $\tau = \tau_0$, when the particle is at rest, we have that $v^\mu = u^\mu$. Hence, at this moment the Hirayama acceleration vector reduces to

$$\alpha^\mu(\tau_0) = a^\mu(\tau_0) - g^\mu. \tag{4.27}$$

This shows that α^μ represents the acceleration of a particle instantaneously at rest relative to an observer permanently at rest in the Rindler frame. It will be called *the Hirayama 4-acceleration.*

The generalized formula for the radiated effect valid in uniformly accelerated frames as well as in inertial ones is

$$P_{RU} = m_0\tau_0\left(1 + \frac{gx}{c^2}\right)^2 \alpha^\mu\alpha_\mu. \tag{4.28}$$

This is not an ad hoc definition but based upon a thorough analysis given in [46] of the near field and far field produced by an accelerated charge as described from a uniformly accelerated reference frame.

4.5. The Freely Falling Charge

For a charge that falls freely from a position x_0 in a uniformly accelerated reference frame one finds for the position and the velocity

$$1+\frac{gx}{c^2}=\frac{1+gx/c^2}{\cosh(gt/c)}, \qquad v=-g\left(1+\frac{gx}{c^2}\right)^2\frac{\sinh(gt/c)}{\cosh^2(gt/c)}. \tag{4.29}$$

The components of the Hirayama 4-acceleration are

$$\alpha^t=\frac{e^{gt/c}\sinh(gt/c)\cosh^2(gt/c)}{g\left(1+gx_0/c^2\right)^2}, \qquad \alpha^x=-\frac{e^{gt/c}\cosh^2(gt/c)}{1+gx_0/c^2}. \tag{4.30}$$

Hence, the acceleration scalar of the charges particle is

$$\alpha^\mu\alpha_\mu=\frac{e^{2gt/c}\cosh(gt/c)}{\left(1+gx_0/c^2\right)^2}. \tag{4.31}$$

Inserting this into the generalized formula for the radiated effect in the uniformly accelerated reference frame (4.24) gives

$$P_{RU}=m_0\tau_0 g^2 e^{2gt/c}. \tag{4.32}$$

The radiated energy is

$$E_R=\int_0^t P_{RU}\,dt=\left(\frac{m_0\tau_0}{2}\right)g\left(e^{2gt/c}-1\right). \tag{4.33}$$

The Schott energy is given by

$$E_S=-m_0\tau_0 v\alpha^x. \tag{4.34}$$

Inserting the expressions for v and α^x from (4.29) and (4.30), respectively, gives

$$E_S=-\left(\frac{m_0\tau_0}{2}\right)g\left(e^{2gt/c}-1\right)=-E_R. \tag{4.35}$$

Hence, for a freely falling charged particle all of the radiated energy comes from the Schott energy.

5. An Electromagnetic Perpetuum Mobile?

Consider a charged particle moving freely in a circular path around the Earth. It has vanishing 4-acceleration. Hence the Abraham 4-vector vanishes, too. This means that it moves along a closed geodesic curve just like a neutral particle. It radiates in accordance with the Larmor formula. This is like a perfect perpetuum mobile [47]. One seems to get radiation energy for nothing from a system working cyclically.

The solution to this problem is that the LAD-equation is valid only in flat spacetime. The LAD-equation was generalized to an equation of motion for radiating charges in curved space by DeWitt and Brehme [48] and by Hobbs [49]. There is a large amount of recent literature on the equation of motion of a charged particle in curved spacetime. An eminent review with a large number of references has been given by Poisson et al. [50].

The generalized equation was applied to a particle moving freely in curved space by B. S. DeWitt and C. M. DeWitt [51]. They found that in the weak field and slow motion approximation the force upon the charge due to the curvature of space can be separated in a conservative part, $\mathbf{f}_C$, and a nonconservative part, $\mathbf{f}_{NC}$. Hence the equation of motion of a radiating charge moving freely in curved space takes the form

$$m_0 \ddot{\mathbf{r}} = \mathbf{f}_{ext} + \mathbf{f}_C + \mathbf{f}_{NC}. \tag{5.1}$$

For a particle moving slowly far outside the Schwarzschild radius of a mass m_0 B. S. DeWitt and C. M. DeWitt found the expressions

$$\begin{aligned} \mathbf{f}_C &= \left(\frac{3}{2}\right) m_0 \tau_0 G M \left(\mathbf{r}/r^4\right), \\ \mathbf{f}_{NC} &= -m_0 \tau_0 G M \left(\frac{\dot{\mathbf{r}}}{r^3} - 3\frac{\mathbf{r}\cdot\dot{\mathbf{r}}}{r^5}\mathbf{r}\right). \end{aligned} \tag{5.2}$$

B. S. DeWitt and C. M. DeWitt write that $\mathbf{f}_C$ arises from the fact that the total mass of the particle is not concentrated at a point but is partly distributed as electric field energy in the space around the particle. The non-conservative force $\mathbf{f}_{NC}$ arises from a back-scatter process in which the Coulomb field of the particle, as it sweeps over the “bumps” in spacetime, receives “jolts” that are propagated back to the particle. This force gives rise to radiation damping even for a charge moving freely in curved spacetime. Hence, the radiated effect comes from the work performed by this force.

Therefore, the radiated effect should be equal to the work performed by this force per unit time,

$$P = \mathbf{f}_{NC} \cdot \dot{\mathbf{r}}. \tag{5.3}$$

The motion of the particle is given by

$$\mathbf{r} = r\mathbf{e}_r, \qquad \mathbf{v} = \dot{\mathbf{r}} = r\omega\mathbf{e}_\theta, \qquad \mathbf{a} = \dot{\mathbf{v}} = -r\omega^2\mathbf{e}_r, \qquad \dot{\mathbf{a}} = -r\omega^3\mathbf{e}_\theta, \tag{5.4}$$

where $a = |\mathbf{a}| = GMr^{-2}$. Hence, according to Larmor’s formula the radiated effect is

$$P = \frac{m_0 \tau_0 G^2 M^2}{r^4}. \tag{5.5}$$

With circular motion the velocity is orthogonal to the radius vector, $\mathbf{r} \cdot \dot{\mathbf{r}} = 0$. Then, (5.3) reduces to

$$\mathbf{f}_{\mathrm{NC}} = m_0 \tau_0 G M \frac{\dot{\mathbf{r}}}{r^3}. \tag{5.6}$$

The work performed by unit time by this force is

$$P_{\mathrm{NC}} = \mathbf{f}_{\mathrm{NC}} \cdot \dot{\mathbf{r}} = -m_0 \tau_0 G M \frac{\dot{\mathbf{r}} \cdot \dot{\mathbf{r}}}{r^3} = -\frac{m_0 \tau_0 G^2 M^2}{r^4}, \tag{5.7}$$

which is minus the radiated effect given in (5.5). This means that due to the non-conservative part of the force due to the curvature of spacetime the charged particle loses energy at the same rate as it radiates energy, and hence energy is conserved.

6. Conclusion

The Schott energy, which is proportional to the product of the velocity and acceleration of the charge, is essential for conservation of energy of a radiating charge. It provides the energy radiated by a uniformly accelerated charge. The Schott energy is increasingly negative if the acceleration has the same direction as the velocity.

The Schott energy can be interpreted as the difference between the bound field energy the charge would have had if it had moved with its current velocity for its entire history and its actual bound field energy.

The existence of electromagnetic radiation is not invariant against a transformation involving acceleration. A freely falling charge radiates as observed by a stationary observer, but not by an inertial observer comoving with the charge.

The usual radiation reaction force vanishes for a charge moving freely in curved spacetime. However, it is replaced by a force due to the curvature of space, which accounts for the energy radiated by the particle.

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References

[1] H. A. Lorentz, *The Theory of Electrons*, based upon lectures given by Lorentz at Columbia University in 1906, Dover, New York, NY, USA, 1952.

[2] M. Abraham, "Zur Theorie der Strahlung und des Strahlungsdruckes," *Annalen der Physik 14*, pp. 236–287, 1904.

[3] P. A. M. Dirac, "Classical theory of radiating electrons," *Proceedings of the Royal Society of London*, vol. 167, pp. 148–169, 1938.

[4] D. V. Gal'tsov and P. A. Spirin, "Radiation reaction reexamined: bound momentum and the Schott term," *Gravitation & Cosmology*, vol. 12, no. 1, pp. 1–10, 2006.

[5] C. Teitelboim, "Splitting of the Maxwell tensor: radiation reaction without advanced fields," *Physical Review*, vol. 1, pp. 1572–1582, 1970.

[6] R. T. Hammond, "Relativistic particle motion and radiation reaction in electrodynamics," *Electronic Journal of Theoretical Physics*, vol. 6, no. 23, pp. 221–258, 2010.

[7] E. Eriksen and Ø. Grøn, "Electrodynamics of hyperbolically accelerated charges. I. The electromagnetic field of a charged particle with hyperbolic motion," *Annals of Physics*, vol. 286, no. 2, pp. 320–342, 2000.

[8] E. Eriksen and Ø. Grøn, "Electrodynamics of hyperbolically accelerated charges. II. Does a charged particle with hyperbolic motion radiate?" *Annals of Physics*, vol. 286, no. 2, pp. 343–372, 2000.

[9] E. Eriksen and Ø. Grøn, "Electrodynamics of hyperbolically accelerated charges. III. Energy-momentum of the field of a hyperbolically moving charge," *Annals of Physics*, vol. 286, no. 2, pp. 373–399, 2000.

[10] E. Eriksen and Ø. Grøn, "Electrodynamics of hyperbolically accelerated charges. IV. Energy-momentum conservation of radiating charged particles," *Annals of Physics*, vol. 297, no. 2, pp. 243–294, 2002.

[11] D. R. Rowland, "Physical interpretation of the Schott energy of an accelerating point charge and the question of whether a uniformly accelerating charge radiates," *European Journal of Physics*, vol. 31, no. 5, pp. 1037–1051, 2010.

[12] G. A. Schott, "On the motion of the lorentz electron," *Philosophical Magazine*, vol. 29, pp. 49–69, 1915.

[13] F. Rohrlich, "The equations of motion of classical charges," *Annals of Physics*, vol. 13, pp. 93–109, 1961.

[14] P. Pearle, "Classical electron models," in *Electromagnetism*, T. Tepliz, Ed., pp. 211–295, Plenum, New York, NY, USA, 1982.

[15] T. Fulton and F. Rohrlich, "Classical radiation from a uniformly accelerated charge," *Annals of Physics*, vol. 9, pp. 499–517, 1960.

[16] E. Eriksen and O. Grøn, "Does preradiation exist?" *Physica Scripta*, vol. 76, no. 1, article 012, pp. 60–63, 2007.

[17] E. Eriksen and O. Grøn, "The significance of the Schott energy in the electrodynamics of charged particles and their fields," *Indian Journal of Physics*, vol. 82, no. 9, pp. 1113–1137, 2008.

[18] Ø. Grøn, "The significance of the Schott energy for energy-momentum conservation of a radiating charge obeying the Lorentz-Abraham-Dirac equation," *American Journal of Physics*, vol. 79, no. 1, pp. 115–122, 2011.

[19] M. Pauri and M. Vallisneri, "Classical roots of the Unruh and Hawking effects," *Foundations of Physics*, vol. 29, no. 10, pp. 1499–1520, 1999.

[20] F. Rohrlich, "The definition of electromagnetic radiation," *Nuovo Cimento*, vol. 21, pp. 811–822, 1961.

[21] V. L. Ginzburg, "Radiation and radiation friction force in uniformly accelerated motion of a charge," *Soviet Physics Uspekhi*, vol. 12, pp. 565–574, 1970.

[22] Ø. Grøn and K. Vøyenli, "On the foundation of the principle of relativity," *Foundations of Physics*, vol. 29, no. 11, pp. 1695–1733, 1999.

[23] Ø. Grøn, "On the relativity of rotation," *Nuovo Cimento B*, vol. 125, no. 7, pp. 861–874, 2010.

[24] P. M. Brown, "Einstein's gravitational field," physics 0204044.

[25] H. Ohanian and R. Ruffini, *Gravitation and Spacetime*, Norton, New York, NY, USA, 1994.

[26] H. Bondi and T. Gold, "The field of a uniformly accelerated charge, with special reference to the problem of gravitational acceleration," *Proceedings of the Royal Society London*, vol. 229, pp. 416–424, 1955.

[27] F. Rohrlich, "The principle of equivalence," *Annals of Physics*, vol. 22, pp. 169–191, 1963.

[28] N. Rosen, "Field of a particle in uniform motion and uniform acceleration," *Annals of Physics*, vol. 17, pp. 269–275, 1962.

[29] A. Kovetz and G. E. Tauber, "Radiation from an accelerated charge and the principle of equivalence," *American Journal of Physics*, vol. 37, pp. 382–385, 1969.

[30] F. Piazzese and G. Rizzi, "Does a charge, falling freely in a static gravitational field, radiate?" *Meccanica*, vol. 20, no. 3, pp. 199–206, 1985.

[31] V. L. Ginzburg and N. Yu. Eroshenko, "Once again about the equivalence principle," *Physics-Uspekhi*, vol. 33, pp. 195–201, 1995.

[32] A. Shariati and M. Khorrami, "Equivalence principle and radiation by a uniformly accelerated charge," *Foundations of Physics Letters*, vol. 12, no. 5, pp. 427–439, 1999.

[33] G. E. A. Matsas, “Do inertial electric charges radiate with respect to uniformly accelerated observers?” *General Relativity and Gravitation*, vol. 26, no. 11, pp. 1165–1169, 1994.
[34] D. G. Boulware, “Radiation from a uniformly accelerated charge,” *Annals of Physics*, vol. 124, no. 1, pp. 169–188, 1980.
[35] C. De Almeida and A. Saa, “The radiation of a uniformly accelerated charge is beyond the horizon: a simple derivation,” *American Journal of Physics*, vol. 74, no. 2, pp. 154–158, 2006.
[36] A. K. Singal, “The equivalence principle and an electric charge in a gravitational field,” *General Relativity and Gravitation*, vol. 27, no. 9, pp. 953–967, 1995.
[37] A. A. Logunov, M. A. Mestvirishvili, and Y. V. Chugreev, “On incorrect formulations of the equivalence principle,” *Physics-Uspekhi*, vol. 39, no. 1, pp. 73–79, 1996.
[38] L. Fabbri, “Free falling electric charge in a static homogeneous gravitational field,” *Annales de la Fondation Louis de Broglie*, vol. 30, no. 1, pp. 87–95, 2005.
[39] S. Lyle, *Uniformly Accelerating Charged Particles: A Threat To the Equivalence Principle*, Springer, New York, NY, USA, 2008.
[40] S. Parrott, “Radiation from a charge uniformly accelerated for all time,” *General Relativity and Gravitation*, vol. 29, no. 11, pp. 1463–1472, 1997.
[41] V. L. Ginzburg, N. Yu. Eroshenko, M. A. Mestvirishvili, and V. Y. Chugreev, “On incorrect formulation of the equivalence principle,” *Physics-Uspekhi*, vol. 39, pp. 81–82, 1996.
[42] M. Kretzschmar and W. Fugmann, “The electromagnetic field of an accelerated charge in the proper reference frame of a noninertial observer,” *Il Nuovo Cimento*, vol. 103, no. 4, pp. 389–412, 1989.
[43] W. Fugmann and M. Kretzschmar, “Classical electromagnetic radiation in noninertial reference frames,” *Il Nuovo Cimento B*, vol. 106, no. 4, pp. 351–373, 1991.
[44] T. Hirayama, “Bound and radiation fields in the Rindler frame,” *Progress of Theoretical Physics*, vol. 106, no. 1, pp. 71–97, 2001.
[45] T. Hirayama, “Classical radiation formula in the Rindler frame,” *Progress of Theoretical Physics*, vol. 108, no. 4, pp. 679–688, 2002.
[46] E. Eriksen and Ø. Grøn, “Electrodynamics of hyperbolically accelerated charges. V. The field of charge in the Rindler space and the Milne space,” *Annals of Physics*, vol. 313, no. 1, pp. 147–196, 2004.
[47] Ø. Grøn and S. K. Næss, “An electromagnetic perpetuum mobile? ,” ArXiv: 0806.0464.
[48] B. S. DeWitt and R. W. Brehme, “Radiation damping in a gravitational field,” *Annals of Physics*, vol. 9, pp. 220–259, 1960.
[49] J. M. Hobbs, “A vierbein formalism of radiation damping,” *Annals of Physics*, vol. 47, no. 1, pp. 141–165, 1968.
[50] E. Poisson, A. Pound, and I. Vega, “The motion of point particles in curved spacetime,” *Living Reviews in Relativity*, vol. 7, p. 1, 2004.
[51] B. S. DeWitt and C. M. DeWitt, “Falling charges,” *Physics 1*, pp. 3–20, 1964.