TOY QUANTUM MECHANICS USING HIDDEN VARIABLES

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An original model of toy quantum mechanics that uses hidden variables but does not violate the well-known Bell theorem is proposed.

1. Introduction

It is used to state that no local theory assuming any kind of evolving-in-time physical field (hidden variables) can reproduce the same predictions as standard quantum mechanics. Bell formulated a corresponding theorem in [1]. An advanced variant of the theorem was proposed in [2]. Feynman, in [3], stated another but analogous idea: not any classical computational device (even when probabilistic algorithms are used) can *exactly* reproduce evolution of a quantum system.

Here, an original model of a toy quantum particle which *does not violate* the theorems in [1, 2] but nevertheless has *hidden variables* is proposed. The reasons of that are explained at the end of the paper.

2. What is toy particle?

The words "toy particle" and "toy quantum mechanics" mean that the model does not describe real physical systems but only demonstrates *principal possibility* to build corresponding theory. It is very important to understand this.

Toy particle represents a single property of real quantum particles: it can make transition from one *registered localized state* to another. The field of hidden variables exists in every point of space, but experiments register only two spatially separated events: radiation of a particle and its fall into registering device.

In other words, the proposed model is *qualitative*; it is of the same class as the famous model of coupled vortexes by Maxwell [5]. (That model was the predecessor of Maxwell's equations of classical electromagnetic field.)

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3. Evolution of a toy quantum particle

The model uses cellular automata (CA), which are traditional instruments for soft modeling. In the case of one spatial dimension, CA is a line of identical cells. The rule that governs the change of a cell's state is local, that is, it takes into account only a few neighbors of a cell.

We assume a one-dimensional CA, the cell states of which belong to the following set: $f_i \in \{$"0"; "1"; "e"; "→"; "←"; "<"; ">"$\}$. These symbols mean the following:

(0) no localized particle in the cell;
(1) a localized particle *may be* in the cell;
(e) a particle expands or diffuses over the space;
(→) a query to the right;
(←) a query to the left;
(>) a refuse to the right;
(<) a refuse to the left.

The change of cells' states ticks synchronously in CA by tradition (since J. von Neumann's times). Our CA violates this tradition. Each elementary step of CA evolution consists of a random choice of a pair of neighboring cells, which change their states. The cells in a pair are not equal: one of them is assumed as *leader*, while the second is *driven*. The leader is randomly selected first, then the driven cell. Such an asynchronous work of CA cells seems more physically realistic than von Neuman's scheme.

Full and formal description of the CA transition rules is in [4], but it is better and easier to understand these rules following the examples of evolution below.

In the beginning, the single particle of our toy world is localized. The CA space looks like this: ...0001000.... *Outer subject* forces a transition "1" → "e": ...000e000. Using the outer subject means only that devices radiating and detecting the particle are macroscopic unstable systems. Their description is *external* in principle for microscopic model.

The sign "e" means "expand"—the particle is delocalized and it begins to "diffuse" in space. *Diffusion* or *expansion* sign expands in both sides with average velocity 1/2 site per time step: ...00$eeee$000....

Each space point with "e" sign can be reset, *also from outside*, to "1," "e" → "1": ...000eee1eee1ee1eee000....

Thus, a number of different cells, *pretending* to be the final localization of the particle, appear in the space. Physically *pretender* means a particle detector put in the cell.

Just after the pretenders appear in the space, they begin their duel to be the single new location of the particle. This means that cell detectors begin certain signal exchange. They send to the left and to the right *queries for localization* "→" and "←." This is depicted by Figure 3.1.

To make this scheme clear, we have to make the next comment.

(1) When two queries from different pretenders, "→" and "←," collide, one of them converts to opposite refuse signal, that is, "<" or ">." The choice of "loser" is random with probability 1/2. Refuse signal "<"(">") expands in the direction where it shows. While propagating, it erases query signals (and their source, which

$$00ee1eeeeee1eee0 \Longrightarrow 0ee \longleftarrow 1 \longrightarrow eeee \longleftarrow 1 \longrightarrow ee0 \Longrightarrow \cdots \Longrightarrow$$

$$00001 \longrightarrow\longrightarrow\longrightarrow\longleftarrow\longleftarrow\longleftarrow 10000 \Longrightarrow 00001 \longrightarrow\longrightarrow < \longleftarrow\longleftarrow\longleftarrow 10000 \Longrightarrow \cdots \Longrightarrow$$

$$00001 \longrightarrow < \longleftarrow\longleftarrow\longleftarrow\longleftarrow 10000 \Longrightarrow \cdots \Longrightarrow 00001 < \longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow 10000 \Longrightarrow$$

$$0000 \longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow 10000 \Longrightarrow 00000 \longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow 10000 \Longrightarrow \cdots \Longrightarrow$$

$$0000000000 \longleftarrow\longleftarrow 10000 \Longrightarrow \cdots \Longrightarrow 00000000000010000.$$

Figure 3.1. Toy particle evolution in 2-detector case.

$$0e1eeeee1eeee1eee0 \Longrightarrow 0 \longleftarrow 1 \longrightarrow eee \longleftarrow 1 \longrightarrow ee \longleftarrow 1 \longrightarrow ee0 \Longrightarrow \cdots \Longrightarrow$$

$$001 \longrightarrow\longrightarrow\longrightarrow\longleftarrow\longleftarrow 1 \longrightarrow\longrightarrow\longleftarrow\longleftarrow 1 \longrightarrow\longrightarrow 00 \Longrightarrow 001 \longrightarrow\longrightarrow\longrightarrow > \longleftarrow 1 \longrightarrow < \longleftarrow\longleftarrow 10000 \Longrightarrow \cdots$$

$$\Longrightarrow 001 \longrightarrow\longrightarrow\longrightarrow\longrightarrow\longrightarrow\longrightarrow < \longleftarrow\longleftarrow 10000 \Longrightarrow 001 < \longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow 10000 \Longrightarrow$$

$$000 \longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow\longleftarrow 10000 \Longrightarrow \cdots \Longrightarrow \cdots 0001000.$$

Figure 3.2. Toy particle evolution in 3-detector case.

is cell-pretender or cell-detector) and changes them to opposite queries, that is, "←" to "→" and vice versa.

(2) When a query signal "→"/"←" reaches "0" in its propagation, it begins to propagate backwards, leaving "0" in front.

(3) It is very important that "→"/"←" and "<"/">" signals propagate in the single direction (where the arrow shows) with average velocity 1 cell per time step, while expansion sign "*e*" expands in both sides with average velocity 1/2 site per time step. That is why the collapse of the particle *always happens*, regardless of how far "*e*" sign has expanded.

It is obvious from "odd-even" considerations that the final result of pretenders' duel will be exactly a single surviving pretender. The particle is localized again in new point in general. That is also correct when the initial number of pretenders is greater than 2 (Figure 3.2).

Pay attention that refuse signals ">"/"<" never collide with each other but only with "←"/"→" or "1." When refuse signal writes appropriate "←"/"→" over pretender-loser "1," the refuse signal disappears.

4. What about Bell's theorem?

We recall the essence of Bell's theorem about hidden variables. Experiments are under analysis when correlated pairs of particles are radiated from one point [2]. For example, a radiation of two photons in two-cascade decay of excited atom is assumed. Radiated photons fly in opposite directions. They are registered with simultaneous measurement of their spins (polarizations).

Assuming that realistic hidden-variable theories are right, the theorem derives some inequalities involving frequencies of different types of registered events. Fulfillment of these inequalities should mean that hidden variables might exist, while the violation of them means that no hidden variables involved in local dynamics are possible in nature.

In the model described above, we have one rather than two particles, but this does not influence the essence of the question discussed. In Bell's theorem experiments, devices that register particles are characterized by two adjustable parameters a and b. If we deal with photons, these are polarization axes directions for polarizers that stand before photo-electronic multipliers registering photons.

The *central idea* of the theorem's proof (i.e., derivation of correlation inequalities) is that statistical probability distribution $\rho(\lambda)$ of hidden variables λ *does not depend* upon macroscopic parameters a and b.

In our model query, signals "→"/"←" and refuse signals "<"/">" constitute what are referred to as hidden variables, while the cells with "1" are analogs of macroscopic adjustable parameters of detecting system. In other words, the *presence* or *absence* of a registering device in the point is by itself a macroscopic adjustable parameter. But it is obvious from the description above that the statistical distribution $\rho(\rightarrow,\leftarrow,>,<,x,t)$ *strongly* depends on the number and location of detectors put into the space, regardless of when and how they are established.

So, the *type of dynamics* of our hidden variables does not principally fall in the region of Bell's theorem applicability. Straightforwardly, this means that appropriate experiments showing the violation of Bell's inequalities *say nothing* about the model described.

5. Discussion

It is interesting to understand the *logical* source of the mentioned assumption made by Bell and the authors of [2] about $\rho(\lambda)$. The matter is in another, *deeper* physical assumption: "suppose now that a statistical correlation of $A(a)$ and $B(b)$ is due to information carried by and localized within each particle..." [2]. Here, $A(a)$ and $B(b)$ are events of particles registering by two detectors with the adjustable parameters a and b mentioned above.

Actually, if we look deeply into this statement, we see that the theorem's proof is based on the theorem's conclusion! Localization of *all* the information in point-like particles means *by itself* the absence of distributed field-like parameters.

What is a particle? We agree that we can "see" only *registered* particles. Without experimental detecting or measurement, the notion "particle" is a *useful abstraction* only. We propose a strict definition: *a quantum particle is the event of its registering by a detector*, which is an unstable macroscopic system.

In terms of the proposed model, the act of registering means arising of combination "010." Configurations "$e1e$," "$e10$," "$01e$," "← 1 →," "← 1e," "e1 →," "01 →," "← 10" *are not* events of registering. In other words, a particle's arrival at a point is a *compound* event—it consists of arising of "0" *on both sides* of "1."

Information duplicating constitutes the core of the model. *Incomplete* information about a particle may propagate over the space without being detected or localized. We can assume that nature allows *only* macroscopic (experimental) detection of events when *duplicated* information arrives. That is why our hidden variables are truly *hidden*.

Essential duplicating of information at the moment of detecting can make clear the notion of superposition of states and the physical sense of wave function in quantum mechanics. Nonduplicated information about a particle's presence may be present elsewhere in space. The *collapse* or *reduction* of wave function in measurement may mean that the single point in space gets *duplicated* information while all other points lose *any* information.

Here is one more topic to discuss. If we apply our theory to a photon, the propagation velocity of photon's signals "→," "←," "<," ">" will turn to be greater than the speed of light! It is hard to believe but this does not contradict any established facts or theories. We must recall that relativistic principles confine only the velocity for transfer of *macroscopically detected* information. In terms of our model, it is the duplicated microscopic information. In other words, *hidden* variables must not obey principles for *detected* variables.

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