

Research Article

Stage-Structured Impulsive *SI* Model for Pest Management

Ruiqing Shi and Lansun Chen

Received 18 April 2007; Revised 16 July 2007; Accepted 5 November 2007

An *SI* epidemic model with stage structure is investigated. In the model, impulsive biological control is taken, that is, we release infected pests to the field at a fixed time periodically. We get a sufficient condition for the global asymptotical stability of the pest-eradication periodic solution $(0, 0, \tilde{I}(t))$, and a condition for the permanence of the system. At last, a brief discussion shows that our results will be helpful for pest management.

Copyright © 2007 R. Shi and L. Chen. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

1. Introduction

According to reports of Food and Agriculture Organization of the United Nations, the warfare between human and pests (such as locust, Aphis, cotton bollworm, etc.) has sustained for thousands of years. With the development of society and the progress of science and technology, man has adopted some advanced and modern weapons such as chemical pesticides, biological pesticides, remote sensing and measuring, computers, atomic energy, and so on. At last, some brilliant achievements have been obtained [1–4]. However, the warfare is not over, and will continue. A great deal of, and a large variety of, pesticides were used to control pests. In all, pesticides are useful because they can quickly kill a significant portion of a pest population and sometimes provide the only feasible method for preventing economic loss. However, pesticide pollution is also recognized as a major health hazard to human beings and beneficial insects. Biological control is, man's use of a suitably chosen living organism, referred as the biological control agent, to control another. Biological control agents can be predators, pathogens or parasites of the organism to be controlled that either kill the harmful organism or interfere with its biological progress. There is a vast amount of literatures on the applications of microbial disease to

suppress pests [1–3], but there are a few papers on the mathematical model of the dynamical of microbial disease in pest control [4–7]. The use of bacteria, fungi, and viruses is potentially one of the most important approaches in pest control. For example, Asian tiger mosquito (*Aedes albopictus*) can transmit viruses which cause dengue fever, Ross River fever, or Japanese encephalitis. To avoid this and to control the spread of mosquito swarms, we could spray with Bt, which is a variety of the bacterium *Bacillus thuringiensis* (Bt). An advantage of using insect pathogens is that they are safer to man and are usually safer to beneficial insects. In this paper, our aim is to control pests.

Impulsive equations are adequate to model processes with abrupt changes. Especially, impulsive differential equations can describe population dynamic models, since many life phenomena and human exploitation are almost impulsive in the natural world. (Some recent and general references on the theory of impulsive equations are [8–16].) There are also many applications of impulsive equations [17–25].

Stage structure is an important notation in epidemiology and demography because there are some kinds of diseases which are only spread or have more opportunities to be spread in children, for example, measles, mumps, chickenpox, scarlet fever, and diphtheria, while other infectious diseases such as gonorrhoea and syphilis are spread only in adults. There are literatures about single species with stage structure [26, 27], while others are about competitive or predator-prey systems with stage structure [28–31]. There are still other models with stage structure [27, 32–36]. In this paper, we consider the case in which the disease spreads only in mature pests, while the immature pests cannot be infected, for the immature are protected by their eggshells. For the purpose of pests control, we take biological control measures, that is, we release infected pests impulsively to the field, then more of the pests will be infected and die. The infected pests may be cultivated artificially in laboratory.

Xiao and Chen [34, 36] have studied *SIS* models with stage structure, but they did not consider the effects of impulse. To our knowledge, there are few papers about stage-structured models with impulse interruption (see [7]). The present paper constructs and analyzes a realistic model of impulsive biological control system for pest management. The paper is organized as follows. In Section 2, the main biological assumptions are formulated, and the model is constructed. In Section 3, by use of the Floquet theory for impulsive differential equations, the small-amplitude perturbation method and comparison techniques, we get the sufficient condition for the global asymptotic stability of the pest-eradication periodic solution and also the sufficient condition for the permanence of the system. Finally, a brief discussion is provided in the last section.

2. Model formulation

The basic stage-structured *SI* model is

$$\begin{aligned} S_1'(t) &= rS_2(t) - \mu S_1(t) - \delta S_1(t), \\ S_2'(t) &= \delta S_1(t) - \mu S_2(t) - S_2^2(t) - \beta S_2(t)I(t), \\ I'(t) &= \beta S_2(t)I(t) - (\mu + \alpha)I(t), \end{aligned} \tag{2.1}$$

where $S_2(t)$ and $I(t)$ denote the densities of susceptible and infected mature populations, respectively, and $S_1(t)$ denotes the density of an immature population which does not contract the disease. All coefficients are positive constants. The models are derived with the following assumptions.

(H₁) All newborns are assumed to be susceptible. We assume that at any time $t > 0$, birth into the immature population is proportional to the existing mature susceptible, with proportionality r , and the infected mature population does not have offsprings.

(H₂) The immature susceptible population has the natural death rate μ . The death rate of the mature susceptible population is proportional both to the existing mature population and to the square of it with proportionality constants μ and 1. The infected mature population has the natural death rate μ and an extra disease-related death rate α .

(H₃) The immature population enters into mature susceptible with proportionality constant δ .

(H₄) It is assumed that the incidence rate is double linear $\beta S_2(t)I(t)$, that is, the rate at which the susceptible mature becomes infected.

In this paper, with an additional assumption (H₅), we construct the following model, and the parameters are the same as those in system (2.1).

(H₅) We release an infected mature population with constant number $p > 0$ periodically with period $\tau > 0$;

$$\begin{aligned} S_1'(t) &= rS_2(t) - \mu S_1(t) - \delta S_1(t), \quad t \neq n\tau, n = 1, 2, 3, \dots, \\ S_2'(t) &= \delta S_1(t) - \mu S_2(t) - S_2^2(t) - \beta S_2(t)I(t), \quad t \neq n\tau, n = 1, 2, 3, \dots, \\ I'(t) &= \beta S_2(t)I(t) - (\mu + \alpha)I(t), \quad t \neq n\tau, n = 1, 2, 3, \dots, \\ \Delta S_1(t) &= 0, \quad \Delta S_2(t) = 0, \quad \Delta I(t) = p, \quad t = n\tau, n = 1, 2, 3, \dots \end{aligned} \tag{2.2}$$

3. Qualitative analysis for system (2.2)

In this section, we will study the effect of impulsive releasing of infected pests to the original model (2.1). To prove our main results, we give the following definition and lemmas.

Definition 3.1. System (2.2) is said to be permanent if there are constants $m, M > 0$ (independent of the initial value), and a finite time T_0 such that for all solutions $(S_1(t), S_2(t), I(t))$ with initial values $S_1(0^+) > 0, S_2(0^+) > 0, I(0^+) > 0, m \leq S_1(t) + S_2(t) \leq M$, and $m \leq I(t) \leq M$ hold for all $t \geq T_0$. Here, T_0 may depend on the initial values $(S_1(0^+), S_2(0^+), I(0^+))$.

LEMMA 3.2 (see [8, 9]). *Suppose $(S_1(t), S_2(t), I(t))$ is a solution of system (2.2) with initial values $S_1(0^+) \geq 0, S_2(0^+) \geq 0$, and $I(0^+) \geq 0$. Then $S_1(t) \geq 0, S_2(t) \geq 0$, and $I(t) \geq 0$ for all $t \geq 0$. And further, $S_1(t) > 0, S_2(t) > 0$, and $I(t) > 0$ if $S_1(0^+) > 0, S_2(0^+) > 0$, and $I(0^+) > 0$.*

LEMMA 3.3 (see [9]). *Let the function $m \in pC'[\mathbb{R}^+, \mathbb{R}]$ satisfy the inequalities*

$$\begin{aligned} m'(t) &\leq p(t)m(t) + q(t), \quad t \geq t_0, t \neq t_k, k = 1, 2, \dots, \\ m(t_k^+) &\leq d_k m(t_k) + b_k, \quad t = t_k, \end{aligned} \tag{3.1}$$

4 Discrete Dynamics in Nature and Society

where $p, q \in PC[\mathbb{R}^+, \mathbb{R}]$, and $d_k \geq 0$, b_k are constants. Then

$$m(t) \leq m(t_0) \left(\prod_{t_0 < t_k < t} d_k \right) e^{\int_{t_0}^t p(s) ds} \\ + \sum_{t_0 < t_k < t} b_k \left[\left(\prod_{t_k < t_j < t} d_j \right) e^{\int_{t_k}^t p(s) ds} \right] + \int_{t_0}^t \left(\prod_{s < t_k < t} d_k \right) e^{\int_s^t p(\sigma) d\sigma} q(s) ds, \quad (3.2)$$

for all $t \geq t_0$.

If $S_1(t) = S_2(t) = 0$, we have the following subsystem of system (2.2):

$$I'(t) = -(\mu + \alpha)I(t), \quad t \neq n\tau, \quad n = 1, 2, \dots \\ I(n\tau^+) = I(n\tau) + p, \quad t = n\tau, \quad n = 1, 2, \dots \quad (3.3)$$

Obviously, $\tilde{I}(t) = pe^{-(\mu+\alpha)(t-n\tau)}/(1-e^{-\alpha\tau})$, $t \in (n\tau, (n+1)\tau]$, $n \in \mathbb{Z}_+ = \{1, 2, 3, \dots\}$, $\tilde{I}(0^+) = p/(1-e^{-(\mu+\alpha)\tau})$, is a positive periodic solution of system (3.3). Therefore, the complete expression for the pest-eradication periodic solution of system (2.2) is obtained as $(0, 0, \tilde{I}(t)) = (0, 0, pe^{-(\mu+\alpha)(t-n\tau)}/(1-e^{-(\mu+\alpha)\tau}))$, $t \in (n\tau, (n+1)\tau]$, $n \in \mathbb{Z}_+$. Since the solution of system (3.3) is $I(t) = (I(0^+) - p/(1-e^{-(\mu+\alpha)\tau}))e^{-(\mu+\alpha)t} + \tilde{I}(t)$, $t \in (n\tau, (n+1)\tau]$, $n \in \mathbb{Z}_+$, so we derive the following.

LEMMA 3.4 (see [8]). *For every solution of system (3.3) with initial condition $I(0^+) > 0$, it follows that $I(t) \rightarrow \tilde{I}(t)$ as $t \rightarrow \infty$.*

LEMMA 3.5. *There exists a constant $M > 0$ such that $S_1(t) \leq M$, $S_2(t) \leq M$, $I(t) \leq M$ for each solution $(S_1(t), S_2(t), I(t))$ of system (2.2) for all t large enough (where t depends on the initial value).*

Proof. Due to the positivity of $S_1(t)$, $S_2(t)$, and $I(t)$, we can select $U(t) = S_1(t) + S_2(t) + I(t)$ as a Liapunov function.

When $t \neq n\tau$, we have

$$D^+U(t) + \mu U(t) = rS_2(t) - S_2^2(t) - \alpha I(t) \\ \leq rS_2(t) - S_2^2(t) \leq \frac{r^2}{4}. \quad (3.4)$$

When $t = n\tau$, $U(n\tau^+) = U(n\tau) + p$.

By Lemma 3.3, for $t \in (n\tau, (n+1)\tau]$, we have

$$U(t) \leq \left(U(0^+) - \frac{r^2}{4\mu} \right) e^{-\mu t} + \frac{p(1-e^{-n\mu\tau})}{1-e^{-\mu\tau}} e^{-\mu(t-n\tau)} + \frac{r^2}{4\mu}. \quad (3.5)$$

So $U(t)$ is uniformly ultimately bounded by a constant, and there exists a constant $M > 0$ such that $S_1(t) \leq M$, $S_2(t) \leq M$, $I(t) \leq M$ for each solution $(S_1(t), S_2(t), I(t))$ of system (2.2) for all t large enough. The proof is complete. \square

In the following, we investigate the stability of the pest eradication periodic solution of system (2.2).

THEOREM 3.6. *Let $(S_1(t), S_2(t), I(t))$ be any solution of system (2.2) with initial values $S_1(0^+) > 0$, $S_2(0^+) > 0$, and $I(0^+) > 0$, then $(0, 0, \tilde{I}(t))$ is locally asymptotically stable, provided that*

$$p > \frac{\tau ab}{\beta}, \tag{3.6}$$

where $a = \mu + \alpha > 0$, $b = (\delta r - \mu(\mu + \delta))/(\mu + \delta) > 0$.

Proof. The local stability of pest-eradication periodic solution may be determined by considering the behaviors of a small amplitude perturbation of the solution. Define $x(t) = S_1(t)$, $y(t) = S_2(t)$, $z(t) = I(t) - \tilde{I}(t)$, then the linearized system of system (2.2) reads as

$$\begin{aligned} x'(t) &= ry(t) - (\mu + \delta)x(t), \quad t \neq n\tau, n = 1, 2, \dots, \\ y'(t) &= \delta x(t) - \mu y(t) - \beta \tilde{I}(t)y(t), \quad t \neq n\tau, n = 1, 2, \dots, \\ z'(t) &= \beta \tilde{I}(t)y(t) - (\mu + \alpha)z(t), \quad t \neq n\tau, n = 1, 2, \dots, \end{aligned} \tag{3.7}$$

$$x(n\tau^+) = x(n\tau), \quad y(n\tau^+) = y(n\tau), \quad z(n\tau^+) = z(n\tau), \quad t = n\tau, n = 1, 2, \dots$$

Let $\Phi(t)$ be the fundamental solution matrix of system (3.7), then $\Phi(t)$ must satisfy

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} -(\mu + \delta) & r & 0 \\ \delta & -\mu - \beta \tilde{I}(t) & 0 \\ 0 & \beta \tilde{I}(t) & -(\mu + \alpha) \end{pmatrix} \Phi(t) \doteq A\Phi(t), \tag{3.8}$$

and $\Phi(0) = I$, the identity matrix. We can easily see that one of the eigenvalues of matrix A is $-(\mu + \alpha)$, and the other two eigenvalues are determined by the 2×2 matrix B , where

$$B = \begin{pmatrix} -(\mu + \delta) & r \\ \delta & -\mu - \beta \tilde{I}(t) \end{pmatrix}. \tag{3.9}$$

Denote the eigenvalues of B as λ_1, λ_2 , then we have $\lambda_1 + \lambda_2 = -(\mu + \delta) - \mu - \beta \tilde{I}(t) < 0$, $\lambda_1 \lambda_2 = (\mu + \delta)(\mu + \beta \tilde{I}(t)) - r\delta = (\mu + \delta)[\mu - r\delta/(\mu + \delta) + \beta \tilde{I}(t)]$. Therefore, by the Floquet theorem, $(0, 0, \tilde{I}(t))$ is locally asymptotically stable, provided that

$$\int_0^\tau \left(\mu + \beta \tilde{I}(t) - \frac{r\delta}{\mu + \delta} \right) dt > 0, \tag{3.10}$$

that is,

$$[r\delta - \mu(\mu + \delta)]\tau < \frac{\beta p(\mu + \delta)}{\mu + \alpha}. \tag{3.11}$$

When $R_0 = r\delta/\mu(\mu + \delta) \leq 1$, the above inequality is satisfied for all $p > 0$; when $R_0 = r\delta/\mu(\mu + \delta) > 1$, the above inequality equals

$$p > \frac{\tau ab}{\beta}. \tag{3.12}$$

The proof is complete. □

6 Discrete Dynamics in Nature and Society

THEOREM 3.7. *If $R_0 \leq 1$, or $R_0 > 1$, and $p > (e^{a\tau} - 1)(b/\beta)$, then $(0, 0, \tilde{I}(t))$ is globally asymptotically stable for system (2.2).*

Proof. Let

$$f(x) = e^x - 1 - x \quad (3.13)$$

because

$$f'(x) = e^x - 1 > 0, \quad \text{whenever } x > 0. \quad (3.14)$$

So we have

$$\begin{aligned} e^{a\tau} - 1 &> a\tau, \\ p &> (e^{a\tau} - 1) \frac{b}{\beta} > a\tau \frac{b}{\beta}. \end{aligned} \quad (3.15)$$

By Theorem 3.6, we know that $(0, 0, \tilde{I}(t))$ is locally asymptotically stable. In the following, we will prove the global attraction of $(0, 0, \tilde{I}(t))$.

Let

$$V(t) = \delta S_1(t) + (\mu + \delta) S_2(t). \quad (3.16)$$

Then

$$V'(t)|_{(2.2)} = [r\delta - \mu(\mu + \delta) - (\mu + \delta)S_2(t) - \beta(\mu + \delta)I(t)]S_2(t). \quad (3.17)$$

When $R_0 \leq 1$, we clearly have

$$V'(t)|_{(2.2)} \leq -\beta(\mu + \delta)I(t)S_2(t) < 0, \quad \forall t \geq 0, \quad (3.18)$$

so $V(t) \rightarrow 0$, $S_1(t) \rightarrow 0$, $S_2(t) \rightarrow 0$, as $t \rightarrow +\infty$, and $(0, 0, \tilde{I}(t))$ is globally attractive.

When $R_0 > 1$ and $p > (e^{a\tau} - 1)(b/\beta)$, we can select $\epsilon > 0$ small enough such that $p > (e^{a\tau} - 1)(b + 2\epsilon)/\beta$.

From system (2.2), we have

$$\begin{aligned} I'(t) &= \beta S_2(t)I(t) - aI(t) \geq -aI(t), \quad t \neq n\tau, n = 1, 2, \dots, \\ I(n\tau^+) &= I(n\tau) + p, \quad t = n\tau, n = 1, 2, \dots \end{aligned} \quad (3.19)$$

By Lemmas 3.3 and 3.4, we know that there exists a $t_1 > 0$ such that $I(t) \geq pe^{-a\tau}/(1 - e^{-a\tau}) - \epsilon/\beta$ for all $t \geq t_1$.

Thus when $t \geq t_1$, we have

$$\begin{aligned} V'(t)|_{(2.2)} &= (\mu + \delta)[b - S_2(t) - \beta I(t)]S_2(t) \\ &\leq -(\mu + \delta)(S_2(t) + \epsilon)S_2(t) < 0. \end{aligned} \quad (3.20)$$

So $V(t) \rightarrow 0$, $S_1(t) \rightarrow 0$, $S_2(t) \rightarrow 0$, and $(0, 0, \tilde{I}(t))$ is globally attractive. The proof is complete. \square

THEOREM 3.8. *If $R_0 > 1$ and $0 < p < b(1 - e^{-a\tau})/\beta$, then system (2.2) is permanent.*

Proof. Suppose $(S_1(t), S_2(t), I(t))$ is any solution of system (2.2) with initial values $S_1(0^+) > 0$, $S_2(0^+) > 0$, and $I(0^+) > 0$. By Lemmas 3.2 and 3.5, we may assume $S_1(t) \leq M$, $S_2(t) \leq M$, and $I(t) \leq M$, for all $t \geq 0$, and $M > b/(1 + \beta)$. Let $m = pe^{-(\mu+\alpha)\tau}/(1 - e^{-(\mu+\alpha)\tau}) - \epsilon_3 > 0$, $\epsilon_3 > 0$. From Lemma 3.3, we clearly have $I(t) \geq m$ for all t large enough. We will next find an $m_1 > 0$ such that $S_1(t) + S_2(t) \geq m_1$ for t large enough. We will do it in the following two steps.

Step 1. Since $R_0 > 1$ and $0 < p < b(1 - e^{-a\tau})/\beta$, we can choose $m_2 > 0$, $\epsilon_1 > 0$ small enough such that

$$0 < p < \frac{(b - m_2 - \epsilon_1)(1 - e^{-(a-\beta m_2)\tau})}{\beta}. \quad (3.21)$$

Denote

$$\begin{aligned} \rho &= b - m_2 - \epsilon_1 - \frac{p\beta}{1 - e^{-(a-\beta m_2)\tau}} > 0, \\ \sigma &= b - M(1 + \beta) < 0. \end{aligned} \quad (3.22)$$

Consider the Liapunov function

$$V(t) = \delta S_1(t) + (\mu + \delta)S_2(t). \quad (3.23)$$

We claim that $V(t) < (\mu + \delta)m_2 = m_3$ cannot hold for all $t \geq 0$. Otherwise, we have that $S_2(t) < m_2$ for all $t \geq 0$. Then, from the third and the sixth equations of system (2.2), we get

$$\begin{aligned} I'(t) &\leq (\beta m_2 - a)I(t), \quad t \neq n\tau, n = 1, 2, \dots, \\ I(n\tau^+) &= I(n\tau) + p, \quad t = n\tau, n = 1, 2, \dots \end{aligned} \quad (3.24)$$

By Lemma 3.3, we know that there exists a $t_1 > 0$ such that $I(t) \leq \bar{u}(t) + \epsilon_1/\beta$, for all $t \geq t_1$, and here, $\bar{u}(t) = pe^{-(a-\beta m_2)(t-n\tau)}/(1 - e^{-(a-\beta m_2)\tau})$, $t \in (n\tau, (n+1)\tau]$, $n \in \mathbb{Z}_+$. So $I(t) \leq p/(1 - e^{-(a-\beta m_2)\tau}) + \epsilon_1/\beta$, for all $t \geq t_1$, and

$$\begin{aligned} V'(t)|_{(2.2)} &= (\mu + \delta)[b - S_2(t) - \beta I(t)]S_2(t) \\ &\geq (\mu + \delta) \left[b - m_2 - \epsilon_1 - \frac{p\beta}{1 - e^{-(a-\beta m_2)\tau}} \right] S_2(t) \\ &= \rho(\mu + \delta)S_2(t) > 0 \end{aligned} \quad (3.25)$$

for all $t > t_1$. Thus

$$\begin{aligned} V'(t)|_{(2.2)} &\geq \rho(\mu + \delta)S_2(t_1) = \epsilon_2 > 0, \\ V(t) &\longrightarrow +\infty, \quad S_2(t) \longrightarrow +\infty, \quad \text{as } t \longrightarrow +\infty. \end{aligned} \quad (3.26)$$

It is a contradiction since $V(t)$ is ultimately bounded by Lemma 3.5. Therefore, there exists $t_1 > 0$ such that $V(t_1) \geq m_3$.

8 Discrete Dynamics in Nature and Society

Step 2. If $V(t) \geq m_3$ hold for all $t \geq t_1$, then we can select $m_1 = m_2$, and our aim is obtained. We consider those solutions which leave the region $D = \{(S_1, S_2, I) \in \mathbb{R}_+^3 : V(t) = \delta S_1(t) + (\mu + \delta)S_2(t) < m_3\}$, and reenter it again. Let $t^* = \inf_{t \geq t_1} \{V(t) < m_3\}$. Then $V(t) \geq m_3$ for $t \in [t_1, t^*]$, and $V(t^*) = m_3$. Suppose $t^* \in [n_1\tau, (n_1 + 1)\tau)$, $n_1 \in N$. Select n_2, n_3 such that

$$\begin{aligned} n_2\tau > T_2 &= -\frac{1}{a - \beta m_2} \ln \frac{\epsilon_1}{M + p}, \\ n_3\tau > T_3 &= \frac{m_3}{\rho_1}, \\ \rho_1 &= \rho(\mu + \delta)S_2((n_1 + 1 + n_2)\tau^+). \end{aligned} \quad (3.27)$$

Let $T = (n_2 + n_3)\tau$. We claim that there must be a $t_2 \in [(n_1 + 1)\tau, (n_1 + 1)\tau + T]$ such that $V(t_2) \geq m_3$. Otherwise, $V(t) < m_3$ and $S_2(t) < m_2$, for all $t \in [(n_1 + 1)\tau, (n_1 + 1)\tau + T]$.

Consider

$$\begin{aligned} u'(t) &\leq (\beta m_2 - a)u(t), \quad t \neq n\tau, n = n_1 + 1, n_1 + 2, \dots, \\ u(n\tau^+) &= u(n\tau) + p, \quad t = n\tau, n = n_1 + 1, n_1 + 2, \dots, \\ u((n_1 + 1)\tau^+) &= I((n_1 + 1)\tau^+). \end{aligned} \quad (3.28)$$

By Lemmas 3.3 and 3.4, we have

$$u(t) = \left(u((n_1 + 1)\tau^+) - \frac{p}{1 - e^{-(a - \beta m_2)\tau}} \right) e^{-(a - \beta m_2)(t - (n_1 + 1)\tau)} + \bar{u}(t) \quad (3.29)$$

for $t \in (n\tau, (n + 1)\tau]$, $n_1 + 1 \leq n \leq n_1 + 1 + n_2 + n_3$. Then

$$|u(t) - \bar{u}(t)| \leq (M + p)e^{-(a - \beta m_2)n_2\tau} < \epsilon_1, \quad (3.30)$$

and by the comparison principle, we have

$$I(t) \leq u(t) \leq \bar{u}(t) + \frac{\epsilon_1}{\beta} \leq \frac{p}{1 - e^{-(a - \beta m_2)\tau}} + \frac{\epsilon_1}{\beta} \quad (3.31)$$

for $(n_1 + 1 + n_2)\tau \leq t \leq (n_1 + 1)\tau + T$, which imply that

$$\begin{aligned} V'(t)|_{(2,2)} &\geq \rho(\mu + \delta)S_2((n_1 + 1 + n_2)\tau^+) = \rho_1 > 0, \\ V((n_1 + 1)\tau + T) &\geq V((n_1 + 1 + n_2)\tau^+) + \rho_1 n_3\tau. \end{aligned} \quad (3.32)$$

When $t \in [t^*, (n_1 + 1 + n_2)\tau]$, we have

$$\begin{aligned} V'(t)|_{(2,2)} &= [r\delta - \mu(\mu + \delta) - (\mu + \delta)S_2(t) - \beta(\mu + \delta)I(t)]S_2(t) \\ &\geq (\mu + \delta)[b - M - \beta M]S_2(t) \\ &= (\mu + \delta)\sigma S_2(t) \geq \sigma V(t). \end{aligned} \quad (3.33)$$

Thus

$$\begin{aligned} V((n_1 + 1 + n_2)\tau) &\geq V(t^*)e^{\sigma(n_2+1)\tau} > 0, \\ V((n_1 + 1)\tau + T) &\geq V((n_1 + 1 + n_2)\tau) + \rho_1 n_3 \tau > m_3. \end{aligned} \quad (3.34)$$

It is a contradiction. Let $\bar{t} = \inf_{t \geq t^*} \{V(t) \geq m_3\}$. Then $V(\bar{t}) \geq m_3$. For $t \in [t^*, \bar{t})$, we have

$$V(t) \geq V(t^*)e^{\sigma(t-t^*)} \geq m_3 e^{\sigma(1+n_1+n_2+n_3)} \doteq m_4. \quad (3.35)$$

For $t > \bar{t}$, the same arguments can be continued since $V(\bar{t}) \geq m_3$. We can select $m_1 = m_4/(\mu + \delta)$, and $S_1(t) + S_2(t) \geq m_1$ for all $t \geq t_1$. The proof is complete. \square

Remark 3.9. From Theorem 3.8, we can see that if the coefficients satisfy the condition $R_0 > 1$, then, without control of releasing infected pests, or the releasing number $p < (1 - e^{-a\tau})(b/\beta)$, the pests will be permanent and they will do harm to the crops. Under this condition, we can increase the releasing amount until $p > (e^{a\tau} - 1)(b/\beta)$, and by Theorem 3.7, we know that the pests will be doomed.

4. Discussion

In this paper, a stage-structured *SI* epidemic model is investigated. In the model, impulsive biological control is taken into consideration, and we analyzed the existence and stability of the pest-eradication periodic solution of the system. We also get a sufficient condition for the permanence of the system. In the real world, we can take impulsive biological control to constrain the pest population. There is still an interesting problem: if $R_0 > 1$ and $0 < p < b(1 - e^{-a\tau})(b/\beta)$, then the system is permanent; if $R_0 > 1$ and $p > (e^{a\tau} - 1)(b/\beta)$, then $(0, 0, \tilde{I}(t))$ is globally asymptotically stable. But we have not considered the case that $R_0 > 1$ and $b(1 - e^{-a\tau})/\beta < p < (e^{a\tau} - 1)(b/\beta)$, how about the dynamical behaviors of the system? In this paper, our aim is to control the pest population, so we omit the above problems. But from a mathematical aspect, it is very interesting, and we leave it as a future work.

Acknowledgments

The authors thank the referees very much for their valuable comments and suggestions. This research is supported by National Natural Science Foundation of China (Grant no. 10471117).

References

- [1] H. D. Burges and N. W. Hussey, Eds., *Microbial Control of Insections and Mites*, Academic Press, New York, NY, USA, 1971.
- [2] P. E. Davis, K. Myers, and J. B. Hoy, "Biological control among vertebrates," in *Theory and Practice of Biological Control*, C. B. Huffaker and P. S. Messenger, Eds., pp. 501–519, Plenum Press, New York, NY, USA, 1976.
- [3] L. A. Falcon, "Problems associated with the use of arthropod viruses in pest control," *Annual Review of Entomology*, vol. 21, pp. 305–324, 1976.

- [4] K. Wickwire, "Mathematical models for the control of pests and infectious diseases: a survey," *Theoretical Population Biology*, vol. 11, no. 2, pp. 182–238, 1977.
- [5] W. Wang, J. Shen, and J. Nieto, "Permanence and periodic solution of predator-prey system with holling type functional response and impulses," *Discrete Dynamics in Nature and Society*, vol. 2007, Article ID 81756, 15 pages, 2007.
- [6] H. Zhang, R. Zhu, and L. Chen, "On a periodic time-dependent impulsive system of strategies for controlling the apple snail in paddy fields," *Journal of Biological Systems*, vol. 15, no. 3, pp. 397–408, 2007.
- [7] H. Zhang, L. Chen, and J. Nieto, "A delayed epidemic model with stage-structure and pulses for pest management strategy," to appear in *Nonlinear Analysis: Real World Applications*.
- [8] D. Bañov and P. Simeonov, Eds., *Impulsive Differential Equations: Periodic Solutions and Applications*, vol. 66 of *Pitman Monographs and Surveys in Pure and Applied Mathematics*, Longman Scientific & Technical, Harlow, UK, 1993.
- [9] V. Lakshmikantham, D. D. Bañov, and P. S. Simeonov, Eds., *Theory of Impulsive Differential Equations*, vol. 6 of *Series in Modern Applied Mathematics*, World Scientific, Teaneck, NJ, USA, 1989.
- [10] S. T. Zavalishchin and A. N. Seseikin, *Dynamic Impulse Systems: Theory and Applications*, vol. 394 of *Mathematics and Its Applications*, Kluwer Academic, Dordrecht, The Netherlands, 1997.
- [11] A. M. Samoilenko and N. A. Perestyuk, *Impulsive Differential Equations*, vol. 14 of *World Scientific Series on Nonlinear Science. Series A: Monographs and Treatises*, World Scientific, River Edge, NJ, USA, 1995.
- [12] J. J. Nieto and R. Rodríguez-López, "Periodic boundary value problem for non-Lipschitzian impulsive functional differential equations," *Journal of Mathematical Analysis and Applications*, vol. 318, no. 2, pp. 593–610, 2006.
- [13] J. J. Nieto, "Periodic boundary value problems for first-order impulsive ordinary differential equations," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 51, no. 7, pp. 1223–1232, 2002.
- [14] L. Chen and J. Sun, "Nonlinear boundary value problem of first order impulsive functional differential equations," *Journal of Mathematical Analysis and Applications*, vol. 318, no. 2, pp. 726–741, 2006.
- [15] Y. Liu, "Further results on periodic boundary value problems for nonlinear first order impulsive functional differential equations," *Journal of Mathematical Analysis and Applications*, vol. 327, no. 1, pp. 435–452, 2007.
- [16] J. Li, J. J. Nieto, and J. Shen, "Impulsive periodic boundary value problems of first-order differential equations," *Journal of Mathematical Analysis and Applications*, vol. 325, no. 1, pp. 226–236, 2007.
- [17] P. Georgescu and G. Moroşanu, "Pest regulation by means of impulsive controls," *Applied Mathematics and Computation*, vol. 190, no. 1, pp. 790–803, 2007.
- [18] S. Gao, L. Chen, J. Nieto, and A. Torres, "Analysis of a delayed epidemic model with pulse vaccination and saturation incidence," *Vaccine*, vol. 24, no. 35–36, pp. 6037–6045, 2006.
- [19] M. Choisy, J.-F. Guégan, and P. Rohani, "Dynamics of infectious diseases and pulse vaccination: teasing apart the embedded resonance effects," *Physica D*, vol. 223, no. 1, pp. 26–35, 2006.
- [20] J. Yan, A. Zhao, and J. J. Nieto, "Existence and global attractivity of positive periodic solution of periodic single-species impulsive Lotka-Volterra systems," *Mathematical and Computer Modelling*, vol. 40, no. 5–6, pp. 509–518, 2004.
- [21] W.-T. Li and H.-F. Huo, "Global attractivity of positive periodic solutions for an impulsive delay periodic model of respiratory dynamics," *Journal of Computational and Applied Mathematics*, vol. 174, no. 2, pp. 227–238, 2005.

- [22] W. Zhang and M. Fan, “Periodicity in a generalized ecological competition system governed by impulsive differential equations with delays,” *Mathematical and Computer Modelling*, vol. 39, no. 4-5, pp. 479–493, 2004.
- [23] X. Zhang, Z. Shuai, and K. Wang, “Optimal impulsive harvesting policy for single population,” *Nonlinear Analysis. Real World Applications*, vol. 4, no. 4, pp. 639–651, 2003.
- [24] A. d’Onofrio, “A general framework for modeling tumor-immune system competition and immunotherapy: mathematical analysis and biomedical inferences,” *Physica D*, vol. 208, no. 3-4, pp. 220–235, 2005.
- [25] S. Gao, Z. Teng, J. J. Nieto, and A. Torres, “Analysis of an SIR epidemic model with pulse vaccination and distributed time delay,” *Journal of Biomedicine and Biotechnology*, vol. 2007, Article ID 64870, 10 pages, 2007.
- [26] W. G. Aiello and H. I. Freedman, “A time-delay model of single-species growth with stage structure,” *Mathematical Biosciences*, vol. 101, no. 2, pp. 139–153, 1990.
- [27] X. Song and L. Chen, “Modelling and analysis of a single-species system with stage structure and harvesting,” *Mathematical and Computer Modelling*, vol. 36, no. 1-2, pp. 67–82, 2002.
- [28] S. Liu, L. Chen, and Z. Liu, “Extinction and permanence in nonautonomous competitive system with stage structure,” *Journal of Mathematical Analysis and Applications*, vol. 274, no. 2, pp. 667–684, 2002.
- [29] X. Song and L. Chen, “Optimal harvesting and stability for a predator-prey system with stage structure,” *Acta Mathematicae Applicatae Sinica*, vol. 18, no. 3, pp. 423–430, 2002.
- [30] W. Wang and L. Chen, “A predator-prey system with stage-structure for predator,” *Computers & Mathematics with Applications*, vol. 33, no. 8, pp. 83–91, 1997.
- [31] Y. Xiao and L. Chen, “Stabilizing effect of cannibalism on a structured competitive system,” *Acta Mathematica Scientia Series A*, vol. 22, no. 2, pp. 210–216, 2002 (Chinese).
- [32] W. G. Aiello, H. I. Freedman, and J. Wu, “Analysis of a model representing stage-structured population growth with state-dependent time delay,” *SIAM Journal on Applied Mathematics*, vol. 52, no. 3, pp. 855–869, 1992.
- [33] S. Liu, L. Chen, and R. Agarwal, “Recent progress on stage-structured population dynamics,” *Mathematical and Computer Modelling*, vol. 36, no. 11–13, pp. 1319–1360, 2002.
- [34] Y. Xiao and L. Chen, “An SIS epidemic model with stage structure and a delay,” *Acta Mathematicae Applicatae Sinica*, vol. 18, no. 4, pp. 607–618, 2002.
- [35] Y. Xiao, L. Chen, and F. ven den Bosch, “Dynamical behavior for a stage-structured SIR infectious disease model,” *Nonlinear Analysis: Real World Applications*, vol. 3, no. 2, pp. 175–190, 2002.
- [36] Y. Xiao and L. Chen, “On an SIS epidemic model with stage structure,” *Journal of Systems Science and Complexity*, vol. 16, no. 2, pp. 275–288, 2003.

Ruiqing Shi: Department of Applied Mathematics, Dalian University of Technology, Dalian, Liaoning 116024, China; School of Mathematics and Computer Science, Shanxi Normal University, Linfen, Shanxi 041004, China
Email address: shirq1979@163.com

Lansun Chen: Department of Applied Mathematics, Dalian University of Technology, Dalian, Liaoning 116024, China
Email address: lschen@math.ac.cn