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Research Article

Extinction and Permanence of a Three-Species Lotka-Volterra System with Impulsive Control Strategies

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A three-species Lotka-Volterra system with impulsive control strategies containing the biological control (the constant impulse) and the chemical control (the proportional impulse) with the same period, but not simultaneously, is investigated. By applying the Floquet theory of impulsive differential equation and small amplitude perturbation techniques to the system, we find conditions for local and global stabilities of a lower-level prey and top-predator free periodic solution of the system. In addition, it is shown that the system is permanent under some conditions by using comparison results of impulsive differential inequalities. We also give a numerical example that seems to indicate the existence of chaotic behavior.

1. Introduction

The mathematical study of a predator-prey system in population dynamics has a long history starting with the work of Lotka and Volterra. The principles of Lotka-Volterra models have remained valid until today and many theoretical ecologists adhere to their principles [1–9]. Thus, we need to consider a Lotka-Volterra-type food chain model which can be described by the following differential equations:

$$\begin{aligned} x'(t) &= x(t)\big(a - bx(t) - cy(t)\big), \\ y'(t) &= y(t)\big(-d_1 + c_1x(t) - e_1z(t)\big), \\ z'(t) &= z(t)\big(-d_2 + e_2y(t)\big), \end{aligned} \tag{1.1}$$

where $x(t)$, $y(t)$, and $z(t)$ are the densities of the lowest-level prey, mid-level predator, and top predator at time t, respectively; $a > 0$ is called intrinsic growth rate of the prey; $b > 0$ is the coefficient of intraspecific competition; $c > 0$ and $e_1 > 0$ are the per-capita rate of predation of the predator; $c_1 > 0$ and $e_2 > 0$ denote the product of the per-capita rate of predation and the conversion rate; $d_1 > 0$ and $d_2 > 0$ denote the death rate of the predators.

Now, we regard $x(t)$ as a pest to establish a new system dealing with impulsive pest control strategies from system (1.1). There are many ways to control pest population. One of the most important methods for pest control is chemical control. A principal substance in chemical control is pesticide. Pesticides are often useful because they quickly kill a significant portion of pest population. However, there are many deleterious effects associated with the use of chemicals that need to be reduced or eliminated. These include human illness associated with pesticide applications, insect resistance to insecticides, contamination of soil and water, and diminution of biodiversity. As a result, we should combine pesticide efficacy tests with other ways of control like biological control. Biological control is another important strategy to control pest population. It is defined as the reduction of the pest population by natural enemies and typically involves an active human role. Natural enemies of insect pests, also known as biological control agents, include predators, parasites, and pathogens. Virtually, all pests have some natural enemies, and the key to successful pest control is to identify the pest and its natural enemies and release them at fixed time for pest control. Such different pest control tactics should work together rather than against each other to accomplish successful pest population control [10–12]. Thus, in this paper, we consider the following Lotka-Volterra-type food chain model with periodic constant releasing natural enemies (mid-level predator) and spraying pesticide at different fixed time:

$$\begin{gathered}
x'(t) = x(t)\big(a - bx(t) - cy(t)\big), \quad t \neq nT,\ t \neq (n+\tau-1)T, \\
y'(t) = y(t)\big(-d_1 + c_1x(t) - e_1z(t)\big), \quad t \neq nT,\ t \neq (n+\tau-1)T, \\
z'(t) = z(t)\big(-d_2 + e_2y(t)\big), \quad t \neq nT,\ t(n+\tau-1)T, \\
x(t^+) = (1-p_1)x(t), \quad t = (n+\tau-1)T, \\
y(t^+) = (1-p_2)y(t), \quad t = (n+\tau-1)T, \\
z(t^+) = (1-p_3)z(t), \quad t = (n+\tau-1)T, \\
x(t^+) = x(t), \quad t = nT, \\
y(t^+) = y(t) + q, \quad t = nT, \\
z(t^+) = z(t), \quad t = nT, \\
\big(x(0^+), y(0^+), z(0^+)\big) = (x_0, y_0, z_0),
\end{gathered} \tag{1.2}$$

where $0 < \tau < 1$, T is the period of the impulsive immigration or stock of the mid-level predator, $0 \leq p_1, p_2, p_3 < 1$ present the fraction of the prey and the predator which die due to the harvesting or pesticides, and q is the size of immigration or stock of the predator. Such system is an impulsive differential equation whose theories and applications were greatly developed by the efforts of Bainov and Simeonov [13] and Lakshmikantham et al. [14]. Also, Nieto and O'Regan. [18] presented a new approach to obtain the existence of solutions to some impulsive problems. Moreover [15–17], the theory of impulsive differential equations is being recognized to be not only richer than the corresponding theory of differential equations without impulses, but also to represent a more natural framework for mathematical modeling of real-world phenomena [18–22].

In recent years, many authors have studied two-dimensional predator-prey systems with impulsive perturbations [23–29]. Moreover, three-species food chain systems with sudden perturbations have been intensively researched, such as those of Holling-type [30, 31] and Beddington-type [32, 33]. However, most researches about food chain systems mentioned above have just dealt with biological control and have only given conditions for extinction of the lowest-level prey and top predator by observing the local stability of lower-level prey and top-predator free periodic solution. For this reason, the main purpose of this paper is to investigate the conditions for the extinction and the permanence of system (1.2).

The organization of the paper is as follows. In the next section, we introduce some notations and lemmas which are used in this paper. In Section 3, we find conditions for local and global stabilities of a lower-level prey and top-predator free periodic solution by applying the Floquet theory and for permanence of system (1.2) by using the comparison theorem. In Section 4, we give some numerical examples including chaotic phase portrait. Finally, we have a conclusion in Section 5.

2. Preliminaries

Now, we will introduce a few notations and definitions together with a few auxiliary results relating to comparison theorem, which will be useful for our main results.

Let $\mathbb{R}_+ = [0, \infty)$ and $\mathbb{R}^3_+ = \{\mathbf{x} = (x(t), y(t), z(t)) \in \mathbb{R}^3 : x(t), y(t), z(t) \geq 0\}$. Denote $\mathbb{N}$ as the set of all nonnegative integers, $\mathbb{R}^*_+ = (0, \infty)$, and $f = (f_1, f_2, f_3)^T$ as the right-hand side of the first three equations in (1.2). We first give the definition of a solution for (1.2).

Definition 2.1 (see [14]). Let $\Omega \subset \mathbb{R}^3$ be an open set and let $D = \mathbb{R}_+ \times \Omega$. A function $\mathbf{x} = (x(t), y(t), z(t)) : (0, a) \to \mathbb{R}^3$, $a > 0$, is said to be a solution of (1.2) if

(1) $\mathbf{x}(0^+) = (x_0, y_0, z_0)$ and $(t, \mathbf{x}(t)) \in D$ for $t \in [0, a)$,

(2) $\mathbf{x}(t)$ is continuously differentiable and satisfies the first three equations in (1.2) for $t \in [0, a)$, $t \neq nT$, and $t \neq (n + \tau - 1)T$,

(3) $0 \leq t < a$; then $\mathbf{x}(t)$ is left continous at $t = (n + \tau - 1)T$ and nT, and

$$\mathbf{x}(t^+) - \mathbf{x}(t) = \begin{cases} \big(-p_1 x(t), -p_2 y(t), -p_3 z(t) \big) & \text{if } t = (n + \tau - 1)T, \\ (0, q, 0) & \text{if } t = nT. \end{cases} \tag{2.1}$$

Now, we introduce another definition to formulate the comparison result. Let $V : \mathbb{R}_+ \times \mathbb{R}^3_+ \to \mathbb{R}_+$, then V is said to be in a class V_0 if

(1) V is continuous on $(nT, (n+1)T] \times \mathbb{R}^3_+$, and $\lim_{(t,y) \to (nT,x), t > nT} V(t, y) = V(nT^+, \mathbf{x})$ exists;

(2) V is locally Lipschitzian in $\mathbf{x}$.

Definition 2.2 (see [14]). For $V \in V_0$, one defines the upper-right Dini derivative of V with respect to the impulsive differential system (1.2) at $(t, \mathbf{x}) \in (nT, (n+1)T] \times \mathbb{R}^3_+$ by

$$D^+ V(t, \mathbf{x}) = \limsup_{h \to 0+} \frac{1}{h} \big[V\big(t + h, \mathbf{x} + h f(t, \mathbf{x})\big) - V(t, \mathbf{x}) \big]. \tag{2.2}$$

Remark 2.3. The smoothness properties of f guarantee the global existence and uniqueness of solutions of system (1.2). (See [13, 14] for details.)

We will use a comparison result of impulsive differential inequalities. We suppose that $g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ satisfies the following hypothesis.

(H) g is continuous on $(nT, (n+1)T] \times \mathbb{R}_+$ and the limit $\lim_{(t,y)\to(nT^+,x)} g(t,y) = g(nT^+, x)$ exists and is finite for $x \in \mathbb{R}_+$ and $n \in \mathbb{N}$.

Lemma 2.4 (see [14]). *Suppose $V \in V_0$ and*

$$\begin{aligned}
D^+V(t,\mathbf{x}) &\le g(t, V(t,\mathbf{x})), \quad t \ne (n+\tau-1)T, nT,\\
V(t,\mathbf{x}(t^+)) &\le \varphi_n^1(V(t,\mathbf{x})), \quad t = (n+\tau-1)T,\\
V(t,\mathbf{x}(t^+)) &\le \varphi_n^2(V(t,\mathbf{x})), \quad t = nT,
\end{aligned} \tag{2.3}$$

where $g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ satisfies (H) and $\varphi_n^1, \varphi_n^2 : \mathbb{R}_+ \to \mathbb{R}_+$ are nondecreasing for all $n \in \mathbb{N}$. Let $r(t)$ be the maximal solution for the impulsive Cauchy problem

$$\begin{aligned}
u'(t) &= g(t, u(t)), \quad t \ne (n+\tau-1)T, nT,\\
u(t^+) &= \varphi_n^1(u(t)), \quad t = (n+\tau-1)T,\\
u(t^+) &= \varphi_n^2(u(t)), \quad t = nT,\\
u(0^+) &= u_0,
\end{aligned} \tag{2.4}$$

defined on $[0,\infty)$. Then $V(0^+, \mathbf{x}_0) \le u_0$ implies that $V(t,\mathbf{x}(t)) \le r(t)$, $t \ge 0$, where $\mathbf{x}(t)$ is any solution of (2.3).

We now indicate a special case of Lemma 2.4 which provides estimations for the solution of a system of differential inequalities. For this, we let $PC(\mathbb{R}_+,\mathbb{R})(PC^1(\mathbb{R}_+,\mathbb{R}))$ denote the class of real piecewise continuous (real piecewise continuously differentiable) functions defined on $\mathbb{R}_+$.

Lemma 2.5 (see [14]). *Let the function $u(t) \in PC^1(\mathbb{R}^+,\mathbb{R})$ satisfy the inequalities*

$$\begin{aligned}
\frac{du}{dt} &\le f(t)u(t) + h(t), \quad t \ne \tau_k,\ t > 0,\\
u(\tau_k^+) &\le \alpha_k u(\tau_k) + \beta_k, \quad k \ge 0,\\
u(0^+) &\le u_0,
\end{aligned} \tag{2.5}$$

where $f, h \in PC(\mathbb{R}_+,\mathbb{R})$ and $\alpha_k \ge 0$, β_k, u_0 are constants, and $(\tau_k)_{k\ge0}$ is a strictly increasing sequence *of positive real numbers. Then, for $t > 0$,*

$$u(t) \le u_0\left(\prod_{0<\tau_k<t}\alpha_k\right)\exp\left(\int_0^t f(s)ds\right) + \int_0^t\left(\prod_{s\le\tau_k<t}\alpha_k\right)\exp\left(\int_s^t f(\gamma)d\gamma\right)h(s)ds$$
$$+\sum_{0<\tau_k<t}\left(\prod_{\tau_k<\tau_j<t}\alpha_j\right)\exp\left(\int_{\tau_k}^t f(\gamma)d\gamma\right)\beta_k. \tag{2.6}$$

Similar results can be obtained when all conditions of the inequalities in Lemmas 2.4 and 2.5 are reversed. Using Lemma 2.5, it is possible to prove that the solutions of the Cauchy problem (2.4) with strictly positive initial value remain strictly positive.

Lemma 2.6. *The positive octant $(\mathbb{R}_+^*)^3$ is an invariant region for system* (1.2).

Proof. Let $(x(t), y(t), z(t)) : [0, t_0) \to \mathbb{R}^3$ be a solution of system (1.2) with a strictly positive initial value (x_0, y_0, z_0). By Lemma 2.5, we can obtain that, for $0 \le t < t_0$,

$$\begin{aligned}
x(t) &= x(0)(1-p_1)^{[t/T]}\exp\left(\int_0^t f_1(s)ds\right),\\
y(t) &= y(0)(1-p_2)^{[t/T]}\exp\left(\int_0^t f_2(s)ds\right),\\
z(t) &= z(0)(1-p_3)^{[t/T]}\exp\left(\int_0^t f_3(s)ds\right),
\end{aligned} \tag{2.7}$$

where $f_1(s) = a - bx(s) - cy(s)$, $f_2(s) = -d_1 + c_1x(s) - e_1z(s)$, and $f_3(s) = -d_2 + e_2y(s)$. Thus, $x(t), y(t)$, and $z(t)$ remain strictly positive on $[0, t_0)$. □

Lemma 2.7. *If $aT + \ln(1-p_1) \le 0$, then $x(t) \to 0$ as $t \to \infty$ for any solution $x(t)$ of the following impulsive differential equation:*

$$\begin{aligned}
x'(t) &= x(t)(a - bx(t)), \quad t \ne nT,\ t \ne (n+\tau-1)T,\\
x(t^+) &= (1-p_1)x(t), \quad t = (n+\tau-1)T,\\
x(t^+) &= x(t), \quad t = nT.
\end{aligned} \tag{2.8}$$

Proof. It is easy to see that for a given initial condition $x(0^+)$,

$$x(t) = \frac{a\exp(a(t-t_0))x(t_0)}{a + bx(t_0)(\exp(a(t-t_0)-1)}, \quad (n+\tau-1)T \le t_0 < t \le (n+\tau)T, \tag{2.9}$$

for a solution $x(t)$ of (2.8). It follows from $aT + \ln(1-p_1) \leq 0$ and (2.9) that

$$
\begin{aligned}
x((n+\tau)T) &= \frac{a(1-p_1)\exp(aT)x((n+\tau-1)T)}{a+b(1-p_1)x((n+\tau-1)T)(\exp(aT)-1)} \\
&\leq \frac{x((n+\tau-1)T)}{1+(b/a)(1-p_1)x((n+\tau-1)T)(\exp(aT)-1)} \\
&\leq x((n+\tau-1)T).
\end{aligned} \tag{2.10}
$$

Thus, we know that the sequence $\{x((n+\tau)T)\}_{n\geq 0}$ is monotonically decreasing and bounded from below by 0, and so it converges to some $L \geq 0$. From (2.10), we obtain that

$$
L = \frac{a(1-p_1)\exp(aT)L}{a+b(1-p_1)L(\exp(aT)-1)}. \tag{2.11}
$$

Since $\ln(1-p_1) + aT \leq 0$, we get $L = 0$. It is from (2.9) that, for $t \in ((n+\tau-1)T, (n+\tau)T]$,

$$
x(t) \leq x((n+\tau-1)T)\exp(aT). \tag{2.12}
$$

Therefore, we have $\lim_{t\to\infty} x(t) = 0$. □

Now, we give the basic properties of another impulsive differential equation as follows:

$$
\begin{aligned}
y'(t) &= -d_1 y(t), \quad t \neq nT,\ t \neq (n+\tau-1)T, \\
y(t^+) &= (1-p_2)y(t), \quad t = (n+\tau-1)T, \\
y(t^+) &= y(t) + q, \quad t = nT, \\
y(0^+) &= y_0 > 0.
\end{aligned} \tag{2.13}
$$

System (2.13) is a periodically forced linear system. It is easy to obtain that

$$
y^*(t) = \begin{cases} \dfrac{q\exp(-d_1(t-(n-1)T))}{1-(1-p_2)\exp(-d_1 T)}, & (n-1)T < t \leq (n+\tau-1)T, \\[2ex] \dfrac{q(1-p_2)\exp(-d_1(t-(n-1)T))}{1-(1-p_2)\exp(-d_1 T)}, & (n+\tau-1)T < t \leq nT, \end{cases} \tag{2.14}
$$

$y^*(0^+) = y^*(nT^+) = q/(1-(1-p_2)\exp(-d_1T))$, and $y^*((n+\tau-1)T^+) = q(1-p_2)\exp(-d_1\tau T)/(1-(1-p_2)\exp(-d_1T))$ is a positive periodic solution of (2.13). Moreover, we can obtain that

$$y(t) = \begin{cases} (1-p_2)^{n-1}\left(y(0^+) - \dfrac{q(1-p_2)e^{-T}}{1-(1-p_2)\exp(-d_1T)}\right)\exp(-d_1t) + y^*(t), \\ \qquad\qquad (n-1)T < t \le (n+\tau-1)T, \\ (1-p_2)^{n}\left(y(0^+) - \dfrac{q(1-p_2)e^{-T}}{1-(1-p_2)\exp(-d_1T)}\right)\exp(-d_1t) + y^*(t), \\ \qquad\qquad (n+\tau-1)T < t \le nT, \end{cases} \tag{2.15}$$

is a solution of (2.13). From (2.14) and (2.15), we get easily the following result.

Lemma 2.8. *All solutions $y(t)$ of (2.13) tend to $y^*(t)$. That is, $|y(t) - y^*(t)| \to 0$ as $t \to \infty$.*

It is from Lemma 2.8 that the general solution $y(t)$ of (2.13) can be synchronized with the positive periodic solution $y^*(t)$ of (2.13).

3. Extinction and permanence

Firstly, we show that all solutions of (1.2) are uniformly ultimately bounded.

Theorem 3.1. *There is an $R > 0$ such that $x(t) \le R$, $y(t) \le R$, and $z(t) \le R$ for all t large enough, where $(x(t), y(t), z(t))$ is a solution of system (1.2).*

Proof. Let $(x(t), y(t), z(t))$ be a solution of (1.2) with $x_0, y_0, z_0 \ge 0$ and let $u(t) = (c_1/c)x(t) + y(t) + (e_1/e_2)z(t)$ for $t \ge 0$. Then, if $t \ne nT$, $t \ne (n+\tau-1)T$, and $t > 0$, then we obtain that

$$\frac{du}{dt} = -\frac{c_1 b}{c}x^2(t) + \frac{c_1 a}{c}x(t) - d_1 y(t) - \frac{e_1 d_2}{e_2}z(t). \tag{3.1}$$

Choosing $0 < \beta_0 < \min\{d_1, d_2\}$, we get

$$\frac{du}{dt} + \beta_0 u(t) \le -\frac{c_1 b}{c}x^2(t) + \frac{c_1}{c}(a+\beta_0)x(t), \quad t \ne nT,\ t \ne (n+\tau-1)T,\ t > 0. \tag{3.2}$$

As the right-hand side of (3.2) is bounded from above by $R_0 = c_1(a+\beta_0)^2/4bc$, it follows that

$$\frac{du(t)}{dt} + \beta_0 u(t) \le R_0, \quad t \ne nT,\ t \ne (n+\tau-1)T,\ t > 0. \tag{3.3}$$

If $t = nT$, then $u(t^+) = u(t) + q$ and if $t = (n+\tau-1)T$, then $u(t^+) \le (1-p)u(t)$, where

$p = \min\{p_1, p_2, p_3\}$. From Lemma 2.5, we get that

$$\begin{aligned} u(t) &\leq u_0(1-p)^{[t/kT]} \exp\left(\int_0^t -\beta_0 ds\right) + \int_0^t (1-p)^{[(t-s)/kT]} \exp\left(\int_s^t -\beta_0 d\gamma\right) R_0 ds \\ &\quad + \sum_{j=1}^{[t/kT]} (1-p)^{[(t-kT)/jT]} \exp\left(\int_{kT}^t -\beta_0 d\gamma\right) q \\ &\leq u_0 \exp(-\beta_0 t) + \frac{R_0}{\beta_0}\left(1 - \exp(-\beta_0 t) + \frac{q\exp(\beta_0 T)}{\exp(\beta_0 T) - 1}\right). \end{aligned} \tag{3.4}$$

Since the limit of the right-hand side of (3.4) as $t \to \infty$ is

$$\frac{R_0}{\beta_0} + \frac{cq\exp(\beta_0 T)}{\exp(\beta_0 T) - 1} < \infty, \tag{3.5}$$

it easily follows that $u(t)$ is bounded for sufficiently large t. Therefore, $x(t), y(t)$, and $z(t)$ are bounded by a constant for sufficiently large t. Hence, there is an $R > 0$ such that $x(t) \leq R$, $y(t) \leq R$, and $z(t) \leq R$ for a solution $(x(t), y(t), z(t))$ with all t large enough. □

Theorem 3.2. *The periodic solution* $(0, y^*(t), 0)$ *is locally asymptotically stable if*

$$\frac{aT + \ln(1-p_1)}{c} < \Gamma < \frac{d_2 T - \ln(1-p_3)}{e_2}, \tag{3.6}$$

where

$$\Gamma = \frac{q((1-(1-p_2)\exp(-d_1 T) - p_2 \exp(-d_1 \tau T)))}{d_1(1-(1-p_2)\exp(-d_1 T))}. \tag{3.7}$$

Proof. The local stability of the periodic solution $(0, y^*(t), 0)$ of system (1.2) may be determined by considering the behavior of small amplitude perturbations of the solution. Let $(x(t), y(t), z(t))$ be any solution of system (1.2). Define $u(t) = x(t)$, $v(t) = y(t) - y^*(t)$, $w(t) = z(t)$. Then they may be written as

$$\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = \Phi(t) \begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix}, \tag{3.8}$$

where $\Phi(t)$ satisfies

$$\frac{d\Phi}{dt} = \begin{pmatrix} a - cy^*(t) & 0 & 0 \\ c_1 y^*(t) & -d_1 & -e_1 y^*(t) \\ 0 & 0 & -d_2 + e_2 y^*(t) \end{pmatrix} \Phi(t) \tag{3.9}$$

and $\Phi(0) = I$, the identity matrix. So the fundamental solution matrix is

$$\Phi(t) = \begin{pmatrix} \exp\left(\int_0^t a - cy^*(s)ds\right) & 0 & 0 \\ \exp\left(c_1\int_0^t y^*(s)ds\right) & \exp(-d_1 t) & \exp\left(-e_1\int_0^t y^*(s)ds\right) \\ 0 & 0 & \exp\left(\int_0^t -d_2 + e_2 y^*(s)ds\right) \end{pmatrix}. \tag{3.10}$$

The resetting impulsive conditions of system (1.2) become

$$\begin{gathered} \begin{pmatrix} u((n+\tau-1)T^+) \\ v((n+\tau-1)T^+) \\ u((n+\tau-1)T^+) \end{pmatrix} = \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1-p_3 \end{pmatrix} \begin{pmatrix} u((n+\tau-1)T) \\ v((n+\tau-1)T) \\ w((n+\tau-1)T) \end{pmatrix}, \\ \begin{pmatrix} u(nT^+) \\ v(nT^+) \\ w(nT^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \\ w(nT) \end{pmatrix}. \end{gathered} \tag{3.11}$$

Note that all eigenvalues of

$$S = \begin{pmatrix} 1-p_1 & 0 & 0 \\ 0 & 1-p_2 & 0 \\ 0 & 0 & 1-p_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Phi(T) \tag{3.12}$$

are $\mu_1 = (1-p_1)\exp(\int_0^T a - cy^*(t)dt)$, $\mu_2 = (1-p_2)\exp(-d_1 T) < 1$, and $\mu_3 = (1-p_3)\exp(\int_0^T - d_2 + e_2 y^*(t)dt)$. Since

$$\int_0^T y^*(t)dt = \frac{q(1-(1-p_2)\exp(-d_1 T) - p_2\exp(-d_1\tau T))}{d_1(1-(1-p_2)\exp(-d_1 T))}, \tag{3.13}$$

the conditions $\mu_1 < 1$ and $\mu_3 < 1$ are equivalent to the equation (3.7). By Floquet theory [13, Chapter 2], we obtain that $(0, y^*(t), 0)$ is locally asymptotically stable. □

Theorem 3.3. *The periodic solution* $(0, y^*(t), 0)$ *is globally stable if*

$$aT + \ln(1-p_1) \le 0, \qquad \Gamma < \frac{d_2 T - \ln(1-p_3)}{e_2}. \tag{3.14}$$

Proof. Suppose that $aT+\ln(1-p_1)\le 0$ and $\Gamma<(d_2T-\ln(1-p_3))/e_2$. Then there are sufficiently small numbers $\epsilon_1,\epsilon_2>0$ such that

$$\phi\equiv(1-p_3)\exp\left(-d_2T+\frac{\Lambda}{(d_1-c_1\epsilon_1)(1-(1-p_2)\exp(-(d_1-c_1\epsilon_1)T)}+e_2\epsilon_2T\right)<1, \tag{3.15}$$

where $\Lambda=e_2q(1-(1-p_2)\exp(-(d_1-c_1\epsilon_1)T)-p_2\exp(-(d_1-c_1\epsilon_1)\tau T)$. It follows from the first equation in (1.2) that $x'(t)=x(t)(a-bx(t)-cy(t))\le x(t)(a-bx(t))$ for $t\ne nT,\ t\ne(n+\tau-1)T$. By Lemma 2.4, $x(t)\le\widetilde{x}(t)$ for $t>0$, where $\widetilde{x}(t)$ is the solution of (2.8). By Lemma 2.7, we get $\widetilde{x}(t)\to 0$ as $t\to\infty$, which implies that there is a $T_1>0$ such that $x(t)\le\epsilon_1$ for $t\ge T_1$. For the sake of simplicity, we suppose that $x(t)\le\epsilon_1$ for all $t>0$. We can infer from the second equation in (1.2) that $y'(t)=y(t)(-d_1+c_1x(t)-e_1z(t))\le y(t)(-d_1+c_1x(t))\le y(t)(-d_1+c_1\epsilon_1)$ for $t\ne nT,\ t\ne(n+\tau-1)T$. Let $\widetilde{y}_1(t)$ be the solution of the following equation:

$$\begin{aligned}
\widetilde{y}_1'(t)&=-(d_1-c_1\epsilon_1)\widetilde{y}_1(t),\quad t\ne nT,\ t\ne(n+\tau-1)T,\\
\widetilde{y}_1(t^+)&=(1-p_2)\widetilde{y}_1(t),\quad t=(n+\tau-1)T,\\
\widetilde{y}_1(t^+)&=\widetilde{y}_1(t)+q,\quad t=nT,\\
\widetilde{y}_1(0^+)&=y_0.
\end{aligned} \tag{3.16}$$

Then we know that $y(t)\le\widetilde{y}_1(t)$ by Lemma 2.4. Thus, from the third equation in (1.2) and Lemma 2.8, we obtain that

$$\begin{aligned}
z'(t)&\le z(t)\left(-d_2+e_2\widetilde{y}_1(t)\right)\\
&\le z(t)\left(-d_2+e_2\widetilde{y}_1^*(t)+e_2\epsilon_2\right),
\end{aligned} \tag{3.17}$$

where

$$\widetilde{y}_1^*(t)=\begin{cases}\dfrac{q\exp(-(d_1-c_1\epsilon_1)(t-(n-1)T))}{1-(1-p_2)\exp(-(d_1-c_1\epsilon_1)T)}, & (n-1)T<t\le(n+\tau-1)T,\\[2ex] \dfrac{q(1-p_2)\exp(-(d_1-c_1\epsilon_1)(t-(n-1)T))}{1-(1-p_2)\exp(-(d_1-c_1\epsilon_1)T)}, & (n+\tau-1)T<t\le nT,\end{cases} \tag{3.18}$$

is the periodic solution of (3.16). Integrating (3.17) on $((n+\tau-1)T,(n+\tau)T]$, we obtain

$$z((n+\tau)T)\le z((n+\tau-1)T^+)\exp\left(\int_{(n+\tau-1)T}^{(n+\tau)T}-d_2+e_2\widetilde{y}_1^*(t)+e_2\epsilon_2dt\right)=z((n+\tau-1)T)\phi. \tag{3.19}$$

Therefore, we have $z((n+\tau)T) \leq z(\tau T)\phi^n \to 0$ as $n \to \infty$. Also, we obtain, for $t \in ((n+\tau-1)T, (n+\tau)T]$,

$$\begin{aligned} z(t) &\leq z\big((n+\tau-1)T^+\big)\exp\left(\int_{(n+\tau-1)T}^{t} -d_2 + e_2\tilde{y}_1^*(t) + e_2\epsilon_1 dt\right) \\ &\leq z\big((n+\tau-1)T\big)\exp\left(\frac{qe_2}{1-(1-p_2)\exp\big(-(d_1-c_1\epsilon_1)T\big)} + e_2\epsilon_1 T\right), \end{aligned} \tag{3.20}$$

which implies that $z(t) \to 0$ as $t \to \infty$. Thus, we may assume that $z(t) \leq \epsilon_2$ for all $t > 0$. It is from the second equation in (1.2) that

$$\begin{aligned} y'(t) &\geq y(t)\big(-d_1 + c_1 x(t) - e_1\epsilon_2\big) \\ &\geq y(t)\big(-d_1 - e_1\epsilon_2\big). \end{aligned} \tag{3.21}$$

Let $\tilde{y}_2(t)$ and $\tilde{y}_2^*(t)$ be the solution and the periodic solution of (2.13), respectively, with d_1 changed into $d_1 + e_1\epsilon_2$ and the same initial value y_0. Then we infer from Lemmas 2.4 and 2.7 that $\tilde{y}_2(t) \leq y(t) \leq \tilde{y}_1(t)$ and $\tilde{y}_1(t)$ and $\tilde{y}_2(t)$ become close to $\tilde{y}_1^*(t)$ and $\tilde{y}_2^*(t)$ as $t \to \infty$, respectively. Note that $\tilde{y}_1^*(t)$ and $\tilde{y}_2^*(t)$ are close to $y^*(t)$ as $\epsilon_1, \epsilon_2 \to 0$. Therefore, we obtain $y(t) \to y^*(t)$ as $t \to \infty$. □

Definition 3.4. System (1.2) is permanent if there exist $M \geq m > 0$ such that, for any solution $(x(t), y(t), z(t))$ of system (1.2) with $x_0, y_0, z_0 > 0$,

$$\begin{aligned} m &\leq \lim_{t\to\infty}\inf x(t) \leq \lim_{t\to\infty}\sup x(t) \leq M, \\ m &\leq \lim_{t\to\infty}\inf y(t) \leq \lim_{t\to\infty}\sup y(t) \leq M, \\ m &\leq \lim_{t\to\infty}\inf z(t) \leq \lim_{t\to\infty}\sup z(t) \leq M. \end{aligned} \tag{3.22}$$

To prove the permanence of system (1.2), we consider the following two subsystems. If the top predator is absent, that is, $z(t) = 0$, then system (1.2) can be expressed as

$$\begin{aligned} x'(t) &= x(t)\big(a - bx(t) - cy(t)\big), \quad t \neq nT,\ t \neq (n+\tau-1)T, \\ y'(t) &= y(t)\big(-d_1 + c_1x(t)\big), \quad t \neq nT,\ t \neq (n+\tau-1)T, \\ x(t^+) &= (1-p_1)x(t), \quad t \neq (n+\tau-1)T, \\ y(t^+) &= (1-p_2)y(t), \quad t \neq (n+\tau-1)T, \\ x(t^+) &= x(t), \quad t = nT, \\ y(t^+) &= y(t) + p, \quad t = nT, \\ (x(0^+), y(0^+)) &= (x_0, y_0). \end{aligned} \tag{3.23}$$

If the prey is extinct, then system (1.2) can be expressed as

$$\begin{aligned}
y'(t) &= y(t)\big(-d_1 - e_1 z(t)\big), \quad t \neq nT,\ t \neq (n+\tau-1)T,\\
z'(t) &= z(t)\big(-d_2 + e_2 y(t)\big), \quad t \neq nT,\ t \neq (n+\tau-1)T,\\
y(t^+) &= (1-p_2)y(t), \quad t \neq (n+\tau-1)T,\\
z(t^+) &= (1-p_3)z(t), \quad t \neq (n+\tau-1)T,\\
y(t^+) &= y(t) + p, \quad t = nT,\\
z(t^+) &= z(t), \quad t = nT,\\
\big(y(0^+), z(0^+)\big) &= (y_0, z_0).
\end{aligned} \tag{3.24}$$

Especially, Liu et al. [23] have given a condition for permanence of subsystem (3.23).

Theorem 3.5 (see [23]). *Subsystem* (3.23) *is permanent if* $\Gamma < (aT + \ln(1-p_1))/c$, *where*

$$\Gamma = \frac{q\big((1-(1-p_2)\exp(-d_1T) - p_2\exp(-d_1\tau T))\big)}{d_1\big(1-(1-p_2)\exp(-d_1T)\big)}. \tag{3.25}$$

Theorem 3.6. *Subsystem* (3.24) *is permanent if* $(d_2T - \ln(1-p_3))/e_2 < \Gamma$, *where*

$$\Gamma = \frac{q\big((1-(1-p_2)\exp(-d_1T) - p_2\exp(-d_1\tau T))\big)}{d_1\big(1-(1-p_2)\exp(-d_1T)\big)}. \tag{3.26}$$

Proof. Let $(y(t), z(t))$ be a solution of subsystem (3.24) with $y(0) > 0$ and $z(0) > 0$. From Theorem 3.1, we may assume that $y(t) \le R$ with $d_1 + R > 0$ and $z(t) \le R/e_1$. Then $y'(t) \ge -(d_1 + R)y(t)$. From Lemmas 2.4 and 2.8, we have $y(t) \ge u^*(t) - \epsilon$ for sufficiently small $\epsilon > 0$, where

$$u^*(t) = \begin{cases} \dfrac{q\exp\big(-(d_1+R)(t-(n-1)T)\big)}{1-(1-p_2)\exp\big(-(d_1+R)T\big)}, & (n-1)T < t \le (n+\tau-1)T,\\[2ex] \dfrac{q(1-p_2)\exp\big(-(d_1+R)(t-(n-1)T)\big)}{1-(1-p_2)\exp\big(-(d_1+R)T\big)}, & (n+\tau-1)T < t \le nT. \end{cases} \tag{3.27}$$

Thus, we obtain that $y(t) \ge (q(\exp(-(d_1+R)T)/(1-(1-p_2)\exp(-(d_1+R)T))) - \epsilon \equiv m_0$ for sufficiently large t. Therefore, we only need to find an $m_2 > 0$ such that $z(t) \ge m_2$ for large enough t. We will do this in the following two steps.

Step 1. From (3.26), we can choose $m_1 > 0$, $\epsilon_1 > 0$ small enough such that

$$\Phi \equiv (1-p_3)\exp\left(-d_2T + \frac{\Delta}{(d_1+e_1m_1)\big(1-(1-p_2)\exp(-(d_1+e_1m_1)T)\big)} - e_2\epsilon_1 T\right) > 1, \tag{3.28}$$

where $\Delta = e_2 q(1-(1-p_2)\exp(-(d_1+e_1m_1)T) - p_2\exp(-(d_1+e_1m_1)\tau T))$. In this step, we will show that $z(t_1) \geq m_1$ for some $t_1 > 0$. Suppose that $z(t) < m_1$ for $t > 0$. Consider the following system:

$$\begin{aligned}
v'(t) &= -(d_1+e_1m_1)v(t), \quad t \neq (n+\tau-1)T,\ nT,\\
w'(t) &= -(d_2-e_2v(t))w(t), \quad t \neq (n+\tau-1)T,\ nT,\\
v(t^+) &= (1-p_2)v(t), \quad t = (n+\tau-1)T,\\
w(t^+) &= (1-p_3)w(t), \quad t = (n+\tau-1)T,\\
v(t^+) &= v(t)+p, \quad t = nT,\\
w(t^+) &= w(t), \quad t = nT,\\
(v(0^+), w(0^+)) &= (y_0, z_0).
\end{aligned} \tag{3.29}$$

Then, by Lemma 2.4, we obtain $z(t) \geq w(t)$. By Lemma 2.8, we have $v(t) \geq v^*(t) - \epsilon_1$, where, for $t \in ((n-1)T, nT]$,

$$v^*(t) = \begin{cases} \dfrac{q\exp\left(-(d_1+e_1m_1)(t-(n-1)T)\right)}{1-(1-p_2)\exp\left(-(d_1+e_1m_1)T\right)}, & (n-1)T < t \leq (n+\tau-1)T,\\[2ex] \dfrac{q(1-p_2)\exp\left(-(d_1+e_1m_1)(t-(n-1)T)\right)}{1-(1-p_2)\exp\left(-(d_1+e_1m_1)T\right)}, & (n+\tau-1)T < t \leq nT. \end{cases} \tag{3.30}$$

Thus, for $t \neq (n+\tau-1)T,\ t \neq nT$,

$$w'(t) \geq \left(-d_2 + e_2(v^*(t)-\epsilon_1)\right)w(t). \tag{3.31}$$

Integrating (3.31) on $((n+\tau-1)T, (n+\tau)T]$, we get

$$w((n+\tau)T) \geq w((n+\tau-1)T^+)\exp\left(\int_{(n+\tau-1)T}^{(n+\tau)T} -d_2 + e_2(v^*(t)-\epsilon_1)dt\right) = w((n+\tau-1)T)\Phi. \tag{3.32}$$

Therefore, $z((n+\tau+k)T) \geq w((n+\tau+k)T) \geq w((n+\tau)T)\Phi^k \to \infty$ as $k \to \infty$ which is a contradiction to the boundedness of $z(t)$.

Step 2. Without loss of generality, we may let $z(t_1) = m_1$. If $z(t) \geq m_1$ for all $t > t_1$, then subsystem (3.24) is permanent. If not, we may let $t_2 = \inf_{t>t_1}\{z(t) < m_1\}$. Then $z(t) \geq m_1$ for $t_1 \leq t \leq t_2$ and, by continuity of $z(t)$, we have $z(t_2) = m_1$ and $t_1 < t_2$. There exists a $t'(> t_2)$ such that $z(t') \geq m_1$ by Step 1. Set $t_3 = \inf_{t>t_2}\{z(t) \geq m_1\}$. Then $z(t) < m_1$ for $t_2 < t < t_3$ and $z(t_3) = m_1$. We can continue this process by using Step 1. If the process is stopped in finite times, we complete the proof. Otherwise, there exists an interval sequence $[t_{2k}, t_{2k+1}],\ k \in \mathbb{N}$, which has the following properties: $z(t) < m_1,\ t \in (t_{2k}, t_{2k+1})$, $t_{2k-1} < t_{2k} \leq t_{2k+1}$, and $z(t_n) = m_1$, where

$k, n \in \mathbb{N}$. Let $T_0 = \sup\{t_{2k+1} - t_{2k} \mid k \in \mathbb{N}\}$. If $T_0 = \infty$, then we can take a subsequence $\{t_{2k_i}\}$ satisfying $t_{2k_i+1} - t_{2k_i} \to \infty$ as $k_i \to \infty$. As in the proof of Step 1, this will lead to a contradiction to the boundedness of $z(t)$. Then we obtain $T_0 < \infty$. Note that

$$\begin{aligned} z(t) &\geq z(t_{2k}) \exp\left(\int_{t_{2k}}^{t} - d_2 + e_2(v^*(s) - \epsilon_1) ds\right) \\ &\geq m_1 \exp(-d_2 T_0) \equiv m_2, \quad t \in (t_{2k}, t_{2k+1}],\ k \in \mathbb{N}. \end{aligned} \tag{3.33}$$

Thus we obtain that $\liminf_{t\to\infty} z(t) \geq m_2$. Therefore, we complete the proof. □

Theorem 3.7. *System* (1.2) *is permanent if*

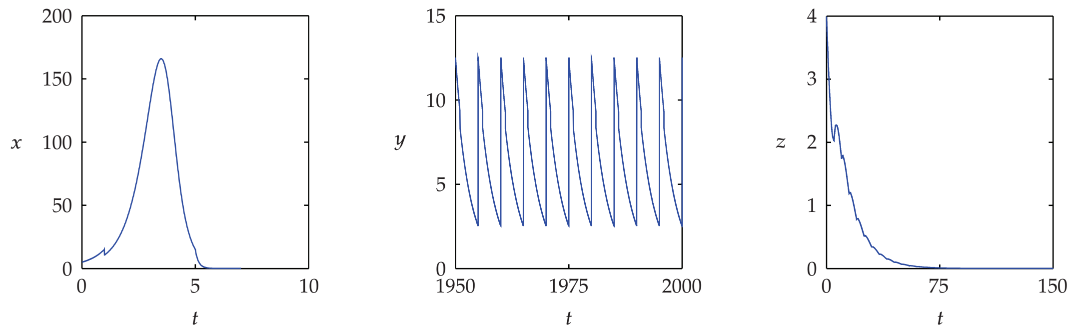
$$\frac{d_2 T - \ln(1 - p_3)}{e_2} < \Gamma < \frac{aT + \ln(1 - p_1)}{c}. \tag{3.34}$$

Proof. Consider the following two subsystems of system (1.2):

$$\begin{aligned} x_1'(t) &= x_1(t)(a - bx_1(t) - cy_1(t)), \quad t \neq nT,\ t \neq (n + \tau - 1)T, \\ y_1'(t) &= y_1(t)(-d_1 + c_1 x_1(t)), \quad t \neq nT,\ t \neq (n + \tau - 1)T, \\ x_1(t^+) &= (1 - p_1)x_1(t), \quad t = (n + \tau - 1)T, \\ y_1(t^+) &= (1 - p_2)y_1(t), \quad t = (n + \tau - 1)T, \\ x_1(t^+) &= x_1(t), \quad t = nT, \\ y_1(t^+) &= y_1(t) + p, \quad t = nT, \\ (x_1(0^+), y_1(0^+)) &= (x_0, y_0); \end{aligned} \tag{3.35}$$

$$\begin{aligned} y_2'(t) &= y_2(t)(-d_1 - e_1 z_2(t)), \quad t \neq nT,\ t \neq (n + \tau - 1)T, \\ z_2'(t) &= z_2(t)(-d_2 + e_2 y_2(t)), \quad t \neq nT,\ t \neq (n + \tau - 1)T, \\ y_2(t^+) &= (1 - p_2)y_2(t), \quad t = (n + \tau - 1)T, \\ z_2(t^+) &= (1 - p_3)z_2(t), \quad t = (n + \tau - 1)T, \\ y_2(t^+) &= y_2(t) + p, \quad t = nT, \\ z_2(t^+) &= z_2(t), \quad t = nT, \\ (y_2(0^+), z_2(0^+)) &= (y_0, z_0). \end{aligned} \tag{3.36}$$

It follows from Lemma 2.4 that $x_1(t) \leq x(t)$, $y_1(t) \geq y(t)$, $y_2(t) \leq y(t)$, and $z_2(t) \leq z(t)$. If $\Gamma < (aT + \ln(1 - p_1))/c$, by Theorem 3.5, subsystem (3.35) is permanent. Thus we can take $T_1 > 0$ and $m_1 > 0$ such that $x(t) \geq m_1$ for $t \geq T_1$. Further, if $(d_2 T - \ln(1 - p_3))/e_2 < \Gamma$, by Theorem 3.6, subsystem (3.36) is also permanent. Therefore, there exist a $T_2 > 0$ and $m_2, m_3 > 0$ such that $y(t) \geq m_2$ and $z(t) \geq m_3$ for $t \geq T_2$. The proof is complete. □



(a) (b) (c)

Figure 1: $a = 2.0$, $b = 0.001$, $c = 0.5$, $c_1 = 0.01$, $d_1 = 0.3$, $d_2 = 0.2$, $e_1 = 0.01$, $e_2 = 0.02$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.01$, $\tau = 0.2$, and $T = 5$. (a)–(c) Time series of system (1.2) when $q = 10$.

4. Numerical examples

In this section, we are concerned with the numerical investigation of some situations covered by Theorems 3.2 and 3.7 which may lead to a chaotic behavior of system (1.2). It is easy to see that the unperturbed three-species food chain system (1.1) has four nonnegative equilibria:

(1) the trivial equilibrium $A(0,0,0)$;

(2) the mid-predator and top-predator free equilibrium $B(d_1/b,0,0)$;

(3) the top-predator free equilibrium $C(d_1/c_1,(ac_1-d_1b)/cc_1,0)\cdot(ac_1-d_1b>0)$;

(4) the positive equilibrium $E^* = (x^*,y^*,z^*)$ if and only if $ae_2c_1 - d_2cc_1 - d_1be_2 > 0$, where

$$x^* = \frac{ae_2 - d_2c}{be_2}, \qquad y^* = \frac{d_2}{e_2}, \qquad z^* = \frac{ae_2c_1 - d_2cc_1 - d_1be_2}{be_1e_2}. \tag{4.1}$$

The stability of equilibrium of system (1.1) has been studied by Zhang and Chen [30].

Lemma 4.1 (see [30]). (1) *If positive equilibrium E^* exists, then E^* is globally stable.*

(2) *If positive equilibrium E^* does not exist and C exists, then C is globally stable.*

(3) *If positive equilibrium E^* and C do not exist, then B is globally stable.*

Throughout this section, we chose $(x(0),y(0),z(0)) = (5,2,4)$ as an initial point.

For $a = 2.0$, $b = 0.001$, $c = 0.5$, $c_1 = 0.01$, $d_1 = 0.3$, $d_2 = 0.2$, $e_1 = 0.01$, $e_2 = 0.02$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.01$, $\tau = 0.2$, and $T = 5$, it follows from Theorem 3.2 that the periodic solution $(0,y^*(t),0)$ is locally stable if $6.3771 < q < 16.6987$. The unperturbed system (1.1) has a globally stable top-predator free equilibrium $C(20,3.94,0)$, but no positive equilibria. The behavior of the trajectories of system (1.2) when $q = 10$ is depicted in Figure 1. Another behavior is illustrated in Figure 2 for $a = 4.0$, $b = 0.001$, $c = 0.5$, $c_1 = 0.01$, $d_1 = 0.3$, $d_2 = 0.02$, $e_1 = 0.01$, $e_2 = 0.02$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.01$, $\tau = 0.2$, and $T = 5$. In this case, the trajectory of system (1.2) tends to a periodic orbit of period T. We know from Theorem 3.7 that system (1.2) is permanent when $1.8194 < q < 12.9901$. The unperturbed system (1.1) has a global stable positive equilibrium $E^* = (3500,1,3470)$. An example of chaotic behavior is

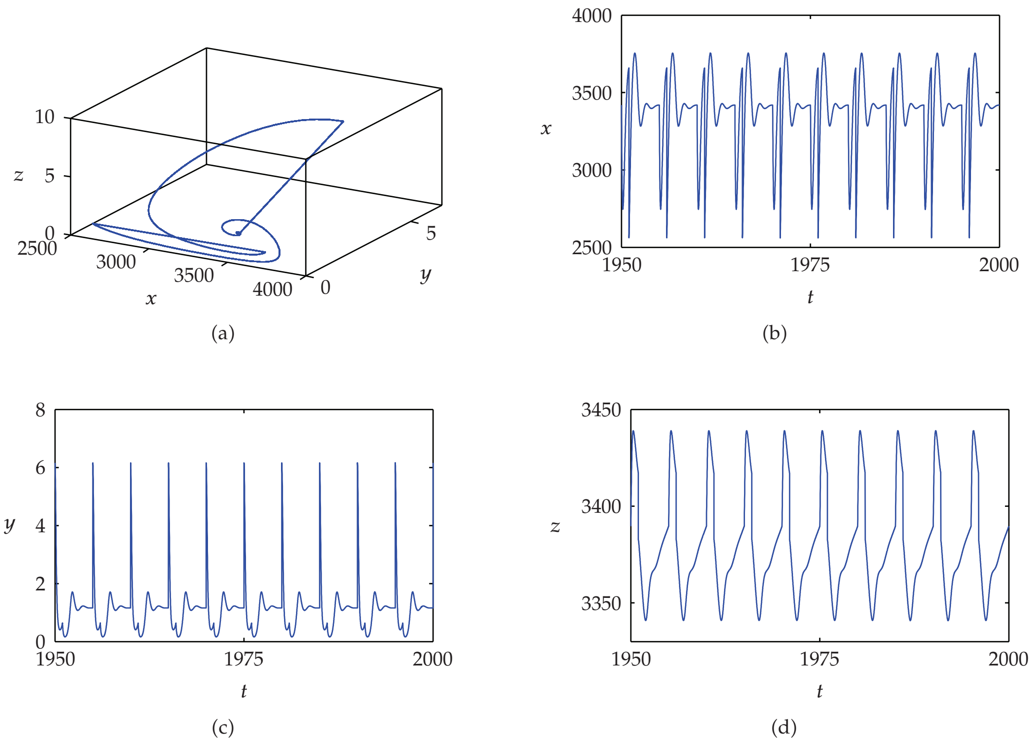


Figure 2: $a = 4.0$, $b = 0.001$, $c = 0.5$, $c_1 = 0.01$, $d_1 = 0.3$, $d_2 = 0.02$, $e_1 = 0.01$, $e_2 = 0.02$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.01$, $\tau = 0.2$, and $T = 5$. (a) The trajectory of system (1.2) when $q = 5$. (b)–(d) Time series.

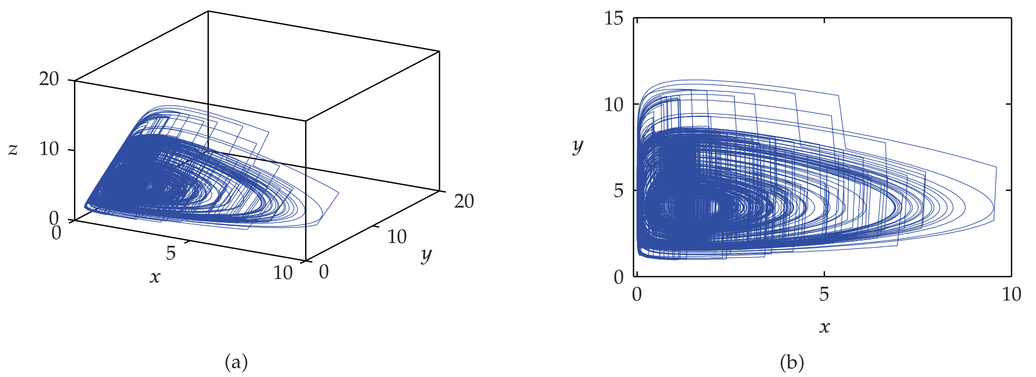


Figure 3: $a = 4.0$, $b = 0.0002$, $c = 1.0$, $c_1 = 0.3$, $d_1 = 0.3$, $d_2 = 0.01$, $e_1 = 0.05$, $e_2 = 0.0005$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.01$, $\tau = 0.2$, and $T = 5$. (a) The trajectory of system (1.2) when $q = 3$. (b) The two-dimensional plot x versus y.

exhibited in Figure 3 for $a = 4.0$, $b = 0.0002$, $c = 1.0$, $c_1 = 0.3$, $d_1 = 0.3$, $d_2 = 0.01$, $e_1 = 0.05$, $e_2 = 0.0005$, $p_1 = 0.3$, $p_2 = 0.1$, $p_3 = 0.01$, $\tau = 0.2$, and $T = 5$. In this case, the unperturbed system (1.1) also has a globally stable top-predator free equilibrium $C(2/3, 3.99, 0)$, but no positive equilibria. By Theorem 3.2, we can figure out that system (1.2) is also locally stable if $6.4951 < q < 39.7113$. Figure 3 indicates that a trajectory may have chaotic behavior.

5. Conclusion

In this paper, we have studied dynamical properties of a food chain system with Lotka-Volterra functional response and impulsive perturbations. We have found sufficient conditions for extinction and permanence of the system by means of the Floquet theory and a comparison theorem. We also have given numerical examples that exhibit a periodic trajectory and a chaotic behavior.

Now, assume that $\Gamma < (d_2T - \ln(1-p_3))/e_2$. It follows from Theorem 3.3 that if p_1 is large enough to make $aT + \ln(1-p_1) \leq 0$ negative (in other words, if we choose strong pesticide to eradicate pests), then the lowest-level prey and top-predator free periodic solution is globally stable, which means that we succeed in controlling pest population. Further, if we only consider biological control in system (1.2), that is, if we take $\tau = 0$, $p_1 = p_2 = p_3 = 0$, then we obtain with the help of Theorems 3.2 and 3.7 the following results.

Theorem 5.1. *Suppose that $\tau = p_1 = p_2 = p_3 = 0$. Then the following statements hold.*

(1) *The periodic solution $(0, y^*(t), 0)$ is locally asymptotically stable if $ad_1T/c < q < d_1d_2T/e_2$.*

(2) *System (1.2) is permanent if $d_1d_2T/e_2 < q < ad_1T/c$.*

Especially, we get Theorem 3.1 in [30] as corollary of Theorem 5.1(1). From Theorem 5.1, we note that if there is no chemical control, global stability of a lower-level prey and top-predator free periodic solution of system (1.2) is not guaranteed. In other words, it is possible to fail to control pest population by using just one control strategy. Theoretically speaking, we need to use more than two different pest control tactics simultaneously to succeed in pest population control.

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