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Review Article

The Generalized Julia Set Perturbed by Composing Additive and Multiplicative Noises

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This paper contrastively researches the structural characteristic and the fission-evolution law of four different kinds of generalized Julia set (generalized J set in short) with different parameter c, which includes the generalized J set without any perturbation, the generalized J set perturbed by additive noises, the generalized J set perturbed by multiplicative noise, and the generalized J set perturbed by composing additive and multiplicative noises, analyzes the effect of random perturbation to the generalized J set, and illuminates the stability of the generalized J set.

1. Introduction

In the area of fractals, deep investigations have been made on the Mandelbrot set (M set in short) of the complex mapping $z \leftarrow z^{\alpha} + c$ ($\alpha = 2$) by using computer technologies. During last twenty years, people studied the generalized M set with $\alpha \in R$ and found that it has a regularity structure [1–8]. In recent years, Argyris et al. discussed the classification and affection of noise in complex dynamical system [9]; Argyris et al. studied the structural characteristic of M-J sets containing noise after importing additive noise and multiplicative noise into the complex map $z_{n+1} = z_n^2 + c$ [10–14]. The authors studied the structural characteristic and fission-evolution law of additive perturbed generalized M-J sets, analyzed the dynamic additive noise perturbed and multiplicative noise perturbed generalized M-J sets [15, 16].

Recently, Negi and Rani put forward a new noise criterion which integrates the dynamic additive noise and dynamic multiplicative noise, and discussed its effect on the usual and superior Mandelbrot maps [17]. Andreadis and Karakasidis proposed a definition for a probabilistic Mandelbrot map and studied the numerical stability of the Mandelbrot and the Julia set of a probabilistic Mandelbrot map [18]. The authors studied the generalized M set perturbed by composing noise of additive and multiplicative and analyzed the effect of random perturbation to the generalized M set [19].

In the present work, we investigate on the structural relationship of the generalized Julia sets with different parameter c perturbed by random noise, study the structural characteristics of the generalized J sets perturbed by additive noise, multiplicative noise, and the composing noise of additive and multiplicative, and analyze the effect of random perturbation to the generalized M set.

2. Theory and Method

According to [19], we give the definition of the generalized J set with composing additive and multiplicative noise. The method in the paper is the same with that of [19].

Definition 2.1. Assume that $f(z_n) = z_n^{\alpha} + W + c$ or $g(z_n) = \gamma(\lambda, k_1, k_2, \mathbf{w}_n)z_n^{\alpha} + c$ ($|\alpha| > 1$) is a complex map in Riemann sphere $\widehat{C}$, $\varphi = \mu f + (1-\mu)g(\mu \in [0,1])$, F_{φ} is a collection of points z whose trajectories do not converge to infinite in C, that is,

$$F_{\varphi} = \left\{ z \in C : \left\{ \left| \varphi^{k}(z) \right| \right\}_{k=1}^{\infty} \text{ is bounded} \right\}. \tag{2.1}$$

Then this set is called the filled generalized J set with composing additive and multiplicative noise corresponding to φ; the boundary of F_{φ} is called the generalized J set of the complex map φ, recorded as J_{φ}, that is,

$$J_{\varphi} = \partial F_{\varphi}. \tag{2.2}$$

3. Experiment and Result

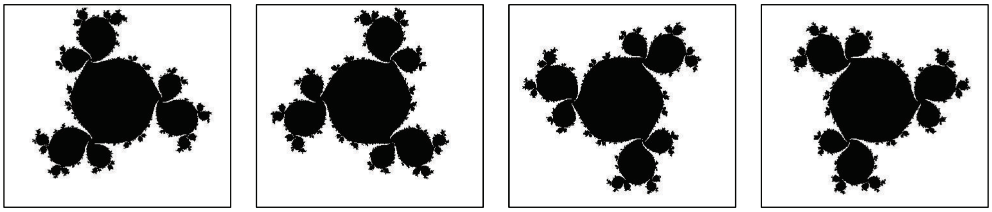
Selecting escape-radius as $R = 30$ and escape-time restriction as $N = 100$, we plot the composing noise perturbed generalized J set from Equation (GN) using escape-time algorithm. When $\alpha > 0$, the black in the figure is the stable region while the white is the escape region; when $\alpha < 0$, the white in the figure is the stable region while the black is the escape region. As to the additive noise perturbed generalized J set, when $|m_1, m_2| < 0.1$ or $|m_1, m_2| > 1$, the change of the structure can be ignored; also as to the multiplicative noise perturbed generalized J set, when $|k_1, k_2| < 0.1$ or $|k_1, k_2| > 1$, the structure almost does not change. Therefore, we select $0.1 \le |m_1, m_2, k_1, k_2| \le 1$ and $\mu = 0.5$. According to the structure characteristics of the generalized J set, we start from the following aspects.

3.1. The Generalized J Set without Any Perturbation

When α is odd, observing the relationship between the generalized J sets without any perturbation and parameter c (Figures 1 and 2), we can find the following properties:

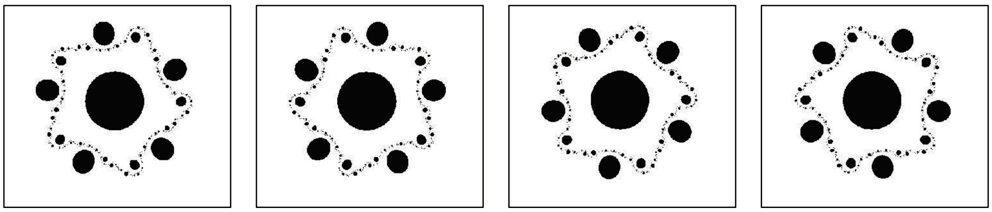
(1) the graph when $c = a + bi$ and the graph when $c = -a + bi$ have mirror symmetry about y-axis;

(2) Clockwise rotating the graph when $c = a + bi$ for 180 degrees we will get the graph when $c = -a - bi$;



(a) $c = -0.5 + 0.5i$ (b) $c = 0.5 + 0.5i$ (c) $c = 0.5 - 0.5i$ (d) $c = -0.5 - 0.5i$

Figure 1: The generalized J sets without any perturbation when $\alpha = 3$.



(a) $c = 0.5 + 0.5i$ (b) $c = -0.5 + 0.5i$ (c) $c = 0.5 - 0.5i$ (d) $c = -0.5 - 0.5i$

Figure 2: The generalized J sets without any perturbation when $\alpha = -5$.

(3) The graph when $c = a + bi$ and the graph when $c = a - bi$ have mirror symmetry about x-axis.

Observing the generalized J sets without any perturbation when α is even (Figure 3), we can find that they have the following properties:

(1) The graph when $c = a + bi$ and the graph when $c = a - bi$ have mirror symmetry about x-axis.

Observing the generalized J sets without any perturbation when α is positive decimal and phase angle $\theta \in [-\pi, \pi)$ (Figure 4), we can find that they have the following property:

(1) the graph when $c = a + bi$ and the graph when $c = a - bi$ have mirror symmetry about x-axis.

Observing Figures 5 and 6, we can find that when α is positive decimal and the value bound of phase angle is $[-\pi/2, 3\pi/2)$ or $[-3\pi/2, \pi/2)$, the generalized J sets have the following property:

(1) the graph when phase angle $\theta \in [-\pi/2, 3\pi/2)$ with $c = a + bi$ and the graph when phase angle $\theta \in [-3\pi/2, \pi/2)$ with $c = a - bi$ have mirror symmetry about x-axis.

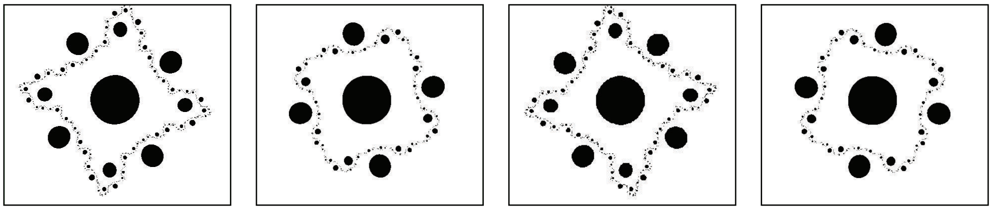
In fact, when α is decimal, the generalized J sets without any perturbation satisfy the following properties:

(1) when phase angle $\theta \in [-\pi, \pi)$, the graph with $c = a+bi$ and the graph with $c = a-bi$ have mirror symmetry about x-axis;

(2) the graph when phase angle $\theta \in [-\pi/2, 3\pi/2)$ with $c = a + bi$ and the graph when phase angle $\theta \in [-3\pi/2, \pi/2)$ with $c = a - bi$ have mirror symmetry about x-axis;

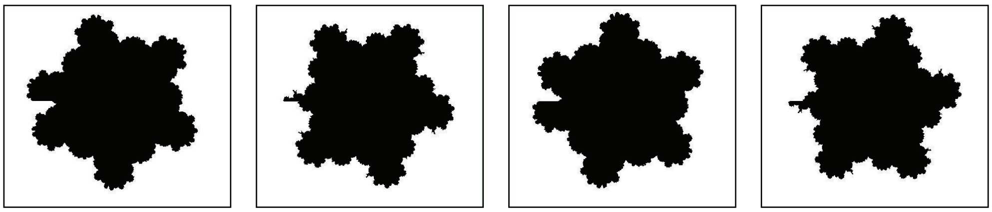
(3) when phase angle $\theta \in [0, 2\pi)$, with different values of parameter c corresponding to different figures, there are basically no corresponding rules as above;

(4) the proof of the above symmetry property of generalized J set can refer to [7, 15].



(a) $c = 0.5 + 0.5i$ (b) $c = -0.5 + 0.5i$ (c) $c = 0.5 - 0.5i$ (d) $c = -0.5 - 0.5i$

Figure 3: The generalized J sets without any perturbation when $\alpha = -4$.



(a) $c = 0.5 + 0.5i$ (b) $c = -0.5 + 0.5i$ (c) $c = 0.5 - 0.5i$ (d) $c = -0.5 - 0.5i$

Figure 4: The generalized J sets without any perturbation when $\alpha = 5.5$, $\theta \in [-\pi, \pi)$.

3.2. The Generalized J Set Perturbed by Additive Noise

Let the order of parameters be (a, b, m_1, m_2); s here a, b are the real part and imaginary part of parameter c, respectively; m_1, m_2 are the intensity coefficients of the real part and imaginary part of additive perturbation W, respectively. Observing the additive perturbed generalized J sets when α is odd (Figure 7), we can get the following properties:

(1) the graph with $c = a + bi$ and additive noise parameter (m_1, m_2) and the graph with $c = -a + bi$ and additive noise parameter $(-m_1, m_2)$ have mirror symmetry about y-axis;

(2) the graph with $c = a + bi$ and additive noise parameter (m_1, m_2) and the graph with $c = a - bi$ and additive noise parameter $(m_1, -m_2)$ have mirror symmetry about x-axis;

(3) rotating the graph with $c = a + bi$ and additive noise parameter (m_1, m_2) by 180 degrees we will get the graph with $c = -a - bi$ and additive noise parameter$(-m_1, -m_2)$.

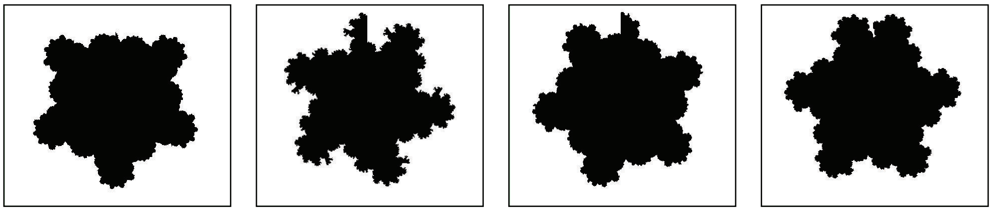
Observing the additive perturbed generalized J sets when α is even (Figure 8), we can get the following property:

(1) the graph with $c = a + bi$ and additive noise parameter (m_1, m_2) and the graph with $c = a - bi$ and additive noise parameter $(m_1, -m_2)$ have mirror symmetry about x-axis.

Next is the property of the generalized J set perturbed by additive noise when α is decimal.

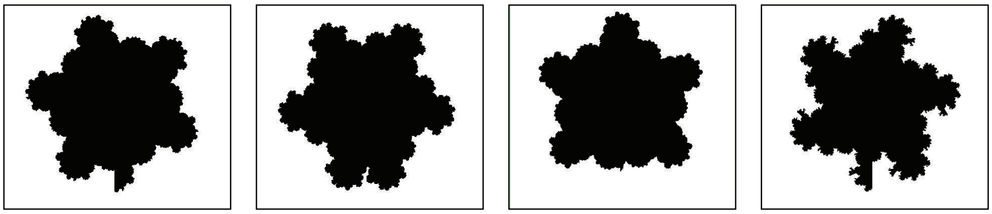
Observing the additive perturbed generalized J sets when $\alpha = 5.5$, $\theta \in [-\pi, \pi)$ (Figure 9), we can get the following property:

the graph with $c = a + bi$ and additive noise parameter (m_1, m_2) and the graph with $c = a - bi$ and additive noise parameter $(m_1, -m_2)$ have mirror symmetry about x-axis.



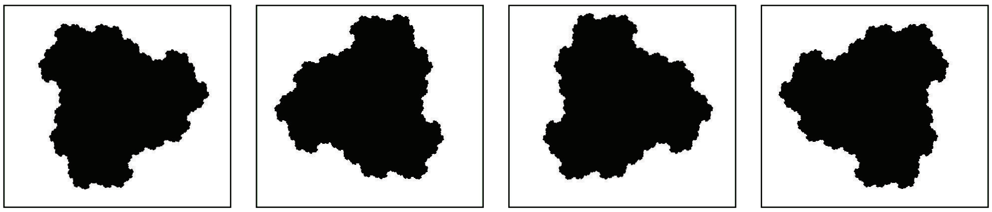
(a) $c = 0.5 + 0.5i$ (b) $c = -0.5 + 0.5i$ (c) $c = 0.5 - 0.5i$ (d) $c = -0.5 - 0.5i$

Figure 5: The generalized J sets without any perturbation when $\alpha = 5.5$, $\theta \in [-\pi/2, 3\pi/2)$.



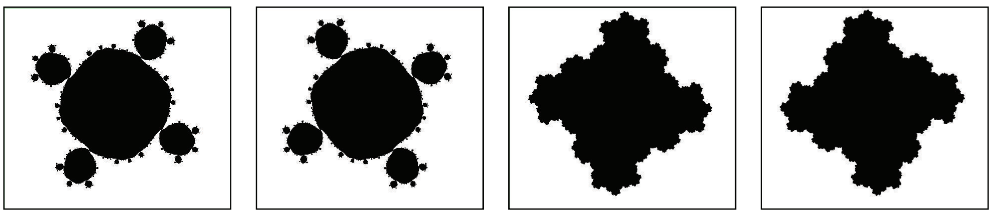
(a) $c = 0.5 + 0.5i$ (b) $c = -0.5 + 0.5i$ (c) $c = 0.5 - 0.5i$ (d) $c = -0.5 - 0.5i$

Figure 6: The generalized J sets without any perturbation when $\alpha = 5.5$, $\theta \in [-3\pi/2, \pi/2)$.



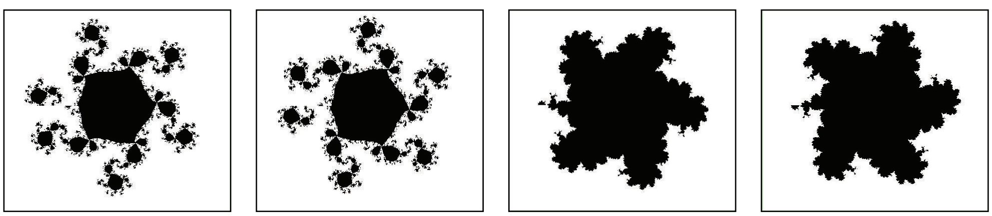
(a) $(-0.5, -0.5, 0.1, 0.3)$ (b) $(0.5, 0.5, -0.1, -0.3)$ (c) $(-0.5, 0.5, 0.1, -0.3)$ (d) $(0.5, -0.5, -0.1, 0.3)$

Figure 7: The generalized J sets perturbed by additive noise when $\alpha = 3$.



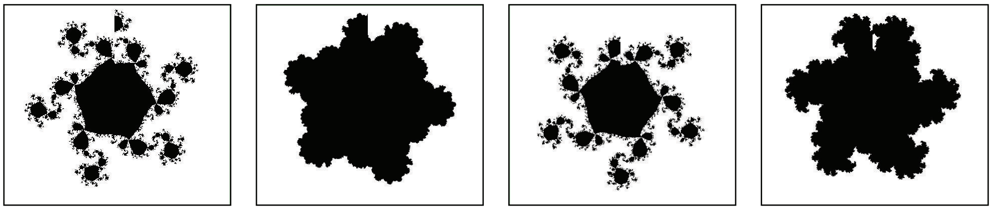
(a) $(0.5, 0.5, -0.1, 0.3)$ (b) $(0.5, -0.5, -0.1, -0.3)$ (c) $(-0.5, 0.5, -0.1, -0.3)$ (d) $(-0.5, -0.5, -0.1, 0.3)$

Figure 8: The generalized J sets perturbed by additive noise when $\alpha = 4$.



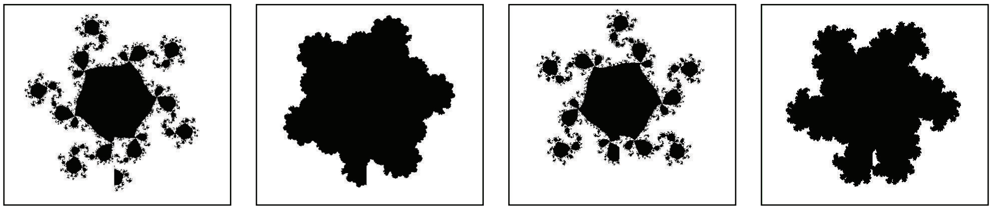
(a) $(0.5, 0.5, 0.1, 0.2)$ (b) $(0.5, -0.5, 0.1, -0.2)$ (c) $(-0.5, 0.5, -0.1, -0.2)$ (d) $(-0.5, -0.5, -0.1, 0.2)$

Figure 9: The generalized J sets perturbed by additive noise when $\alpha = 5.5$, $\theta \in [-\pi, \pi)$.



(a) (0.5, −0.5, 0.1, −0.2) (b) (−0.5, 0.5, −0.1, −0.2) (c) (0.5, 0.5, 0.1, 0.2) (d) (−0.5, −0.5, −0.1, 0.2)

Figure 10: The generalized J sets perturbed by additive noise with different parameters (a, b, m_1, m_2) when $\alpha = 5.5$, $\theta \in [-\pi/2, 3\pi/2)$.



(a) (0.5, 0.5, 0.1, 0.2) (b) (−0.5, −0.5, −0.1, 0.2) (c) (0.5, −0.5, 0.1, −0.2) (d) (−0.5, 0.5, −0.1, −0.2)

Figure 11: The generalized J sets perturbed by additive noise with different parameters (a, b, m_1, m_2) when $\alpha = 5.5$, $\theta \in [-3\pi/2, \pi/2)$.

Observe the additive perturbed generalized J sets when $\alpha = 5.5$, $\theta \in [-\pi/2, 3\pi/2)$ and when $\alpha = 5.5$, $\theta \in [-3\pi/2, \pi/2)$ (Figures 10 and 11), we can get the following property:

The graph when phase angle $\theta \in [-\pi/2, 3\pi/2)$ with $c = a + bi$ and additive noise parameter (m_1, m_2) and the graph when phase angle $\theta \in [-3\pi/2, \pi/2)$ with $c = a - bi$ and additive noise parameter $(m_1, -m_2)$ have mirror symmetry about x-axis.

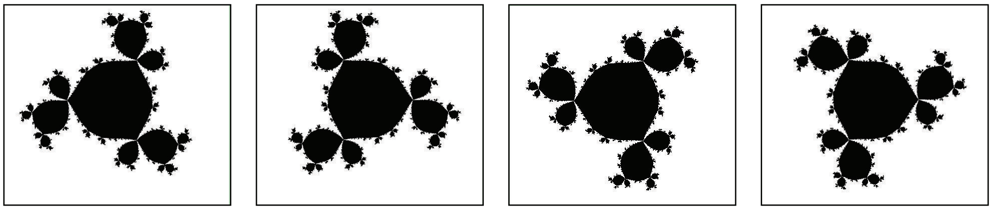
When α is decimal and phase angle $\theta \in [0, 2\pi)$, different c values corresponding to different graphs, basically there are no obvious rules among the additive noise perturbed generalized J sets with different parameter c.

3.3. The Generalized J Set Perturbed by Multiplicative Noise

Let the order of the parameters be (a, b, k_1, k_2); here a, b are the real part and the imaginary part of parameter c,respectively; k_1, k_2 are the intensity coefficients of entrance perturbation noise $\mathbf{w}_n$ in the direction of x axis and y axis of multiplicative perturbation. Observing the generalized J sets perturbed by multiplicative noise when α is odd (Figure 12), we can find the following rules:

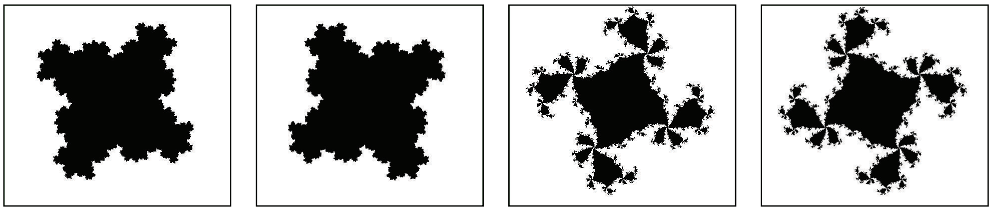
(1) the graph when $c = a+bi$ with multiplicative noise parameter (k_1, k_2) and the graph when $c = -a + bi$ with the same multiplicative noise parameter (k_1, k_2) have mirror symmetry about y-axis;

(2) the graph when $c = a+bi$ with multiplicative noise parameter (k_1, k_2) and the graph when $c = a - bi$ with the same multiplicative noise parameter (k_1, k_2) have mirror symmetry about x-axis;



(a) $(0.5, 0.5, 0.1, 0.1)$ (b) $(-0.5, 0.5, 0.1, 0.1)$ (c) $(0.5, -0.5, 0.1, 0.1)$ (d) $(-0.5, -0.5, 0.1, 0.1)$

Figure 12: The generalized J sets perturbed by multiplicative noise when $\alpha = 3$.



(a) $(0.5, 0.5, 0.1, 0.1)$ (b) $(0.5, -0.5, 0.1, 0.1)$ (c) $(-0.5, 0.5, -0.1, -0.1)$ (d) $(-0.5, -0.5, -0.1, -0.1)$

Figure 13: The generalized J sets perturbed by multiplicative noise when $\alpha = 4$.

(3) Clockwise rotating the graph when $c = a + bi$ with multiplicative noise parameter (k_1, k_2) for 180-degree, we will get the graph when $c = -a - bi$ with the same multiplicative noise parameter (k_1, k_2).

Observing the generalized J sets perturbed by multiplicative noise when α is even (Figure 13), we can find the following rule:

(1) The multiplicative noise perturbed graph with $c = a + bi$ and the multiplicative noise perturbed graph with $c = a - bi$ of corresponding parameters have mirror symmetry about x-axis.

Next is the property of the generalized J set perturbed by multiplicative noise when α is decimal.

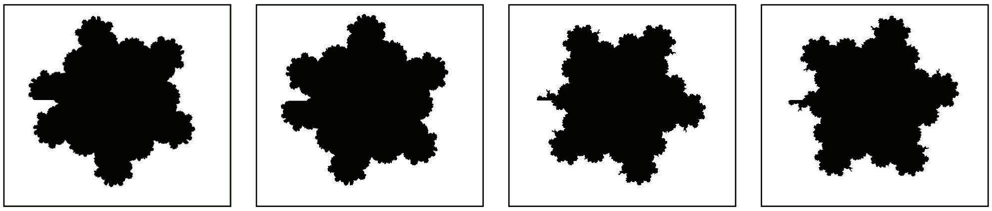
Observing the generalized J sets perturbed by multiplicative noise when α is decimal and phase angle $\theta \in [-\pi, \pi)$ (Figure 14), we can find the following rule:

the multiplicative noise perturbed graph with $c = a + bi$ and the multiplicative noise perturbed graph with $c = a - bi$ of corresponding parameters have mirror symmetry about x-axis.

Observing the generalized J sets perturbed by multiplicative noise when α is decimal and phase angle $\theta \in [-\pi/2, 3\pi/2)$ and phase angle $\theta \in [-3\pi/2, \pi/2)$ (Figures 15 and 16), we can find the following rule:

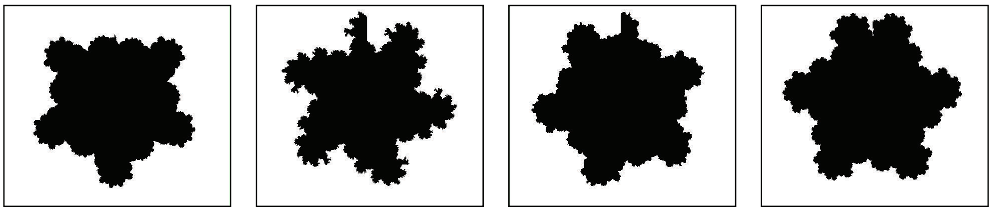
when α is decimal, the graph when phase angle $\theta \in [-\pi/2, 3\pi/2)$ with $c = a + bi$ and the graph with corresponding multiplicative parameters when phase angle $\theta \in [-3\pi/2, \pi/2)$ with $c = a - bi$ have mirror symmetry about x-axis.

When α is decimal and phase angle $\theta \in [0, 2\pi)$, as to the multiplicative noise perturbed generalized J sets, different c values correspond to different graphs; basically the changes of the graphs are only related to the values of c, but have little connection with the intensity coefficients (k_1, k_2) of the multiplicative noise; the general outside shape is determined by the value of c, while the mini changes in detail are related to multiplicative parameters.



(a) (0.5, 0.5, 0.1, 0.1) (b) (0.5, −0.5, 0.1, 0.1) (c) (−0.5, 0.5, 0.1, 0.1) (d) (−0.5, −0.5, 0.1, 0.1)

Figure 14: The generalized J sets perturbed by multiplicative noise when $\alpha = 5.5$, $\theta \in [-\pi, \pi)$.



(a) (0.5, 0.5, 0.1, 0.1) (b) (−0.5, 0.5, −0.1, 0.1) (c) (0.5, −0.5, 0.1, −0.1) (d) (−0.5, −0.5, −0.1, −0.1)

Figure 15: The generalized J sets perturbed by multiplicative noise when $\alpha = 5.5$, $\theta \in [-\pi/2, 3\pi/2)$.

3.4. The Generalized J Set Perturbed by the Composing Noise of Additive and Multiplicative

Let the order of parameters be $(\mu, m_1, m_2, k_1, k_2)$; here μ is the proportion coefficient in the composing noise of Equation (GN); m_1, m_2 are the intensity coefficients of the real part and imaginary part of additive perturbation W, respectively; k_1, k_2 are the intensity coefficients of entrance perturbation noise $\mathbf{w}_n$ in the direction of x axis and y axis of multiplicative perturbation.

(1) When α is even

Observe that the generalized J sets perturbed by the composing noise when α is even (Figures 17 and 18) have the following property:

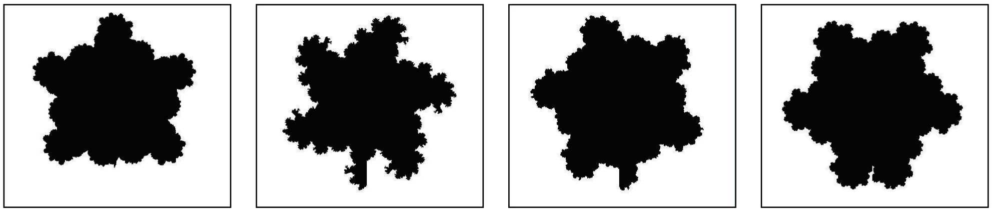
the graph with $c = a + bi$ and the composing noise parameters (m_1, m_2, k_1, k_2) and the graph with $c = a - bi$ and composing noise parameters $(m_1, -m_2, k_1, k_2)$ have mirror symmetry about x-axis.

Property 1. Assume that $f(z_n) = z_n^\alpha + W + c$ or $g(z_n) = \gamma(\lambda, k_1, k_2, \mathbf{w}_n) z_n^\alpha + c (|\alpha| > 1)$ is a complex map in Riemann sphere $\widehat{C}$; $\varphi = \mu f + (1 - \mu) g \quad (\mu \in [0, 1])$ is the composed mapping of f and g; constructing the generalized J sets perturbed by the composing noise of additive and multiplicative noise with it, when α is integer, there is

$$\varphi^k(z, W, c, \gamma) = \overline{\varphi^k\left(\overline{z}, \overline{W}, \overline{c}, \gamma\right)} \quad (k = 1, 2, \ldots, N). \tag{3.1}$$

Proof. Use mathematical induction: because

$$f(z, W, c) = z^\alpha + W + c, \qquad f\left(\overline{z}, \overline{W}, \overline{c}\right) = (\overline{z})^\alpha + \overline{W} + \overline{c} = \overline{z^\alpha + W + c}, \tag{3.2}$$



(a) (0.5, −0.5, 0.1, 0.1) (b) (−0.5, −0.5, −0.1, 0.1) (c) (0.5, 0.5, 0.1, −0.1) (d) (−0.5, 0.5, −0.1, −0.1)

Figure 16: The generalized J sets perturbed by multiplicative noise when $\alpha = 5.5$, $\theta \in [-3\pi/2, \pi/2)$.

then

$$f(z, W, c) = \overline{f(\overline{z}, \overline{W}, \overline{c})}. \tag{3.3}$$

Suppose

$$f^{k-1}(z, W, c) = \overline{f^{k-1}(\overline{z}, \overline{W}, \overline{c})}, \tag{3.4}$$

then there is

$$\begin{aligned} f^{k}(z, W, c) &= f^{k-1}(f(z, W, c), W, c) = f^{k-1}\left(\overline{f\left(\overline{z}, \overline{W}, \overline{c}\right)}, W, c\right) \\ &= \overline{f^{k-1}\left(f\left(\overline{z}, \overline{W}, \overline{c}\right), \overline{W}, \overline{c}\right)} = \overline{f^{k}(\overline{z}, \overline{W}, \overline{c})}. \end{aligned} \tag{3.5}$$

Again Use mathematical induction: for

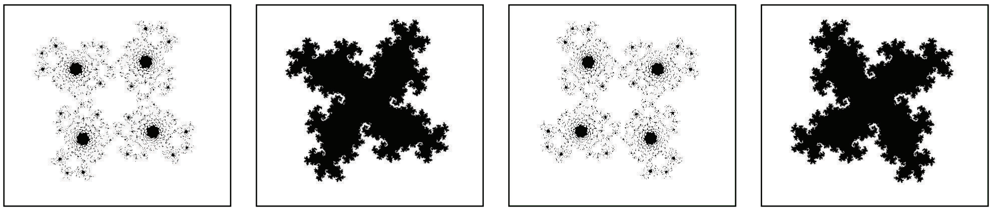
$$g^{1}(z, c, \gamma) = \gamma z^{\alpha} + c, \qquad g^{1}(\overline{z}, \overline{c}, \gamma) = \gamma(\overline{z})^{\alpha} + \overline{c}. \tag{3.6}$$

γ is inlet perturbations of F on the x-axis and y-axis, where $\gamma_x = \lambda_1 + k_1\mathbf{w}_n$ and $\gamma_y = \lambda_2 + k_2\mathbf{w}_n$. So $\gamma_x = \overline{\gamma}_x$, $\gamma_y = \overline{\gamma}_y$. Hence

$$g(z, c, \gamma) = \overline{g(\overline{z}, \overline{c}, \gamma)}. \tag{3.7}$$

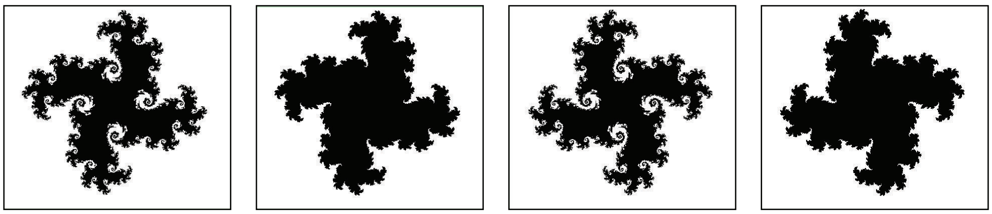
Suppose

$$g^{k-1}(z, c, \gamma) = \overline{g^{k-1}(\overline{z}, \overline{c}, \gamma)}, \tag{3.8}$$



(a) (0.5, 0.1, 0.1, 0.1, 0.1) (b) (0.5, 0.1, 0.1, 0.1, −0.1) (c) (0.5, 0.1, −0.1, 0.1, 0.1) (d) (0.5,0.1,−0.1,0.1,−0.1)

Figure 17: The generalized J sets perturbed by the composing noise when $\alpha = 4$, here Figures (a) and (b) are the graphs with $c = 0.5 + 0.5i$, Figures (c) and (d) are the graphs with $c = 0.5 - 0.5i$.



(a) (0.5,0.1,−0.1,−0.1,0.1) (b) (0.5,0.1,−0.1,−0.1,−0.1) (c) (0.5, 0.1, 0.1, −0.1, 0.1) (d) (0.5,0.1,0.1,−0.1,−0.1)

Figure 18: The generalized J sets perturbed by the composing noise when $\alpha = 4$, here Figures (a) and (b) are the graphs with $c = -0.5 + 0.5i$, Figures (c) and (d) are the graphs with $c{=}{-0.5}{-}0.5i$.

then,

$$\begin{aligned} g^k(z,c,\gamma) &= g^{k-1}(g(z,c,\gamma),c,\gamma) = g^{k-1}\left(\overline{g(\overline{z},\overline{c},\gamma)},c,\gamma\right) \\ &= \overline{g^{k-1}(g(\overline{z},\overline{c},\gamma),\overline{c},\gamma)} = \overline{g^k(\overline{z},\overline{c},\gamma)}, \\ \varphi^k(z,W,c,\gamma) &= \mu f^k(z,W,c) + (1-\mu)g^k(z,c,\gamma) = \mu\overline{f^k(\overline{z},\overline{W},\overline{c})} + (1-\mu)\overline{g^k(\overline{z},\overline{c},\gamma)} \\ &= \overline{\varphi^k(\overline{z},\overline{W},\overline{c},\gamma)}. \end{aligned} \tag{3.9}$$

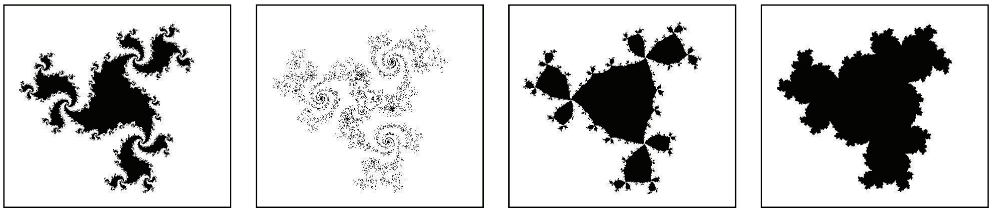
The proposition is tenable. Property 1 indicates that when α is integer, as to the composing noise perturbed generalized J sets, the graph with parameter c and composing noise parameters (m_1, m_2, k_1, k_2) and the graph with parameter $\overline{c}$ and composing noise parameters $(m_1, -m_2, k_1, k_2)$ have mirror symmetry about x-axis.

(2) When α is odd

Observe that the generalized J sets perturbed by the composing noise when α is odd (Figures 19, 20, 21, and 22) have the following properties

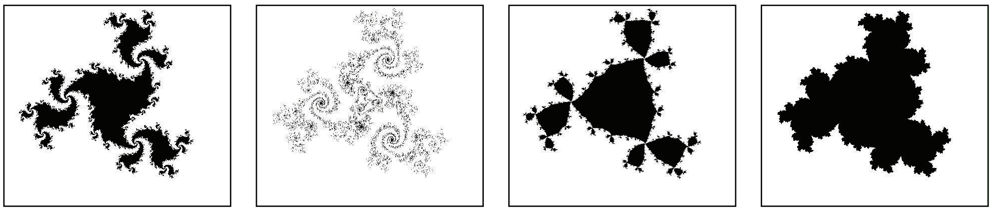
(1) the graph with $c = a + bi$ and the composing noise parameters (m_1, m_2, k_1, k_2) and the graph with $c = -a + bi$ and the composing noise parameters $(-m_1, m_2, k_1, k_2)$ have mirror symmetry about y-axis;

(2) the graph with $c = a + bi$ and the composing noise parameters (m_1, m_2, k_1, k_2) and the graph with $c = a - bi$ and the composing noise parameters $(m_1, -m_2, k_1, k_2)$ have mirror symmetry about x-axis (see proof of Property 1);



(a) (0.5, 0.1, 0.1, 0.1, 0.1) (b) (0.5,−0.1,−0.1,0.1,0.1) (c) (0.5, 0.1, −0.1, 0.1, 0.1) (d) (0.5, −0.1, 0.1, 0.1, 0.1)

Figure 19: The generalized J sets perturbed by the composing noise when $\alpha = 3$ and $c = 0.5 - 0.5i$.



(a) (0.5, 0.1, −0.1, 0.1, 0.1) (b) (0.5, −0.1, 0.1, 0.1, 0.1) (c) (0.5, 0.1, 0.1, 0.1, 0.1) (d) (0.5,−0.1,−0.1,0.1,0.1)

Figure 20: The generalized J sets perturbed by the composing noise when $\alpha = 3$ and $c = 0.5 + 0.5i$.

(3) rotating the graph with $c = a + bi$ and the composing noise parameters (m_1, m_2, k_1, k_2) for 180-degree will get the graph with $c = -a - bi$ and composing noise parameters $(-m_1, -m_2, k_1, k_2)$. □

Property 2. Assume that $f(z_n) = z_n^\alpha + W + c$ or $g(z_n) = \gamma(\lambda, k_1, k_2, \mathbf{w}_n) z_n^\alpha + c (|\alpha| > 1)$ is a complex map in Riemann sphere $\widehat{C}$; $\varphi = \mu f + (1-\mu) g \quad (\mu \in [0,1])$ is the composed mapping of f and g; constructing the generalized J sets perturbed by the composing noise of additive and multiplicative noise with it, when $\alpha = 2j+1$ $(j = \pm 1, \pm 2, \ldots)$, there is

$$\varphi^k(z, W, c, \gamma) = -\overline{\varphi^k\left(-\overline{z}, -\overline{W}, -\overline{c}, \gamma\right)} \quad (k = 1, 2, \ldots, N). \tag{3.10}$$

Proof. Use mathematical induction: because

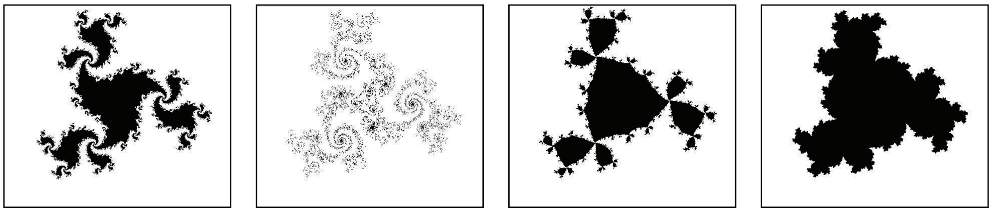
$$f(z, W, c) = z^\alpha + W + c, \qquad f\left(-\overline{z}, -\overline{W}, -\overline{c}\right) = (-\overline{z})^\alpha - \overline{W} - \overline{c}, \tag{3.11}$$

when $\alpha = 2j+1$ $(j = \pm 1, \pm 2, \ldots)$, there is

$$f(z, W, c) = -\overline{f(-\overline{z}, -\overline{W}, -\overline{c})}. \tag{3.12}$$

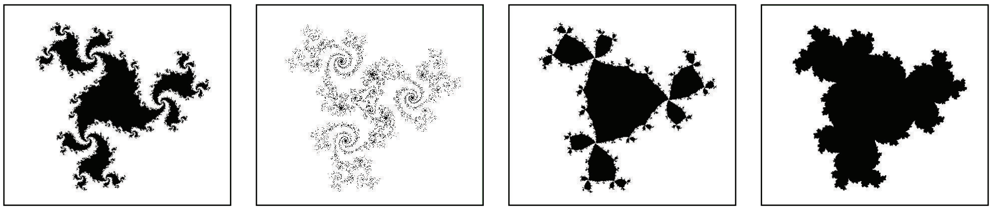
Suppose

$$f^{k-1}(z, W, c) = -\overline{f^{k-1}(-\overline{z}, -\overline{W}, -\overline{c})}, \tag{3.13}$$



(a) (0.5,−0.1,−0.1,0.1,0.1) (b) (0.5, 0.1, 0.1, 0.1, 0.1) (c) (0.5, −0.1, 0.1, 0.1, 0.1) (d) (0.5, 0.1, −0.1, 0.1, 0.1)

Figure 21: The generalized J sets perturbed by the composing noise when $\alpha = 3$ and $c = -0.5 + 0.5i$.



(a) (0.5, −0.1, 0.1, 0.1, 0.1) (b) (0.5, 0.1, −0.1, 0.1, 0.1) (c) (0.5,−0.1,−0.1,0.1,0.1) (d) (0.5, 0.1, 0.1, 0.1, 0.1)

Figure 22: The generalized J sets perturbed by the composing noise when $\alpha = 3$ and $c = -0.5 - 0.5i$.

then there is

$$\begin{aligned} f^k(z, W, c) &= f^{k-1}(f(z, W, c), W, c) = f^{k-1}\left(\overline{-f(-\bar{z}, -\overline{W}, -\bar{c})}, W, c\right) \\ &= \overline{-f^{k-1}(f(-\bar{z}, -\overline{W}, -\bar{c}), -\overline{W}, -\bar{c})} = \overline{-f^k(-\bar{z}, -\overline{W}, -\bar{c})}. \end{aligned} \tag{3.14}$$

Again Use mathematical induction: for

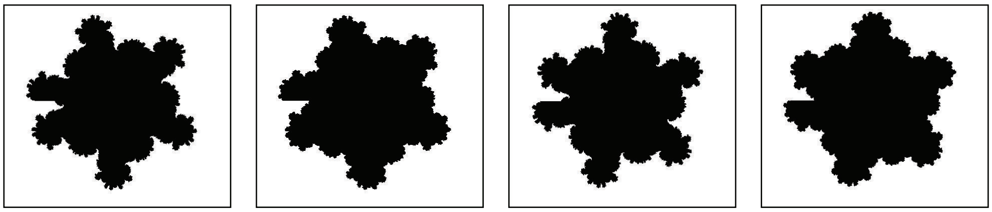
$$g^1(z, c, \gamma) = \gamma z^\alpha + c, \qquad g^1(-\bar{z}, -\bar{c}, \gamma) = \gamma(-\bar{z})^\alpha - \bar{c}. \tag{3.15}$$

γ is inlet perturbations of F on the x-axis and y-axis, where $\gamma_x = \lambda_1 + k_1\mathbf{w}_n$ and $\gamma_y = \lambda_2 + k_2\mathbf{w}_n$. So $\gamma_x = \overline{\gamma}_x$, $\gamma_y = \overline{\gamma}_y$. Hence, when $\alpha = 2j + 1$ $(j = \pm 1, \pm 2, \ldots)$, there is

$$g(z, c, \gamma) = \overline{-g(-\bar{z}, -\bar{c}, \gamma)}. \tag{3.16}$$

Suppose

$$g^{k-1}(z, c, \gamma) = \overline{-g^{k-1}(-\bar{z}, -\bar{c}, \gamma)}, \tag{3.17}$$



(a) (0.5, 0.1, 0.1, 0.1, 0.1) (b) (0.5,−0.1,0.1,0.1,−0.1) (c) (0.5, 0.1, −0.1, 0.1, 0.1) (d) (0.5,−0.1,−0.1,0.1,−0.1)

Figure 23: The generalized J sets perturbed by the composing noise when $\alpha = 5.5$, $\theta \in [-\pi, \pi)$, here Figures (a) and (b) are the graphs with $c = 0.5 + 0.5i$; Figures (c) and (d) are the graphs with $c = 0.5 - 0.5i$.

then,

$$\begin{aligned} g^k(z,c,\gamma) &= g^{k-1}(g(z,c,\gamma),c,\gamma) = g^{k-1}\left(-\overline{g(-\overline{z},-\overline{c},\gamma)},c,\gamma\right) \\ &= -\overline{g^{k-1}(g(-\overline{z},-\overline{c},\gamma),-\overline{c},\gamma)} = -\overline{g^k(-\overline{z},-\overline{c},\gamma)}. \\ \varphi^k(z,W,c,\gamma) &= \mu f^k(z,W,c) + (1-\mu)g^k(z,c,\gamma) \\ &= \mu\left(-\overline{f^k(-\overline{z},-\overline{W},-\overline{c})}\right) + (1-\mu)\left(-\overline{g^k(-\overline{z},-\overline{c},\gamma)}\right) = -\overline{\varphi^k(-\overline{z},-\overline{W},-\overline{c},\gamma)}. \end{aligned} \tag{3.18}$$

The proposition is tenable. Property 2 indicates that when α is odd, as to the composing noise perturbed generalized J sets, the graph with parameter c and composing noise parameters (m_1, m_2, k_1, k_2) and the graph with parameter $-\overline{c}$ and composing noise parameters $(-m_1, m_2, k_1, k_2)$ have mirror symmetry about y-axis. □

Property 3. Assume that $f(z_n) = z_n^\alpha + W + c$ or $g(z_n) = \gamma(\lambda, k_1, k_2, \mathbf{w}_n)z_n^\alpha + c(|\alpha| > 1)$ is a complex map in Riemann sphere $\widehat{C}$; $\varphi = \mu f + (1-\mu)g$ $(\mu \in [0,1])$ is the composed mapping of f and g; constructing the generalized J sets perturbed by the composing noise of additive and multiplicative noise with it, when $\alpha = 2j+1$ $(j = \pm 1, \pm 2, \ldots)$, there is

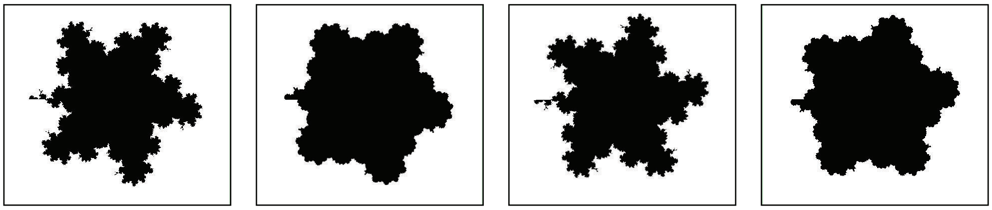
$$\varphi^k(z,W,c,\gamma) = -\overline{\overline{\varphi^k\left(\overline{-\overline{z}},\overline{-\overline{W}},\overline{-\overline{c}},\gamma\right)}} \quad (k = 1, 2, \ldots, N). \tag{3.19}$$

The proof is similar to Property 2. Property 3 indicates that when α is odd, as to the composing noise perturbed generalized J sets, rotating the graph when $c = (x, y)$ and composing noise parameters are (m_1, m_2, k_1, k_2) for 180-degree will get the graph when $c' = (-x, -y)$ and composing noise parameters are $(-m_1, -m_2, k_1, k_2)$.

(3) When α is decimal

Observe the generalized J sets perturbed by the composing noise when α is decimal and phase angle $\theta \in [-\pi, \pi)$ has the following property (Figures 23 and 24):

(1) the graph with $c = a + bi$ and the composing noise parameters (m_1, m_2, k_1, k_2) and the graph with $c = a - bi$ and the composing noise parameters $(m_1, -m_2, k_1, k_2)$ have mirror symmetry about x-axis.



(a) (0.5,–0.1,0.1,0.1,–0.1) (b) (0.5,0.1,–0.1,0.1,0.1) (c) (0.5,–0.1,–0.1,0.1, –0.1) (d) (0.5,0.1,0.1,0.1,0.1)

Figure 24: The generalized J sets perturbed by the composing noise when $\alpha = 5.5$, $\theta \in [-\pi, \pi)$, here Figures (a) and (b) are the graphs with $c = -0.5 + 0.5i$; Figures (c) and (d) are the graphs with $c = -0.5 - 0.5i$.

Property 4. Assume that $f(z_n) = z_n^\alpha + W + c$ or $g(z_n) = \gamma(\lambda, k_1, k_2, \mathbf{w}_n) z_n^\alpha + c$ $(|\alpha| > 1)$ is a complex map in Riemann sphere $\widehat{C}$; $\varphi = \mu f + (1-\mu) g$ $(\mu \in [0,1])$ is the composed mapping of f and g; constructing the generalized J sets perturbed by the composing noise of additive and multiplicative noise with it, when α is decimal and phase angle $\theta \in [-\pi, \pi)$, there is

$$\varphi^k(z, W, c, \gamma) = \overline{\varphi^k(\overline{z}, \overline{W}, \overline{c}, \gamma)} \quad (k = 1, 2, \ldots, N). \tag{3.20}$$

Proof. Use mathematical induction: select the phase angle as $\theta \in [-\pi, \pi)$; let

$$z = |z|e^{i\theta}, \qquad \overline{z} = |z|e^{-i\theta}, \qquad c = |c|e^{i\theta}, \qquad \overline{c} = |c|e^{-i\theta}. \tag{3.21}$$

Since

$$f(z, W, c) = \left(|z|e^{i\theta}\right)^\alpha + W + |c|e^{i\theta}, \qquad f\left(\overline{z}, \overline{W}, \overline{c}\right) = \left(|z|e^{-i\theta}\right)^\alpha + \overline{W} + |c|e^{-i\theta}, \tag{3.22}$$

there is

$$f(z, W, c) = \overline{f(\overline{z}, \overline{W}, \overline{c})}. \tag{3.23}$$

Assume

$$f^{k-1}(z, W, c) = \overline{f^{k-1}(\overline{z}, \overline{W}, \overline{c})} \tag{3.24}$$

is tenable, then there is

$$\begin{aligned} f^k(z, W, c) &= f^{k-1}(f(z, W, c), W, c) = f^{k-1}\left(\overline{f\left(\overline{z}, \overline{W}, \overline{c}\right)}, W, c\right) \\ &= \overline{f^{k-1}\left(f\left(\overline{z}, \overline{W}, \overline{c}\right), \overline{W}, \overline{c}\right)} = \overline{f^k\left(\overline{z}, \overline{W}, \overline{c}\right)}. \end{aligned} \tag{3.25}$$

Because

$$\gamma_x = \overline{\gamma}_x, \quad \gamma_y = \overline{\gamma}_y,$$

$$g(z,c,\gamma) = \gamma z^{\alpha} + c = \gamma\left(|z|e^{i\theta}\right)^{\alpha} + |z|e^{i\theta}, \qquad g^1(\overline{z},\overline{c},\gamma) = \gamma(\overline{z})^{\alpha} + \overline{c} = \gamma\left(|z|e^{-i\theta}\right)^{\alpha} + |c|e^{-i\theta}. \tag{3.26}$$

Hence

$$g(z,c,\gamma) = \overline{g(\overline{z},\overline{c},\gamma)}. \tag{3.27}$$

Assume

$$g^{k-1}(z,c,\gamma) = \overline{g^{k-1}(\overline{z},\overline{c},\gamma)} \tag{3.28}$$

is tenable, then there is

$$\begin{aligned} g^k(z,c,\gamma) &= g^{k-1}(g(z,c,\gamma),c,\gamma) = g^{k-1}\left(\overline{g(\overline{z},\overline{c},\gamma)},c,\gamma\right) = \overline{g^{k-1}(g(\overline{z},\overline{c},\gamma),\overline{c},\gamma)} = \overline{g^k(\overline{z},\overline{c},\gamma)}. \\ \varphi^k(z,W,c,\gamma) &= \mu f^k(z,W,c) + (1-\mu)g^k(z,c,\gamma) = \mu\overline{f^k(\overline{z},\overline{W},\overline{c})} + (1-\mu)\overline{g^k(\overline{z},\overline{c},\gamma)} \\ &= \overline{\varphi^k(\overline{z},\overline{W},\overline{c},\gamma)}. \end{aligned} \tag{3.29}$$

The proposition is tenable. Property 4 indicates that when α is decimal and $\theta \in [-\pi,\pi)$, as to the composing noise perturbed generalized J sets, the graph with parameter c and composing noise parameters (m_1,m_2,k_1,k_2) and the graph with parameter $\overline{c}$ and composing noise parameters $(m_1,-m_2,k_1,k_2)$ have mirror symmetry about x-axis.

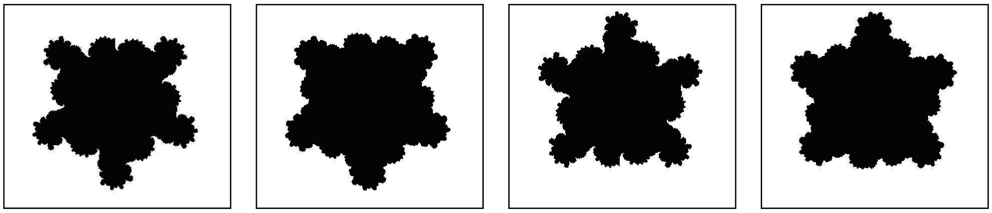
The following analyses the graphs when α is decimal and phase angle $\theta \in [-\pi/2, 3\pi/2)$ or $\theta \in [-3\pi/2,\pi/2)$.

Observing the graphs when α is decimal and phase angle $\theta \in [-\pi/2, 3\pi/2)$ or $\theta \in [-3\pi/2,\pi/2)$ (Figures 25 and 26) we will find the following rule:

when α is decimal, as to the generalized J sets perturbed by the composing noise, the graph when phase angle $\theta \in [-\pi/2, 3\pi/2)$ with $c = a + bi$ and composing noise parameters (m_1,m_2,k_1,k_2) and the graph when phase angle $\theta \in [-3\pi/2,\pi/2)$ with $c = a - bi$ and composing noise parameters $(m_1,-m_2,k_1,k_2)$ have mirror symmetry about x-axis. □

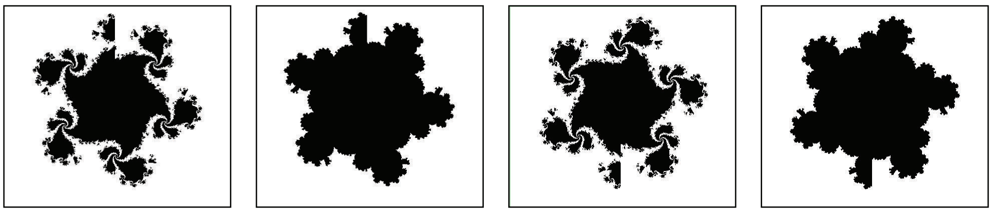
Property 5. Assume that $f(z_n) = z_n^{\alpha} + W + c$ or $g(z_n) = \gamma(\lambda,k_1,k_2,\mathbf{w}_n)z_n^{\alpha} + c(|\alpha| > 1)$ is a complex map in Riemann sphere $\widehat{C}$; $\varphi = \mu f + (1-\mu)g \ \ (\mu \in [0,1])$ is the composed mapping of f and g; constructing the generalized J sets perturbed by the composing noise of additive and multiplicative noise with it, when α is decimal and phase angle $\theta \in [-3\pi/2,\pi/2)$ or $\theta \in [-\pi/2, 3\pi/2)$, there is

$$\varphi^k(z,W,c,\gamma)\Big|_{\theta\in[-3\pi/2,\pi/2)} = \overline{\varphi^k(\overline{z},\overline{W},\overline{c},\gamma)}\Big|_{\theta\in[-\pi/2,3\pi/2)} \quad (k = 1,2,\ldots,N). \tag{3.30}$$



(a) (0.5, 0.1, 0.1, 0.1, 0.1) (b) (0.5,–0.1,0.1,0.1,–0.1) (c) (0.5, 0.1, –0.1, 0.1, 0.1) (d) (0.5,–0.1,–0.1, 0.1,–0.1)

Figure 25: The generalized J sets perturbed by the composing noise, here Figures (a) and (b) are the graphs when α =5.5, $\theta \in [-\pi/2, 3\pi/2)$ with $c = 0.5 + 0.5i$; Figures (c) and (d) are the graphs when α =5.5, $\theta \in [-3\pi/2, \pi/2)$ with $c = 0.5 - 0.5i$.



(a) (0.5, 0.1, 0.1, 0.1, 0.1) (b) (0.5,–0.1,–0.1,0.1,0.1) (c) (0.5, 0.1, –0.1, 0.1, 0.1) (d) (0.5, –0.1, 0.1, 0.1, 0.1)

Figure 26: The generalized J sets perturbed by the composing noise, here Figures (a) and (b) are the graphs when α =5.5, $\theta \in [-\pi/2, 3\pi/2)$ with $c = -0.5 + 0.5i$, Figures (c) and (d) are the graphs when α =5.5, $\theta \in [-3\pi/2, \pi/2)$ with $c = -0.5 - 0.5i$.

Proof. Because the selection of $\theta \in [-\pi/2, 3\pi/2)$ and $\theta \in [-3\pi/2, \pi/2)$ is symmetrical about x-axis, thus when selecting $\theta \in [-\pi/2, 3\pi/2)$, if $z = |z|e^{i\theta}|_{\theta\in[-3\pi/2,\pi/2)}$, $c = |c|e^{i\theta}|_{\theta\in[-\pi/2,3\pi/2)}$, then when selecting $\theta \in [-3\pi/2, \pi/2)$, its conjugate can be denoted as $\overline{z} = |z|e^{-i\theta}|_{\theta\in[-\pi/2,3\pi/2)}$, $\overline{c} = |c|e^{-i\theta}|_{\theta\in[-3\pi/2,\pi/2)}$. From the proof process of Property 4, we can easily get Property 5.

Property 5 indicates that when α is decimal, as to the generalized J sets perturbed by the composing noise, the graph when phase angle $\theta \in [-\pi/2, 3\pi/2)$ with $c = a + bi$ and composing noise parameters (m_1, m_2, k_1, k_2) and the graph when phase angle $\theta \in [-3\pi/2, \pi/2)$ with $c = a - bi$ and composing noise parameters $(m_1, -m_2, k_1, k_2)$ have mirror symmetry about x-axis.

As to the generalized J sets perturbed by the composing noise when α is decimal and phase angle $\theta \in [0, 2\pi)$, the graphs of different c values with different noise parameters have different changes. However, generally , the changes of the composing noise perturbed generalized J sets have a big relation with the changes of the additive noise parameters (m_1, m_2), which decides the general shape of the graph, while the changes brought by the multiplicative parameters (k_1, k_2) seem to be smaller, which only changes the mini details of the graph; however, sometimes there are exceptions, that is, although the values of the additive noise parameters are the same, the changes brought by the difference of the multiplicative noise parameters are kind of large. But this kind of exception is very merely, at most situations the effect of the additive noise is in the dominated manner. □

4. Conclusions

In the above-mentioned we present the generalized J sets without any perturbation, perturbed by the additive noise, and perturbed by the multiplicative noise, perturbed by the composing noise of the additive and multiplicative noise. From the above analyses, we can find that different perturbations bring different effects to the generalized J sets; the shape of the graph varies differently. But there is one thing, no matter how the graph shape changes, the number of the petals, the distribution of the petals, and the symmetry among different parameters of the perturbed generalized J sets do not change, which further illuminates the stability of the generalized J set. Comparing the properties of the graphs for random noise perturbed generalized Mandelbrot sets and the random noise perturbed generalized Julia sets, we can find that the properties of the generalized Mandelbrot sets are the embodiment of the generalized Julia sets, which indicates that the perturbed generalized Mandelbrot set is the parameter space of the perturbed generalized Julia set.

Acknowledgments

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