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Research Article

Permanence and Global Stability of Positive Periodic Solutions of a Discrete Competitive System

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We consider the dynamic behaviors of a discrete competitive system. A good understanding of the permanence, existence, and global stability of positive periodic solutions is gained. Numerical simulations are also presented to substantiate the analytical results.

1. Introduction

In biomathematics, one of the most challenging aspects of mathematical biology is competition modeling. Although the mathematical idea is simple [1], this type of modeling is so difficult to carry out in any generality since there are so many ways for a population to compete; many classical competitive models have been established to describe the relationships between species and the outer environment, and the connections between different species. Lotka-Volterra competitive model of two species communities is probably the best known model of mathematical ecology. In 1934, Gause [2] found out the competitive exclusion theory, which states that two species that compete for the exact same resources cannot stably coexist. One of the two competitors will always have an ever so slight advantage over the other that leads to extinction of the second competitor in the long run. Since then the competitive model has increasingly won attention as an important and fundament model in biomathematics. The dynamic relationship between species and their competitors has long been one of the dominant theses in both ecology and mathematical ecology. As a consequence, many excellent results concerned with permanence, extinction, stability and hopf bifurcations, and existence and global stability of positive periodic solutions of Lotka-Volterra competitive system are obtained (see [3–16]).

Although much progress has been seen for Lotka-Volterra competitive systems, such systems are not well studied in the sense that most results are continuous time versions related. Many authors [17–21] have argued that the discrete-time models governed by difference equations are more appropriate than the continuous ones when populations have a short life expectancy, nonoverlapping generations in the real word. Discrete-time models can provide efficient computational models of continuous models for numerical simulations. So it is reasonable to study discrete-time competitive systems governed by difference equations.

In this paper, we will consider the dynamic behavior of a discrete-time competitive system. Let us first introduce its continuous time version which is motivated in [22]

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)\left[r_1(t) - a_1(t)x_1(t) - \frac{c_2(t)x_2(t)}{1+x_2(t)}\right], \\ \dot{x}_2(t) &= x_2(t)\left[r_2(t) - a_2(t)x_2(t) - \frac{c_1(t)x_1(t)}{1+x_1(t)}\right], \end{aligned} \tag{1.1}$$

where $x_1(t)$, $x_2(t)$ are the population densities of two competing species; $r_1(t)$, $r_2(t)$ are the intrinsic growth rates of species; $a_1(t)$, $a_2(t)$ are the rates of intraspecific competition of the first and second species, respectively; $c_1(t)$, $c_2(t)$ are the rates of interspecific competition of the first and second species, respectively. All the coefficients above are continuous and bounded above and below by positive constants.

Following the same idea and method in [21], one can easily derive the discrete analogue of system (1.1), which takes the form of

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left[r_1(n) - a_1(n)x_1(n) - \frac{c_2(n)x_2(n)}{1+x_2(n)}\right], \\ x_2(n+1) &= x_2(n)\exp\left[r_2(n) - a_2(n)x_2(n) - \frac{c_1(n)x_1(n)}{1+x_1(n)}\right], \end{aligned} \quad n = 0,1,2\ldots. \tag{1.2}$$

The exponential form of system (1.2) is more biologically reasonable than that directly derived by replacing the differential by difference in system (1.1) because this exponential form can assure $x_i(n+1) > 0$ if $x_i(0) > 0$ $(i = 1,2)$. Here $x_i(n)$ represent the densities of species x_i at the nth generation, $r_i(n)$ are the intrinsic growth rates of species x_i at the nth generation, $a_i(n)$ measure the intraspecific effects of the nth generation of species x_i on own population, and $c_i(n)$ stand for the interspecific effects of the nth generation of species x_i on species x_j $(i,j = 1,2;\ i \neq j)$.

It is well known that, compared to the continuous time systems, the discrete-time ones are more difficult to deal with. The principle aim of this paper is to explore the permanence, existence, and global stability of positive periodic solutions of system (1.2). To the best of our knowledge, no work has been done for system (1.2).

For the sake of simplicity and convenience in the following discussion, the notations below will be used through this paper:

$$f^U = \sup_{n\in\mathbb{N}} f(n), \qquad f^L = \inf_{n\in\mathbb{N}} f(n), \tag{1.3}$$

where $\{f(n)\}$ is a bounded sequence and $\mathbb{N}$ is the set of nonnegative integer numbers.

For biological reasons, in system (1.2) we only consider the solution $\{x_1(n), x_2(n)\}$ with the initial value $\{x_1(0), x_2(0)\} > 0$.

The organization of this paper is as follows. In the next section, we establish the permanence of system (1.2). In Section 3, we obtain sufficient conditions which ensure the existence and global stability of positive periodic solutions of system (1.2). Numerical simulations are present to illustrate the feasibility of our main results in final section.

2. Permanence

In this section, we will establish sufficient conditions for the permanence of system (1.2).

Definition 2.1. System (1.2) is said to be permanent if there exist positive constants m_i and M_i such that each positive solution $\{x_1(n), x_2(n)\}$ of system (1.2) satisfies

$$m_i \le \liminf_{n\to+\infty} x_i(n) \le \limsup_{n\to+\infty} x_i(n) \le M_i, \quad i = 1, 2. \tag{2.1}$$

Proposition 2.2. *Any positive solution* $\{x_1(n), x_2(n)\}$ *of system* (1.2) *satisfies*

$$\limsup_{n\to+\infty} x_i(n) \le M_i \stackrel{def}{=} \frac{[\exp(r_i^U - 1)]}{a_i^L}, \quad i = 1, 2. \tag{2.2}$$

Proof. To prove Proposition 2.2, we consider Case 1 and Case 2.

Case 1. Assume that there exists an $n_0 \in \mathbb{N}$ such that $x_1(n_0+1) \ge x_1(n_0)$, from the first equation of system (1.2), it follows that

$$r_1(n_0) - a_1(n_0)x_1(n_0) - \frac{c_2(n_0)x_2(n_0)}{1 + x_2(n_0)} \ge 0, \tag{2.3}$$

which implies,

$$x_1(n_0) \le \frac{r_1(n_0)}{a_1(n_0)} \le \frac{r_1^U}{a_1^L}. \tag{2.4}$$

Then

$$\begin{aligned} x_1(n_0 + 1) &= x_1(n_0) \exp\left[r_1(n_0) - a_1(n_0)x_1(n_0) - \frac{c_2(n_0)x_2(n_0)}{1 + x_2(n_0)}\right] \\ &\le x_1(n_0) \exp\left[r_1^U - a_1^L x_1(n_0)\right] \\ &\le \frac{[\exp(r_1^U - 1)]}{a_1^L} \\ &\stackrel{\text{def}}{=} M_1, \end{aligned} \tag{2.5}$$

where we use the fact that $\max_{x\in\mathbb{R}}[x\exp(a-bx)] = [\exp(a-1)]/b$ for $a,b>0$, and $\mathbb{R}$ is the set of all real numbers. Hence $x_1(n_0) \le M_1$.

We claim that $x_1(n) \le M_1$ for all $n \ge n_0$. By way of contradiction, assume that there exists a $p_0 > n_0$ such that $x_1(p_0) > M_1$, then $p_0 \ge n_0 + 2$. Let

$$\widehat{p}_0 = \min\{p_0 : p_0 \ge n_0 + 2,\ x_1(p_0) > M_1\}, \tag{2.6}$$

that is, $x_1(\widehat{p}_0) > M_1$ and $\widehat{p}_0 \ge n_0 + 2$, then $x_1(\widehat{p}_0) > M_1 \ge x_1(\widehat{p}_0 - 1)$. It is easy to obtain that $x_1(\widehat{p}_0) \le M_1$ from the above argument, which is a contradiction. Therefore, $x_1(n) \le M_1$ for all $n \ge n_0$, then $\limsup_{n\to+\infty} x_1(n) \le M_1$. This proves the claim.

Case 2. Suppose that $x_1(n+1) < x_1(n)$ for all $n \in \mathbb{N}$. In particular, $\lim_{n\to+\infty} x_1(n)$ exists, denoted by $\overline{x}_1$, we will prove $\overline{x}_1 \le r_1^U/a_1^L$ by way of contradiction as follows. Assume that $\overline{x}_1 > r_1^U/a_1^L$, taking limit in the first equation of system (1.2), which leads to

$$\lim_{n\to+\infty}\left[r_1(n) - a_1(n)x_1(n) - \frac{c_2(n)x_2(n)}{1+x_2(n)}\right] = 0, \tag{2.7}$$

however,

$$\begin{aligned}\lim_{n\to+\infty}\left[r_1(n) - a_1(n)x_1(n) - \frac{c_2(n)x_2(n)}{1+x_2(n)}\right] &\le \lim_{n\to+\infty}[r_1(n) - a_1(n)x_1(n)]\\ &\le r_1^U - a_1^L\overline{x}_1\\ &< 0,\end{aligned} \tag{2.8}$$

which is a contradiction. This proves the claim. By the fact that $\min_{x\in\mathbb{R}^+}\{[\exp(x-1)]/x\} = 1$, we obtain that $\overline{x}_1 \le r_1^U/a_1^L \le r_1^U/a_1^L \cdot \exp(r_1^U - 1)/r_1^U \overset{\text{def}}{=} M_1$. Therefore,

$$\limsup_{n\to+\infty} x_1(n) \le M_1 = \frac{[\exp(r_1^U - 1)]}{a_1^L}. \tag{2.9}$$

Analogously,

$$\limsup_{n\to+\infty} x_2(n) \le M_2 = \frac{[\exp(r_2^U - 1)]}{a_2^L}. \tag{2.10}$$

This completes the proof of Proposition 2.2. □

Proposition 2.3. *Suppose that system* (1.2) *satisfies the following assumptions:*

$$r_1^L > c_2^U, \qquad r_2^L > c_1^U. \tag{2.11}$$

Then any positive solution $\{x_1(n), x_2(n)\}$ *of system* (1.2) *satisfies*

$$\liminf_{n\to+\infty} x_i(n) \geq m_i \overset{def}{=} \frac{r_i^L - c_j^U}{a_i^U} \exp\left(r_i^L - a_i^U M_i - c_j^U\right), \quad i \neq j;\ i,j = 1,2. \tag{2.12}$$

Proof. By Proposition 2.2, since $\limsup_{n\to+\infty} x_1(n) \leq M_1$ for each $\varepsilon > 0$, then there exists an $n^* \in \mathbb{N}$ such that $x_1(n) \leq M_1 + \varepsilon$ for $n \geq n^*$. In order to prove Proposition 2.3, there are two cases to be considered as follows.

Case 1. Assume that there exists an $n_0 \geq n^*$ such that $x_1(n_0+1) \leq x_1(n_0)$, by the first equation of system (1.2), it derives that

$$\begin{aligned} x_1(n_0+1) &= x_1(n_0)\exp\left[r_1(n_0) - a_1(n_0)x_1(n_0) - \frac{c_2(n_0)x_2(n_0)}{1+x_2(n_0)}\right] \\ &\geq x_1(n_0)\exp\left[r_1^L - a_1^U x(n_0) - c_2^U\right], \end{aligned} \tag{2.13}$$

therefore,

$$r_1^L - a_1^U x(n_0) - c_2^U \leq 0. \tag{2.14}$$

It follows from the inequality (2.11) that

$$x_1(n_0) \geq \frac{(r_1^L - c_2^U)}{a_1^U} > 0. \tag{2.15}$$

By (2.13) and (2.15) we have

$$\begin{aligned} x_1(n_0+1) &= x_1(n_0)\exp\left[r_1(n_0) - a_1(n_0)x_1(n_0) - \frac{c_2(n_0)x_2(n_0)}{1+x_2(n_0)}\right] \\ &\geq \frac{r_1^L - c_2^U}{a_1^U}\exp\left[r_1^L - a_1^U(M_1+\varepsilon) - c_2^U\right] \\ &> 0. \end{aligned} \tag{2.16}$$

Hence $x_1(n_0) \geq x_1^{\varepsilon}$, where $x_1^{\varepsilon} \overset{\text{def}}{=} ((r_1^L - c_2^U)/a_1^U)\ \exp[r_1^L - a_1^U(M_1+\varepsilon) - c_2^U]$.

In the following we will prove $x_1(n) \geq x_1^{\varepsilon}$ for all $n \geq n_0$. By way of contradiction, assume that there exists a $p_0 > n_0$ such that $x_1(p_0) < x_1^{\varepsilon}$, then $p_0 \geq n_0 + 2$. Let

$$\tilde{p}_0 = \min\{p_0 : p_0 \geq n_0 + 2,\ x_1(p_0) < x_1^{\varepsilon}\}, \tag{2.17}$$

that is, $x_1(\widetilde{p}_0) < x_1^{\varepsilon}$ and $\widetilde{p}_0 \geq n_0+2$, then $x_1(\widetilde{p}_0) < x_1^{\varepsilon} \leq x_1(\widetilde{p}_0-1)$, the above argument produces that $x_1(\widetilde{p}_0) \geq x_1^{\varepsilon}$, which is a contradiction. Therefore, $x_1(n) \geq x_1^{\varepsilon}$ for all $n \geq n_0$, since ε can be sufficiently small, it gives that

$$x_1(n) \geq \frac{r_1^L - c_2^U}{a_1^U} \exp\left(r_1^L - a_1^U M_1 - c_2^U\right) \overset{\text{def}}{=} m_1 > 0, \tag{2.18}$$

then $\liminf_{n\to+\infty} x_1(n) \geq m_1$. This proves the claim.

Case 2. Assume that $x_1(n+1) > x_1(n)$ for a sufficiently large $n \geq n^*$. In this case, $\lim_{n\to+\infty} x_1(n)$ exists, denoted by $\underline{x}_1$. For the sake of proving $x_1 \geq (r_1^L - c_2^U)/a_1^U$, by way of contradiction, assume that $\underline{x}_1 < (r_1^L - c_2^U)/a_1^U$, taking limit in the first equation of system (1.2), it follows that

$$\lim_{n\to+\infty}\left[r_1(n) - a_1(n)x_1(n) - \frac{c_2(n)x_2(n)}{1+x_2(n)}\right] = 0, \tag{2.19}$$

however,

$$\begin{aligned}\lim_{n\to+\infty}\left[r_1(n) - a_1(n)x_1(n) - \frac{c_2(n)x_2(n)}{1+x_2(n)}\right] &\geq \lim_{n\to+\infty}\left[r_1(n) - a_1(n)x_1(n) - c_2(n)\right] \\ &\geq r_1^L - a_1^U \underline{x}_1 - c_2^U \\ &> 0,\end{aligned} \tag{2.20}$$

which is a contradiction. It implies that $\underline{x}_1 \geq (r_1^L - c_2^U)/a_1^U$. By the fact that $\min_{x\in\mathbb{R}^+}\{[\exp(x-1)]/x\} = 1$, we obtain that $M_1 = [\exp(r_1^U - 1)]/a_1^L \geq r_1^U/a_1^L \geq r_1^L/a_1^U$. Thus $\underline{x}_1 > (r_1^L - c_1^U)/a_1^U \geq x_1^{\varepsilon}$. Therefore, $\liminf_{n\to+\infty} x_1(n) \geq x_1^{\varepsilon}$. Since ε can be sufficiently small, we have

$$\liminf_{n\to+\infty} x_1(n) \geq m_1 \overset{\text{def}}{=} \frac{r_1^L - c_2^U}{a_1^U} \exp\left(r_1^L - a_1^U M_1 - c_2^U\right) > 0. \tag{2.21}$$

Analogously, by the second inequality in (2.11), we can obtain that

$$\liminf_{n\to+\infty} x_2(n) \geq m_2 \overset{\text{def}}{=} \frac{r_2^L - c_1^U}{a_2^U} \exp\left(r_2^L - a_2^U M_2 - c_1^U\right) > 0. \tag{2.22}$$

This completes the proof of Proposition 2.3.

□

Now, we are in a position to state Theorem 2.4 whose proof is a direct consequence of Propositions 2.2 and 2.3.

Theorem 2.4. *If the inequalities in* (2.11) *hold, then system* (1.2) *is permanent.*

3. Existence and Global Stability of Positive Periodic Solutions

In this section, we suppose system (1.2) is a periodic system, and then we investigate the existence and global stability of positive periodic solutions of such system. To do this, assume that all the coefficients of system (1.2) are ω-periodic, in other words,

$$r_i(n+\omega) = r_i(n), \quad a_i(n+\omega) = a_i(n), \quad c_i(n+\omega) = c_i(n), \quad i = 1,2. \tag{3.1}$$

Theorem 3.1. *If the inequalities in* (2.11) *hold, then system* (1.2) *has at least one strictly positive ω-periodic solution, denoted by $\{x_1^*(n), x_2^*(n)\}$.*

Proof. We know that $K = [m_1, M_1] \times [m_2, M_2]$ is an invariant set of system (1.2) from Propositions 2.2 and 2.3. Define the continuous mapping F on K

$$F\{x_1^*(0), x_2^*(0)\} = \{x_1^*(\omega), x_2^*(\omega)\}, \quad \text{for } \{x_1^*(0), x_2^*(0)\} \in K. \tag{3.2}$$

Obviously, F depends continuously on $\{x_1^*(0), x_2^*(0)\}$, then F is continuous and maps the compact set $K = [m_1, M_1] \times [m_2, M_2]$ into itself. Therefore, F has a fixed point $\{x_1^*, x_2^*\}$. It is easy to see that the solution $\{x_1^*(n), x_2^*(n)\}$ which passes through $\{x_1^*, x_2^*\}$ is an ω-periodic solution of system (1.2). The proof is complete. □

Next, we derive sufficient conditions which guarantee that the positive periodic solution of system (1.2) is globally stable. We first give the definition of global stability.

Definition 3.2. A positive periodic solution $\{x_1^*(n), x_2^*(n)\}$ of system (1.2) is globally stable if each other solution $\{x_1(n), x_2(n)\}$ of system (1.2) with positive initial value defined for all $n > 0$ satisfies

$$\lim_{n\to+\infty} \left|x_1(n) - x_1^*(n)\right| = 0, \qquad \lim_{n\to+\infty} \left|x_2(n) - x_2^*(n)\right| = 0. \tag{3.3}$$

Now, we give the main result in this section.

Theorem 3.3. *In addition to* (2.11), *assume further that the following assumptions*

$$\lambda_1 \stackrel{def}{=} \max\left\{\left|1 - a_1^L m_1\right|, \left|1 - a_1^U M_1\right|\right\} + c_2^U < 1, \tag{3.4}$$

$$\lambda_2 \stackrel{def}{=} \max\left\{\left|1 - a_2^L m_2\right|, \left|1 - a_2^U M_2\right|\right\} + c_1^U < 1 \tag{3.5}$$

hold. Then the positive periodic solution of system (1.2) *is globally stable.*

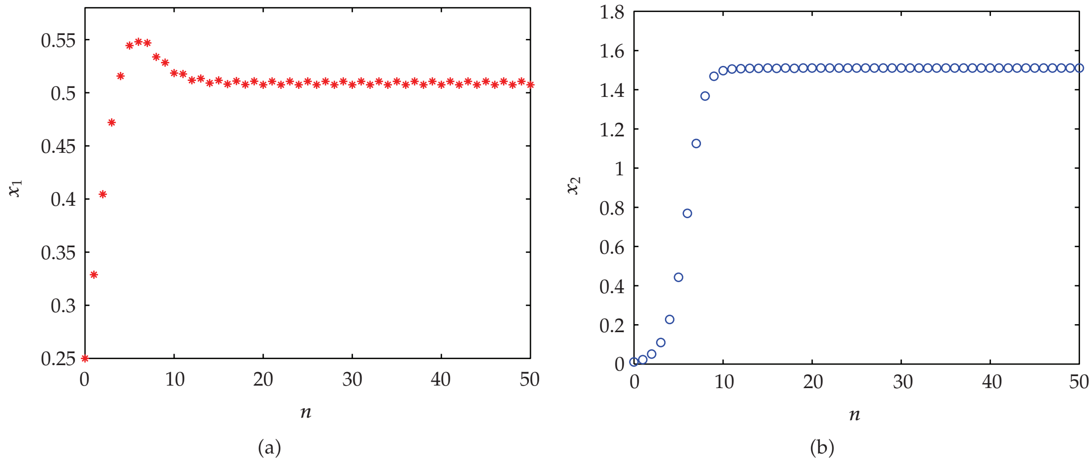

Figure 1: Permanence of system (1.2) with initial value $(0.25, 0.01)$. (a) Time series of x_1 for $n \in [0, 50]$. (b) Time series of x_2 for $n \in [0, 50]$.

Proof. Let $\{x_1^*(n), x_2^*(n)\}$ be a positive periodic solution of system (1.2). We make the change of variables $x_1(n) = x_1^*(n) \exp u(n)$ and $x_2(n) = x_2^*(n) \exp v(n)$, then system (1.2) is rewritten as

$$u(n+1) - u(n) = a_1(n)x_1^*(n)\left[1 - \exp u(n)\right] + c_2(n)\left\{\frac{1}{1 + x_2^*(n)\exp[v(n)]} - \frac{1}{1 + x_2^*(n)}\right\},$$
$$v(n+1) - v(n) = a_2(n)x_2^*(n)\left[1 - \exp v(n)\right] + c_1(n)\left\{\frac{1}{1 + x_1^*(n)\exp[u(n)]} - \frac{1}{1 + x_1^*(n)}\right\}. \tag{3.6}$$

By the mean-value theorem, it derives that

$$u(n+1) = u(n)\{1 - a_1(n)x_1^*(n)\exp[\theta_1 u(n)]\} - c_2(n)v(n)\frac{x_2^*(n)\exp[\theta_2 v(n)]}{\{1 + x_2^*(n)\exp[\theta_2 v(n)]\}^2},$$
$$v(n+1) = v(n)\{1 - a_2(n)x_2^*(n)\exp[\theta_3 v(n)]\} - c_1(n)u(n)\frac{x_1^*(n)\exp[\theta_4 u(n)]}{\{1 + x_1^*(n)\exp[\theta_4 u(n)]\}^2}, \tag{3.7}$$

where the constants $\theta_1, \theta_2, \theta_3, \theta_4 \in (0, 1)$.

Now, by (3.4) and (3.5), we choose the constant ε sufficiently small such that

$$\lambda_1^\varepsilon = \max\left\{\left|1 - a_1^L(m_1 - \varepsilon)\right|, \left|1 - a_1^U(M_1 + \varepsilon)\right|\right\} + c_2^U < 1,$$
$$\lambda_2^\varepsilon = \max\left\{\left|1 - a_2^L(m_2 - \varepsilon)\right|, \left|1 - a_2^U(M_2 + \varepsilon)\right|\right\} + c_1^U < 1. \tag{3.8}$$

In view of Propositions 2.2 and 2.3, there exists an $n_0 \in N$ such that $n \geq n_0$, we have

$$\begin{aligned} 0 < m_1 - \varepsilon \leq x_1^*(n) \leq M_1 + \varepsilon, \qquad 0 < m_1 - \varepsilon \leq x_1(n) \leq M_1 + \varepsilon, \\ 0 < m_2 - \varepsilon \leq x_2^*(n) \leq M_2 + \varepsilon, \qquad 0 < m_2 - \varepsilon \leq x_2(n) \leq M_2 + \varepsilon. \end{aligned} \tag{3.9}$$

Since $\theta_1, \theta_2, \theta_3, \theta_4 \in (0,1)$, both $x_1^*(n)\exp[\theta_1 u(n)]$ and $x_1^*(n)\exp[\theta_4 u(n)]$ are between $x_1(n)$ and $x_1^*(n)$. Meanwhile, both $x_2^*(n)\exp[\theta_2 v(n)]$ and $x_2^*(n)\exp[\theta_3 v(n)]$ are between $x_2(n)$ and $x_2^*(n)$. From the first equation of system (3.7), it follows that

$$\begin{aligned} |u(n+1)| &\leq |u(n)|\left|1 - a_1(n)x_1^*(n)\exp[\theta_1 u(n)]\right| + \left|c_2(n)v(n)\frac{x_2^*(n)\exp[\theta_2 v(n)]}{1 + x_2^*(n)\exp[\theta_2 v(n)]}\right| \\ &\leq \max\left\{\left|1 - a_1^L(m_1 - \varepsilon)\right|, \left|1 - a_1^U(M_1 + \varepsilon)\right|\right\}|u(n)| + \max\{|c_2(n)v(n)|\} \\ &\leq \max\left\{\left|1 - a_1^L(m_1 - \varepsilon)\right|, \left|1 - a_1^U(M_1 + \varepsilon)\right|\right\}|u(n)| + c_2^U|v(n)| \\ &\leq \lambda_1^\varepsilon \max\{|u(n)|, |v(n)|\}. \end{aligned} \tag{3.10}$$

Similarly, it follows from (3.5) that

$$|v(n+1)| \leq \lambda_2^\varepsilon \max\{|u(n)|, |v(n)|\}. \tag{3.11}$$

Denote $\lambda^\varepsilon = \max\{\lambda_1^\varepsilon, \lambda_2^\varepsilon\}$, then $\lambda^\varepsilon < 1$. Therefore, when $n \geq n_0$,

$$\max\{|u(n+1)|, |v(n+1)|\} \leq \lambda^\varepsilon \max\{|u(n)|, |v(n)|\} \leq (\lambda^\varepsilon)^{n-n_0} \max\{|u(n_0)|, |v(n_0)|\}, \tag{3.12}$$

as a consequence, $\lim_{n\to+\infty}|x_1(n) - x_1^*(n)| = 0$, $\lim_{n\to+\infty}|x_2(n) - x_2^*(n)| = 0$. By using Definition 3.2, it follows that the positive periodic solution $\{x_1^*(n), x_2^*(n)\}$ of system (1.2) is globally stable. This completes the proof. □

Remark 3.4. Theorem 3.3 shows that $\{x_1^*(n), x_2^*(n)\}$ is the global attractor of all positive solutions of system (1.2), then $\{x_1^*(n), x_2^*(n)\}$ is the unique positive periodic solution of system (1.2).

4. Example and Numerical Simulation

In this paper, we have investigated the permanence and global stability of positive periodic solutions of a discrete competitive system. Each species is not isolated from its living environment, but competes with the other for the same resource. Sufficient conditions which guarantee the permanence, existence and global stability of positive periodic solutions are established, respectively. The theoretical results are confirmed by the following examples and their numerical results.

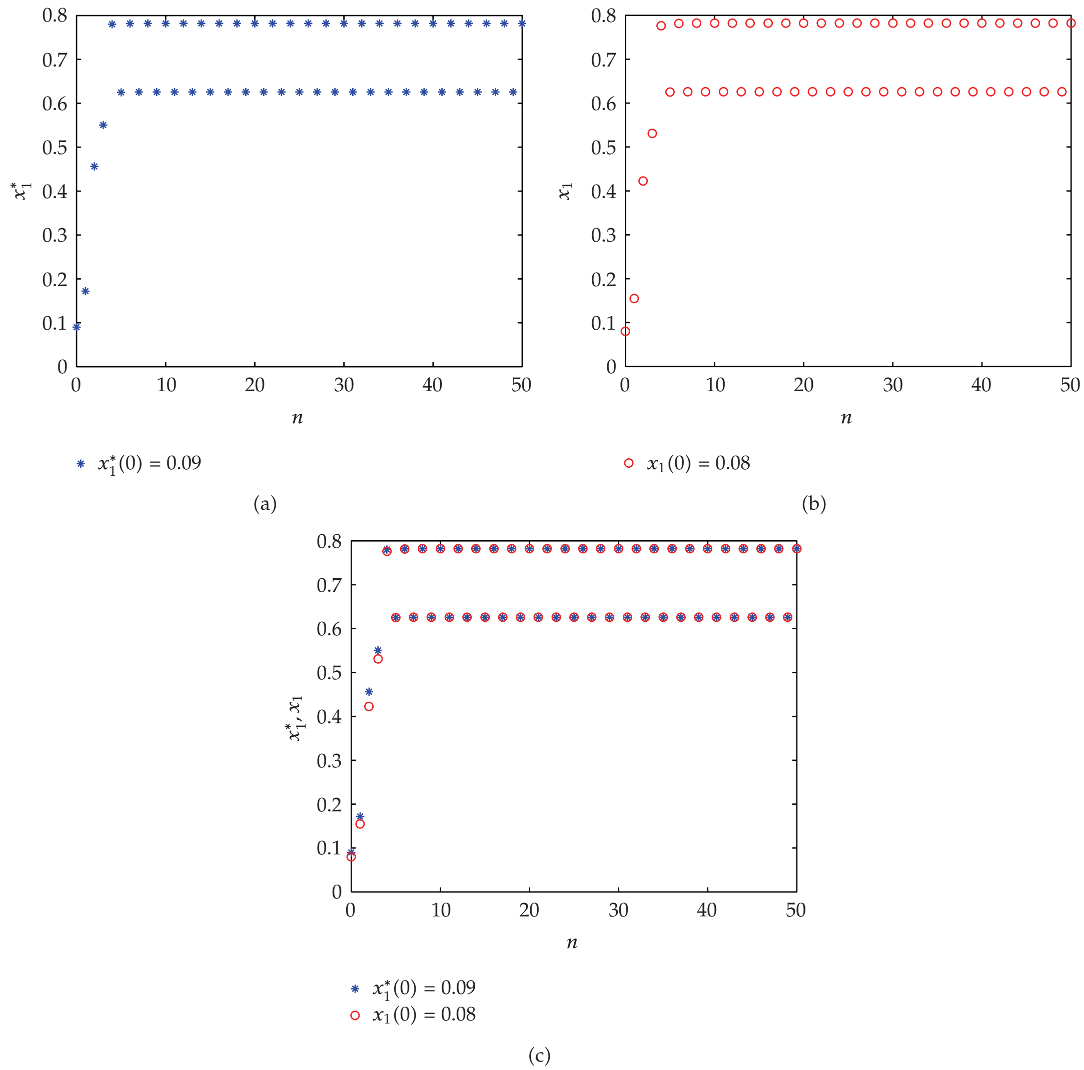

Figure 2: (a) Time series of x_1^* with $x_1^*(0) = 0.09$ for $n \in [0, 50]$. (b) Time series of x_1 with $x_1(0) = 0.08$ for $n \in [0, 50]$. (c) Time series of x_1^* and x_1 for $n \in [0, 50]$ in the same coordinate system.

To verify the sufficient conditions for permanence of system (1.2), we assume that $r_1(n) = 0.46 + 0.01 \sin \pi n$, $r_2(n) = 0.88 - 0.02 \sin \pi n$, $a_1(n) = 0.75 + 0.01 \cos \pi n$, $a_2(n) = 0.52 + 0.01 \sin \pi n$, $c_1(n) = 0.28 + 0.02 \sin \pi n$, $c_2(n) = 0.13 - 0.01 \sin \pi n$, and the initial condition $(x_1(0), x_2(0)) = (0.25, 0.01)$. Clearly, (2.11) in Theorem 2.4 are satisfied, and hence system (1.2) is permanent (see Figure 1).

Now, we further verify the sufficient conditions for the existence and global stability of positive periodic solutions of periodic system (1.2). Let us assume that all the coefficients of system (1.2) are periodic and listed in Table 1.

Besides, we choose the positive periodic solution with initial values (0.09, 0.01), denoted by (x_1^*, x_2^*), and the positive solution with initial value (0.08, 0.02) denoted by (x_1, x_2). By Theorem 3.3, a simple calculation shows that the assumptions in (3.4) and (3.5) hold.

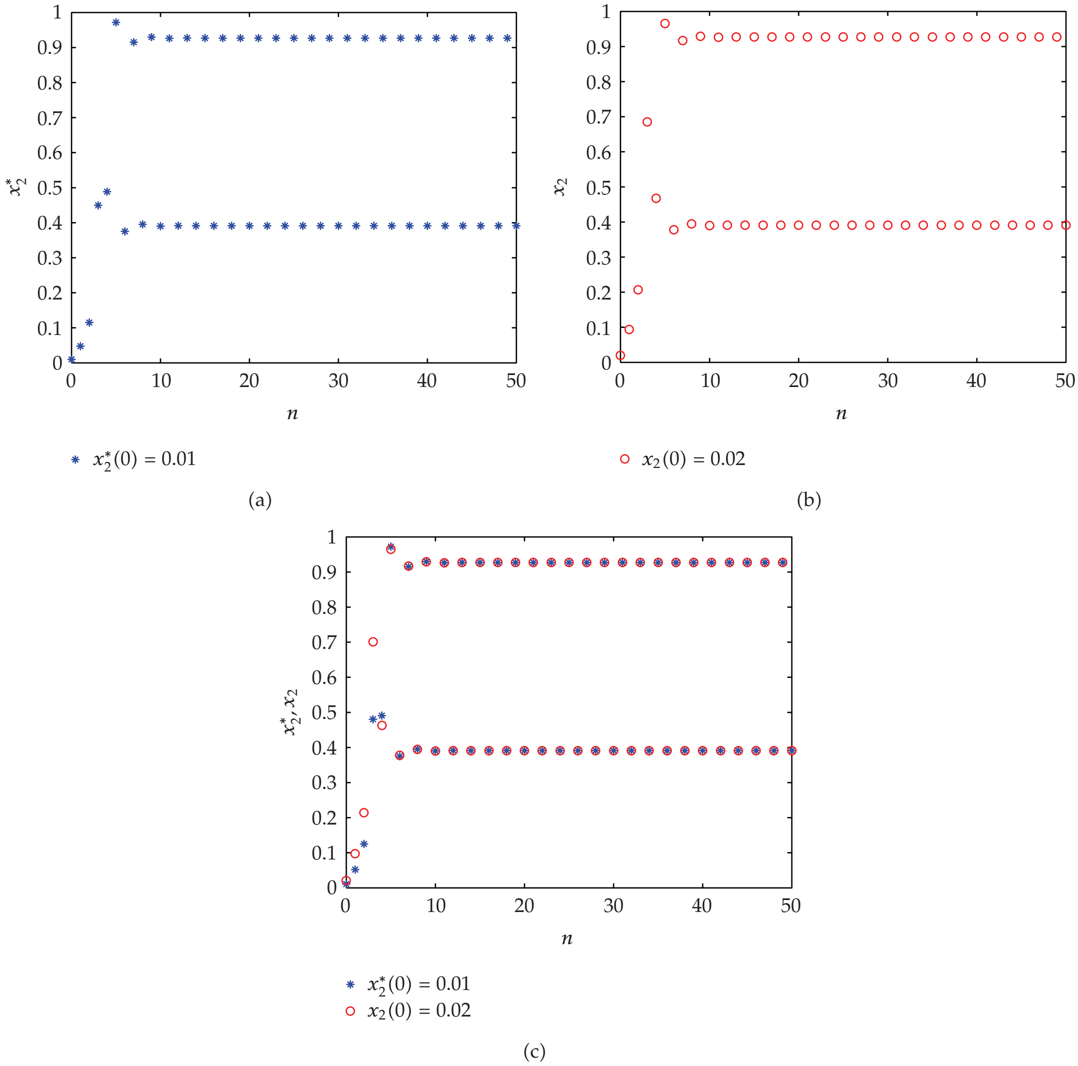

Figure 3: (a) Time series of x_2^* with $x_2^*(0) = 0.01$ for $n \in [0, 50]$. (b) Time series of x_2 with $x_2(0) = 0.02$ for $n \in [0, 50]$. (c) Time series of x_2^* and x_2 for $n \in [0, 50]$ in the same coordinate system.

Table 1: The coefficient values when **n** is under different conditions.

n	$r_1(n)$	$r_2(n)$	$a_1(n)$	$a_2(n)$	$c_1(n)$	$c_2(n)$
Odd number	0.76	1.58	1.25	1.80	0.03	0.02
Even number	1.26	0.98	1.65	1.98	0.02	0.01

So from Theorem 3.3 we know that periodic system (1.2) has a positive 2-periodic solution which is globally stable. From Figures 2(a), 2(b), and 2(c), we see that x_1 with $x_1(0) = 0.08$ will tend to x_1^* with $x_1^*(0) = 0.09$. Similarly, from Figures 3(a), 3(b), and 3(c), we see that x_2 with $x_2(0) = 0.02$ will tend to x_2^* with $x_2^*(0) = 0.01$. Furthermore, Figures 4(a) and 4(b) show the phase portrait of periodic system (1.2) with $(x_1^*(0), x_2^*(0)) = (0.09, 0.01)$ for $n \in [0, 50]$

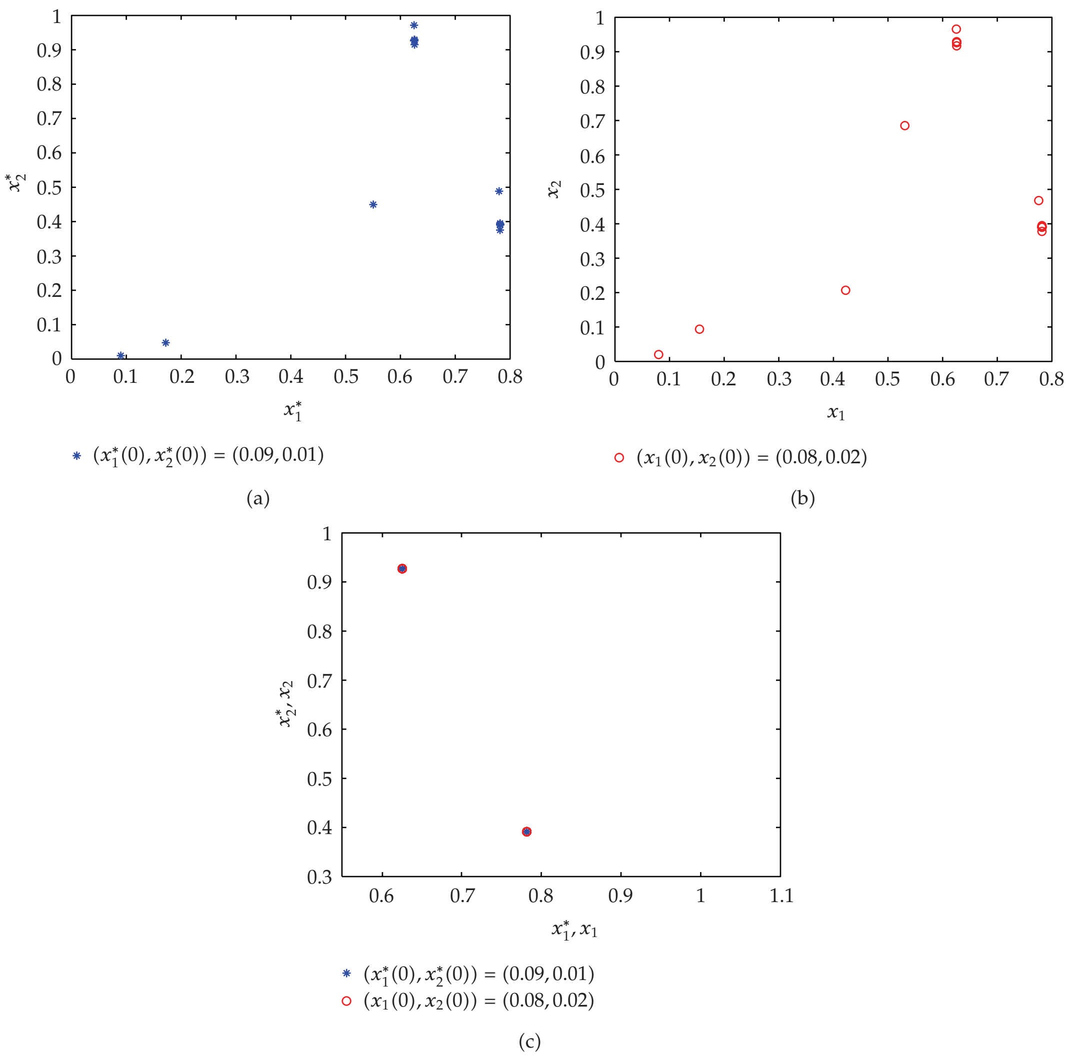

Figure 4: (a) Phase portrait of x_1^* and x_2^* with initial value $(0.09, 0.01)$ for $n \in [0, 50]$. (b) Phase portrait of x_1 and x_2 with initial value $(0.08, 0.02)$ for $n \in [0, 50]$. (c) Phase portraits of x_1^*, x_2^* and x_1, x_2 for $n \in [30, 50]$ in the same coordinate system.

and $(x_1(0), x_2(0)) = (0.08, 0.02)$ for $n \in [0, 50]$, respectively. From Figure 4(c), we can see that periodic system (1.2) has a positive 2-periodic solution which is globally stable.

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