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Research Article

Subprime Risk and Insurance with Regret

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This paper investigates some of the risk and insurance issues related to the subprime mortgage crisis. The discussion takes place in a discrete-time framework with a subprime investing bank being considered to be regret and risk averse before and during the mortgage crisis, respectively. In particular, we investigate the bank's investment choices related to risky subprime structured mortgage products and riskless treasuries. We conclude that if the bank takes regret into account, it will be exposed to higher risk when the difference between the expected returns on subprime structured mortgage products and treasuries is small. However, there is low-risk exposure when this difference is high. Furthermore, we assess how regret can influence the bank's view of a rate of return guarantee from monoline insurers. We find that before the crisis, regret decreased the investment bank's preparedness to forfeit on returns when its structured product portfolio was considered to be safe. Alternatively, risk- and regret-averse banks forfeit the same returns when their structured mortgage product portfolio is considered to be risky. We illustrate the aforementioned findings about structured mortgage products and monoline insurance via appropriate examples.

1. Introduction

The 2007–2009 subprime mortgage crisis (SMC) was preceded by a period of favorable macroeconomic conditions with strong growth and low inflation combining with low default rates, high profitability, strong capital ratios, and strong innovation involving structured financial products in the banking sector. These conditions contributed to the SMC in that they led to overconfidence and increased *regret aversion* among investors such as subprime investing banks. (Regret is defined as the disutility of failing to choose the expost optimal alternative. Regret aversion reflects an aversion to expost comparisons of its realized outcome with outcomes that could have been achieved had it chosen differently. Alternatively, regret aversion mirrors a disproportionate distaste for large regrets and for a given menu of acts. Such regret aversion distorts the agent's choice behavior relative to the behavior of an expected utility maximizer.) In the search for yield, the growth in structured notes would have been nigh impossible without these banks' strong demand for high-margin, higher-risk assets such as securities backed by subprime mortgages. Such securitization involves the pooling of mortgages that are subsequently repackaged into interest-bearing securities.

The first step in the securitization process involves subprime originators that extend mortgages that are subsequently removed from their balance sheets and pooled into reference mortgage portfolios. Originators then sell these portfolios to special purpose vehicles (SPVs)—entities set up by financial institutions—specifically to purchase mortgages and realize their off-balance-sheet treatment for legal and accounting purposes. Next, the SPV finances the acquisition of subprime reference mortgage portfolios by issuing tradable, interest-bearing securities that are sold to, for instance, subprime investing banks. They receive fixed or floating rate coupons from the SPV account funded by cash flows generated by reference mortgage portfolios. In addition, servicers service the mortgage portfolios, collect payments from the original mortgagors, and pass them on—less a servicing fee—directly to the SPV. The interest and principal payments from the reference mortgage portfolio are passed through to credit market investors. The risks associated with mortgage securitization are transferred from subprime originators to SPVs and securitized mortgage bond holders such as subprime investing banks. The distribution of reference mortgage portfolio losses are structured into *tranches*. As in Figure 1, we consider three such tranches, namely, the *senior* (usually AAA rated; abbreviated as sen), *mezzanine* (usually AA, A, BBB rated; abbreviated as mezz), and *junior (equity)* (usually BB, B rated or unrated; abbreviated as jun) tranches, in order to contractually specify claim priority. In particular, losses from this portfolio are applied first to the most junior tranches until the principal balance of that tranche is completely exhausted.

In the sequel, *structured mortgage products* (SMPs) will be the collective term used to refer to structured residential mortgage notes such as residential mortgage-backed securities (RMBSs) and collateralized debt obligation (CDOs) as well as their respective tranches. A diagrammatic overview of mortgage securitization is given as follows.

It is clear that, especially before and during the SMC, mortgage securitization represented an alternative and diversified source of housing finance based on the transfer of credit risk (and possibly also credit counterparty and tranching risk). (We consider "before the SMC" to be the period prior to July 2007 and "during the SMC" to be the period between July 2007 and December 2009. "After the SMC" is the period subsequent to December 2009.). In this process, some agents assumed risks beyond their capabilities and capital base and found themselves in an unsustainable position once investors became *risk averse*. Because of the aforementioned discussions, we cast our subsequent analysis of subprime mortgage securitization in a risk and regret framework. At this stage, the location and extent of subprime risk cannot be clearly described. This is due to the chain of interacting securities that cause the risk characteristics to be opaque. Other contributing factors are the credit derivatives that resulted in negative basis trades moving CDO risk and that created additional long exposure to subprime mortgages. Determining the extent of the risk is also difficult because the effects on expected mortgage losses depend on house prices as the first-order risk factor. Simulating the effects of this through the chain of interlinking securities is very difficult. Despite this interlinking enabling the risk to be spread among many subprime agents, it caused a loss of transparency with regard to the

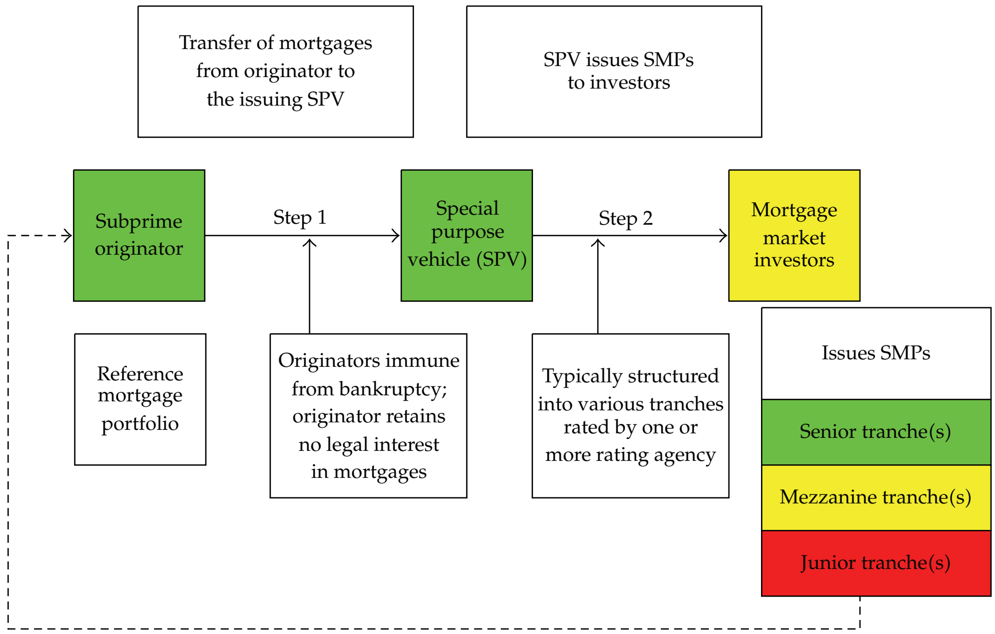

Figure 1: Diagrammatic overview of mortgage securitization.

destination of the aforementioned risks (compare with the IDIOM hypothesis postulated in [1]).

With the unravelling of the SMC in 2007, subprime SMP bonds became distressed. The impact on structured mortgage markets had devastating consequences for monoline insurers. In this regard, Radian Group which insured structured mortgage products was worst hit with shares in Radian Group falling by over 67% in a short space of time. The tumbling share price reflected the almost ninefold increase in the cost of protecting subprime investing banks from SMP default. At this time, monoline insurers were highly leveraged having small capital bases compared to the volume of SMP bonds insured. In this regard, credit rating agencies have come under increasing scrutiny by regulators for their methods as monoline insurers lent their high credit ratings to SMPs issued by others in return for a fee (see, e.g., [2]). When the housing market declined, defaults soared to record levels on subprime mortgages and innovative adjustable rate mortgages such as interest-only, option-adjustable rate, stated-income, and NINJA (No Income, No Job, or Asset) mortgages which had been issued in anticipation of continued rises in house prices. Monoline insurers suffered losses as insured SMPs backed by subprime mortgages defaulted (see, e.g., [2]).

For the sake of readability, in the sequel, we replace the terms credit rating agency (CRA), subprime interbank lender (SIL), subprime originator (SOR), subprime dealer bank (SDB), and subprime investing bank (SIB) with *rating agency*, *lender*, *originator*, *dealer,* and *investor*, respectively. However, the abbreviations displayed above are used in some figures and tables to save on space. For instance, in this case, we make use of the abbreviations SIV and MB to denote *structured investment vehicle* and *mortgage broker*, respectively. Also, unless otherwise stated, the terms *mortgage*, *mortgage loan,* and *residential mortgage loan* (RML) will have the same meaning.

1.1. Literature Review

In this subsection, we briefly review pertinent contributions related to subprime mortgage models (including mortgages and their securitization), risk and regret, as well as monoline insurance.

1.1.1. Literature Review on Subprime Mortgage Models

Paper [3] examines the different factors that have contributed to the SMC. These include the lack of market transparency (see, e.g., [4]), the limitation of extant valuation models (see, e.g., Sections 2.1 and 2.2), agency problems (compare with Sections 3.1 and 3.2), lax underwriting standards, rating agency incentive problems, poor risk management by financial institutions (compare with the discussions in Section 4.1 and [5]), the complexity of financial instruments (see, e.g., Section 4.2), the search for yield enhancement and investment management (see Section 4.3 for a numerical example), and the failure of regulators to understand the implications of the changing environment for the financial system (see, also, [1, 6]). In the main, the aforementioned contributions discuss subprime issues and offer recommendations to help avoid future crises (see, also, [7, 8]).

Our contribution has close connections with [9], where the key structural features of a typical subprime mortgage securitization, how rating agencies assign credit ratings to asset-backed securities (ABSs), and how these agencies monitor the performance of mortgage pools are presented (see, the examples in Sections 4.2 and 4.3). Furthermore, this paper discusses RMBS and CDO architecture and is related to [10] that illustrates how misapplied bond ratings caused RMBSs and ABS CDO market disruptions. In [11], it is shown that the subprime (securitized) mortgage market deteriorated considerably subsequent to 2007 (see, also, [1]). We believe that mortgage screening standards became slack because securitization gave rise to moral hazard, since each link in the mortgage securitization chain made a profit while transferring associated credit risk to the next link (see, e.g., Sections 2.1 and 2.2 as well as [12]). At the same time, some originators retained many mortgages which they originated, thereby retaining credit risk and so were less guilty of moral hazard (see, e.g., [13]). The increased distance between originators and the ultimate bearers of risk potentially reduced the former's incentives to screen and monitor mortgagors (see [1] for more details). The increased complexity of RMBSs and credit markets also reduces investor's ability to value them correctly, where the value depends on the correlation structure of default events (see, e.g., [6, 13]). Reference [14] considers parameter uncertainty and the credit risk of ABS CDOs (see, also, [1, 7, 8]).

1.1.2. Literature Review on Subprime Risk and Regret

Editorial [15] mentions a number of contributions that are related to subprime risk. These include financial regulation and risk management (see Figure 2), contagion, securitization, and risk management (see, e.g., Sections 2.1 and 2.2), bank risk management and stability (see, e.g., Sections 3.1 and 3.2), as well as liquidity risk and SMPs (compare with Figure 7). Article [16] is concerned with the risk management of subprime mortgage portfolios and their relation with default correlation in measuring that risk. (Default correlation is a measure of the dependence among risks. Along with default rates and recovery rates, it is a necessary input in the estimation of the value of the portfolio at risk due to credit. In general, the concept of default correlation incorporates the fact that systemic events cause the default event to cluster. Coincident movements in default among borrowers may be triggered by common underlying factors.) Using a large portfolio of subprime mortgages from an anonymous originator, they show that default correlation can be substantial. In particular, the significance of this correlation increases as the internal credit rating declines (see, e.g., Figure 2).

Journal article [17] discusses subprime risk with an emphasis on operational risk issues underlying the SMC. The paper identifies the fact that the components, mortgage origination, and securitization, investors and markets embed risk (see, e.g., Sections 2.1 and 2.2). In particular, mortgage origination as it pertains to the underwriting of new mortgages embeds credit risk (mortgage quality) and operational risk (documentation, background checks, and mortgage process integrity), while securitization embeds reputational and operational risk (e.g., misselling, valuation, and investor issues) and liquidity risk (cash shortages). Moreover, [17] claims that investors carry credit, market, and operational risks (mark-to-market issues, structured mortgage products worth when sold in volatile markets, uncertainty involved in investment payoffs, and the design and intricacy of structured products). Also, the market reactions described in [17] include market, operational (increased volatility leading to behavior that can increase operational risk such as unauthorized trades, dodgy valuations, and processing issues), and credit risk (possibility of bankruptcies if originators, subprime dealer banks, and subprime investing banks cannot raise funds).

Paper [18] analyzes the systemic elements that transformed the SMC into a global crisis. The author explains the role of mortgage securitization in the US as a mechanism for allocating risks from real estate investments and discusses what has gone wrong and why (see, e.g., Sections 2.1 and 2.2). Also, [18] discusses the incidence of credit, maturity mismatch, interest rate, and systemic risk in this crisis (see Section 1.2.3 for more details). According to [19], declines in asset-backed securities exchange (ABX) prices exposed the shock to valuation from subprime risk. Although it did not reveal the location of these risks, the uncertainty caused a loss of confidence in mortgage markets. During the SMC, this was evidenced by the disruption in the arbitrage foundation of the ABX indices. (The ABX indices played several important roles in the panic. Starting in January 2006, the indices were the only place, where a subprime-related instrument traded in a transparent way, aggregating and revealing information about the value of subprime RMBSs. Other subprime-related instruments, RMBS bonds, CDO tranches, structured investment vehicle liabilities, and so on do not trade in visible markets, and there are no secondary markets. Also, the ABX allowed for hedging subprime risk. These two markets are linked by an arbitrage relationship, but this breaks down during the crisis, an indication of the disappearance of the repo market for subprime-related instruments.). The behavior of the basis—the difference in spreads between the ABX index and the underlying cash bonds—showed that the concern about the location of the risks led to fear of counterparty default, especially in the repo markets, where defaults would curtail the sale of bonds. These repo problems are significant because the US repurchase agreement market is estimated to be worth $12 trillion and is central to the "shadow banking system" which is the nexus of SPVs that issue bonds into capital markets (see [6] for more information). This short-term financing market became very illiquid during the SMC, and an increase in repo haircuts (the initial margin) caused massive deleveraging. The extreme stress in the repo market was seen in the US government securities market, where the instances of "repo fails" where borrowed securities were not returned on time reached record levels (see, also, [20]).

Our analysis is set in a regret-theoretic framework that was developed in [21] (see, also, [22–25]). More recently, regret theory has been used in [26] to investigate risk mitigation and the pricing of assets in a complete market setting (see, e.g., Sections 2.1 and 2.2 and their corresponding discussions in Sections 5.1.1 and 5.1.2, resp.). In the current paper, we consider preferences about regret avoidance for which the investor maximizes its regret-theoretical expected utility function (see, e.g., [27] for more details on expected utility functions). To our knowledge, except for [1], very little (if any) research has focused on how behavior compatible with such a utility structure arises in the banking industry.

1.1.3. Literature Review on Monoline Insurance

In the SMC, as the net worth of banks and other financial institutions deteriorated because of losses related to subprime mortgages, the likelihood increased that those selling monoline insurances would have to pay their counterparties (see Section 3.1 and [2] for further discussion). This created system uncertainty as investors wondered which companies would be required to pay to cover SMP defaults and what forfeits on returns they would face (compare with Section 3.2 and [1]). (The term *forfeit on returns* refers to the fact that monoline insurance with a guarantee results in the shrinking of the risk premium on SMP bonds with an ultimate reduction in investor's rate of return. In this case, the investor has to forfeit a part of its SMP rate of return, r^P, in order to ensure that the SPV pays the monoline insurance premium.) This situation was exacerbated by the fact that monoline insurers are largely not regulated. The volume of monoline insurance outstanding increased 100-fold from 1998 to 2008, with estimates of the debt covered by such insurance, as of November 2008, ranging from $33 to $47 trillion (see [2]). As of 2008, there was no central clearinghouse to honor monoline insurance in the event that an insurance counterparty was unable to perform its obligations under the monoline insurance contract. Companies such as American International Group (AIG), Municipal Bond Insurance Association (MBIA), and Ambac faced ratings downgrades because widespread mortgage defaults increased their potential exposure to losses (see, e.g., Sections 3.1 and 3.2 as well as [1]).

1.2. Preliminaries about Risk, Insurance, and Regret

The main agents in our model are insurers and rating agencies as well as subprime mortgagors, originators, SPVs (monoline insurance protection buyer), and investors in SMPs. Each participant except the investor—allowed to be risk averse—is risk neutral. All events take place in period t that begins at time instant 0 and ends at time 1.

1.2.1. Preliminaries about Subprime Mortgage Securitization

We introduce a subprime mortgage model with default to explain the key aspects of mortgage securitization.

Figure 2 presents a subprime mortgage model involving nine subprime agents, four subprime banks, and three types of markets. As far as subprime agents are concerned, we note that circles **2a**, **2b**, **2c**, and **2d** represent flawed independent assessments by house appraisers, mortgage brokers, rating agencies rating SPVs, and monoline insurers being rated by rating agencies, respectively. Regarding the former agent, the process of subprime mortgage origination is flawed with house appraisers not performing their duties with integrity and independence. According to [17], this type of fraud is the "linchpin of the house

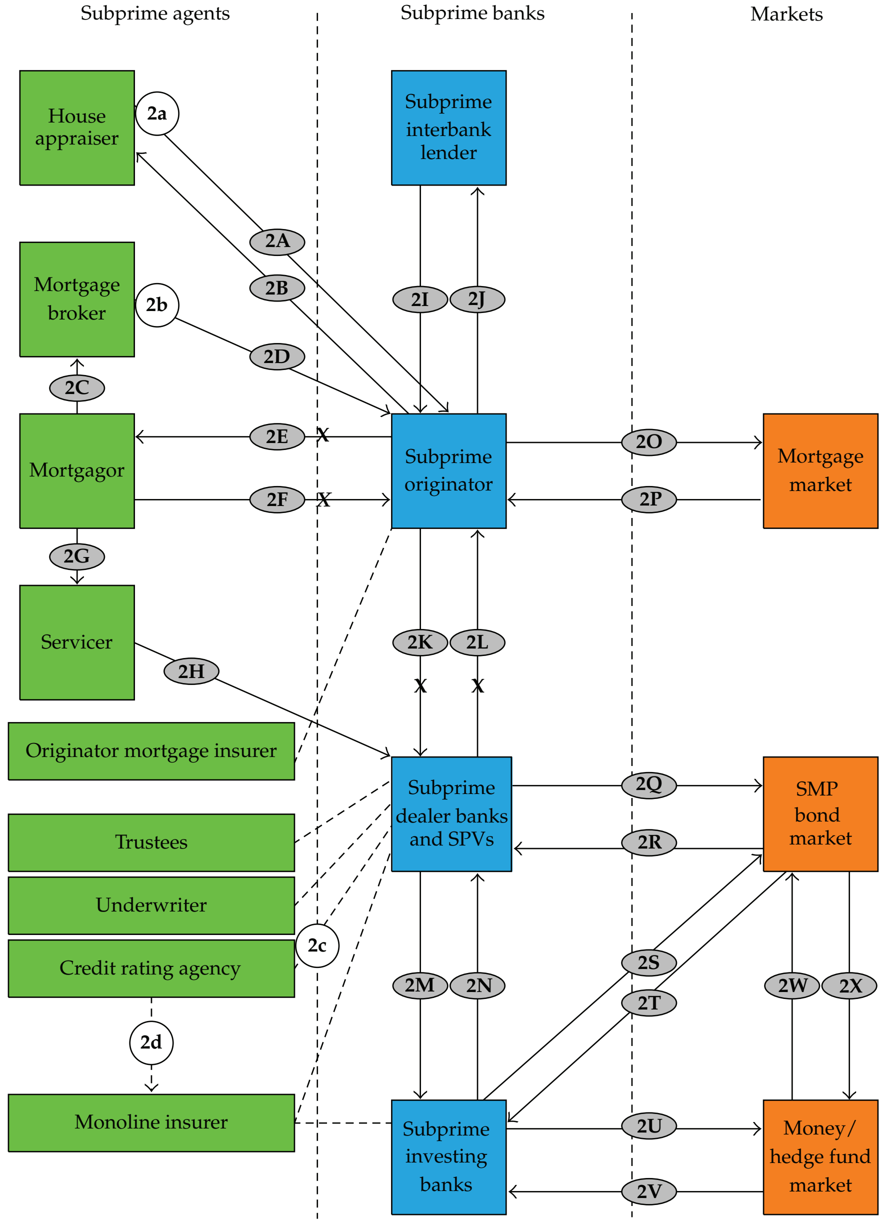

Figure 2: A subprime mortgage model with default.

buying transaction" and is an example of operational risk. Also, the symbol **X** indicates that the cash flow stops as a consequence of defaults. Before the SMC, appraisers estimated house values based on data that showed that the house market would continue to grow (compare with **2A** and **2B**). In steps **2C** and **2D**, independent mortgage brokers arrange mortgage deals and perform checks of their own, while originators originate mortgages in **2E**. Subprime mortgagors generally pay high mortgage interest rates to compensate for their increased risk from poor credit histories (compare with **2F**). Next, the servicer collects monthly payments from mortgagors and remits payments to dealers and SPVs. In this regard, **2G** is the mortgage interest rate paid by mortgagors to the servicer of the reference mortgage portfolios, while the interest rate **2H** (mortgage interest rate minus the servicing fee) is passed by the servicer to the SPV for the payout to investors. Originator mortgage insurers compensate originators for losses due to mortgage defaults. Several subprime agents interact with the SPV. For instance, the trustee holds or manages and invests in mortgages and SMPs for the benefit of another. Also, the underwriter is a subprime agent who assists the SPV in underwriting new SMPs. Monoline insurers guarantee investors' timely repayment of bond principal and interest when an SPV defaults. In essence, such insurers provide guarantees to SPVs, often in the form of credit wraps, that enhance the credit rating of the SPV. They are so named because they provide services to only one industry. These insurance companies first began providing wraps for municipal bond issues but now provide credit enhancement for other types of SMP bonds, such as RMBSs and CDOs. In so doing, monoline insurers act as credit enhancement providers that reduce the risk of mortgage securitization.

The originator has access to subprime mortgage investments that may be financed by borrowing from the lender, represented by **2I**. The lender, acting in the interest of risk-neutral shareholders, invests its deposits either in treasuries or in the originator's subprime mortgage projects. In return, the originator pays interest on these investments to the lender, represented by **2J**. Next, the originator deals with the mortgage market represented by **2O** and **2P**, respectively. Also, the originator pools its mortgages and sells them to dealers and/or SPVs (see **2K**). The dealer or SPV pays the originator an amount which is slightly greater than the value of the reference mortgage portfolios as in **2L**. An SPV is an organization formed for a limited purpose that holds the legal rights over mortgages transferred by originators during securitization. In addition, the SPV divides this pool into sen, mezz, and jun tranches which are exposed to different levels of credit risk. Moreover, the SPV sells these tranches as securities backed by subprime mortgages to investors (see **2N**) that is paid out at an interest rate determined by the mortgage default rate, prepayment, and foreclosure (see **2M**). Also, SPVs deal with the SMP bond market for investment purposes (compare with **2Q** and **2R**). Furthermore, originators have securitized mortgages on their balance sheets, which have connections with this bond market. Investors invest in this bond market, represented by **2S**, and receive returns on SMPs in **2T**. The money market and hedge fund market are secondary markets, where previously issued marketable securities such as SMPs are bought and sold (compare with **2W** and **2X**). Investors invest in these short-term securities (see **2U**) to receive profit, represented by **2V**. During the SMC, the model represented in Figure 2 was placed under major duress as house prices began to plummet. As a consequence, there was a cessation in subprime agent activities, and the cash flows to the markets began to dry up, thus, causing the whole subprime mortgage model to collapse.

We note that the traditional mortgage model is embedded in Figure 2 and consists of mortgagors, lenders, and originators as well as the mortgage market. In this model, the lender lends funds to the originator to fund mortgage originations (see **2I** and **2J**). Home valuation as well as income and credit checks were done by the originator before issuing the mortgage. The originator then extends mortgages and receives repayments that are represented by **2E** and **2F**, respectively. The originator also deals with the mortgage market in **2O** and **2P**. When a mortgagor defaults on repayments, the originator repossesses the house.

1.2.2. Preliminaries about Structured Mortgage Products

The face value of SMPs will be denoted by P and rate of return by r^P. In period t, investors invests a proportion of its funds in a subprime SMP portfolio with stochastic returns, r_t^P.

On the other hand, investors have the option of investing in treasuries at the deterministic rate, $r^{\mathrm{T}}(t) \leq r_t^P$. Investment in subprime SMPs enables the originator to expand its subprime mortgage origination activities. In making its risk allocation choice, the investor takes into account that it may regret its choice if the investment proves to be suboptimal after the expiry of the SMP contract. The following is an important assumption throughout our discussion.

Assumption 1.1 (investor's regret aversion). The investor avoids deleterious consequences of a result that is worse than the best that could be achieved had knowledge of investment losses been known exante.

This assumption implies that, if the investor invests heavily in subprime SMPs and then incurs a large loss, it would experience some additional disutility of not having invested less in such SMPs. The following assumption makes the cash flow dynamics related to SMP investment easier to follow.

Assumption 1.2 (investor's normalized fund supply). We assume that the aggregate supply of funds by the investor to the SPV in exchange for SMP interest and principal payments is fixed and normalized to unity.

For the face value of the investor's subprime SMP portfolio the following assumption is important.

Assumption 1.3 (distribution of SMP rate and success probability). We assume that a subprime SMP rate, r^{P}, is distributed according to the two-point distribution

$$r^P = \begin{cases} P & \text{with probability } q(P,\mathtt{m}), \\ 0 & \text{with probability } 1 - q(P,\mathtt{m}), \end{cases} \tag{1.1}$$

where $\mathtt{m} \in [0,1]$ is a stochastic i.i.d. variable representing a random variable related to the level of macroeconomic activity, distributed over the interval [0, 1] with a continuous density function $f(\mathtt{m})$, and a cumulative distribution function, $F(\mathtt{m}), F(1) = 1$. For the sake of simplicity, we assume that the functional form for the probability of success is given by

$$q(P,\mathtt{m}) = \mathtt{m}q(P). \tag{1.2}$$

The assumption enables the expected returns to be written as

$$\mathbf{E}\left[r^P\right] = \xi q(P)P, \tag{1.3}$$

with $q(P) \in C^2$ and

$$\xi \equiv \int_0^1 \mathtt{m}\widetilde{\varphi}(\mathtt{m})d\mathtt{m} < 1, \tag{1.4}$$

where $\tilde{\varphi}$ is chosen such that the higher the realization of $\mathtt{m}$, the higher the expected returns, $\mathbf{E}[r^P]$, for any given choice P. Also, the higher the realization of $\mathtt{m}$, the higher the probability of success, $q(P)$. We assume that a higher P is associated with a lower probability of success q. This means that $q'(P) < 0$. In addition, to avoid corner solutions with infinite risk, we assume that $q''(P) \leq 0$, so that (1.3) is strictly concave in the control variable P and that there exists a $\overline{P} < \infty$, such that $q(\overline{P}) = 0$. Furthermore, we also assume that

$$P \geq \underline{P} = \frac{r^{\mathrm{T}}}{\xi} \tag{1.5}$$

with $q(\underline{P}) = 1$ and $q'(\underline{P}) > -1/\underline{P}$. In reality, the value of P depends on the level of macroeconomic activity, $\mathtt{m}$, where $q'(P,\mathtt{m}) < 0$ with $q(P,\mathtt{m})$ being the probability of success. Before the SMC, q was high because of minimal default rates on reference mortgage portfolios. In turn, this prompted rating agencies to assign high ratings to subprime SMPs (and monoline insurers) which drove investors to hold large quantities of such SMPs. During the SMC, mortgagors started to default, and this increased the probability of failure, $1 - q(P,\mathtt{m})$, which led many investors to charge higher interest rates. As this situation worsened, they started to invest their funds in riskless assets such as treasuries. The behavior of these investors exacerbated the financial crisis. In particular, due to the decisions taken by investors, the global mortgage market froze. From the above, the following result is immediate.

Proposition 1.4 (investor returns from treasuries and SMPs). *The investor's riskless treasuries are dominated in expected returns by (at least) some risky SMP portfolio.*

1.2.3. Preliminaries about Subprime Risks

The main risks that arise when dealing with SMPs are credit (including counterparty and default), market (including interest rate, price, and liquidity), operational (including house appraisal, valuation, and compensation), tranching (including maturity mismatch and synthetic), and systemic (including maturity transformation) risks. For the sake of argument, risks falling in the categories described above are cumulatively known as *subprime risks*. In Figure 3, we provide a diagrammatic overview of the aforementioned subprime risks.

The most fundamental of the above risks is *credit* and *market risk* (refer to Sections 2.1, 3.1, and 5.1). The former involves originators' risk of loss from a mortgagor who does not make scheduled payments and its securitization equivalent. This risk category generally includes *counterparty risk* that, in our case, is the risk that a banking agent does not pay out on a bond, credit derivative, or credit insurance contract (see, e.g., Sections 3.1 and 5.1 for an example of this from monoline insurance). It refers to the ability of banking agents—such as originators, mortgagors, servicers, investors, SPVs, trustees, underwriters, and depositors—to fulfill their obligations towards each other (see Section 2.1 for more details). During the SMC, even banking agents who thought that they had hedged their bets by buying insurance—via credit default swap contracts or monoline insurance—still faced the risk that the insurer will be unable to pay (see, e.g., Sections 3.1 and 3.2 for monoline insurance).

In our case, *market risk* is the risk that the value of the mortgage portfolio will decrease mainly due to changes in the value of securities prices and interest rates. *Interest rate risk* arises from the possibility that subprime SMP interest rates will change. Subcategories of interest

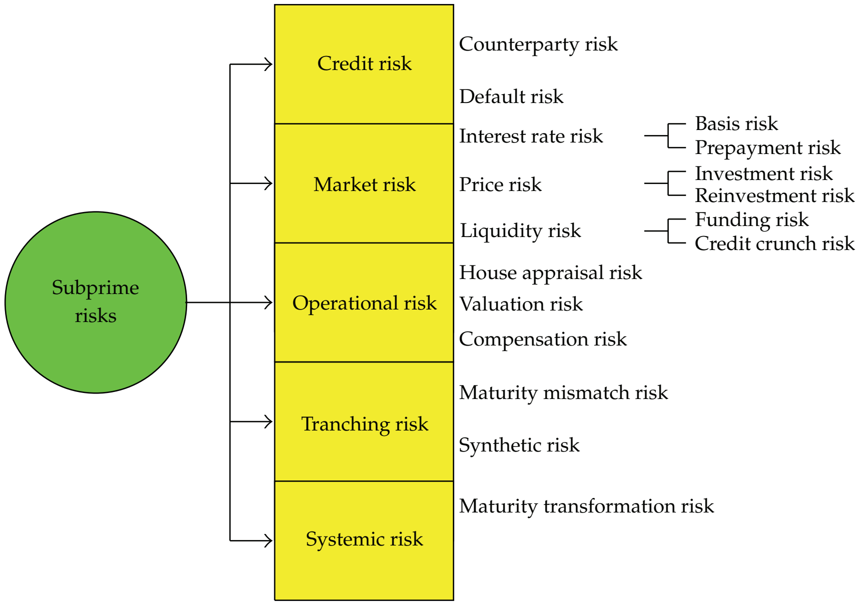

Figure 3: Diagrammatic overview of subprime risks.

rate risk are basis and prepayment risk. The former is the risk associated with yields on SMPs and costs on deposits which are based on different bases with different rates and assumptions (discussed in Section 2.2). *Prepayment risk* results from the ability of mortgagors to voluntarily (refinancing) and involuntarily (default) prepay their mortgages under a given interest rate regime. *Liquidity risk* arises from situations in which a banking agent interested in selling (buying) SMPs cannot do it because nobody in the market wants to buy (sell) those SMPs (see, e.g., Sections 4.1, 5.1, and 5.3). Such risk includes funding and credit crunch risk. *Funding risk* refers to the lack of funds or deposits to finance mortgages, and *credit crunch risk* refers to the risk of tightened mortgage supply and increased credit standards. We consider *price risk* to be the risk that SMPs will depreciate in value, resulting in financial losses, markdowns, and possibly margin calls that is discussed in Sections 4.1 and 5.1. Subcategories of price risk are valuation risk (resulting from the valuation of long-term SMP investments) and reinvestment risk (resulting from the valuation of short-term SMP investments).

Valuation issues are a key concern that must be dealt with if the capital markets are to be kept stable, and they involve a great deal of operational risk (see, e.g., Section 5.3). *Operational risk* is the risk of incurring losses resulting from insufficient or inadequate procedures, processes, systems, or improper actions taken (see, also, Sections 2.1, 3.1, 4.1, and 5.1). As we have commented before, for subprime mortgage origination, operational risk involves documentation, background checks, and progress integrity. Also, mortgage securitization embeds operational risk via misselling, valuation, and investor issues (see, also, Sections 2.2, 4.2, and 5.3). Operational risk related to mortgage origination and securitization results directly from the design and intricacy of mortgages and related structured products. Moreover, investors carry operational risk associated with mark-to-market issues, the worth of securitized mortgages when sold in volatile markets, and uncertainty involved in investment payoffs (see Section 4.2). Also, market reactions include increased volatility leading to behavior that can increase operational risk such as

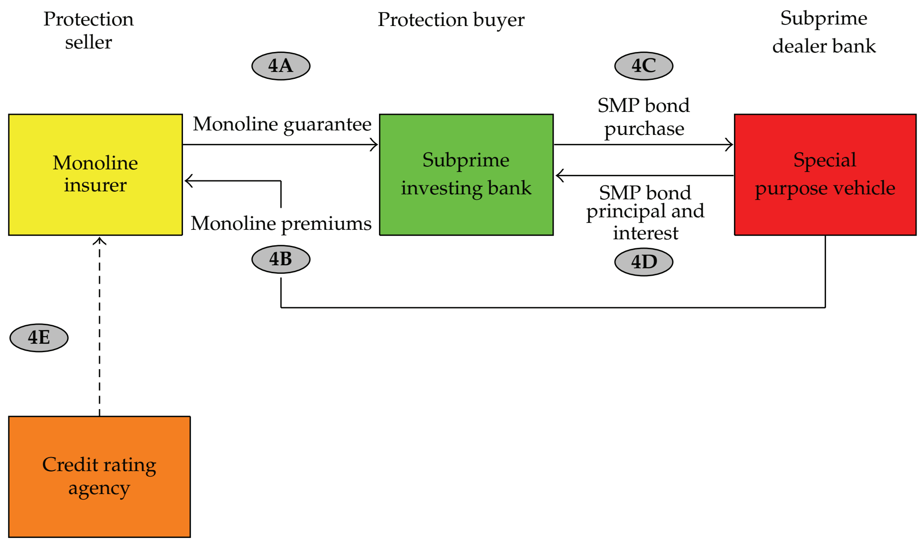

Figure 4: Structured mortgage products wrapped by monoline insurance.

unauthorized trades, dodgy valuations, and processing issues. Often additional operational risk issues such as model validation, data accuracy, and stress testing lie beneath large market risk events (see, e.g., [17]).

Tranching risk is the risk that arises from the intricacy associated with the slicing of securitized mortgages into tranches in securitization deals (refer to Sections 4.2 and 5.3). Prepayment, interest rate, price, and tranching risk are also discussed in Section 5.1, where the intricacy of subprime SMPs is considered. Another tranching risk that is of issue for SMPs is *maturity mismatch risk* that results from the discrepancy between the economic lifetimes of SMPs and the investment horizons of investors. *Synthetic risk* can be traded via credit derivatives (like CDSs) referencing individual subprime RMBS bonds, synthetic CDOs or via an index linked to a basket of such bonds. Synthetic risk is discussed in Section 5.3.

In banking, *systemic risk* is the risk that problems at one bank will endanger the rest of the banking system (compare with Sections 2.1, 3.1, and 2.2). In other words, it refers to the risk imposed by interlinkages and interdependencies in the system, where the failure of a single entity or cluster of entities can cause a cascading effect which could potentially bankrupt the banking system or market (see, e.g., Sections 4.1, 5.1, and 5.3).

In Table 1, we identify the links in the chain of subprime risks with comments about the information created and the agents involved.

1.2.4. Preliminaries about Monoline Insurance

In this subsection, a diagrammatic overview of SMPs being wrapped by monoline insurance is provided.

The monoline insurance model in Figure 4 allows for (senior tranches of) SMPs to be wrapped by monoline insurance. In this process, monoline insurers offer investors a guarantee on returns from SMP bonds (refer to **4A**). There are many reasons why such guarantees are viable in the financial sector. Differences in access to information and in demand for credit risk are but two of them. To make this possible, the SPV—the protection

Table 1: Chain of subprime risk and securitization; compare with [6].

Step in chain	Information generated	Agents involved
Mortgage origination	Underwriting standards, mortgage risk characteristics, credit risk (mortgage quality), operational risk (documentation, creditworthiness, origination process)	SORs and MBs
Mortgage securitization	Reference mortgage portfolio Selected, RMBS structured credit (reference portfolio) risk, market (valuation, liquidity) risk, operational (misselling, SIB issues) risk, tranching (maturity mismatch) risk, systemic (maturity transformation) risk,	SDBs, SRs, CRAs, SIBs buying deal
Securitization of ABSs, RMBSs, CMBSs into ABS CDOs	ABS portfolio selected, manager selected, cdo structured credit (reference portfolio) risk, market (valuation, liquidity) risk, operational (misselling, SIB issues) risk, tranching (maturity mismatch) risk, systemic (maturity transformation) risk	SDBs, CDO managers, CRAs, SIBs buying deal
CDO risk transfer via MLIs in negative basis trade	CDOs and tranche selected, credit risk in the form of market (basis) risk credit (counterparty) risk	SDBs, banks with balance sheets, CDOs
CDO tranches sale to SIVs and other vehicles	CDOs and Tranche selected for SIV portfolio market (price and interest rate) risk	SIV manager, SIV investors buy SIV liabilities
Investment in SIV liabilities by money market funds	Choice of SIV and seniority	Only agents directly involved: buyer and seller
CDO tranches sale to money market funds via liquidity puts	CDOs and tranche selected	Dealer banks, money market funds, put writers
Final destination of cash RMBS tranches, cash CDO tranches and synthetic risk	Location of risk	Only agents directly involved: buyer and seller

buyer—makes a regular stream of premium payments to the monoline insurer (see **4B**) in order to guarantee principal and interest payments on the SMP bonds issued to the investor (see **4C**). Monoline insurance with a guarantee results in the shrinking of the risk premium on the SMP bond with a reduction in the investor's rate of return (refer to **4D**). In fact, in the sequel, we view the investor's forfeit on returns as a part of its SMP rate of return, r^P, that ensures that the SPV pays the monoline insurance premium. Monoline insurers generally carry enough capital to earn AAA ratings from rating agencies and as a result often do not have to post collateral (refer to **4E**). As a consequence, monoline insurance increases the marketability of SMPs, as investor analysis is simplified since credit risk is essentially that of highly rated monoline insurers.

1.3. Main Problems and Outline of Paper

In this subsection, we state the main problems and provide an outline of the paper.

1.3.1. Main Problems

Our general objective is to investigate aspects of the securitization of subprime mortgages and their associated risk as well as their connections with the SMC. In this regard, specific research objectives are listed as follows.

Problem 1 (utility function of investor funds under regret). Can we choose a utility function that incorporates investor's risk allocation preferences in a regret framework (see Section 2.1)?

Problem 2 (investor optimization problem with risk and regret). Can we solve an investor optimization problem that determines the optimal allocation of funds between subprime SMPs and treasuries under risk and regret (see Theorem 2.1 in Section 2.2)?

Problem 3 (monoline insurance). How much risk- and regret-averse investors are prepared to forfeit for a rate of return guarantee by monoline insurers (see Theorem 3.3 in Section 3.2)?

Problem 4 (risk, insurance, and the SMC). How does investors' aforementioned risk and insurance problems relate to the SMC (Section 5)?

1.3.2. Outline of the Paper

The current section is introductory in nature. In Section 2.1 of Section 2, we present pertinent facts about subprime SMPs and treasuries with regret, risk allocation spreads, and regret utility functions. More specifically, Section 2.1.1 analyzes the interplay between the subprime SMPs rate of return, r^P, and treasuries rate, r^T. In particular, it gives the mathematical formulation of the expost optimal final level of funds, that is, the fund level that investor could have attained if it had made the optimal choice with respect to the realized state of the economy. Section 2.1.2 illustrates situations, where the risk allocation spread is low and high. In Section 2.1.3, we construct a regret utility function that incorporates both risk and regret. In Section 2.2, Theorem 2.1 proves that a regret-averse investor will always allocate away from $\pi^{\rho*} = 0$ and $\pi^{\rho*} = 1$, where $\pi^{\rho*}$ denotes the optimal fraction of available investor funds invested in subprime SMPs. The next important result shows the existence of a treasuries rate at which regret has no impact on investor's optimal proportion invested in subprime SMPs (see Corollary 2.2). Also, Proposition 2.3 in Section 2.2 proposes that higher regret amplifies the effect of the investor hedging its bets.

Monoline insurance is discussed in Section 3.1. In Section 3.2, we suggest a way of mitigating risk and regret via monoline insurance. Theorem 3.3 in Section 3.2 shows that when the fraction of available funds invested in the subprime SMPs is low, a regret-averse investor values monoline insurance guarantees less than its risk-averse counterpart. On the other hand, both risk- and regret-averse investors forfeit the same SMP return when their SMP portfolio is considered to be risky. Sections 4.1, 4.2, and 4.3 in Section 4 provide numerical and illustrative examples involving risk and insurance with regret.

In Section 5, we analyze the main risk, insurance, and regret issues and their connections with the SMC. A discussion on mortgage securitization in a risk and regret framework is presented in Section 5.1. In particular, Section 5.1.2 discusses liquidity risk and its effects in relation to the SMC. In particular, we consider the impact of risk allocation away from subprime SMPs towards treasuries to the economy within the context of the SMC.

Monoline insurance guarantees and its function of mitigating risk and regret introduced in Section 3 is discussed in Section 5.2. The analysis of the examples presented in Section 4 is provided in Section 5.3.

Section 6 offers a few concluding remarks, while Appendices A–E provide further details about regret theory and contains full proofs of Theorems 2.1 and 3.3, Proposition 2.3, as well as Corollary 2.2.

2. Risk and Regret

In this section, we provide a few key results involving risk and regret in banking. In the sequel, the subprime SMPs that we restrict our discussion to are (senior tranches of) SMPs wrapped by monoline insurance. Except for issues related to this type of insurance, the arguments presented below will work equally well for any risky subprime residential mortgage product.

2.1. Risk, Regret, and Structured Mortgage Products

In this subsection, we discuss subprime SMPs and treasuries in a regret framework as well as the associated risk allocation spread $\xi q(P)P - r^{\mathrm{T}}$. Finally, we consider appropriate utility functions.

2.1.1. Subprime Structured Mortgage Products and Treasuries with Regret

In the sequel, we make a distinction between the cases, where the interest rate earned by investors on the subprime SMPs, r^P, exceeds the treasuries rate, $r^P \geq r^{\mathrm{T}}$. For some SMP portfolios, this possibility is guaranteed by Proposition 1.4. However, in reality, the opposite situation may also arise; that is, $r^P < r^{\mathrm{T}}$ (compare with Proposition 1.4). In the first instance, for optimal returns, the regret-averse investor would have wanted to invest all available funds in the subprime SMPs. On the other hand, in the second case, it would have been optimal to invest all funds in the treasuries. Symbolically, we can express this as

$$f^{\max} = \begin{cases} f_0\left(1 + r^P\right), & \text{if } r^P \geq r^{\mathrm{T}}, \\ f_0\left(1 + r^{\mathrm{T}}\right), & \text{if } r^P < r^{\mathrm{T}}, \end{cases} \tag{2.1}$$

where $f^{\max}$ is the value of the expost optimal final level of funds, that is, the fund level that the investor could have attained if it had made the optimal choice with respect to the realized state of the economy. Also, we have that

$$f = f_0\left(1 + \pi r^P + (1 - \pi) r^{\mathrm{T}}\right) \tag{2.2}$$

is the actual final fund level. In reality, the expost optimal final level of funds will always be greater than the actual final fund level.

2.1.2. Risk Allocation Spread

The investment decisions between the subprime SMPs and treasuries will partly be based on their allocation spread

$$\xi q(P)P - r^{\mathrm{T}} \tag{2.3}$$

whose realized value is not known in advance (compare with (1.3)). Moreover, we will show a particular interest in the situations, where

$$q(P) = \frac{r^{\mathrm{T}}}{\xi P}, \tag{2.4}$$

$$q(P) = \frac{r^{\mathrm{T}}\mathbf{E}\left[U'\left(f_0\left(1+r^P\right)\right)\right] + \operatorname{cov}\left[-r^P, U'\left(f_0\left(1+r^P\right)\right)\right]}{\xi I\mathbf{E}\left[U'\left(f_0\left(1+r^P\right)\right)\right]}. \tag{2.5}$$

In this paper, (2.4) represents the case where the risk allocation spread $\xi q(P)P - r^{\mathrm{T}}$ is zero, whereas (2.5) corresponds to the case where the spread is high. A motivation for considering a special form for the right-hand side of (2.5) is given as follows. If the risk allocation spread is nonnegative; that is, $\xi q(P)P - r^{\mathrm{T}} > 0$, so that $q(P) > r^{\mathrm{T}}/\xi P$, then $f^{\max} = f_0(1+r^P)$. In this regard, for $U' > 0$, with $q(P)$ given by (2.5), we are guaranteed that the risk allocation spread will be high. Note that we will sometimes use the notation $q(P) \gg r^{\mathrm{T}}/\xi P$ when referring to (2.5). Because the subprime SMPs are riskier than treasuries, the risk allocation spread should generally be nonnegative which makes scenario (2.5) more realistic than (2.4).

2.1.3. Regret-Theoretical Expected Utility Function

Expected utility theory is a major paradigm in investment theory (for more details see [21–23] as well as Appendix A). In our contribution, we choose a regret-theoretical expected utility of the form

$$\int \left[U(f_{\mathtt{m}}) - \rho \cdot g\left(U\left(f_{\mathtt{m}}^{\max}\right) - U(f_{\mathtt{m}})\right)\right] dF(\mathtt{m}), \tag{2.6}$$

where $F(\mathtt{m})$ is a cumulative distribution function that incorporates institutional views about macroeconomic states, $\mathtt{m}$, where $f_{\mathtt{m}}$ is the result in state, $\mathtt{m}$, of action f being taken. With this in mind, we investigate the impact of regret on the investor's exante risk allocation by representing its preferences as a two-component Bernoulli utility function, $U^{\rho} : \mathbf{R}^{+} \to \mathbf{R}$, given by

$$U^{\rho}(f) = U(f) - \rho \cdot g\left(U\left(f^{\max}\right) - U(f)\right), \tag{2.7}$$

where $U : \mathbf{R}^{+} \to \mathbf{R}$ is the traditional Bernoulli utility (value) function over funding positions. (A Bernoulli utility function refers to a decision maker's utility over wealth. Interestingly, it was Bernoulli who originally proposed the idea that a system's internal, subjective value for an amount of money was not necessarily equal to the physical value of that money.)

In the above, regret aversion corresponds to the convexity of g, and the investor's preference is assumed to be representable by maximization subject to U. The second term in (2.7) is concerned with the prospect of investor regret. The function $g(\cdot)$ measures the amount of regret that the investor experiences, which depends on the difference between the value it assigns to the expost optimal fund level, $f^{\max}$, that it could have achieved, and the value that it assigns to its actual final level of funds, f. The parameter $\rho \geq 0$ measures the weight of the regret attribute with respect to the first attribute that is indicative of risk aversion. The expost optimal funds level should be greater than the actual final level of funds; that is, $f^{\max} > f$. The first term in (2.7) relates to risk aversion and involves the investor's utility function $U(\cdot)$ with $U'(\cdot) > 0$ and $U''(\cdot) < 0$. Therefore, the utility function of expost optimal funds level is greater than the utility function of actual final level of funds; that is, $U(f^{\max}) > U(f)$ because $U(\cdot)$ is an increasing function. In the sequel, for $\rho > 0$, it is necessary that $U(f^{\max}) > U(f)$. In this case, the investor's utility function includes some compensation for regret, and we call the investor *regret averse*. Throughout the paper, $g(\cdot)$ is increasing and strictly convex; that is, $g'(\cdot) > 0$ and $g''(\cdot) > 0$, which also implies regret aversion. For $\rho = 0$, investor's utility function does not include regret, and we call the investor *risk averse*. In particular, the investor would be a maximizer of risk-averse expected utility, which means that $U^0(\cdot) = U(\cdot)$. The mathematical conditions which imply risk aversion are $U'(\cdot) > 0$ and $U''(\cdot) < 0$.

2.2. Investor Optimization Problem with Risk and Regret

In this section, we consider how the investor's optimal risk allocation is influenced by regret theoretic issues in a stylized framework. Let π^ρ denote the fraction of available investor funds invested in the subprime SMPs with regret parameter $\rho \geq 0$. For the case, where π^ρ is optimal (denoted by $\pi^{\rho *}$), we have that π_0^* denotes the optimal fraction invested in the subprime SMPs by the risk-averse investor. For the two-attribute Bernoulli utility function (2.7), the objective function is given by

$$J(\pi) = \mathbf{E}\left[U^\rho(f(\pi))\right]. \tag{2.8}$$

In order to determine the optimal risk allocation, $\pi^{\rho *}$, we consider the *set of admissible controls* given by

$$\mathcal{A} = \{\pi^\rho : 0 \leq \pi^\rho \leq 1,\ (2.8) \text{ has a finite value}\}. \tag{2.9}$$

Also, if π is the proportion of available investor funds invested in subprime SMPs, the *value function* is given by

$$V(\pi) = \max_{\pi \in \mathcal{A}} \mathbf{E}\left[U^\rho(f(\pi))\right] = \max_{\pi \in \mathcal{A}} \mathbf{E}\left[U(f(\pi)) - \rho \cdot g\left(U(f^{\max}) - U(f(\pi))\right)\right]. \tag{2.10}$$

The optimal risk allocation problem with regret may be formally stated as follows.

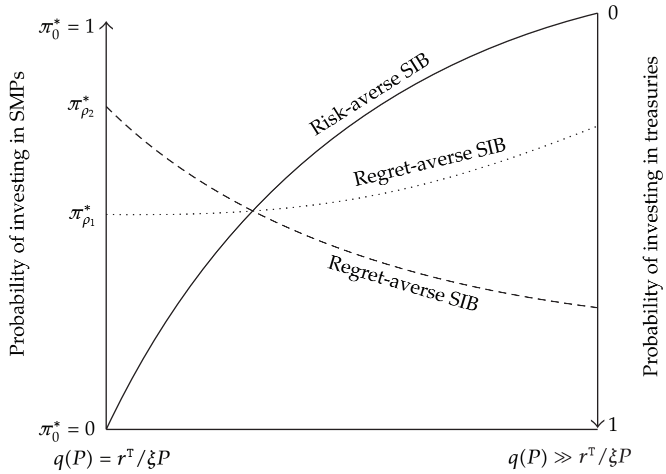

Figure 5: Investor's optimal risk and regret.

Problem 5 (optimal investment in subprime SMPs and treasuries). Suppose that the Bernoulli utility function, U^{ρ}, objective function, J, and admissible class of control laws, $\mathcal{A} \neq \emptyset$, are described by (2.7), (2.8), and (2.9), respectively. In this case, characterize $V(\pi)$ in (2.10) and the optimal control law, π^*, if it exists.

The ensuing optimization result demonstrates that a regret-averse investor (before the SMC) will always allocate away from $\pi^{\rho*} = 0$ and $\pi^{\rho*} = 1$. In other words, by comparison with risk-averse investors (during the SMC), regret-averse investors will commit to a riskier allocation if the difference $\xi q(P)P - r^{\mathrm{T}}$ is low and a less risky allocation if $\xi q(P)P - r^{\mathrm{T}}$ is high. In the years leading up to the SMC, SMP investment by the majority of investors—considered to be regret averse—was driven by high spreads. Spread size was an indication that risk was perceived to be low. This encouraged many investors to invest more in SMP portfolios. However, during the SMC, when mortgagors failed to make repayments, the value of SMPs as well as the spread declined. In this period, risk was considered to have increased, with many investors becoming risk averse and preferring investment in safer assets such as treasuries.

Theorem 2.1 (optimal investment in subprime SMPs and treasuries). *Suppose that Assumption 1.2 holds and that U^{ρ} is the two-attribute Bernoulli utility function defined by* (2.7)*. Regret-averse investors always invest funds in subprime SMPs even if the risk allocation spread is zero as in* (2.4)*. However, risk-averse investors would hold only treasuries in its portfolio in that case. Moreover, for a sufficiently large risk allocation spread as in* (2.5)*, regret-averse investors always invest a positive amount in treasuries, whereas risk-averse investors hold only subprime SMPs in their portfolio.*

Proof. The proof is contained in Appendix B. □

Theorem 2.1 suggests that holding only treasuries will expose investors to the likelihood of severe regret if SMPs perform well as was the case before the SMC. Also, if investors only hold SMPs, it will feel less regret if SMPs perform well but will feel some regret if they perform badly as was the case during the SMC. Theorem 2.1 can be illustrated as shown in Figure 5.

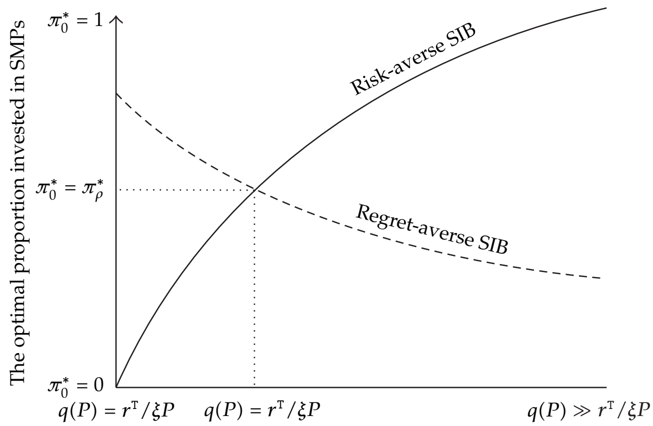

Figure 6: The Certainty equivalent.

We use Theorem 2.1 to show that the next corollary holds.

Corollary 2.2 (risk allocation of risk- and regret-averse investors). *Suppose that ξ is given as in* (1.4)*. Furthermore, assume that the value of the investor's subprime SMP portfolio, the probability of realizing P, and optimal proportion invested in subprime SMPs are denoted by P, $q(P)$, and π^*, respectively. In this case, there exists a treasuries rate, $\widetilde{r}^{\mathrm{T}}$, and therefore a level $\xi q(P)P - \widetilde{r}^{\mathrm{T}}$, for which regret does not affect π^*. At this specific $\xi q(P)P - \widetilde{r}^{\mathrm{T}}$, the SMP allocation for a regret-averse investor will correspond to that of a risk-averse investor.*

Proof. The proof is contained in Appendix C. □

Corollary 2.2 suggests the existence of a treasuries rate, where the allocation of risk will be the same for regret- and risk-averse investors. It may therefore be that, before and during the SMC, a point was reached, where risky asset allocation was independent of whether the investor was regret or risk averse. The results of Corollary 2.2 can be represented graphically as shown in Figure 6.

In the following proposition, we show that higher regret exacerbates the effect of the investor hedging its bets.

Proposition 2.3 (hedging against subprime risk). *Suppose that the investor is more regret- than risk averse (as measured by ρ). Then, under* (2.4)*, it invests more in subprime SMPs, whereas under* (2.5) *it invests less in SMPs. In particular, the more regret averse the investor, the more likely it will be to hold subprime SMPs in its portfolio as long as the risk allocation spread is zero. Conversely, it will hold less SMPs when the risk allocation spread is high.*

Proof. The proof is contained in Appendix D. □

Proposition 2.3 suggests that, before the SMC, in the case where the risk allocation spread is zero, regret-averse investors are more likely to hold subprime SMPs in their portfolios. Conversely, during the SMC, these banks will hold less subprime SMPs when the risk allocation spread is high.

3. Monoline Insurance

In this section, we discuss monoline insurance and its relationship with regret.

3.1. *Monoline Insurance with Regret*

In principle, monoline insurance guarantees may help to alleviate the regret experienced by investors, by protecting their SMP returns when macroeconomic activity is depressed. This is especially true in the case where investors have high levels of SMP investment with the potential to cause regret. The effect of monoline insurance—having the character of a guarantee—is that the risk premium on the SMP bond shrinks thus reducing the return investors receive from SMPs. Also, the SPV has to pay a price for protecting SMP returns by paying the monoline insurance premium. Before the SMC, given the low-perceived risk of SMPs, monoline insurers generally had very high leverage, with outstanding guarantees often amounting to 150 times capital. In this type of insurance, default risk is transferred from the bondholders—in our case investors—to monoline insurers. Investors are only left with the residual risk that the monoline insurer will default. As a result, the analysis of this insurer is closely connected with the analysis of the default risk of all bonds they insured.

In the sequel, we consider a rate-of-return monoline insurance guarantee that involves the guaranteed repayment of investors' investment in SMPs. However, a monoline guarantee also comes at an additional cost for investors. This cost depends on how much investment risk is borne by investors. The guarantee becomes more costly for investors as the risk associated with the RMPs increases. As explained before, the cost of guaranteeing SMP returns for regret-averse investors involves a forfeit on returns. In particular, in the sequel, we compare regret- and risk-averse investors' preparedness to forfeit returns by examining how they value a monoline insurance guarantee.

3.2. *The Main Monoline Insurance Result*

The following assumption about monoline insurers guaranteed rate of return is important.

Assumption 3.1 (investor guaranteed rate of return). *We assume that $r^{Pg} \geq 0$ is the investor's guaranteed rate of return from monoline insurance that is paid on the fixed face value of the insured SMP portfolio, π^f.*

In Assumption 3.1, the investor's portfolio allocation is assumed to be fixed in order to sidestep the moral hazard problem resulting from portfolio reshuffling under guarantee. In the situation where the SPV buys no protection against risk related to the returns on subprime SMP portfolios, the investor's forfeit should be zero; that is, $r^{Pg} = 0$. In the case where no credit event takes place, the monoline insurer pays nothing. In this case, investors will receive the normal rate of return on subprime SMPs, r^P, throughout the SMP term. If a credit event occurs, the monoline insurer will pay R^{Pg} given by

$$R^{Pg} = \max\left(r^P, r^{Pg}\right). \tag{3.1}$$

In the sequel, the monoline insurance contract does not alter the expost optimal level of funds, $f^{\max}$. Therefore, the expost optimal preference is for the investor to invest all its available funds in subprime SMPs, in the event that the realized return, r^P, is above the treasuries rate, r^{T}, and all of it in the treasuries otherwise. Mathematically, this may be expressed as

$$f^{\max} = f_0\left(1 + \max\left(r^P, r^{\mathrm{T}}\right)\right). \tag{3.2}$$

Furthermore, suppose that $c^\rho(r^{Pg}, \pi^f)$ is the maximum forfeit by an investor with regret parameter $\rho \geq 0$ for guarantees on π^f. In this case, the size of the forfeit is dependent on the guaranteed rate of return, r^{Pg}. For instance, in the case of a very risky investment, r^{Pg} is likely to be very high, which will force the investor to make a large forfeit. In this case, the investor's forfeit is governed by the indifference equation

$$\begin{aligned}
&\mathbf{E}\left[U^\rho\left(f_0\left(1 + \pi^f r^P + \left(1 - \pi^f\right) r^{\mathrm{T}}\right)\right)\right] \\
&\quad = \mathbf{E}\left[U^\rho\left(\left(f_0 - c^\rho\left(r^{Pg}, \pi^f\right)\right)\left(1 + \pi^f R^{Pg} + \left(1 - \pi^f\right) r^{\mathrm{T}}\right)\right)\right].
\end{aligned} \tag{3.3}$$

The right-hand side of (3.3) describes the situation where no credit protection is bought, while the left-hand side incorporates the cash flow on the monoline insurance contract purchased by the investor. In the case where no credit protection is bought; that is, $r^{Pg} = 0$, the investor's forfeit for the monoline guarantee is zero. This means that

$$c^\rho\left(0, \pi^f\right) = 0, \quad \forall 0 \leq \pi^f \leq 1. \tag{3.4}$$

If we rewrite (3.3), then the coming result follows immediately.

Lemma 3.2 (hedging against investor subprime risk via monoline insurance). *For R^{Pg} given by* (3.1), *if one puts*

$$\aleph\left(R^{Pg}, \pi^f\right) = 1 + \pi^f R^{Pg} + \left(1 - \pi^f\right) r^{\mathrm{T}}, \tag{3.5}$$

then

$$\mathbf{E}\left[U^\rho\left(f_0 \aleph\left(r^P, \pi^f\right)\right)\right] = \mathbf{E}\left[U^\rho\left(\left(f_0 - c^\rho\left(r^{Pg}, \pi^f\right)\right)\aleph\left(R^{Pg}, \pi^f\right)\right)\right]. \tag{3.6}$$

Of course, if all the investor's funds were allocated to treasuries, its monoline insurance forfeit should be zero, so that

$$c^\rho\left(r^{Pg}, 0\right) = 0, \quad \forall 0 \leq r^{Pg} \leq r^{\mathrm{T}}. \tag{3.7}$$

In the following theorem, we consider the ramifications of the proportion of available funds invested in subprime SMPs being low. Also, we consider the case where the proportion of investment of subprime SMPs in the portfolio is high.

Theorem 3.3 (risk mitigation via monoline insurance). *Suppose that r^{Pg}, π^f, $c^\rho(r^{Pg},\pi^f)$, and $c^0(r^{Pg},\pi^f)$ denote the guaranteed rate of return, fixed face value of the protected SMPs, the maximum forfeit by the investor with regret parameter $\rho \geq 0$ for monoline insurance, and $c^\rho = c^0$, with $\rho = 0$, respectively. In this case, one has that*

$$c^\rho\left(r^{Pg},\pi^f\right) < c^0\left(r^{Pg},\pi^f\right) \tag{3.8}$$

for low levels of π^f and all r^{Pg}. On the other hand, it is true that

$$c^\rho\left(r^{Pg},\pi^f\right) = c^0\left(r^{Pg},\pi^f\right) \tag{3.9}$$

for high levels of π^f and low levels of r^{Pg}.

Proof. The proof is contained in Appendix E. □

Theorem 3.3 intimates in inequality (3.8) that, if the portfolio contains a low proportion of SMPs, the regret-averse investor would forfeit less for the monoline insurance guaranteed rate of return than is the case for a risk-averse investor. This is typical of the situation during the SMC, where a relatively low proportion of SMPs was held in investor portfolios. In particular, during this time, empirical evidence shows that risk-averse investors forfeit more for monoline insurance guarantees than their regret-averse counterparts. Inequality (3.9) tells a contrasting story for high levels of π^f and low levels of r^{Pg}. Further analysis of Theorem 3.3 follows in Section 5.1.2.

4. Examples Involving Risk, Insurance, and Regret

In this section, we provide an illustrative and numerical example involving subprime residential mortgage products.

4.1. Numerical Example Involving Risk, Insurance, and Regret

In Table 2, we provide parameter values for a numerical example to illustrate important features of the discussions on subprime mortgage products and their risks in this section.

Equation (1.3) is solved as follows. The functional form for the probability of success is given by

$$q(P,\mathtt{m}) = \mathtt{m}q(P) = 0.5 \times 0.4 = 0.2, \tag{4.1}$$

so that expected returns can be written as

$$\mathbf{E}\left[r^P\right] = \xi q(P)P = 0.102 \times 0.4 \times 10 = 0.408, \tag{4.2}$$

Table 2: Numerical example involving subprime structured mortgage products.

Parameter	Period t	Parameter	Period t
r^P	0.08	P	\$10
$\mathtt{m}$	0.5	$q(P)$	0.4
$\tilde{\varphi}(\mathtt{m})$	0.204	r^R	0.5
π	0.5	f_0	\$1
$U(f(\pi))$	0.08	$U(f^{\max})$	0.1
$U(f(\pi_\rho))$	0.09	$c^\rho(r^{Pg}, \pi^f)$	0.1917
$\mathbf{E}[U'(f_0(1+r^P))]$	0.5	$U'(f_0(1+r^P))$	0.5
r^{T}	0.036	$\rho \cdot g(U(f^{\max}) - U(f(\pi)))$	0.035
π^f	0.6	$\rho \cdot g(U(f^{\max}) - U(f(\pi_\rho)))$	0.04

with $q(P) \in C^2$ and

$$\xi \equiv \int_0^1 0.5 \times 0.204 \, d\mathtt{m} = 0.102 < 1. \tag{4.3}$$

$f^{\max}$ can be expressed as

$$f^{\max} = \begin{cases} 1 \times (1 + 0.08) = 1.08, & \text{if } r^P \geq r^{\mathrm{T}}, \\ 1 \times (1 + 0.036) = 1.036, & \text{if } r^P < r^{\mathrm{T}}. \end{cases} \tag{4.4}$$

Thus, in our case, $f^{\max} = 1.08$. Also, we have that

$$f = 1 \times (1 + 0.5 \times 0.08 + (1 - 0.5) \times 0.036) = 1.058. \tag{4.5}$$

The investment decisions between SMPs and treasuries are partly based on their allocation spread

$$\xi q(P) P - r^{\mathrm{T}} = 0.102 \times 0.4 \times 10 - 0.036 = 0.372. \tag{4.6}$$

Moreover, when the risk allocation spread is zero, we look at (2.4):

$$q(P) = \frac{0.036}{0.102 \times 10} = 0.035, \tag{4.7}$$

whereas we look at (2.5) when the risk allocation spread is high:

$$q(P) = \frac{0.036 \times \mathbf{E}[U'(1.08)] + \operatorname{cov}[-0.08, 0.5]}{0.102 \times 10 \times \mathbf{E}[U'(1.08)]} = \frac{0.036 \times 0.5 + 0}{0.102 \times 10 \times 0.5} = 0.035. \tag{4.8}$$

The Bernoulli utility function is given by

$$U^{\rho}(f) = 0.08 - 0.035 = 0.045. \tag{4.9}$$

The payout on monoline insurance is given by

$$R^{Pg} = \max(0.08, 0.5) = 0.5. \tag{4.10}$$

The optimal level of funds to be invested in subprime SMPs is

$$f^{\max} = 1 \times (1 + \max(0.08, 0.036)) = 1 \times (1 + 0.08) = 1.08. \tag{4.11}$$

The investor's forfeit is governed by the indifference equation (3.3):

$$\begin{aligned}&\mathbf{E}[U^{\rho}(1 \times (1 + 0.6 \times 0.08 + (1 - 0.6) \times 0.036))]\\&\quad = \mathbf{E}[U^{\rho}((1 - 0.1917)(1 + 0.6 \times 0.5 + (1 - 0.6) \times 0.036))] = \mathbf{E}[U^{\rho}(1.0624)].\end{aligned} \tag{4.12}$$

According to (3.6), hedging against the investor's subprime risk via monoline insurance implies that if

$$\aleph\left(R^{Pg}, \pi^{f}\right) = 1 + 0.6 \times 0.5 + (1 - 0.6) \times 0.036 = 1.3144, \tag{4.13}$$

thus,

$$\aleph\left(r^{P}, \pi^{f}\right) = 1 + 0.6 \times 0.08 + (1 - 0.6) \times 0.036 = 1.0624, \tag{4.14}$$

then

$$\mathbf{E}[U^{\rho}(1 \times 1.0624)] = \mathbf{E}[U^{\rho}((1 - 0.1917))1.3144] = \mathbf{E}[U^{\rho}(1.0624)]. \tag{4.15}$$

The objective function is given by (2.8):

$$J(\pi) = \mathbf{E}\left[U^{\rho}(f(\pi))\right] = \mathbf{E}[0.045] = 0.045. \tag{4.16}$$

Also, the value function is given by (2.10):

$$\begin{aligned}V(\pi) &= \max_{\pi \in \mathcal{A}} \mathbf{E}\left[U^{\rho}(f(\pi))\right] = \mathbf{E}\left[U(f(\pi_{\rho})) - \rho \cdot g\left(U(f^{\max}) - U(f(\pi_{\rho}))\right)\right]\\&= \mathbf{E}[0.09 - 0.04] = \mathbf{E}[0.05] = 0.05,\end{aligned} \tag{4.17}$$

where

$$\mathcal{A} = \{\pi^{\rho} : 0 \leq \pi^{\rho} \leq 1,\ (2.8)\ \text{has a finite value}\}. \tag{4.18}$$

4.2. Illustrative Example Involving Structured Mortgage Product Complexity

The following example illustrates the complexity and information loss problems associated with subprime mortgage securitization (compare with [1, 6]). The simplified example ignores the dynamic aspects of such securitization and discounting but only considers simple tranching. We consider payouts from a single subprime mortgage, a sen/subtranche RMBS securitization of this mortgage, and a sen/subtranche RMBS CDO, which has purchased the sen tranche of the RMBS. In our example, the transactions takes place in a single period t with all payouts taking place at the end of period t at time 1.

In our example, the mortgage has a face value of M. At the end of period t at time 1, the mortgage experiences a stepup rate and will either be refinanced or not. If it is not refinanced, then it defaults, in which case the originator will recover R_1. Therefore, the originator will suffer a loss of S_1 which is given by $S_1 = M - R_1$, where S_1 and R_1 are the mortgage losses and recovery at time 1, respectively. In the case where no default occurs, the new mortgage is expected to be worth M at time 1. If we assume no dependence of R_1 and M on house prices, the payout to the originator is given by $\Pi_1 = \max[M, R_1]$, where M is the value of the new mortgage after refinancing. If $M < R_1$, then the originator does not refinance and mortgages default.

The originator finances mortgages via securitization in which case the mortgage is sold at par of M. The originator can still exercise the option of refinancing in the manner previously described, with the securitization either receiving M or R_1. The subprime RMBSs transaction has two tranches, namely, the first tranche attaches at 0 and detaches at N_1; the second tranche attaches at N_1 and detaches at the end value M. (The phrase "first tranche attaches at 0 and detaches at N_1" means that the loss $N_1 - 0$ is borne (absorbed) by the first tranche. Similarly, the phrase "second tranche attaches at N_1 and detaches at M" means that the loss $M - N_1$ is borne (absorbed) by the second tranche.). The face value of the sen tranche is the difference between the face value of mortgage and the first loss to be absorbed by the jun tranche, that is, $M - N_1$. It then follows that the losses that may occur on a RMBS sen tranche at time 1 are given by

$$S_1^s = \max[S_1 - N_1, 0], \tag{4.19}$$

where N_1 is the value at which the first RMBS tranche detaches at time 1. Here, the payout on the sen tranche at time 1 takes the form

$$\Pi_1^s = \min\begin{cases} \max[M - N_1, 0], \\ M - N_1 - S_1^s. \end{cases} \tag{4.20}$$

In this case, because $M - N_1 > 0$, we have that $\max[M - N_1, 0] = M - N_1$. Furthermore, since it is always true that $\Pi_1^s \geq 0$, it follows that

$$M - N_1 - S_1^s \leq M - N_1. \tag{4.21}$$

This implies that

$$\Pi_1^s = M - N_1 - S_1^s \tag{4.22}$$

which, in turn, implies that

$$\Pi_1^s = \min[M - N_1, M - S_1]. \tag{4.23}$$

By way of illustration, for $\Pi_1^s = \$50$ and $N_1 = \$30$, if the mortgage is not refinanced and defaults, then the sen tranche will suffer a \$20 loss because the first loss tranche only absorbs the first \$30 loss. In this case the Π_1^s of the sen tranche is \$50.

Next, we consider a situation in which the sen tranche of the subprime RMBS is sold to a CDO, which has two tranches, namely, the first tranche attaches at 0 and detaches at N_1^c; the second tranche attaches at N_1^c and detaches at the end value $M - N_1$. We note that the size of the CDO is $M - N_1$ since it only purchases the sen tranche of the subprime RMBS. Moreover, $N_1^c < N_1$ since the CDO portfolio is smaller. However, the subtranche of the CDO could be larger in percentage terms. In this case, we have that the loss on the sen tranche is

$$S_1^c = \max\left[\min\left[S_1^s, M - N_1\right] - N_1^c, 0\right]. \tag{4.24}$$

Furthermore, in this case, the payoff on this RMBS CDO tranche is given by

$$\Pi_1^c = \min\begin{cases} \max\left[M - N_1 - N_1^c, 0\right], \\ M - N_1 - N_1^c - S_1^c. \end{cases} \tag{4.25}$$

If we substitute (4.24) into (4.25), then Π_1^c takes the form

$$\Pi_1^c = \min\begin{cases} \max\left[M - N_1 - N_1^c, 0\right], \\ N_1 - N_1^c - \max\left[\min\left[S_1^s, M - N_1\right] - N_1^c, 0\right]. \end{cases} \tag{4.26}$$

Finally, substituting (4.19), we obtain

$$\Pi_1^c = \min\begin{cases} \max\left[M - N_1 - N_1^c, 0\right], \\ N_1 - N_1^c - \max\left[\min\left[\max\left[\min\left[S_1^s, M - N_1\right] - N_1^c, 0\right], M - N_1\right] - N_1^c, 0\right]. \end{cases} \tag{4.27}$$

Next, we illustrate the issues raised in the above discussion numerically (see [6]). Suppose that $M = \$100$, the size of RMBS subtranche is \$20, and the size of the sen RMBS tranche is \$80. Furthermore, the subprime RMBS tranche is sold to a CDO, which only buys this tranche, so that the size of the CDO is \$80. Let \$15 be the size of the CDO subtranche, so that the sen tranche's size is \$65. If we keep these parameters constant and vary the recovery amount, Table 3 shows the loss on the sen RMBS tranche, the payout from the sen RMBS tranche, the loss on the sen CDO tranche, and the payout from the sen CDO tranche at the end of the period.

Table 3: Computational results, source [6].

	Parameters							
R_1 (\$)	**90**	**70**	**60**	**50**	**40**	**30**	**20**	**10**
	Results							
S_1^s	0	10	20	30	40	50	60	70
Π_1^s	80	70	60	50	40	30	20	10
Π_1^s as % of 80	100%	87.5%	75%	62.5%	50%	37.5%	25%	12.5%
S_1^c	0	0	5	15	25	35	45	55
Π_1^c	65	65	60	50	40	30	20	10
Π_1^c as % of 65	100%	100%	92.3%	76.9%	61.5%	46.2%	30.8%	15.4%

4.3. Numerical Example Involving Structured Mortgage Product Returns

The ensuing example illustrates issues involving mortgage securitization and its connections with profit and capital. Here, we view the originator as an investor and consider some of its incentives to securitize its subprime mortgages. We note that the type of forfeiture mentioned in this example—although the principle is the same—is distinct from the monoline insurance forfeits mentioned earlier. Also, we discuss how the securitization process can go wrong as was evidenced by the events during the SMC. In the sequel, f^{Σ} denotes the fraction of the face value of the originator's mortgages, M, that is securitized.

The sale of the original mortgage portfolio via securitization is intended to save economic capital K^e, where $K^e \geq E$.(Economic capital is the amount of risk capital (equity, E) which the originator requires to mitigate against risks such as market, credit, operational, tranching, and subprime risk. It is the amount of money which is needed to secure survival in a worst case scenario.), K^e differs from regulatory capital, K, in the sense that the latter is the mandatory capital that regulators require to be maintained while K^e is the best estimate of required capital that originators use internally to manage their own risk and the cost of maintaining K. In this example, we assume that the originator's profit from mortgage securitization, $\Pi^{\Sigma} \propto \Pi$, where Π is expressed as return on equity (ROE; denoted by r^E) (ratio of profit after tax to equity capital employed.) and return on assets (ROA; denoted by r^A) (ratio which measures the return the originator generates from its total assets).

This subsection contains the following discussions. The purpose of the analysis in Section 4.3.1 is to determine the costs and benefits of securitization and to assess the impact on ROE. The three steps involved in this process are the description of the originator's unsecuritized mortgage portfolio, calculation of the originator's weighted cost of funds for on-balance sheet items, $c^{M\omega}$, as well as the originator's weighted cost of funds for securitization, $c^{M\Sigma\omega}$. In Section 4.3.2, the influence on ROE results from both lower-level E and reduced $c^{M\Sigma\omega}$. The value gained is either the present value or an improvement of annual margins averaged over the life of the securitization. In our example, capital saving, K^{es}, is calculated with a preset forfeit percentage of 4% used as an input. In this regard, the impact on ROE follows in Section 4.3.3. Under a forfeit valuation of capital as a function of $f^{\Sigma}M$, we are able to determine whether securitization enhances r^E, by how much and what the constraints are. Under full economic capital analysis, K^e results from a direct calculation involving a mortgage portfolio model with and without securitization. The enhancement issue involves finding out whether the securitization enhances the risk-return profile of the original mortgage portfolio and, more practically, whether postsecuritization $r^{E\Sigma}$ is higher or lower than the presecuritization r^E.

4.3.1. Cost of Funds

In the sequel, we show how K^{es} results from securitization, where $K^{es} \propto f^{\Sigma}M$ is valid for K under Basel capital regulation. For the sake of illustration, we assume that the originator's balance sheet can be written as

$$M_t = D_t + n_t E_{t-1} + O_t. \tag{4.28}$$

For the original mortgage portfolio, $M = 10000$, suppose that the weight $\omega^M = 0.5$ and the market cost of equity, $c^E = 0.25$ before tax. In the case where $\rho = 0.08$ (see [1] for more information), regulatory capital is given by

$$K_t = n_t E_{t-1} + O_t = 1.25 \times 200 + 150 = 400 = 0.08 \times 0.5 \times 10000, \tag{4.29}$$

with $n_t = 1.25$. Furthermore, we assume that c^E is equivalent to a minimum accounting $r^E = 0.25$, and the originator considers mortgage securitization for $M = 6500$, while K includes subordinate debt with $r^O = 0.101$ and deposits (liabilities) cost $c^D = 0.101$. We suppose that the original mortgage portfolio, M, has an effective duration of 7 years despite its 10-year theoretical duration due to early voluntary prepayments as before and during the SMC. The return nett of statistical losses and direct monitoring and transaction costs is $r^M = 0.102$. Consider $K^{es} = 260 = 0.04 \times 6500$, where the K^{es} calculation uses a 4% forfeit applied to $f^{\Sigma}M$. K^{es} is constituted by 130 equity and 130 subordinate debt and is the marginal risk contribution of $f^{\Sigma}M$ as evaluated with a portfolio model. The resulting K^{es} would depend on the selection of $f^{\Sigma}M$ and its correlation with M.

In Table 4, we provide an example of a subprime mortgage securitization with two classes of tranches, namely, sen (AAA rating) and sub (including mezz and jun tranches; BBB rating). Given such ratings, the required rate of return for subtranches is $r^{B\text{Sub}} = 0.1061$ and that of sen tranches is $r^{BS} = 0.098 \leq c^D = 0.101$. However, in order to obtain a BBB rating, the rating agency imposes that the subtranches $B^{\text{sub}} \geq 0.101 \times f^{\Sigma}M$. The direct costs include the initial cost of organizing the structure, c^i, plus the servicing fees, f^s. The annual servicing fees, $f^s = 0.002 \times f^{\Sigma}M$.

For $f^{\Sigma}M = 6500$, the sen tranches fund 5850 and subtranches fund 650 with M decreasing from 10000 to 3500, where the weighted mortgages $M^{\omega} = 1750 = 0.5 \times 3500$. The capital required against this portfolio is $K = 140 = 0.08 \times 1750$. With an initial $K^e = 400$, the deal saves and frees $K^{es} = 260$ for further utilization. This is shown in Table 4.

The cost of funds structure consists of K at 4%—divided into 2% E at $c^E = 0.25$ and 2% O at $c^{SD} = 0.102$—as well as 96% deposits at $c^D = 0.101$. We consider the face value of D by using book values for weights, so that the weighted cost of funds is

$$\overline{c}^{M\omega} = 0.96 \times 0.101 + 0.08 \times 0.5 \times (0.25 \times 0.5 + 0.102 \times 0.5) = 0.104, \tag{4.30}$$

where $\overline{c}^{M\omega}$ is consistent with $r^E = 0.25$ before tax, where r^E is the ROE. If the original mortgage portfolio fails to generate this required return, then r^E adjusts. Since the original

Table 4: Original mortgage portfolio.

Current funding	
Cost of equity (c^E)	0.25
Cost of subordinate debt (c^{SD})	0.102
Cost of deposits (c^D)	0.101
Structure	
Cost of sen tranches (c^{BS})	0.098
Weight of sen tranches (ω^{BS})	0.9
Maturity of sen tranches (m^{BS})	10 years
Cost of subtranches ($c^{B\mathrm{Sub}}$)	0.1061
Weight of subtranches ($\omega^{B\mathrm{Sub}}$)	0.1
Maturity of subtranches ($m^{B\mathrm{Sub}}$)	10 years
Direct costs of the structure (c^{Σ})	0.002
Original assets	
Reference mortgage portfolio rate of return (r^M)	0.102
Reference mortgage portfolio duration (y^M)	7 years
Outstanding balances	
Outstanding mortgage value (M)	10 000
Securitized mortgage value ($f^{\Sigma}M$)	6500

Table 5: Required capital before and after securitization.

Outstanding balances	Value		Required capital
Original mortgage portfolio (M)	10 000		400
Reference mortgage portfolio ($f^{\Sigma}M$)	6500		260
Sen tranches (B^S)	5850	Sold	
Subtranches (B^{Sub})	650	Sold	
Final mortgage portfolio ($(1 - f^{\Sigma})M$)	3500		140
Total mortgages ($(1 - f^{\Sigma})M$)	3500	—	
Total weighted mortgages (M^{ω})	1750		140

mortgage portfolio generates only $r^M = 0.102 < \overline{c}^{M\omega} = 0.104$, the actual $r^E < 0.25$. The return actually obtained by shareholders is such that the average cost of funds is identical to $r^A = 0.102$. If $r^A = \overline{c}^{M\omega}$, then effective r^E may be computed as in

$$r^A = 0.96 \times 0.101 + 0.08 \times 0.5 \times \left(r^E \times 0.5 + 0.102 \times 0.5\right). \tag{4.31}$$

After calculation, $r^E = 0.15 < 0.25$ before tax. In this case, it would be impossible to raise new capital since the portfolio return does not compensate its risk. Therefore, the originator cannot originate any additional mortgages without securitization. In addition, the securitization needs to improve the return to shareholders from $(1 - f^{\Sigma})M$.

The potential benefit of securitization is a reduction in $\overline{c}^{M\omega}$. The cost of funds via securitization, $\overline{c}^{M\Sigma\omega}$, is the weighted cost of the sen and subtranches (denoted by $c^{BS\omega}$ and $c^{B\mathrm{Sub}\omega}$, resp.) plus any additional cost of the structure $c^{\Sigma} = 0.002$. Without considering differences in duration, the cost of sen notes is $c^{BS\omega} = 9.8\%$ and that of sub notes is

Table 6: Costs and benefits from mortgage securitization.

$\overline{c}^{M\omega}$	0.102
$c^{B\mathrm{Sub}}$	0.1061
$\overline{c}^{M\Sigma\omega}$	0.0988
$\overline{c}^{M\Sigma\omega A}$	0.1008
Reference mortgage portfolio value at $\overline{c}^{M\omega}$	1
Reference mortgage portfolio value at $\overline{c}^{M\Sigma\omega A}$	1.0084
K^{es}	0.0084

Table 7: Effect of securitization on return on capital.

Presecuritization of 6500	Balances	Returns and costs (%)	Returns and costs
Mortgages	10000	0.102	1020
Deposits	9600	−0.101	−969.6
Equity capital	200		30
Subordinate debt	200	−0.101	−20.2
Return on capital			0.15
Mortgages	3500	0.102001292	357.0
Deposits	3360	−0.101	−339.36
Equity capital	70		
Subordinate debt	70	−0.101	−7.07
Return on capital			0.20263
Mortgages	3500	0.102001292	357.0
Deposits	3360	−0.101	−339.36
Equity capital	190		29.9991
Subordinate debt	190	−0.101	−19.19
Return on capital			0.15789

$c^{B\mathrm{Sub}\omega} = 10.61\%$. The weighted average is $\overline{c}^{M\Sigma\omega}$ before monitoring and transaction costs, so that

$$\overline{c}^{M\Sigma\omega} = 0.09881 = 0.9 \times 0.098 + 0.1 \times 0.1061. \tag{4.32}$$

The overall cost of securitization, $\overline{c}^{M\Sigma\omega A}$, is the sum of $\overline{c}^{M\Sigma\omega}$ and the annual $c^{\Sigma} = 0.002$ averaged over the life of the deal. The overall costs, $\overline{c}^{M\Sigma\omega A}$, become the aggregate weighted cost of funds via securitization so that

$$\overline{c}^{M\Sigma\omega A} = 0.10081 = \overline{c}^{M\Sigma\omega} + 0.002 = 0.09881 + 0.002. \tag{4.33}$$

From the above, we draw the following preliminary conclusions. Firstly, we note that $\overline{c}^{M\Sigma\omega A} = 0.10081 < \overline{c}^{M\omega} = 0.104$. This is sufficient to make mortgage securitization an attractive option. Also, $\overline{c}^{M\Sigma\omega A} = 0.10081 < r^M = 0.102$. Therefore, selling the reference mortgage portfolio to SPV generates a capital gain which improves the originator's profitability. However, the change in r^E remains to be quantified.

4.3.2. Return on Equity

The value of the reference mortgage portfolio, $f^{\Sigma}M$, is the discounted value of cash flows calculated at a rate equal to the required return to originators, r^{B}. The average required return to originators who buy SPV's securities is $\overline{r}^{B} = 0.10081 = \overline{c}^{M\Sigma\omega A}$. For lenders and shareholders, the average return on the unsecuritized mortgage portfolio is $r^{ls} = 0.102$. Nevertheless, existing shareholders would like to have $r^{E} = 0.25$ instead of $r^{E} = 0.15$ resulting from insufficient $r^{A} = 0.102$. In order to obtain $r^{E} = 0.25$, the originator's ROA should be higher and reach $r^{A} = 0.104$. The present value of $f^{\Sigma}M$ for the originators is the discounted value of future flows at $\delta = 0.1008$. The value of this mortgage portfolio for those who fund it results from discounting the same cash flows at $\delta = 0.102$, either with the current effective $r^{E} = 0.15$ or $\delta = 0.104$, with a required $r^{E} = 0.25$. In both cases, $\overline{c}^{M\Sigma\omega} > \overline{c}^{M\omega}$. Therefore, the price of the originator's reference mortgage portfolio, at the $\delta = 0.1008$ discount rate required by originators, will be higher than the price calculated with either $\delta = 0.102$ or $\delta = 0.104$. The difference is a capital gain for the originator's existing shareholders. Since the details of projected cash flows generated by $f^{\Sigma}M$ are unknown, an accurate calculation of its present value is not feasible. In practice, a securitization model generates the entire cash flows, with all interest received from the reference mortgage portfolio, r^{M}, voluntary and involuntary prepayments, as well as recoveries.

For this example, we simplify the entire process by circumventing model intricacy for capital structure. The *duration formula* offers an easier way to get a valuation for the originator's reference mortgage portfolio. We know that the discounted value of future flows generated by the reference mortgage portfolio at $r^{A} = 0.102$ is exactly 1000 because its return is 0.102. With another discount rate, the present value differs from this face value. An approximation of this new value can be obtained from the duration formula via

$$\frac{p^{1}f^{\Sigma}M - 100}{100} = \frac{-\text{Duration}}{(1+i)\left(\delta - r^{M}\right)}, \tag{4.34}$$

where the present value of the reference mortgage portfolio is denoted by $p^{1}f^{\Sigma}M$. In this case, the rate of return from $f^{\Sigma}M$ is r^{M}, and the discount rate is δ, while the ratio $(p^{1}f^{\Sigma}M - 100)/100$ provides this value as a percentage of the face value, M. The duration formula provides $p^{1}f^{\Sigma}M$ given all three other parameters so that

$$p^{1}f^{\Sigma}M(\%\text{ of Face Value}) = 100\% + \text{Duration}\left(r^{M}\% - \delta\%\right). \tag{4.35}$$

Since $r^{M} = 0.102$, the value of $f^{\Sigma}M$ at the discount rate $\delta = 10.08\%$ is

$$p^{1}f^{\Sigma}M = 1 + 7 \times (0.102 - 0.1008) = 1.0084. \tag{4.36}$$

This means that the sale of mortgages to the SPV generates $K^{es} = 0.0084$ over an amount of 6500 or 54.6 in value. The sale of the reference mortgage portfolio will generate a capital gain only when $\overline{c}^{M\Sigma\omega A} < \mu^{M} = 0.102$, so that

$$\overline{c}^{M\Sigma\omega A} < \overline{c}^{M\omega}. \tag{4.37}$$

In this case, the capital gain from the sale of $f^{\Sigma}M$ will effectively increase revenues, thereby increasing the average r^A on the balance sheet. This is a sufficient condition to improve r^E under present assumptions. The reason is that the effective ROE remains a linear function of the effective r^M inclusive of capital gains from the sale of $f^{\Sigma}M$ to SPV, as long as the weights used to calculate it from r^A as a percentage of the original mortgage portfolio remain approximately constant. This relation remains

$$r^A = 0.96 \times 0.101 + 0.08 \times 0.5 \times \left(r^E \times 0.5 + 0.102 \times 0.5\right). \tag{4.38}$$

This is true as long as $E \propto f^{\Sigma}M$, which is the case in this example. However, in general, $K^e \not\propto f^{\Sigma}M$, and the linear relationship collapses. One has to take uncertainty into account if one is required to determine the effective r^E. Note that current $\overline{c}^{M\omega} = 0.102$, by definition, since it equates r^M with the weighted average cost of capital: effective percentage of r^A of effective percentage of $\overline{c}^{M\omega}$. The implied return to shareholders is $r^{Is} = 0.15$. Whenever, $\overline{c}^{M\Sigma\omega A} < r^M$, it is by definition lower than the effective $\overline{c}^{M\omega}$. If the shareholders obtain $r^E = 0.25$, instead of the effective $r^E = 0.15$ only, then $\overline{c}^{M\omega} = 0.104$. This securitization would be profitable as long as $\overline{c}^{M\Sigma\omega A} < 0.104$. Since, in this case, $\overline{c}^{M\Sigma\omega A} = 0.1008$, the deal meets both conditions. However, the first one only is sufficient to generate capital gain. Using the current effective $r^E = 0.15$, we find that the originator's capital gain from selling mortgages is 0.0084 as shown in Table 6.

It is possible to convert K^{es} from securitization into an additional annualized margin obtained over the life of the deal. A simple proxy for this annual margin is equal to the instantaneous capital gain averaged over the life of the deal (ignoring the time value of money). The gain is $K^{es} = 54.6 = 0.0084 \times 6500$. This implies that subsequent to securitization, the reference mortgage portfolio provides $r^M = 0.102$ plus an annual return of $K^{es} = 0.0084$ applicable only to $f^{\Sigma}M = 6500$. Once the original mortgage portfolio has been securitized, the size of the balance sheet drops to 3500 that still provides $r^M = 0.102$. There is an additional return due to the capital gain. Since this annualized capital gain is $K^{es} = 0.0084$ of 6500, it is $(0.0084/6500) \times 3500$ in percentage of $(1-f^{\Sigma})M$ or 0.000001292 applicable to 3500. Accordingly $r^M = 0.102$ increases to $r^M = 0.102001292$ after mortgage securitization. This increased r^M also implies a higher r^E (see Section 4.3.3).

4.3.3. Enhancing Return on Equity via Securitization

Under a forfeit valuation of capital as a function of $f^{\Sigma}M$, it is relatively easy to determine whether the securitization enhances r^E, by how much and what the limitations are. Under full economic analysis, the capital results from a direct calculation of the reference and unsecuritized mortgage portfolios. The enhancement issue consists of finding out whether the securitization enhances the risk-return profile of the original mortgage portfolio and, more practically, whether postsecuritization r^E is higher or lower than presecuritization r^E. We address both these issues in the sequel.

Table 7 shows the income statement under the originator's reference and unsecuritized mortgage portfolios. The deposits, subordinate debt, and equity represent the same percentages of the original mortgage portfolio, namely, 96%, 2%, and 2%, respectively. Their costs are identical to the above.

Table 8: Effect of regret on risk allocation and liquidity.

Cases	Behavior with respect to	Risk-averse SIB	Regret-averse SIB
Case 1: $q(P) = \dfrac{r^{\mathrm{T}}}{\xi P}$	Probability of investing in subprime SMPs	$= 0$ $\therefore\ \pi_0^* = 0$	> 0 $\therefore\ \pi_\rho^* > 0$
Case 2: $q(P) \gg \dfrac{r^{\mathrm{T}}}{\xi P}$	Probability of investing in treasuries	$= 0$ $\therefore\ \pi_0^* = 1$	> 0 $\therefore\ \pi_\rho^* < 1$
Case 1: $q(P) = \dfrac{r^{\mathrm{T}}}{\xi P}$	Liquidity effect	High	Low
Case 2: $q(P) > \dfrac{r^{\mathrm{T}}}{\xi P}$	Liquidity effect	Low	Low if $\rho \gg 0$
$q(P) \gg \dfrac{r^{\mathrm{T}}}{\xi P}$		Lower	Higher if $\rho \gg 0$

Before the original mortgage portfolio is securitized, $r^M = 0.102$, while thereafter $r^M = 0.102001292$. This gain influences the ROE directly with an increase from $r^E = 0.15$ to $r^E = 0.20263$. In general, an increase in r^M causing an increase in r^E is not guaranteed since K^{es} is the marginal risk contribution of $f^{\Sigma}M$. Therefore, an increase in r^M due to K^{es} from the sale of the reference mortgage portfolio to SPV might not increase the r^E if K^{es} is lower. For instance, if K decreases to 190 and subordinate debt does also, the remaining deposits being the complement to 9000 or 9620, then the same calculations would show that the new ROE becomes $r^E = 0.15789 = 30/190$. It is necessary to determine K^e before and after the original mortgage portfolio is securitized in order to determine the size of K^{es} and to perform return calculations on new capital subsequent to securitization. Once K^e is determined and converted into a percentage of the original mortgage portfolio, we have the same type of formula as in the above.

5. Risk, Insurance, Regret, and the SMC

In this section, we provide an analysis of risk, regret, and monoline insurance as well as an investor optimization problem and their relationship with the SMC. As far as the former is concerned, the main risks that are discussed in the sequel are credit, maturity mismatch, basis, counterparty, liquidity, synthetic, prepayment, interest rate, price, tranching, and systemic risks.

5.1. Risk, Regret, and the SMC

In this subsection, we discuss the relationship between risk, regret and monoline insurance, and the SMC.

5.1.1. Risk, Regret, Structured Mortgage Products, and the SMC

Our interest in Section 2.1.2 is in the risk allocation spread given by (2.4) and (2.5). Here, we have that the investor's rate of return on subprime SMPs, r^P, is a function of the subprime mortgage rate that is defined as the sum of the index rate and risk premium. The premium is an indication of perceived subprime risk associated with SMPs. When the premium is low (high), investors will receive a relatively low (high) return on their SMP investment. Before the SMC, the average difference between prime and subprime mortgage interest rates (the *subprime markup*) declined quite dramatically. In other words, the risk premium required to originate subprime mortgages declined. This continued to occur during the SMC even though the level of macroeconomic activity and the quality of mortgages, both, declined. A maturity mismatch problem arises from the fact that the investor takes deposits which are very liquid and invests it in illiquid subprime SMPs.

5.1.2. Investor Optimization Problem with Risk and Regret and the SMC

In this subsection, we briefly discuss the solutions emanating from the optimization problems solved in Section 2.2 and their relationships with the SMC. For this optimization problem, we recall that the objective function, set of admissible controls, and value function are given by (2.8), (2.9), and (2.10), respectively.

The primary message conveyed by Theorem 2.1 is that regret aversion—as reflected by the convexity of g in (2.7)—results in the suboptimality of extreme decisions. This result shows that for (2.4), the regret-averse investor invests more in subprime SMPs, whereas the risk-averse investor would allocate its available funds to treasuries. If (2.5) holds, the risk-averse investor makes a decision of investing all of its funds in subprime SMPs. However, the regret-averse investor would choose to invest less in subprime SMPs and the rest in treasuries. In particular, when (2.5) holds, the investment strategies of both risk- and regret-averse investors are likely to produce lower and higher liquidity in the secondary mortgage market, respectively. (In our case, *liquidity* refers to the degree to which SMPs can be bought or sold in the secondary market without affecting their prices. Liquidity is characterized by a high level of trading in SMPs in this market. The complexity of SMPs conceal risk and reduce liquidity. In circumstances where mortgage default rates increase, SMPs that have blended varying types of credit risk in a complex transactions network become "contaminated." Contamination spreads across the banking sector, the wholesale markets, the retail markets, insurance companies, the asset management industry, and into the household. The situation further deteriorates when holders of SMPs have trouble finding other investors to buy these as the secondary mortgage market runs short of liquidity. Those holding SMPs, and who took out financing to do so, may have margin calls that force them to trade, at a discount, what illiquid underlying investments are. Liquidity is further restricted because financially distressed investors will hold on to cash as an insurance against further chargeoffs of irrecoverable mortgages. In the sequel, we comment on the possible liquidity problems that arise from the allocation of funds by the investor under risk and regret.) In the case where (2.4) is true, the allocating strategies of risk- and regret-averse investors are likely to result in higher and lower liquidity in the market, respectively. In the first case, if every investor is risk averse, there will be a slowing of investment in mortgage markets. Similarly, in the second instance, that is, (2.4), if every bank is regret averse, we expect a boom in the economy. Proposition 2.3 shows that, when the investor is more regret averse, under (2.4) it will invest more in subprime SMPs, whereas for (2.5) it chooses to invest less proportion of its funds in subprime SMPs. Inline with this discussion, we recall that, before the SMC, institutions were more willing to lend, which led to more investment in subprime SMPs. This is related to the previous analysis on investment strategies for regret- and risk-averse investors under (2.4) and (2.5). During the SMC, the same investors switched from investing in subprime SMPs to

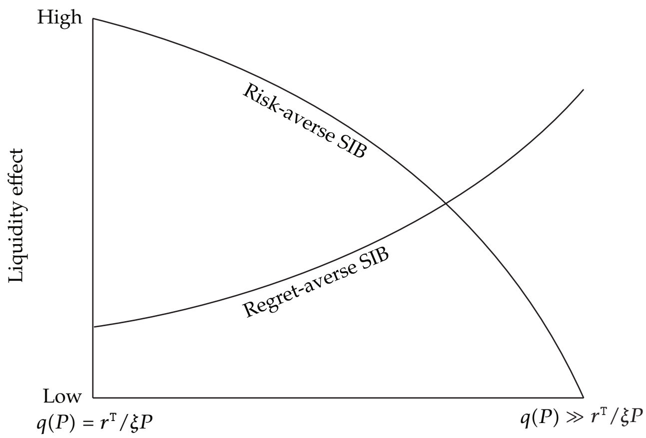

Figure 7: The liquidity effect.

treasuries. This can also be linked with (2.4) and (2.5) for risk- and regret-averse investors, respectively. The investment away from subprime SMPs to treasuries was a root cause of the lack of liquidity in secondary mortgage markets. Figure 7 illustrates the behavior of the investor and its association with liquidity.

Corollary 2.2 claims that, for some intermediate level of $\xi q(P)P - r^{\mathrm{T}}$, a regret-averse investor can choose a risk allocation as if regret was not considered. As the level of regret aversion rises, that is, the value of ρ increases, the amount of available funds invested in subprime SMPs increases. With a relatively large $\xi q(P)P - r^{\mathrm{T}}$, the risk-averse investor allocates all of its available funds to the subprime SMPs, while the regret-averse investor invests some money in treasuries, $\mathtt{T}$. As the level of regret aversion increases, with a high $\xi q(P)P - r^{\mathrm{T}}$, the amount of available funds invested in the subprime SMPs decreases. Hence, the certainty equivalent is the point $q(P) = \tilde{r}^{\mathrm{T}}/\xi P$, where a regret-averse investor chooses an optimal risk allocation as if regret was not considered. In Proposition 2.3, investors that weigh regret aversion heavily, that is, have large values for ρ, are more likely to hold subprime SMPs in its portfolio under the assumption that $\xi q(P)P - r^{\mathrm{T}}$ is low.

5.2. Monoline Insurance and the SMC

In this subsection, we discuss monoline insurance with regret and the main monoline insurance result in relation to the SMC.

5.2.1. Monoline Insurance with Regret and the SMC

In Section 3.1, we discuss monoline insurance with regret. Prior to 2007, not a single monoline insurer had ever filed for bankruptcy or been downgraded (see, e.g., [2]). On 7 November 2007, ACA, the only single-A-rated insurer, reported a $1 billion loss that wiped out equity and resulted in negative nett worth [27]. On 19 November 2007, ACA noted that if downgraded below A, collateral would have to be posted to comply with standard insurance agreements and that, based on fair values, they would not have the ability to post such collateral. On December 13, ACA's stock was delisted from the NYSE due to low market price and negative net worth, but ACA retained its A rating (see, e.g., [28]). Finally, on 19 December 2007, it was downgraded to CCC by S&P (see [29]).

On 18 January 2008, Ambac Financial Group Inc's rating was reduced from AAA to AA by Fitch Ratings (see, e.g., [30]). Because the rating of the monoline insurer is endowed upon the RMP, the downgrade of a major monoline triggered a simultaneous downgrade of bonds from over 100 000 municipalities and institutions totalling more than $500 billion. Rating agencies placed the other monoline insurers under review. CDS markets quoted rates for monoline default protection more typical for less than investment grade credits (see, e.g., [31]). Structured credit issuance ceased, and many municipal bond issuers spurned bond insurance, as the market was no longer willing to pay the traditional premium for monoline-backed paper (see, e.g., [32]). New players such as Warren Buffett's Berkshire Hathaway Assurance entered the market (see [33]). The illiquidity of the over-the-counter market in default insurance is illustrated by Berkshire taking four years (2003–2006) to unwind 26 000 undesirable swap positions in calm market conditions, losing $400 million in the process (see, e.g., [2]). By January 2008, many municipal and institutional bonds were trading at prices as if they were uninsured, effectively discounting monoline insurance completely. The tardy reaction of ratings agencies in detecting this resulted in the slow downgrading of subprime mortgage debt a year earlier. Besides this, rating agencies have been criticized for giving monoline insurers inflated ratings that enables them to charge a substantial fee to endow SMP bonds with these ratings, even when the bonds were issued by superior credits. On Thursday, 19 June 2008, Moody's also downgraded Ambac and MBIA from Aaa to Aa3 and A2, respectively (see, e.g., [34]). The stock prices for the publicly traded monolines, like AMBAC and MBIA, fell dramatically. AMBAC had climbed from the teens in the early 1990s to a price of 96 in 2007 but deteriorated to about 1 dollar by mid 2008. MBIA had a similar fate: climbing to the 60s by 2007, but by 2009 trading at about 6 dollars (see, e.g., [2]).

In 2009, the New York State Insurance Department introduced several new regulations regarding CDSs, CDOs, monolines, and other entities involved in the SMC. These regulations were described in the document entitled "Circular Letter No. 19 (2008)" (see, e.g., [35]). In 2010, the Wisconsin insurance commissioner took over Ambac's CDS contracts, with the plan to pay about 25 cents on the dollar to the "counterparties" that are owed (see, e.g., [2]). In 2009, Berkshire Hathaway suspended operations involving municipal bond insurance. On 8 November 2010, Ambac filed for Chapter 11 bankruptcy (see, e.g., [2]).

5.2.2. The Main Monoline Insurance Result and the SMC

Under the indifference equations (3.3) and (3.6), Theorem 3.3 presented in Section 3.2 demonstrates that, if the proportion of available funds invested in the subprime SMPs is low, the regret-averse investor would be willing to forfeit less for the monoline insurer's rate of return guarantee (compare with Section 3.1 for rate of return) than would a risk-averse investor. In this case, the benefits from monoline insurance in mitigating regret are small, and the additional regret cost through the price weighs more as in (3.8).

On the other hand, for (3.9), when investment in the subprime SMPs is large and the forfeit is small (see (3.1) for more information), the benefits of regret mitigation would be large and would outweigh its cost. Under these conditions, both risk- and regret-averse investors would forfeit the same. Here we note the importance of Lemma 3.2 in setting the hypothesis of Theorem 3.3.

5.3. Examples Involving Structured Mortgage Products and the SMC

In this subsection, we discuss the relationship between the SMC and the examples involving subprime residential mortgage products.

5.3.1. Numerical Example Involving Risk, Insurance, and Regret and the SMC

The numerical example in Section 4.1 determines values for the functional form for the probability of success, expected returns, as well as investment decisions between SMPs and treasuries that are partly based on their allocation spread. Furthermore, the corresponding payout from monoline insurance and optimal level of funds to be invested in subprime SMPs is computed.

Section 4.1 reinforces that the monoline insurer assumes the credit risk that SPV does not wish to bear in exchange for periodic premiums. In this regard, SPV will be exposed to counterparty risk rather than credit risk. Here, counterparty risk refers to the risk that the monoline insurer will not be able to make a payment to the investor if SPVs default. This could happen because monoline insurers are over-the-counter and unregulated, and the contracts often get traded so much that it is hard to know who stands at each end of a transaction. There is the possibility that the monoline insurer (and, indeed, the SPV) may not have the financial strength to abide by the insurance contract's provisions, making it difficult to value the contracts. The leverage involved in many monoline insurance transactions and the possibility that a widespread downturn in the market could cause massive SPV defaults and challenge the ability of monoline insurers to pay their obligations, which adds to the uncertainty. During the SMC, as the net worth of banks and other financial institutions deteriorated because of losses related to subprime mortgages and SMPs, the likelihood increased that monoline insurers would have to pay their counterparties.

5.3.2. Illustrative Example of Structured Mortgage Product Complexity and the SMC

The example in Section 4.2 resonates with the IDIOM hypothesis postulated in [1] that the SMC was largely caused by the intricacy and design of subprime agents, mortgage origination, and securitization that led to information problems (loss, asymmetry, and contagion), valuation opaqueness, and ineffective risk mitigation. Some risks underlying this complexity is prepayment, interest rate, and price risk. Prepayments of mortgage principal include both voluntary and involuntary (default) prepayments. The dynamics of involuntary prepayments over time has been described under assumptions about the level and timing of mortgage losses. As a consequence, assumptions about the prepayment curve really relate to the severity of loss from defaulted mortgages (in order to identify the number of involuntary prepayments) and the number of voluntary prepayments. Interest rate risk can be associated with mortgage refinancing problems related to the change in interest rates from, for instance, the teaser to the stepup rate. In the context of subprime SMPs, it is a source of considerable uncertainty in the analysis of cash flows which can lead to the proliferation of price risk.

From (4.25) and (4.26), it is clear that Π_1^c is dependent on the structure of the securitization, N_1, and on the losses, S_1^s, incurred by the sen tranche of an RMBS (see formulas (4.19) and (4.24) for exact formulations). In addition, we note that S_1^s is dependent on house price, H, appreciation. M does not appear in Π_1^c, because, if the mortgage is refinanced at time 1, it is fully paid out, and there are no losses (compare with (4.24)). Therefore, M should be under the expectation operator. Also, the relationship between H and Π_1^c only appears through the recovery value of the house if default occurs. In reality, refinancing results in M being paid into the securitization. This cash allocation should follow the priority rules and triggers which govern the amortization. The example in Section 4.2 does not make provision for this situation.

In essence, the valuation of Π_1^c requires integrating the above expression over a distribution of house prices (compare with formula (4.27) for the CDO payoff). Two practical problems arise from this situation. First, the dependence on house prices creates a practical valuation problem—even if the distribution of house prices is known. For instance, as in the computational example in Section 4.2, the subprime securitization has four portfolios, each consisting of many mortgages. The CDO has purchased 100 tranches from different securitizations, including, say, twenty sen subprime tranches from different deals. In principle, the issue is how to evaluate the sen CDO tranche (even ignoring all the overcollateralization (OC) tests and other complications of the CDO structure). Besides the fact that this is very difficult to accomplish, interlinking the three structures together in a meaningful way is virtually impossible. In principle, an investor who actually purchased a particular CDO tranche or subprime RMBS tranche would receive trustee reports and would, therefore, have some knowledge about the reference mortgage portfolios. However, since the computational complexity is very high, it remains difficult for an investor in subprime RMBSs to look through the reference portfolios and determine the value of such tranches. The second problem involves accounting for all the structure. Despite the fact that there are vendor-provided packages that model the structure of structured products, the valuation is based on (point estimate) assumptions that are input by the user, rather than simulation of the performance of the reference mortgage portfolios.

From the above, we can conclude that mortgage securitization via tranching makes subprime mortgage securitization deals very complex and risky. Besides the problems posed by estimation of the reference mortgage portfolio's loss distribution, tranching requires in-depth, deal-specific documentation to ensure that the desired characteristics, such as the seniority ordering the various tranches, will be delivered in all situations. In addition, complexity may be amplified by the involvement of regret-averse asset managers and other parties, whose own incentives to act in the interest of some investor classes at the expense of others may need to be curtailed. With increased complexity, less-sophisticated investors have more difficulty understanding SMP tranching and thus a diminished capacity to make informed investment decisions about related structured financial products. For instance, tranches from the same deal may have different risk, reward, and/or maturity characteristics. Modeling the performance of tranched transactions based on historical performance may have led to the overrating (by rating agencies) and underestimation of risks (by investors) of asset-backed securities with high-yield debt as their underlying assets. These factors have contributed towards the SMC.

5.3.3. Numerical Example Involving Structured Mortgage Product Returns and the SMC

The part of our example of mortgage securitization in Section 4.3.1 does not suggest that it is generally true that all outcomes will be favorable, because it is possible that $\overline{c}^{M\Sigma\omega} < \overline{c}^{M\omega}$, but is still higher than the reference mortgage portfolio return, r^M, thereby generating a loss when selling mortgages to SPVs. Even if $\overline{c}^{M\Sigma\omega} > \overline{c}^{M\omega}$, there would be room to improve the originator's portfolio r^E because of K^{es}. As a consequence, the discussion presented in Section 4.3.1 is not representative of all possible situations.

In Section 4.3.2, we note that the influence on r^E is from both lower-level E and reduced $\overline{c}^{M\Sigma\omega A}$. The gain value is either present value or an improvement of annual margins averaged over the life of the deal. In the example, K^{es} is a present forfeit percentage of 4% used as an input. When considering the SMC, this same percentage of mortgages could result from modeling K^e and could be an output of a mortgage portfolio model. In both cases, an analysis of the securitization economics should strive to determine whether securitization improves the risk-return profile of the original mortgage portfolio. Enhancing the risk-return profile means optimizing the efficient frontier or increasing r^E for the reference mortgage portfolio. We may ascertain whether this is true by calculating r^E and $r^{E\Sigma}$ as well as comparing them.

From Section 4.3.3, if securitization improves r^E, the originator might be inclined to increase $f^\Sigma M$. Potentially, the originator could benefit even more from the good relationship between $f^\Sigma M$ and r^E—known as the *leverage effect of securitization*. Leverage is positive as long as $\overline{c}^{M\Sigma\omega} = 0.1008$ remains fixed with a higher $f^\Sigma M$ leading to a higher final r^E subsequent to securitization. For instance, using the example in Section 4.3.3, securitizing 2000 instead of 1000, and keeping the same proportions of mortgages to D and K, would automatically increase r^E. This increase does not result from an additional capital gain in $f^\Sigma M$, since this gain remains 0.00833 of mortgages. Instead it results from the fact that the additional annualized rate of return, r^a, is proportional to the ratio of mortgages before and after securitization. In the example, with $f^\Sigma M = 1000$, r^a as a percentage of $f^\Sigma M$ is

$$r^a = 0.000926 = 0.00833 \times \frac{1000}{9000} \tag{5.1}$$

of mortgages. Should the originator sell 2000, the same percentage of $f^\Sigma M$ would increase to $0.001851 = 0.00833 \times 2000/9000$, the earnings before tax (EBT) would become $r^e = 0.33346$, and the return on capital (now 160) would be $r^K = 0.0020841 = 0.33346/160$. Another simulation will demonstrate that $f^\Sigma M = 5000$ would provide an $r^e = 0.23965$ and $r^K = 0.23965$. In fact, $f^\Sigma M = 5553$ would allow hitting the 25% target return on $(1 - f^\Sigma)M$. This is the leverage effect of securitization, which is more than proportional to $f^\Sigma M$. We note that there are limits to this leverage effect. Firstly, an originator securitizing all mortgages (i.e., $f^\Sigma = 1$ as in true-sales securitization) changes its core operations by becoming a pure OR reselling new business to SPVs. As in the OTH model, origination and lending, collecting deposits, and providing financial services to customers are the core business of commercial banking. Keeping mortgages on the balance sheet is part of the core business. This is a first reason for OR not going to the extreme by securitizing its entire balance sheet. Secondly, ORs need to replenish the portfolio of securitized receivables. In order to do so, they need to keep assets on the balance sheet. This happens, for instance, when securitizing credit cards that amortize much quicker than mortgages. In such cases, the pool of credit card receivables rolls over with time and fluctuates depending on the customers' behavior. The originator needs a pool of such receivables to replenish its reference mortgage portfolio. Thirdly, increasing securitization would result in significant changes in operations and might change teh originators' perception by, for instance, modifying its cost of funds. This may or may not be true depending on how K^{es} is utilized.

6. Conclusions and Future Directions

This paper shows how regret can influence the risk allocation behavior of investors. Our outcomes show that an investor with regret-averse attributes will select risk allocations that are less extreme than those predicted by conventional expected utility. If very risky subprime SMPs were selected by a purely risk-averse investor, its regret-averse counterpart will choose less risky subprime SMPs. Conversely, when the purely risk-averse investor picks a riskless portfolio, the regret-averse investor would prefer a riskier portfolio. In essence, regret-averse investors tend to hedge their bets, taking into account the possibility that their preferences may turn out to be suboptimal after the maturity of the subprime SMP contract.

More specifically, from Proposition 2.3, we conclude that investors that weigh regret aversion more strongly than risk aversion (as measured by ρ) are more likely to hold subprime SMPs in their portfolio when $\xi q(P)P - r^{\mathrm{T}}$ is low. Conversely, investors will hold less subprime SMPs if $\xi q(P)P - r^{\mathrm{T}}$ is high. Corollary 2.2 claims that for $q(P) = \tilde{r}^{\mathrm{T}}/\xi P$, a regret-averse investor chooses an asset portfolio allocation as if regret was not considered. We also commented on how much a regret-averse investor is willing to pay for credit protection via monoline insurance, given a fixed asset portfolio allocation. Theorem 2.1 shows that regret allows investor decisions to move away from $\pi^{\rho*} = 0$ and $\pi^{\rho*} = 1$, if no credit protection is bought. This means that investors who take regret into account will hold more subprime SMPs when $\xi q(P)P - r^{\mathrm{T}}$ is low but less when $\xi q(P)P - r^{\mathrm{T}}$ is high. However, under credit protection, Theorem 3.3 shows that regret-averse investors value monoline insurance contracts less than purely risk-averse investors, when the investment in subprime SMPs is small. On the other hand, both risk- and regret-averse investors will forfeit the same when the proportion of available funds invested in subprime SMPs is high. Table 8 gives a summary of the main results obtained here.

The other main thrust of the paper is the discussion of credit (including counterparty and default), market (including interest rate, price, and liquidity), operational (including house appraisal, valuation, and compensation), tranching (including maturity mismatch and synthetic), and systemic (including maturity transformation) risks.

A shortcoming of this paper is that it does not provide a complete description of what would happen if the economy were to deteriorate or improve from one period to the next. This is especially interesting in the light of the fact that in the real economy one has yield curves that are not flat and so describe changes in the dynamics of the SMP market. More specifically, we would like to know how this added structure will affect the results obtained in this paper. This is a question for future consideration.

Appendices

In this section, we provide more details about regret theory, prove Theorems 2.1 and 3.3, Proposition 2.3, as well as Corollary 2.2.

A. More Details about Regret Theory

The utility function in (2.7) is derived from regret theory as pioneered in [21–25]. In the main, these contributions discuss decision makers that optimize the expected value of a utility function of the type

$$u(x, y) = v(x) + g[v(x) - v(y)], \tag{A.1}$$

where $u(\cdot), v(\cdot)$, and $g(\cdot)$ are defined in an analogous manner to the variables subsequent to (2.7). In essence, x is the chosen outcome—the outcome on which an individual has bet—and y is the foregone outcome. Also, $g(\cdot)$ represents the regret or rejoicing that the decision maker experiences as a result of receiving x as opposed to y. If $x > y$, then the decision maker made a beneficial choice and gains some additional utility by having foregone the alternative. If

$x < y$, then the decision maker experiences disutility from having foregone the possibility of performing better with a choice of the foregone alternative. We conclude that regret theory not only supposes that decision-makers experience regret, but also that the anticipation of experiencing regret impacts the decision-making processes (see. e.g., [36]).

By contrast to the regret theoretic formulation in (A.1), where two outcomes of a lottery are discussed, our work involves preferences among actions. It is straightforward to convert the regret utility function in (A.1) into the expected utility function

$$\mathbf{E}[U(x,y)] = \int [v(x_\theta) - k \cdot g(v(x_\theta) - v(y_\theta))]\, dF(\theta), \tag{A.2}$$

where $F(\theta)$ is a cumulative distribution function that reflects the subjective beliefs about realizations of states of the world θ. In this regard, x_θ (y_θ) is the outcome in state of the world θ that accrues when the chosen action x (y) is chosen. This expected regret utility emanates from [27]—comparing preferences for actions, assuming subjective probabilities—rather than the work of von Neumann-Morgenstern that proposes bets on lotteries given known probabilities.

It has been noted that (A.3) does not satisfy the Axiom of Independence—the payoff of the foregone action affects the value of the chosen one. As a consequence, the Axioms for Subjective Expected Utility in [27] cannot be represented by (A.3). In recent literature (see, e.g., [26]), this problem is solved by utilizing a regret-theoretical expected utility (RTEU) function of the form

$$\mathbf{E}^r[U(x,y)] = \int [v(w_\theta) - k \cdot g(v(w_\theta^{\max}) - v(w_\theta))]\, dF(\theta), \tag{A.3}$$

that is, consistent with the Axioms of Regret Theory of [25] and the Axiom of Irrelevance of Statewise Dominated Alternatives (ISDA) proposed by [24]. The latter requires the decision maker to render irrelevant any actions in the feasible set that are statewise dominated by other actions in this set. Of considerable consequence in Quiggin's ISDA is that if a decision maker's preferences are consistent with ISDA and the Sugden axioms, then the regret associated with a given action in a particular state of nature depends only on the actual outcome and the best possible outcome that the individual could have attained in that same state of nature. Hence, $\mathbf{E}^r[U(x,y)]$ from (A.3) embeds preferences that are consistent with these axioms. Since the Sugden axioms are a rejigging of those of [27], we have a normative basis for $\mathbf{E}^r[U(x,y)]$ that enables the use of the model to analyze monoline insurance choices. As a consequence, we are able to focus exclusively on regret and its associated disutility. (This is in contrast to erstwhile definitions of regret theory that also allow for *rejoicing* when the *better* outcome is chosen for the eventual state of the world. Indeed, because $g(0) = 0$ and because one can never do better than the best possible outcome, we have eliminated *rejoicing* from the regret/rejoice model altogether. We can then restrict $k \geq 0$ as measure of the influence of regret on the decision.).

B. Proof of Theorem 2.1

The statement of Theorem 2.1 is equivalent to

(1) if (2.4) holds, then $\pi^{\rho *} > 0$ for all $\rho > 0$, with $\pi_0^* = 0$;

(2) if (2.5) holds, then $\pi^{\rho *} < 1$ for all $\rho > 0$, with $\pi_0^* = 1$.

In this regard, we use standard maximization arguments to prove the above results. In particular, we must show that the first derivative of (2.10) with respect to π, at $\pi_\rho^* = 0$ and $\pi_\rho^* = 1$, does not vanish. In this regard, we have that

$$f(\pi) = f_0\left(1 + \pi r^P + (1-\pi) r^{\mathrm{T}}\right), \qquad f^{\max} = f_0\left(1 + \max\left(r^P, r^{\mathrm{T}}\right)\right) \tag{B.1}$$

denote the investor's final fund level and expost optimal fund level, respectively. The first- and second-order conditions for (2.10) are

$$\frac{d\mathbf{E}\left[U^\rho(f(\pi))\right]}{d\pi} = 0, \tag{B.2}$$

$$\frac{d^2\mathbf{E}\left[U^\rho(f(\pi))\right]}{d\pi^2} < 0, \tag{B.3}$$

respectively. But

$$\begin{aligned}
\frac{d\mathbf{E}\left[U^\rho(f(\pi))\right]}{d\pi} &= \mathbf{E}\left[f_0\left(r^P - r^{\mathrm{T}}\right)U'(f(\pi))\left(1 + \rho g'\left(U\left(f^{\max}\right) - U(f(\pi))\right)\right)\right], \\
\frac{d^2\mathbf{E}\left[U^\rho(f(\pi))\right]}{d\pi^2} &= \mathbf{E}\left[f_0^2\left(r^P - r^{\mathrm{T}}\right)^2 U''(f(\pi))\left(1 + \rho g'\left(U\left(f^{\max}\right) - U(f(\pi))\right)\right)\right] \\
&\quad - \mathbf{E}\left[f_0^2\left(r^P - r^{\mathrm{T}}\right)^{\mathbf{2}} \rho U'^2(f(\pi)) g''\left(U\left(f^{\max}\right) - U(f(\pi))\right)\right].
\end{aligned} \tag{B.4}$$

It then follows that (B.2) and (B.3) take the forms

$$\frac{d\mathbf{E}\left[U^\rho(f(\pi))\right]}{d\pi} = \mathbf{E}\left[f_0\left(r^P - r^{\mathrm{T}}\right)U'(f(\pi))\left(1 + \rho \cdot g'\left(U\left(f^{\max}\right) - U(f(\pi))\right)\right)\right] = 0, \tag{B.5}$$

$$\begin{aligned}
\frac{d^2\mathbf{E}\left[U^\rho(f(\pi))\right]}{d\pi^2} &= \mathbf{E}\left[f_0^2\left(r^P - r^{\mathrm{T}}\right)^2 U''(f(\pi))\left(1 + \rho \cdot g'\left(U\left(f^{\max}\right) - U(f(\pi))\right)\right)\right] \\
&\quad - \mathbf{E}\left[f_0^2\left(r^P - r^{\mathrm{T}}\right)^{\mathbf{2}} \rho U'^2(f(\pi)) g''\left(U\left(f^{\max}\right) - U(f(\pi))\right)\right] < 0,
\end{aligned} \tag{B.6}$$

respectively. This implies that $\mathbf{E}[U^{\rho}(f(\pi))]$ is strictly concave in π, so that any solution of the first-order condition (B.5) uniquely fixes the global maximum. Furthermore, in this case, a decomposition of (B.5) may be given by

$$\begin{aligned}
&\frac{d\mathbf{E}[U^{\rho}(f(\pi))]}{d\pi} \\
&\quad= \frac{d\mathbf{E}[U^{0}(f(\pi))]}{d\pi} + \int_{-1}^{r^{\mathrm{T}}} \rho f_0\left(r^P - r^{\mathrm{T}}\right)U'(f(\pi))g'(U(f(0)) - U(f(\pi)))dF\left(r^P\right) \\
&\qquad+ \int_{r^{\mathrm{T}}}^{\infty} \rho f_0\left(r^P - r^{\mathrm{T}}\right)U'(f(\pi))g'(U(f(1)) - U(f(\pi)))dF\left(r^P\right).
\end{aligned} \tag{B.7}$$

If we evaluate this first derivative at $\pi = 0$ and $\pi = 1$, then we obtain

$$\begin{aligned}
\left.\frac{d\mathbf{E}[U^{\rho}(f(\pi))]}{d\pi}\right|_{\pi=0} &= \left.\frac{d\mathbf{E}[U^{0}(f(\pi))]}{d\pi}\right|_{\pi=0} + \rho f_0 U'(f(0))g'(0)\int_{-1}^{r^{\mathrm{T}}}\left(r^P - r^{\mathrm{T}}\right)dF\left(r^P\right) \\
&\quad+ \rho f_0 U'(f(0))\int_{r^{\mathrm{T}}}^{\infty}\left(r^P - r^{\mathrm{T}}\right)g'(U(f(1)) - U(f(0)))dF\left(r^P\right) \\
&> \left.\frac{d\mathbf{E}[U^{0}(f(\pi))]}{d\pi}\right|_{\pi=0} + \rho f_0 U'(f(0))g'(0)\int_{-1}^{r^{\mathrm{T}}}\left(r^P - r^{\mathrm{T}}\right)dF\left(r^P\right) \\
&= f_0 U'(f(0))\left(\mathbf{E}\left[r^P\right] - r^{\mathrm{T}}\right)(1 + \rho g'(0)) \\
&= f_0 U'(f(0))\left(\xi q(P)P - r^{\mathrm{T}}\right)(1 + \rho g'(0)), \\
\left.\frac{d\mathbf{E}[U^{\rho}(f(\pi))]}{d\pi}\right|_{\pi=1} &= \left.\frac{d\mathbf{E}[U^{0}(f(\pi))]}{d\pi}\right|_{\pi=1} \\
&\quad+ \int_{-1}^{r^{\mathrm{T}}} \rho f_0\left(r^P - r^{\mathrm{T}}\right)U'(f(1))g'(U(f(0)) - U(f(1)))dF\left(r^P\right) \\
&\quad+ \int_{r^{\mathrm{T}}}^{\infty} \rho f_0\left(r^P - r^{\mathrm{T}}\right)U'(f(1))g'(0)dF\left(r^P\right) \\
&< \left.\frac{d\mathbf{E}[U^{0}(f(\pi))]}{d\pi}\right|_{\pi=1} + \rho f_0 g'(0)\int_{r^{\mathrm{T}}}^{\infty}\left(r^P - r^{\mathrm{T}}\right)U'(f(1))dF\left(r^P\right) \\
&= f_0\mathbf{E}\left[\left(r^P - r^{\mathrm{T}}\right)U'\left(f_0\left(1 + r^P\right)\right)\right](1 + \rho g'(0)) \\
&= \left\{\mathbf{E}\left[r^P U'\left(f_0\left(1 + r^P\right)\right)\right] - r^{\mathrm{T}}\mathbf{E}\left[U'\left(f_0\left(1 + r^P\right)\right)\right]\right\}f_0(1 + \rho g'(0)),
\end{aligned} \tag{B.8}$$

respectively. As a result of this, if (2.4) holds, then

$$\left.\frac{d\mathbf{E}\left[U^{\rho}(f(\pi))\right]}{d\pi}\right|_{\pi=0} > 0 \tag{B.9}$$

for all $\rho > 0$. On the other hand, if (2.5) holds, that is,

$$q(P) = \frac{r^{\mathrm{T}}\mathbf{E}\left[U'\left(f_0\left(1+r^P\right)\right)\right] + \operatorname{cov}\left[-r^P, U'\left(f_0\left(1+r^P\right)\right)\right]}{\xi I\mathbf{E}\left[U'\left(f_0\left(1+r^P\right)\right)\right]}, \tag{B.10}$$

and taking into account that

$$r^{\mathrm{T}} = \frac{\mathbf{E}\left[r^P U'\left(f_0\left(1+r^P\right)\right)\right]}{\mathbf{E}\left[U'\left(f_0\left(1+r^P\right)\right)\right]}, \tag{B.11}$$

then

$$\left.\frac{d\mathbf{E}\left[U^{\rho}(f(\pi))\right]}{d\pi}\right|_{\pi=1} < 0 \tag{B.12}$$

for all $\rho > 0$. This implies, in the former instance, that $\pi^{\rho *} > 0$ for all $\rho > 0$ and $\pi^{\rho *} < 1$ for all $\rho > 0$ in the second situation.

C. Proof of Corollary 2.2

In Corollary 2.2, we must show that there exists a treasuries rate, $\tilde{r}^{\mathrm{T}}$, such that

$$0 < \xi q(P)P - \tilde{r}^{\mathrm{T}} < \frac{\operatorname{cov}\left(-r^P, U'\left(f_0\left(1+r^P\right)\right)\right)}{\mathbf{E}\left(U'\left(f_0\left(1+r^P\right)\right)\right)}, \tag{C.1}$$

and, for all $\rho > 0$, we have that $\pi^{\rho *} = \pi_0^*$.

We have proved in Theorem 2.1, for any fixed $\rho > 0$, that

$$\begin{aligned} &\pi^{\rho *} > 0, \quad \pi_0^* = 0, \quad \text{if (2.4) holds,} \\ &\pi^{\rho *} < 1, \quad \pi_0^* = 1, \quad \text{if (2.5) holds.} \end{aligned} \tag{C.2}$$

Furthermore, the Intermediate Value Theorem suggests the existence of a treasuries rate, $\tilde{r}^{\mathrm{T}}$, with the property that

$$\xi q(P)P > \tilde{r}^{\mathrm{T}} > \frac{\mathbf{E}\left[r^P U'\left(f_0\left(1+r^P\right)\right)\right]}{\mathbf{E}\left[U'\left(f_0\left(1+r^P\right)\right)\right]} \tag{C.3}$$

and $\pi^{\rho*} = \pi_0^*$. The first-order derivative conditions

$$\left.\frac{d\mathbf{E}[U(f(\pi))]}{d\pi}\right|_{\pi=\pi_0^*} = \mathbf{E}\left[f_0\left(r^P - \tilde{r}^{\mathrm{T}}(\rho)\right)U'\left(f\left(\pi^{0*}\right)\right)\right] = 0 \tag{C.4}$$

and (B.5) at $\pi = \pi^{\rho*}$; that is,

$$\begin{aligned}&\left.\frac{d\mathbf{E}[U^{\rho}(f(\pi))]}{d\pi}\right|_{\pi=\pi^{\rho*}}\\&\quad= \mathbf{E}\left[f_0\left(r^P - \tilde{r}^{\mathrm{T}}(\rho)\right)U'(f(\pi_0^*))(1+\rho g'(U(f^{\max}) - U(f(\pi_0^*))))\right] = 0.\end{aligned} \tag{C.5}$$

It then follows that

$$\begin{aligned}&\mathbf{E}\left[f_0\left(r^P - \tilde{r}^{\mathrm{T}}(\rho)\right)U'(f(\pi_0^*))\right]\\&\quad= \mathbf{E}\left[f_0\left(r^P - \tilde{r}^{\mathrm{T}}(\rho)\right)U'(f(\pi_0^*))(1+\rho g'(U(f^{\max}) - U(f(\pi_0^*))))\right].\end{aligned} \tag{C.6}$$

Assuming a continuous-time environment, we write

$$\begin{aligned}&\int_{-1}^{\infty} f_0\left(r^P - \tilde{r}^{\mathrm{T}}(\rho)\right)U'(f(\pi_0^*))\,dF\left(r^P\right)\\&\quad= \int_{-1}^{\infty} f_0\left(r^P - \tilde{r}^{\mathrm{T}}(\rho)\right)U'(f(\pi_0^*))(1+\rho g'(U(f^{\max}) - U(f(\pi_0^*))))\,dF\left(r^P\right).\end{aligned} \tag{C.7}$$

The above expression holds if and only if

$$f_0\left(r^P - \tilde{r}^{\mathrm{T}}(\rho)\right)U'(f(\pi_0^*)) = f_0\left(r^P - \tilde{r}^{\mathrm{T}}(\rho)\right)U'(f(\pi_0^*))(1+\rho g'(U(f^{\max}) - U(f(\pi_0^*)))), \tag{C.8}$$

which simply means that

$$f_0\left(r^P - \tilde{r}^{\mathrm{T}}(\rho)\right)U'(f(\pi_0^*))\rho g'(U(f^{\max}) - U(f(\pi_0^*))) = 0. \tag{C.9}$$

Therefore, we have that

$$r^P - \tilde{r}^{\mathrm{T}}(\rho) = 0, \tag{C.10}$$

since $f_0 > 0$, $\rho > 0$, $U'(\cdot) > 0$, and $g'(\cdot) > 0$. Thus, we conclude that for all $\rho > 0$, we have $\tilde{r}^{\mathrm{T}}(\rho) = r^P$.

D. Proof of Proposition 2.3

In Proposition 2.3, we required to show that

$$\frac{\partial \pi^{\rho *}}{\partial \rho}\begin{cases} > 0, & \text{if (2.4) holds,} \\ < 0, & \text{if (2.5) holds.}\end{cases} \tag{D.1}$$

Taking the total differential of the first-order condition (B.5) with respect to π and ρ yields

$$d\left[\frac{d\mathbf{E}[U^{\rho}(f(\pi))]}{d\pi}\right]\Bigg|_{\pi=\pi^{\rho *}} = \frac{\partial^2 \mathbf{E}[U^{\rho}(f(\pi))]}{\partial \pi^2}\Bigg|_{\pi=\pi^{\rho *}} \cdot d\pi + \frac{\partial^2 \mathbf{E}[U^{\rho}(f(\pi))]}{\partial \pi \partial \rho}\Bigg|_{\pi=\pi^{\rho *}} \cdot d\rho = 0. \tag{D.2}$$

In this case, we therefore have that

$$\frac{\partial \pi^{\rho *}}{\partial \rho} = -\frac{\left(\partial^2 \mathbf{E}[U^{\rho}(f(\pi))]/\partial \pi \partial \rho\right)\big|_{\pi=\pi^{\rho *}}}{\left(\partial^2 \mathbf{E}[U^{\rho}(f(\pi))]/\partial \pi^2\right)\big|_{\pi=\pi^{\rho *}}}. \tag{D.3}$$

Since it is true that

$$\frac{\partial^2 \mathbf{E}[U^{\rho}(f(\pi))]}{\partial \pi^2}\Bigg|_{\pi=\pi^{\rho *}} < 0, \tag{D.4}$$

we may conclude that

$$\operatorname{sign}\left(\frac{\partial \pi^{\rho *}}{\partial \rho}\right) = \operatorname{sign}\left(\frac{\partial^2 \mathbf{E}[U^{\rho}(f(\pi))]}{\partial \pi \partial \rho}\Bigg|_{\pi=\pi^{\rho *}}\right). \tag{D.5}$$

We observe that the mixed partial derivative yields

$$\frac{\partial^2 \mathbf{E}[U^{\rho}(f(\pi))]}{\partial \pi \partial \rho}\Bigg|_{\pi=\pi^{\rho *}} = \mathbf{E}\left[f_0\left(r^P - r^{\mathrm{T}}\right)U'(f(\pi^{\rho *}))\rho g'\left(U(f^{\max}) - U(f(\pi^{\rho *}))\right)\right]. \tag{D.6}$$

Furthermore, from the first-order condition (B.5), it follows that

$$\frac{d\mathbf{E}[U^{\rho}(f(\pi))]}{d\pi}\Bigg|_{\pi=\pi^{\rho *}} = \frac{d\mathbf{E}[U^{0}(f(\pi))]}{d\pi}\Bigg|_{\pi=\pi^{\rho *}} + \rho \cdot \frac{\partial^2 \mathbf{E}[U^{\rho}(f(\pi))]}{\partial \pi \partial \rho}\Bigg|_{\pi=\pi^{\rho *}}. \tag{D.7}$$

Since we have that

$$\frac{d\mathbf{E}[U^{\rho}(f(\pi))]}{d\pi}\Bigg|_{\pi=\pi^{\rho *}} = 0, \tag{D.8}$$

we can deduce that

$$\operatorname{sign}\left(\frac{\partial \pi^{\rho *}}{\partial \rho}\right) = \operatorname{sign}\left(\left.\frac{\partial^2 \mathbf{E}[U^{\rho}(f(\pi))]}{\partial \pi \partial \rho}\right|_{\pi=\pi^{\rho *}}\right) = -\operatorname{sign}\left(\left.\frac{d\mathbf{E}[U^{0}(f(\pi))]}{d\pi}\right|_{\pi=\pi^{\rho *}}\right). \tag{D.9}$$

Our conclusion is that if (2.4) holds then $\pi^{\rho *} > 0$ for all $\rho > 0$ and $\pi_0^* = 0$ according to Theorem 2.1. This implies that

$$\left.\frac{d\mathbf{E}[U^{0}(f(\pi))]}{d\pi}\right|_{\pi=\pi^{\rho *}>0} < 0, \tag{D.10}$$

and, as a consequence, we have

$$\frac{\partial \pi^{\rho *}}{\partial \rho} > 0 \tag{D.11}$$

as suggested by (D.9).

If, on the other hand, (2.5) holds, then $\pi^{\rho *} < 1$ for all $\rho > 0$ and $\pi_0^* = 1$ according to Theorem 2.1. By the method used in the above, this implies that

$$\left.\frac{d\mathbf{E}[U^{0}(f(\pi))]}{d\pi}\right|_{\pi=\pi^{\rho *}<1} > 0, \tag{D.12}$$

and thus

$$\frac{\partial \pi^{\rho *}}{\partial \rho} < 0 \tag{D.13}$$

by (D.9).

E. Proof of Theorem 3.3

The investor's forfeit is implicitly defined through the conditional indifference equation (3.6). A regret-averse investor is willing to make a smaller forfeit for the monoline insurance guarantee than a risk-averse investor; that is, (3.8) holds for all r^{Pg}, if and only if

$$\begin{aligned} \mathbf{E}\left[U\left(\left(f_0 - c^{\rho}\left(r^{Pg}, \pi^f\right)\right) \aleph\left(R^{Pg}, \pi^f\right)\right)\right] &> \mathbf{E}\left[U\left(\left(f_0 - c^{0}\left(r^{Pg}, \pi^f\right)\right) \aleph\left(R^{Pg}, \pi^f\right)\right)\right] \\ &= \mathbf{E}\left[U\left(f_0 \aleph\left(r^{P}, \pi^f\right)\right)\right] \end{aligned} \tag{E.1}$$

for all r^{Pg}. Define the function $h : [0,1] \to \mathbf{R}$ as

$$h\left(\pi^f\right) = \mathbf{E}\left[U\left(\left(f_0 - c^{\rho}\left(r^{Pg}, \pi^f\right)\right) \aleph\left(R^{Pg}, \pi^f\right)\right)\right] - \mathbf{E}\left[U\left(f_0 \aleph\left(r^{P}, \pi^f\right)\right)\right]. \tag{E.2}$$

A first observation is that for $\pi^f = 0$, we have $h(0) = 0$. In order to prove that (3.8) holds for small π^f and all r^{Pg}, we thus have to show that $h'(0) > 0$. Finding the derivative of h with respect to π^f yields

$$\begin{aligned}h'\left(\pi^f\right) = &-\frac{\partial c^\rho\left(r^{Pg},\pi^f\right)}{\partial \pi^f}\mathbf{E}\left[U'\left(\left(f_0 - c^\rho\left(r^{Pg},\pi^f\right)\right)\aleph\left(R^{Pg},\pi^f\right)\right)\aleph\left(R^{Pg},\pi^f\right)\right]\\ &+\mathbf{E}\left[\left(f_0 - c^\rho\left(r^{Pg},\pi^f\right)\right)\left(R^{Pg}-r^{\mathrm{T}}\right)U'\left(\left(f_0 - c^\rho\left(r^{Pg},\pi^f\right)\right)\aleph\left(R^{Pg},\pi^f\right)\right)\right]\\ &-\mathbf{E}\left[f_0\left(r^P - r^{\mathrm{T}}\right)U'\left(f_0\aleph\left(r^P,\pi^f\right)\right)\right],\end{aligned} \tag{E.3}$$

and thus

$$h'(0) = U'\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\left[-\left.\frac{\partial c^\rho\left(r^{Pg},\pi^f\right)}{\partial \pi^f}\right|_{\pi^f=0}\left(1+r^{\mathrm{T}}\right) + f_0\mathbf{E}\left[R^{Pg}-r^P\right]\right]. \tag{E.4}$$

If we differentiate (3.6) with respect to π^f, we obtain

$$\begin{aligned}&\mathbf{E}\left[f_0\left(r^P - r^{\mathrm{T}}\right)U'\left(f_0\aleph\left(r^P,\pi^f\right)\right)\left(1+\rho g'\left(U\left(f^{\max}\right) - U\left(f_0\aleph\left(r^P,\pi^f\right)\right)\right)\right)\right]\\ &\quad= -\frac{\partial c^\rho\left(r^{Pg},\pi^f\right)}{\partial \pi^f}\mathbf{E}\left[\aleph\left(r^P,\pi^f\right)U'\left(\left(f_0 - c^\rho\left(r^{Pg},\pi^f\right)\right)\aleph\left(r^P,\pi^f\right)\right)\right.\\ &\qquad\qquad\left.\times\left(1+\rho g'\left(U\left(f^{\max}\right) - U\left(\left(f_0 - c^\rho\left(r^{Pg},\pi^f\right)\right)\aleph\left(R^{Pg},\pi^f\right)\right)\right)\right)\right]\\ &\qquad+\mathbf{E}\left[\left(f_0 - c^\rho\left(r^{Pg},\pi^f\right)\right)\left(R^{Pg}-r^{\mathrm{T}}\right)U'\left(\left(f_0 - c^\rho\left(r^{Pg},\pi^f\right)\right)\aleph\left(R^{Pg},\pi^f\right)\right)\right.\\ &\qquad\qquad\left.\times\left(1+\rho g'\left(U\left(f^{\max}\right) - U\left(\left(f_0 - c^\rho\left(r^{Pg},\pi^f\right)\right)\aleph\left(R^{Pg},\pi^f\right)\right)\right)\right)\right].\end{aligned} \tag{E.5}$$

If we set $\pi^f = 0$, it follows that

$$\begin{aligned}&f_0U'\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\mathbf{E}\left[\left(r^P - r^{\mathrm{T}}\right)\left(1+\rho g'\left(U\left(f^{\max}\right) - U\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\right)\right)\right]\\ &\quad= -\left.\frac{\partial c^\rho\left(r^{Pg},\pi^f\right)}{\partial \pi^f}\right|_{\pi^f=0}\left(1+r^{\mathrm{T}}\right)U'\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\mathbf{E}\left[\left(1+\rho g'\left(U\left(f^{\max}\right) - U\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\right)\right)\right]\\ &\qquad+ f_0U'\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\mathbf{E}\left[\left(R^{Pg}-r^{\mathrm{T}}\right)\left(1+\rho g'\left(U\left(f^{\max}\right) - U\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\right)\right)\right]\end{aligned} \tag{E.6}$$

which, in turn, implies that

$$\left.\frac{\partial c^{\rho}(r^{Pg},\pi^f)}{\partial \pi^f}\right|_{\pi^f=0} = \frac{f_0\mathbf{E}\left[(R^{Pg}-r^P)(1+\rho g'(U(f^{\max})-U(f_0(1+r^{\mathrm{T}}))))\right]}{(1+r^{\mathrm{T}})\mathbf{E}\left[1+\rho g'(U(f^{\max})-U(f_0(1+r^{\mathrm{T}})))\right]}. \tag{E.7}$$

If we substitute (E.7) into (E.4), we may conclude that

$$\begin{aligned} h'(0) &= f_0U'\left(f_0\left(1+r^{\mathrm{T}}\right)\right) \\ &\quad\times\left(-\frac{(1+r^{\mathrm{T}})\mathbf{E}\left[(R^{Pg}-r^P)(1+\rho g'(U(f^{\max})-U(f_0(1+r^{\mathrm{T}}))))\right]}{(1+r^{\mathrm{T}})\mathbf{E}\left[1+\rho g'(U(f^{\max})-U(f_0(1+r^{\mathrm{T}})))\right]}+\mathbf{E}\left[R^{Pg}-r^P\right]\right) \\ &= -\frac{f_0U'(f_0(1+r^{\mathrm{T}}))\rho}{\mathbf{E}\left[1+\rho g'(U(f^{\max})-U(f_0(1+r^{\mathrm{T}})))\right]} \\ &\quad\cdot\operatorname{cov}\left(R^{Pg}-r^P, g'\left(U(f^{\max})-U\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\right)\right). \end{aligned} \tag{E.8}$$

Also, we observe that

$$\begin{aligned} &\operatorname{cov}\left(R^{Pg}-r^P, g'\left(U(f^{\max})-U\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\right)\right) \\ &\quad=\operatorname{cov}\left(R^{Pg}-r^P, g'\left(U\left(f_0\left(1+\max\left(r^P,r^{\mathrm{T}}\right)\right)\right)-U\left(f_0\left(1+r^{\mathrm{T}}\right)\right)\right)\right)<0. \end{aligned} \tag{E.9}$$

In this case, we may conclude that $h'(0)>0$, which implies that $h(\pi^f)>0$ for small π^f since $h(0)=0$. From this, we can deduce that (3.8) holds for low levels of π^f and all r^{Pg}.

Next, we would like to show that (3.9) holds for high levels of π^f and small r^{Pg}. This inequality holds if and only if $h(\pi^f)=0$ for π^f and small r^{Pg} (see (E.2) for the definition of $h(\cdot)$). A first observation is that

$$h(1)=\mathbf{E}\left[U\left(\left(f_0-c^{\rho}\left(r^{Pg},1\right)\right)\left(1+R^{Pg}\right)\right)\right]-\mathbf{E}\left[U\left(f_0\left(1+r^P\right)\right)\right]. \tag{E.10}$$

A further observation is that, at $r^{Pg}=0$, we have that $h(1)|_{r^{Pg}=0}=0$. If we differentiate $h(1)$ with respect to r^{Pg}, we obtain

$$\frac{\partial h(1)}{\partial r^{Pg}}=-\left.\frac{\partial c^{\rho}(r^{Pg},\pi^f)}{\partial r^{Pg}}\right|_{\pi^f=1}\mathbf{E}\left[\left(1+R^{Pg}\right)U'\left(\left(f_0-c^{\rho}\left(r^{Pg},1\right)\right)\left(1+R^{Pg}\right)\right)\right]. \tag{E.11}$$

Determining the value at $r^{Pg}=0$ yields

$$\frac{\partial h(1)}{\partial r^{Pg}}=-\left.\frac{\partial c^{\rho}(r^{Pg},\pi^f)}{\partial r^{Pg}}\right|_{\pi^f=1,r^{Pg}=0}\mathbf{E}\left[\left(1+r^P\right)U'\left(f_0\left(1+r^P\right)\right)\right]. \tag{E.12}$$

Furthermore, if we differentiate (3.6) with respect to r^{Pg}, it follows that

$$-\frac{\partial c^{\rho}(r^{Pg},\pi^f)}{\partial r^{Pg}}\mathbf{E}\Big[\aleph\left(R^{Pg},\pi^f\right)U'\left(\left(f_0-c^{\rho}\left(r^{Pg},\pi^f\right)\right)\aleph\left(R^{Pg},\pi^f\right)\right)$$
$$\times\left(1+\rho g'\left(U(f^{\max})-U\left(\left(f_0-c^{\rho}\left(r^{Pg},\pi^f\right)\right)\aleph\left(R^{Pg},\pi^f\right)\right)\right)\right)\Big]=0. \tag{E.13}$$

For $\pi^f=1$, we obtain

$$-\frac{\partial c^{\rho}(r^{Pg},\pi^f)}{\partial r^{Pg}}\bigg|_{\pi^f=1}\mathbf{E}\Big[\left(1+R^{Pg}\right)u'\left(\left(f_0-c^{\rho}\left(r^{Pg},1\right)\right)\left(1+R^{Pg}\right)\right)$$
$$\times\left(1+\rho g'\left(u(f^{\max})-U\left(\left(f_0-c^{\rho}\left(r^{Pg},1\right)\right)\left(1+R^{Pg}\right)\right)\right)\right)\Big]=0. \tag{E.14}$$

If we evaluate at $r^{Pg}=0$, then it follows that

$$\frac{\partial c^{\rho}(r^{Pg},\pi^f)}{\partial r^{Pg}}\bigg|_{\pi^f=1,r^{Pg}=0}=0, \tag{E.15}$$

and as a consequence

$$\frac{\partial h(1)}{\partial r^{Pg}}\bigg|_{r^{Pg}=0}=0. \tag{E.16}$$

If we differentiate again we obtain

$$\frac{\partial^2 h(1)}{\partial (r^{Pg})^2}=-\frac{\partial^2 c^{\rho}(r^{Pg},\pi^f)}{\partial (r^{Pg})^2}\bigg|_{\pi^f=1}\mathbf{E}\Big[\left(1+R^{Pg}\right)U'\left(\left(f_0-c^{\rho}\left(r^{Pg},1\right)\right)\left(1+R^{Pg}\right)\right)\Big]$$
$$+\left(\frac{\partial c^{\rho}(r^{Pg},\pi^f)}{\partial r^{Pg}}\bigg|_{\pi^f=1}\right)^2\mathbf{E}\Big[\left(1+R^{Pg}\right)^2U''\left(\left(f_0-c^{\rho}\left(r^{Pg},1\right)\right)\left(1+R^{Pg}\right)\right)\Big]. \tag{E.17}$$

Recall that when $r^{Pg}=0$, we have

$$\frac{\partial c^{\rho}(r^{Pg},\pi^f)}{\partial r^{Pg}}\bigg|_{\pi^f=1}=0, \tag{E.18}$$

and thus

$$\frac{\partial^2 h(1)}{\partial (r^{Pg})^2}\bigg||_{r^{Pg}=0}=-\frac{\partial^2 c^{\rho}(r^{Pg},\pi^f)}{\partial (r^{Pg})^2}\bigg||_{\pi^f=1,r^{Pg}=0}\mathbf{E}\Big[\left(1+r^P\right)U'\left(f_0\left(1+r^P\right)\right)\Big]. \tag{E.19}$$

If we differentiate (E.14) with respect to r^{Pg} and determine a value at $r^{Pg} = 0$, then it follows that

$$-\left.\frac{\partial^2 c^{\rho}\left(r^{Pg}, \pi^f\right)}{\partial\left(r^{Pg}\right)^2}\right|_{\pi^f=1, r^{Pg}=0} = 0. \tag{E.20}$$

From this, it follows that

$$\left.\frac{\partial^2 h(1)}{\partial\left(r^{Pg}\right)^2}\right|_{r^{Pg}=0} = 0. \tag{E.21}$$

Since we have

$$\left.\frac{\partial^2 h(1)}{\partial\left(r^{Pg}\right)^2}\right|_{r^{Pg}=0} = 0, \qquad \left.\frac{\partial h(1)}{\partial r^{Pg}}\right|_{r^{Pg}=0} = 0, \tag{E.22}$$

it follows that $h(1) = 0$ for small forfeits. This, in turn, confirms that (3.9) holds for large π^f and small r^{Pg}.

References

[1] M. A. Petersen, M. C. Senosi, and J. Mukuddem-Petersen, *Subprime Mortgage Models*, Nova, New York, NY, USA, 2010.
[2] Wikipedia: The Free Encyclopedia, "Subprime mortgage crisis," December 2010, http://en.wikipedia.org/wiki/Subprime_mortgage_crisis.
[3] D. Crouhy, P. Turnbull, and H. Jarrow, "The subprime credit crisis of 07'," Working Paper, University of Houston, Natixis and Cornell University, 2008.
[4] G. P. Dwyer and P. Tkac, "The financial crisis of 2008 in fixed-income markets," *Journal of International Money and Finance*, vol. 28, no. 8, pp. 1293–1316, 2009.
[5] A. B. Ashcraft, P. Goldsmith-Pinkham, and J. I. Vickery, "MBS ratings and the mortgage credit boom," FRB of New York Staff Report 449, 2010, http://ssrn.com/abstract=1615613.
[6] G. B. Gorton, "The subprime panic," Yale ICF Working Paper 08, 25, 2008, http://ssrn.com/abstract=1276047.
[7] Financial Services Authority, "Mortgage market review: responsible lending," Consultation Paper 10/16, July 2010, http://www.fsa.gov.uk/.
[8] M. L. Shapiro, "Testimony Concerning the State of the Financial Crisis. Before the Financial Crisis Inquiry Commission," U.S. Securities and Exchange Commission, January 2010, http://www.sec.gov/news/testimony/2010/ts011410mls.htm.
[9] A. B. Ashcraft and T. Schuermann, *Understanding the Securitization of SubprimeMortgage Credit*, Now Publishers, New York, NY, USA, 2008.
[10] J. Mason and J. Rosner, "Where did the risk go? How misapplied bond ratings cause mortgage backed securities and collateralized debt obligation market disruptions," Working Paper, Drexel University, 2007.
[11] B. De Waal, M. A. Petersen, J. Mukuddem-Petersen, and M. P. Mulaudzi, "Subprime residential mortgage funding and liquidity risk," to appear in *Quantitative Finance*.
[12] M. A. Petersen, M. P. Mulaudzi, J. Mukuddem-Petersen, and I. Schoeman, "A note on the subprime mortgage crisis: dynamic modelling of bank leverage profit under loan securitization," *Applied Economics Letters*, vol. 17, no. 15, pp. 1469–1474, 2010.
[13] H. H. Fouche, J. Mukuddem-Petersen, M. A. Petersen, and M. C. Senosi, "Bank valuation and its connections with the subprime mortgage crisis and basel II capital accord," *Discrete Dynamics in Nature and Society*, vol. 2008, Article ID 740845, 44 pages, 2008.

[14] E. Heitfeld, "Parameter uncertainty and the credit risk of collateralized debt obligations," Working Paper, Federal Reserve Board, 2008.

[15] S. Claessens, A. Demirgüç-Kunt, and F. Moshirian, "Global financial crisis, risk analysis and risk measurement," *Journal of Banking and Finance*, vol. 33, no. 11, pp. 1949–1952, 2009.

[16] A. M. Cowan and C. D. Cowan, "Default correlation: an empirical investigation of a subprime lender," *Journal of Banking and Finance*, vol. 28, no. 4, pp. 753–771, 2004.

[17] P. Cagan, "What lies beneath operational risk issues underlying the subprime crisis," *The Risk Management Association (RMA) Journal*, pp. 96–99, 2008.

[18] M. F. Hellwig, "Systemic risk in the financial sector: an analysis of the subprime-mortgage financial crisis," *Economist*, vol. 157, no. 2, pp. 129–207, 2009.

[19] G. Gorton, "Information, liquidity, and the (ongoing) panic of 2007," *American Economic Review*, vol. 99, no. 2, pp. 567–572, 2009.

[20] J. Mukuddem-Petersen, M. P. Mulaudzi, M. A. Petersen, and I. M. Schoeman, "Optimal mortgage loan securitization and the subprime crisis," *Optimization Letters*, vol. 4, no. 1, pp. 97–115, 2009.

[21] D. E. Bell, "Regret in decision making under uncertainty," *Operations Research*, vol. 30, no. 5, pp. 961–981, 1982.

[22] D. E. Bell, "Risk premiums for decision regret," *Management Science*, vol. 39, no. 10, pp. 1156–1166, 1983.

[23] G. Loomes and R. Sugden, "Regret theory: an alternative theory of rational choice under uncertainty," *Economic Journal*, vol. 92, no. 368, pp. 805–824, 1982.

[24] J. Quiggin, "Regret theory with general choice sets," *Journal of Risk and Uncertainty*, vol. 8, no. 2, pp. 153–165, 1994.

[25] R. Sugden, "An axiomatic foundation for regret theory," *Journal of Economic Theory*, vol. 60, no. 1, pp. 159–180, 1993.

[26] C. Gollier and B. Salanie, "Individual decisions under risk, risk-sharing and asset prices with regret," Working Paper, Toulouse School of Economics, 2006.

[27] L. J. Savage, *The Foundations of Statistics*, John Wiley & Sons, New York, NY, USA, 1954.

[28] J. Keehner, "NYSE moves to delist ACA Capital Holdings," December 2007, http://www.reuters.com/article/idUSN1325383920071213.

[29] R. Ackermann, "ACA downgrade unsettles investors," December 2007, http://www.forbes.com/2007/12/19/aca-bond-insurers-markets-equity-cx_ra_1219markets33.html.

[30] C. Richard, "AMBAC's insurance unit cut to AA from AAA by Fitch ratings," January 2008, http://www.bloomberg.com/apps/news?pid=newsarchive&sid=asLtTQyLRQQs.

[31] K. Brettell, "MBIA debt protection costs rise on negative outlook," December 2007, http://www.reuters.com/article/idUSN1740166120071217.

[32] D. Preston, "Muni insurance worthless as borrowers shun AMBAC," December 2007, http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aUinEIdJ4QBo.

[33] G. Serkin, "Buffet starts U.S. municipal bond insurer, WSJ says," December 2007, http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aIy4h.TX4q3k.

[34] W. Witkowski, "Moody's downgrades Aaa rating of Ambac, MBIA," June 2008, http://www.marketwatch.com/story/moodys-downgrades-aaa-rating-of-ambac-mbia.

[35] D. Hakim, "New York to regulate credit default swaps," *The New York Times*, September 2008, http://www.nytimes.com/2008/09/23/business/23swap.html?_r=1.

[36] R. P. Larrick and T. L. Boles, "Avoiding regret in decisions with feedback: a negotiation example," *Organizational Behavior and Human Decision Processes*, vol. 63, no. 1, pp. 87–97, 1995.