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Research Article

The Asymptotic Behavior for Second-Order Neutral Stochastic Partial Differential Equations with Infinite Delay

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By establishing two Lemmas, the exponential stability and the asymptotical stability for mild solution to the second-order neutral stochastic partial differential equations with infinite delay are obtained, respectively. Our results can generalize and improve some existing ones. Finally, an illustrative example is given to show the effectiveness of the obtained results.

1. Introduction

The neutral stochastic differential equations can play an important role in describing many sophisticated dynamical systems in physical, biological, medical, chemical engineering, aero-elasticity, and social sciences [1–3], and the qualitative dynamics such as the existence and uniqueness, stability, and controllability for first-order neutral stochastic partial differential equations with delays have been extensively studied by many authors; see, for example, the existence and uniqueness for neutral stochastic partial differential equations under the non-Lipschitz conditions was investigated by using the successive approximation [4–6]; in [7], Caraballo et al. have considered the exponential stability of neutral stochastic delay partial differential equations by the Lyapunov functional approach; in [8], Dauer and Mahmudov have analyzed the existence of mild solutions to semilinear neutral evolution equations with nonlocal conditions by using the fractional power of operators and Krasnoselski-Schaefer-type fixed point theorem; in [9], Hu and Ren have established the existence results for impulsive neutral stochastic functional integrodifferential equations with infinite delays by means of the Krasnoselskii-Schaefer-type fixed point theorem; some sufficient conditions ensuring the controllability for neutral stochastic functional differential inclusions with infinite delay in the abstract space with the help of the Leray-Schauder nonlinear alterative have been given by Balasubramaniam and Muthukumar in [10]; Luo and Taniguchi, in [11], have studied the asymptotic stability for neutral stochastic partial differential equations with infinite delay by using the fixed point theorem. Very recently, in [12], the author has discussed the exponential stability for mild solution to neutral stochastic partial differential equations with delays by establishing an integral inequality.

Although there are many valuable results about neutral stochastic partial differential equations, they are mainly concerned with the first-order case. In many cases, it is advantageous to treat the second-order stochastic differential equations directly rather than to convert them to first-order systems. The second-order stochastic differential equations are the right model in continuous time to account for integrated processes that can be made stationary. For instance, it is useful for engineers to model mechanical vibrations or charge on a capacitor or condenser subjected to white noise excitation through a second-order stochastic differential equations. The studies of the qualitative properties about abstract deterministic second-order evolution equation governed by the generator of a strongly continuous cosine family was proposed in [13–15]. Recently, Mahmudov and McKibben, in [16], have established the approximate controllability of second-order neutral stochastic evolution equations; the existence and uniqueness for mild solution to second-order neutral impulsive stochastic evolution equations with delay under the non-Lipschitz condition was considered by the successive approximation [17]; Balasubramaniam and Muthukumar in [10] have also discussed the approximate controllability of second-order neutral stochastic distributed implicit functional differential equations with infinite delay; Sakthivel et al. in [18] have studied the asymptotic stability of second-order neutral stochastic differential equations by the fixed point theorem.

However, the work done by Sakthivel et al. [18] is mainly in connection with no heredity case. Since many systems arising from realistic models heavily depend on histories [19] (i.e., there is the effect of infinite delay on state equations), there is a real need to proceed with studying the second-order neutral stochastic partial differential equations with infinite delay. Although Sakthivel et al. [18] have applied the fixed point theorem to discuss the asymptotic stability for mild solution to the second-order neutral stochastic partial differential equations, the method proposed by Sakthivel et al. [18] is not suitable for such equations with infinite delay. Obviously, the Lyapunov functional method utilized by Caraballo et al. [7] fails to deal with the asymptotic behavior for mild solution to the second-order neutral stochastic partial differential equations with infinite delay since the mild solutions do not have stochastic differentials. Besides, to the best of author's knowledge, there is no paper which is involved with the exponential stability and the asymptotic stability for mild solution to second-order neutral stochastic partial differential equations with infinite delay. So, in this paper, we will make the first attempt to close this gap.

The format of this work is organized as follows. In Section 2, some necessary definitions, notations, and Lemmas used in this paper are introduced; in Section 3, the main results in this paper are given. Finally, an illustrative example is provided to demonstrate the effectiveness of our obtained results.

2. Preliminaries

Let X and Y be two real, separable Hilbert spaces and $L(Y, X)$ the space of bounded linear operators from Y to X. For the sake of convenience, we will use the same notation $\|\cdot\|$ to denote the norms in X, Y and $L(Y, X)$ when no confusion possibly arises. Let $(\Omega, \Im, \{\Im_t\}_{t\geq 0}, P)$ be a complete probability space equipped with some filtration $\Im_t$ $(t \geq 0)$ satisfying the usual conditions, that is, the filtration is right continuous and $\Im_0$ contains all P-null sets. Let $C((-\infty, 0], X)$ be the space of all bounded and continuous functions φ from $(-\infty, 0]$ to X with the sup-norm $\|\cdot\|_C = \sup_{-\infty<\theta\leq 0}\|\varphi(\theta)\|$, and the space $\mathfrak{B}$ present the family of all $\Im_t$ $(t \geq 0)$-measurable and $C((-\infty, 0], X)$-valued random variables.

Let $\beta_n(t)$ $(n = 1, 2, \ldots)$ be a sequence of real-valued one-dimensional standard Brownian motions mutually independent over $(\Omega, \Im, \{\Im_t\}_{t\geq 0}, P)$. Set $w(t) = \sum_{n=1}^{+\infty} \sqrt{\lambda_n}\beta_n(t)e_n,\ t \geq 0$, where $\lambda_n \geq 0$ $(n = 1, 2, \ldots)$ are nonnegative real numbers and $\{e_n\}$ $(n = 1, 2, \ldots)$ is a complete orthonormal basis in Y. Let $Q \in L(Y, Y)$ be an operator defined by $Qe_n = \lambda_n e_n$ with finite trace $\mathrm{tr}Q = \sum_{n=1}^{+\infty} \lambda_n < +\infty$. Then, the above Y-valued stochastic process $w(t)$ is called a Q-Wiener process.

Definition 2.1 (see [20]). Let $\sigma \in L(Y, X)$ and define

$$\|\sigma\|_{L_2^0}^2 := \mathrm{tr}(\sigma Q\sigma^*) = \left\{\sum_{n=1}^{+\infty}\left\|\sqrt{\lambda_n}\sigma e_n\right\|^2\right\}. \tag{2.1}$$

If $\|\sigma\|_{L_2^0}^2 < +\infty$, then σ is called a Q-Hilbert—Schmidt operator, and let $L_2^0(Y, X)$ denote the space of all Q-Hilbert—Schmidt operators $\sigma : Y \to X$.

Now, for the definition of an X-valued stochastic integral of an $L_2^0(Y, X)$-valued and $\Im_t$-adapted predictable process $\Phi(t)$ with respect to the Q-Wiener process $w(t)$, the readers can refer to [20].

In this paper, we consider the following second-order neutral stochastic partial differential equations with infinite delay:

$$\begin{gathered} d\left[x'(t) - f_0(t, x_t)\right] = \left[Ax(t) + f_1(t, x_t)\right]dt + f_2(t, x_t)dw(t), \quad t \in [0, +\infty), \\ x_0(\cdot) = \varphi, \\ x'(0) = \xi, \end{gathered} \tag{2.2}$$

where $\varphi \in \mathfrak{B}$ and ξ is also an $\Im_0$-measurable X-valued random variable independent of the Wiener process $w(t)$. $A : D(A) \subset X \to X$ is the infinitesimal generator of a strongly continuous cosine family on X; $f_i : [0, +\infty) \times \mathfrak{B} \to X$ $(i = 0, 1)$, $f_2 : [0, +\infty) \times \mathfrak{B} \to L_2^0(Y, X)$ are three approximate mappings. In this sequel, the history $x_t : (-\infty, 0] \to X$, $x_t(\theta) = x(t + \theta)$ $(t \geq 0)$ belongs to the space $\mathfrak{B}$.

At the end of this section, let us introduce the following Lemmas and definitions that are useful for the development of our results. The one parameter cosine family $\{C(t) :\ t \in R\} \subset L(X, X)$ satisfying

(i) $C(0) = I$,

(ii) $C(t)x$ is in continuous in t on R for all $x \in R$,

(iii) $C(t + s) + C(t - s) = 2C(t)C(s)$ for all $t, s \in R$

is called a strongly continuous cosine family.

The corresponding strongly continuous sine family $\{S(t) :\ t \in R\} \subset L(X)$ is defined by $S(t)x = \int_0^t C(s)x\, ds, t \in R, x \in X$. The generator $A : X \to X$ of $\{C(t) : t \in R\}$ is given by

$Ax = (d^2/dt^2)C(t)x|_{t=0}$ for all $x \in D(A) = \{x \in X : C(\cdot)x \in C^2(R, X)\}$. It is well known that the infinitesimal generator A is a closed, densely defined operator on X. Such cosine and the corresponding sine families and their generators satisfy the following properties.

Lemma 2.2 (see [21]). *Suppose that A is the infinitesimal generator of a cosine family of operators $\{C(t) : t \in R\}$. Then, the following holds:*

(i) *there exists $M^* \geq 1$ and $\alpha \geq 0$ such that $\|C(t)\| \leq M^* e^{\alpha t}$ and hence $\|S(t)\| \leq M^* e^{\alpha t}$,*

(ii) $A \int_s^r S(u)x\,du = [C(r) - C(u)]x$ *for all $0 \leq s \leq r < +\infty$,*

(iii) *there exists $N^* \geq 1$ such that $\|S(s) - S(r)\| \leq N^* |\int_r^s e^{\alpha|\theta|} d\theta|$ for all $0 \leq r \leq s < +\infty$.*

Lemma 2.3 (see [20]). *For any $r \geq 1$ and for arbitrary $L_2^0(Y, X)$-valued predictable process $\phi(\cdot)$ such that*

$$\sup_{s\in[0,t]} E\left\|\int_0^s \phi(u)dw(u)\right\|^{2r} \leq C_r \left(\int_0^t \left(E\|\phi(s)\|_{L_2^0}^{2r}\right)^{1/r} ds\right)^r, \quad t \in [0, +\infty), \tag{2.3}$$

where $C_r = (r(2r-1))^r$.

Definition 2.4. An X-value stochastic process $x(t)$ $(t \in R)$ is called a mild solution of the system (2.2) if

(i) $x(t)$ is adapted to $\Im_t$ $(t \geq 0)$ and has càdlàg path on $t \geq 0$ almost surely,

(ii) for arbitrary $t \in [0, +\infty)$, $P\{\omega : \int_0^t \|x(t)\|^2 dt < +\infty\} = 1$ and almost surely

$$\begin{aligned} x(t) = {} & C(t)\varphi + S(t)\left(\xi - f_0(0, x_0)\right) + \int_0^t C(t-s) f_0(s, x_s)ds \\ & + \int_0^t S(t-s) f_1(s, x_s)ds + \int_0^t S(t-s) f_2(s, x_s)dw(s), \end{aligned} \tag{2.4}$$

where $x_0(\cdot) = \varphi \in \mathfrak{B}$.

Definition 2.5. The solution of integral equation (2.4) is said to be exponentially stable in p $(p \geq 2)$ moment, if there exists a pair of positive constants $\gamma > 0$ and $M_1 > 0$ such that

$$E\|x(t)\|^p \leq M_1 e^{-\gamma t}, \quad t \geq 0, \; p \geq 2, \tag{2.5}$$

for any initial value $\varphi \in \mathfrak{B}$.

Definition 2.6. The solution of integral equation (2.4) is said to be stable in p $(p \geq 2)$ moment, if for arbitrarily given $\varepsilon > 0$, there exists a $\delta > 0$ such that $E\|\xi\|_C^2 < \delta$ guarantees that

$$E\left\{\sup_{t\geq 0} \|x(t)\|^p\right\} < \varepsilon, \quad p \geq 2. \tag{2.6}$$

Definition 2.7. The solution of integral equation (2.4) is said to be asymptotically stable in p $(p \geq 2)$ moment, if it is stable in mean square and for any $\varphi \in \mathfrak{B}$, a.s., we have

$$\lim_{T\to+\infty} E\left\{\sup_{t\geq T}\|x(t)\|^p\right\} = 0, \quad p \geq 2. \tag{2.7}$$

3. Main Results

In order to obtain our main results, we need the following assumptions.

(H_1) The cosine family of operators $\{C(t) : t \geq 0\}$ on X and the corresponding sine family $\{S(t) : t \geq 0\}$ satisfy the conditions $\|C(t)\| \leq Me^{-bt}$ and $\|S(t)\| \leq Me^{-at}$, $t \geq 0$ for some constants $M \geq 1$, $a > 0$ and $b > 0$.

(H_2) The mappings f_i $(i = 0,1,2)$ satisfy the following conditions: there exist three positive constants $C_i > 0$ $(i = 0,1,2)$ and a function: $k : (-\infty,0] \to [0,+\infty)$ with two important properties: $\int_{-\infty}^0 k(t)dt = 1$ and $\int_{-\infty}^0 k(t)e^{-\nu t}dt < +\infty$ $(\nu > 0)$, such that

$$\begin{gathered}
\|f_i(t,x) - f_i(t,y)\| \leq C_i\int_{-\infty}^0 k(\theta)\|x(t+\theta) - y(t+\theta)\|d\theta, \quad f_i(t,0) = 0,\ i = 0,1, \\
\|f_2(t,x) - f_2(t,y)\|_{\mathfrak{L}_2^0} \leq C_2\int_{-\infty}^0 k(\theta)\|x(t+\theta) - y(t+\theta)\|d\theta, \quad f_2(t,0) = 0,
\end{gathered} \tag{3.1}$$

for any $x, y \in \mathfrak{B}$ and $t \geq 0$.

(H_3) $5^{p-1}M^p\left[b^{-p}C_0^p + a^{-p}C_1^p + C_2^p a^{-(p/2)}\left(\dfrac{2(p-1)}{p-2}\right)^{1-(p/2)}\left(\dfrac{p(p-1)}{2}\right)^{p/2}\right] < 1, \quad (p \geq 2).$

Remark 3.1. Obviously, under the conditions: (H_1)-(H_2), the existence and uniqueness of mild solution to the system (2.2) can be shown by using the Picard iterative method, and the proof is very similar to that proposed in [4, 17]. Here, we omit it. In particular, the system (2.2) has one unique trivial mild solution when the initial value $\varphi = 0$.

Lemma 3.2. *For $\gamma_1, \gamma_2 \in (0,\nu]$, there exist some positive constants: $\lambda_i > 0$ $(i = 1,2,3,4)$ and a function $y : (-\infty,+\infty) \to [0,+\infty)$. If $(\lambda_3/\gamma_1) + (\lambda_4/\gamma_2) < 1$, the following inequality:*

$$y(t) \leq \begin{cases} \lambda_1 e^{-\gamma_1 t} + \lambda_2 e^{-\gamma_2 t} + \lambda_3\displaystyle\int_0^t e^{-\gamma_1(t-s)}\int_{-\infty}^0 k(\theta)y(s+\theta)d\theta\, ds & \\ \quad +\lambda_4\displaystyle\int_0^t e^{-\gamma_2(t-s)}\int_{-\infty}^0 k(\theta)y(s+\theta)d\theta\, ds, & t \geq 0, \\ \lambda_1 e^{-\gamma_1 t} + \lambda_2 e^{-\gamma_2 t}, & t \in (-\infty,0], \end{cases} \tag{3.2}$$

holds. Then, one has $y(t) \leq M_2 e^{-\mu t}$, $t \in (-\infty,+\infty)$, where $\mu \in (0,\gamma_1 \wedge \gamma_2)$ is a positive root of the algebra equation: $((\lambda_3/(\gamma_1-\mu))+(\lambda_4/(\gamma_2-\mu)))\int_{-\infty}^0 k(\theta)e^{-\mu\theta}d\theta = 1$ and $M_2 = \max\{\lambda_1+\lambda_2, (\lambda_1(\gamma_1-\mu)/(\lambda_3\int_{-\infty}^0 k(\theta)e^{-\mu\theta}d\theta)), (\lambda_2(\gamma_2-\mu)/(\lambda_4\int_{-\infty}^0 k(\theta)e^{-\mu\theta}d\theta))\} > 0$.

Proof. Letting $F(\lambda) = ((\lambda_3/(\gamma_1-\lambda)) + (\lambda_4/(\gamma_2-\lambda)))\int_{-\infty}^{0} k(\theta)e^{-\lambda\theta}d\theta - 1$, we have $F(0)F(\gamma-) < 0$ holds, that is, there exists a positive constant $\mu \in (0, \gamma_1 \wedge \gamma_2)$, such that $F(\mu) = 0$.

For any $\varepsilon > 0$ and letting

$$C_\varepsilon = \max\left\{\lambda_1 + \lambda_2 + \varepsilon, (\lambda_1+\varepsilon)\frac{\gamma_1-\mu}{\lambda_3\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta}, (\lambda_2+\varepsilon)\frac{\gamma_2-\mu}{\lambda_4\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta}\right\} > 0. \tag{3.3}$$

Now, in order to show this Lemma, we only claim that (3.2) implies

$$y(t) \le C_\varepsilon e^{-\mu t}, \qquad t \in (-\infty, +\infty). \tag{3.4}$$

It is easily seen that (3.4) holds for any $t \in (-\infty, 0]$. Assume, for the sake of contradiction, that there exists a $t_1 > 0$ such that

$$y(t) < C_\varepsilon e^{-\mu t}, \quad t \in (-\infty, t_1), \quad y(t_1) = C_\varepsilon e^{-\mu t_1}. \tag{3.5}$$

Then, it from (3.2) follows that

$$\begin{aligned} y(t_1) \le{}& \lambda_1 e^{-\gamma_1 t_1} + \lambda_2 e^{-\gamma_2 t_1} + \lambda_3 C_\varepsilon \int_0^{t_1} e^{-\gamma_1(t_1-s)}\int_{-\infty}^{0} k(\theta)e^{-\mu(s+\theta)}ds \\ &+ \lambda_4 C_\varepsilon \int_0^{t_1} e^{-\gamma_2(t_1-s)}\int_{-\infty}^{0} k(\theta)e^{-\mu(s+\theta)}d\theta\, ds \\ ={}& \left(\lambda_1 - \frac{C_\varepsilon\lambda_3}{\gamma_1-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta\right)e^{-\gamma_1 t_1} + \left(\lambda_2 - \frac{C_\varepsilon\lambda_4}{\gamma_2-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta\right)e^{-\gamma_2 t_1} \\ &+ \left(\frac{\lambda_3}{\gamma_1-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta + \frac{\lambda_4}{\gamma_2-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta\right)C_\varepsilon e^{-\mu t_1}. \end{aligned} \tag{3.6}$$

From the definitions of μ and C_ε, we obtain

$$\begin{gathered} \frac{\lambda_3}{\gamma_1-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta + \frac{\lambda_4}{\gamma_2-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta = 1, \\ \lambda_1 - \frac{\lambda_3 C_\varepsilon}{\gamma_1-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta = \lambda_1 - \frac{\lambda_3}{\gamma_1-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta(\lambda_1+\varepsilon)\frac{\gamma_1-\mu}{\lambda_3\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta} < 0, \\ \lambda_2 - \frac{C_\varepsilon\lambda_4}{\gamma_2-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta = \lambda_2 - \frac{\lambda_4}{\gamma_2-\mu}\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta(\lambda_2+\varepsilon)\frac{\gamma_2-\mu}{\lambda_4\int_{-\infty}^{0} k(\theta)e^{-\mu\theta}d\theta} < 0. \end{gathered} \tag{3.7}$$

Thus, (3.6) yields

$$y(t_1) < C_\varepsilon e^{-\mu t_1}, \tag{3.8}$$

which contradicts (3.5), that is, (3.4) holds.

As $\varepsilon > 0$ is arbitrarily small, in view of (3.4), it follows

$$y(t) \le M_2 e^{-\mu t}, \quad t \ge 0, \tag{3.9}$$

where $M_2 = \max\{\lambda_1 + \lambda_2, (\lambda_1(\gamma_1 - \mu)/(\lambda_3 \int_{-\infty}^{0} k(\theta)e^{-\mu\theta} d\theta)), (\lambda_2(\gamma_2 - \mu)/(\lambda_4 \int_{-\infty}^{0} k(\theta)e^{-\mu\theta} d\theta))\} > 0$. The proof of this Lemma is completed. □

Lemma 3.3. *For $\gamma_1, \gamma_2 > 0$, there exist some positive constants: $\lambda_i > 0$ $(i = 1, 2, 3, 4)$ and a function $y : (-\infty, +\infty) \to [0, +\infty)$. If $(\lambda_3/\gamma_1) + (\lambda_4/\gamma_2) < 1$, the following inequality:*

$$y(t) \le \begin{cases} \lambda_1 e^{-\gamma_1 t} + \lambda_2 e^{-\gamma_2 t} + \lambda_3 \displaystyle\int_0^t e^{-\gamma_1(t-s)} \int_{-\infty}^{0} k(\theta) y(s+\theta) d\theta\, ds \\ \quad + \lambda_4 \displaystyle\int_0^t e^{-\gamma_2(t-s)} \int_{-\infty}^{0} k(\theta) y(s+\theta) d\theta\, ds, & t \ge 0, \\ \lambda_1 + \lambda_2, & t \in (-\infty, 0], \end{cases} \tag{3.10}$$

holds. Then, one has $\lim_{t\to+\infty} y(t) = 0$.

Proof. In order to show the conclusion of this Lemma, the proof is divided into two steps as follows.

Step 1. We show that there exists a positive constant $d_\varepsilon > 0$ such that

$$y(t) \le d_\varepsilon, \tag{3.11}$$

for any $t \in (-\infty, +\infty)$. Firstly, for for all $\varepsilon > 0$, letting

$$d_\varepsilon = \max\left\{ \lambda_1 + \lambda_2 + \varepsilon, \frac{\gamma_1}{\lambda_3}(\lambda_1 + \varepsilon), \frac{\gamma_2}{\lambda_4}(\lambda_2 + \varepsilon) \right\} > 0. \tag{3.12}$$

It is obviously seen that $y(t) \le d_\varepsilon$ for any $t \in (-\infty, 0]$. Assumed that there exists a $t_1 > 0$ such that

$$y(t) < d_\varepsilon, \quad t \in (-\infty, t_1), \quad y(t_1) = d_\varepsilon. \tag{3.13}$$

Then, it from (3.10) implies that

$$\begin{aligned} y(t_1) &\leq \lambda_1 e^{-\gamma_1 t_1} + \lambda_2 e^{-\gamma_2 t_1} + \lambda_3 d_\varepsilon \int_0^{t_1} e^{-\gamma_1 (t_1 - s)} ds + \lambda_4 d_\varepsilon \int_0^{t_1} e^{-\gamma_2 (t_1 - s)} ds \\ &= \left(\lambda_1 - \frac{\lambda_3 d_\varepsilon}{\gamma_1}\right) e^{-\gamma_1 t_1} + \left(\lambda_2 - \frac{\lambda_4 d_\varepsilon}{\gamma_2}\right) e^{-\gamma_2 t_1} + \left(\frac{\lambda_3}{\gamma_1} + \frac{\lambda_4}{\gamma_2}\right) d_\varepsilon. \end{aligned} \tag{3.14}$$

From the definition of d_ε, we have

$$\begin{aligned} \lambda_1 - \frac{\lambda_3 d_\varepsilon}{\gamma_1} &= \lambda_1 - \frac{\lambda_3}{\gamma_1} \frac{\gamma_1}{\lambda_3} (\lambda_1 + \varepsilon) < 0, \\ \lambda_2 - \frac{\lambda_4 d_\varepsilon}{\gamma_2} &= \lambda_2 - \frac{\lambda_4}{\gamma_2} \frac{\gamma_2}{\lambda_4} (\lambda_2 + \varepsilon) < 0. \end{aligned} \tag{3.15}$$

Thus, (3.14) yields

$$y(t_1) < d_\varepsilon, \tag{3.16}$$

which contradicts (3.13), that is, (3.11) holds.

As $\varepsilon > 0$ is arbitrarily small, in view of (3.11), it follows

$$y(t) \leq d, \quad t \in (-\infty, +\infty), \tag{3.17}$$

where $d = \max\{\lambda_1 + \lambda_2, (\gamma_1/\lambda_3)\lambda_1, (\gamma_2/\lambda_4)\lambda_2\}$.

Step 2. We prove that $\lim_{t \to +\infty} y(t) = 0$.

From the inequality (3.17), it has shown that $y(t)$ is a bounded function defined on the interval $(-\infty, +\infty)$. Thus, as $t \to +\infty$, the upper limit (denoted by $l \geq 0$) of $y(t)$ exists, namely,

$$\overline{\lim}_{t \to +\infty} y(t) = l, \tag{3.18}$$

the remaining work is to prove $l = 0$.

Supposed that $l > 0$. From (3.18), there must exist arbitrary positive scalar $\varepsilon > 0$ and constant $T_1 > 0$ such that

$$y(t) < l + \varepsilon, \quad \forall t \geq T_1. \tag{3.19}$$

On the other hand, since $\int_{-\infty}^{0} k(s) ds = 1$, there must exist $T_2 > 0$ such that

$$\int_{-\infty}^{-T_2} k(\theta) d\theta < \varepsilon, \quad \forall t \geq T_2. \tag{3.20}$$

Letting $T = \max\{T_1, T_2\}$, (3.19) and (3.20) hold for $t > T$. Thus, it from (3.10) follows that

$$
\begin{aligned}
y(t) \leq{}& \lambda_1 e^{-\gamma t} + \lambda_2 e^{-\gamma_2 t} + \lambda_3 \int_0^t e^{-\gamma_1(t-s)} \int_{-\infty}^{s-T} k(u-s) y(u) du\, ds \\
&+ \lambda_3 \int_0^t e^{-\gamma_1(t-s)} \int_{s-T}^{s} k(u-s) y(u) du\, ds \\
&+ \lambda_4 \int_0^t e^{-\gamma_2(t-s)} \int_{-\infty}^{s-T} k(u-s) y(u) du\, ds \\
&+ \lambda_4 \int_0^t e^{-\gamma_2(t-s)} \int_{s-T}^{s} k(u-s) y(u) du\, ds \\
\leq{}& \left[\lambda_1 + \lambda_3 \int_0^{2T} e^{\gamma_1 s} \int_{s-T}^{s} k(u-s) y(u) du\, ds\right] e^{-\gamma_1 t} \\
&+ \left[\lambda_2 + \lambda_4 \int_0^{2T} e^{\gamma_2 s} \int_{s-T}^{s} k(u-s) y(u) du\, ds\right] e^{-\gamma_2 t} \\
&+ \left(\frac{\lambda_3 d}{\gamma_1} + \frac{\lambda_4 d}{\gamma_2}\right)\varepsilon + \left(\frac{\lambda_3}{\gamma_1} + \frac{\lambda_4}{\gamma_2}\right)(l+\varepsilon).
\end{aligned}
\tag{3.21}
$$

By virtue of (3.18), we have

$$
l \leq \left(\frac{\lambda_3 d}{\gamma_1} + \frac{\lambda_4 d}{\gamma_2}\right)\varepsilon + \left(\frac{\lambda_3}{\gamma_1} + \frac{\lambda_4}{\gamma_2}\right)(l+\varepsilon).
\tag{3.22}
$$

From the arbitrary property of ε, it follows $l \leq ((\lambda_3/\gamma_1) + (\lambda_4/\gamma_2))l$, that is,$(\lambda_3/\gamma_1) + (\lambda_4/\gamma_2) \geq 1$, which contradicts the condition: $(\lambda_3/\gamma_1) + (\lambda_4/\gamma_2) < 1$. Thus, $l = 0$. The proof of this Lemma is completed.

□

Theorem 3.4. *Suppose that the conditions: (H_1)–(H_3) are satisfied and $a, b \in (0, \nu]$, then the mild solution to system* (2.2) *is exponentially stable in p ($p \geq 2$) moment.*

Proof. In view of (2.4) and the elementary inequality, we have

$$
\begin{aligned}
\|x(t)\|^p = \Bigg\| & C(t)\varphi + S(t)\left(\xi - f_0(0, x_0)\right) + \int_0^t C(t-s) f_0(s, x_s) ds \\
&+ \int_0^t S(t-s) f_1(s, x_s) ds + \int_0^t S(t-s) f_2(s, x_s) dw(s) \Bigg\|^p
\end{aligned}
$$

$$\leq 5^{p-1}M^p\|\varphi\|^p e^{-bt}+5^{p-1}M^p\|\xi-f_0(0,x_0)\|^p e^{-at}+5^{p-1}M^p b^{1-p}\int_0^t e^{-b(t-s)}\|f_0(s,x_s)\|^p ds$$
$$+5^{p-1}M^p a^{1-p}\int_0^t e^{-a(t-s)}\|f_1(s,x_s)\|^p ds+5^{p-1}\left\|\int_0^t S(t-s)f_2(s,x_s)dw(s)\right\|^p. \tag{3.23}$$

From the condition (H_2), it from (3.23) concludes that

$$\begin{aligned}E\|x(t)\|^p &\leq 5^{p-1}M^pE\|\varphi\|^p e^{-bt}+5^{p-1}M^pE\|\xi-f_0(0,x_0)\|^p e^{-at}\\
&\quad+5^{p-1}M^pb^{1-p}C_0^p\int_0^t e^{-b(t-s)}E\left(\int_{-\infty}^0 k(\theta)\|x(s+\theta)\|d\theta\right)^p ds\\
&\quad+5^{p-1}M^pa^{1-p}C_1^p\int_0^t e^{-a(t-s)}E\left(\int_{-\infty}^0 k(\theta)\|x(s+\theta)\|d\theta\right)^p ds\\
&\quad+5^{p-1}E\left\|\int_0^t S(t-s)f_2(s,x_s)dw(s)\right\|^p.\end{aligned} \tag{3.24}$$

From Lemma 2.3, we obtain

$$\begin{aligned}&E\left\|\int_0^t S(t-s)f_2(s,x_s)dw(s)\right\|^p\\
&\quad\leq M^p\left(\int_0^t\left(e^{-ap(t-s)}E\|f_2(s,x_s)\|_{L_2^0}^p\right)^{2/p}ds\right)^{p/2}\left(\frac{p(p-1\ )}{2}\right)^{p/2}\\
&\quad= M^p\left(\int_0^t e^{-2a(t-s)}\left(E\|f_2(s,x_s)\|_{L_2^0}^p\right)^{2/p}ds\right)^{p/2}\left(\frac{p(p-1)}{2}\right)^{p/2}\\
&\quad\leq M^p\left(\int_0^t e^{-(2a(p-1)/(p-2))(t-s)}ds\right)^{(p/2)-1}\int_0^t e^{-a(t-s)}E\|f_2(s,x_s)\|_{L_2^0}^p ds\left(\frac{p(p-1)}{2}\right)^{p/2}\\
&\quad\leq M^pC_2^p\left(\frac{2a(p-1)}{p-2}\right)^{1-(p/2)}\left(\frac{p(p-1)}{2}\right)^{p/2}\int_0^t e^{-a(t-s)}E\left(\int_{-\infty}^0 k(\theta)\|x(s+\theta)\|d\theta\right)^p ds.\end{aligned} \tag{3.25}$$

Substituting (3.25) into (3.24), it follows

$$\begin{aligned}&E\|x(t)\|^p\\
&\leq 5^{p-1}M^pE\|\varphi\|^p e^{-bt}+5^{p-1}M^pE\|\xi-f_0(0,x_0)\|^p e^{-at}\\
&\quad+5^{p-1}M^pb^{1-p}C_0^p\int_0^t e^{-b(t-s)}\int_{-\infty}^0 k(\theta)E\|x(s+\theta)\|^p d\theta\, ds\end{aligned}$$

$$+ 5^{p-1}M^p a^{1-p}C_1^p \int_0^t e^{-a(t-s)} \int_{-\infty}^0 k(\theta)E\|x(s+\theta)\|^p d\theta\, ds$$
$$+ 5^{p-1}M^p C_2^p \left(\frac{2a(p-1)}{p-2}\right)^{1-(p/2)} \left(\frac{p(p-1)}{2}\right)^{p/2} \int_0^t e^{-a(t-s)} \int_{-\infty}^0 k(\theta)E\|x(s+\theta)\|^p d\theta\, ds. \tag{3.26}$$

And it is easily verified that there exists two positive number $M' > 0$ and $M'' > 0$ such that $E\|x(t)\|^2 \le M'e^{-bt} + M''e^{-at}$, for any $t \in (-\infty, 0]$.

By Lemma 3.2, we can derive that $E\|x(t)\|^2 \le M_1 e^{-\mu t}$, $t \in [0, +\infty)$ $(\mu \in (0, \gamma_1 \wedge \gamma_2))$, where

$$M_1 = \Bigg\{ 5^{p-1}M^p\left(E\|\varphi\|^p + E\|\xi - f_0(0, x_0)\|^p\right), 5^{p-1}M^p b^{-p}C_0^p, 5^{p-1}M^p\left(a^{-p}C_1^p + C_2^p a^{-p/2}\right)$$
$$\times \left(\frac{2(p-1)}{p-2}\right)^{1-(p/2)} \left(\frac{p(p-1)}{2}\right)^{p/2} \Bigg\} > 0. \tag{3.27}$$

The proof of this Theorem is completed. □

Theorem 3.5. *Suppose that the conditions: (H_1)–(H_3) are satisfied, then the mild solution to system* (2.2) *is asymptotically stable in p $(p \ge 2)$ moment.*

Proof. Similarly, we can obtain the conclusion as follows:

$$\begin{aligned}
&E\|x(t)\|^p \\
&\le 5^{p-1}M^p E\|\varphi\|^p e^{-bt} + 5^{p-1}M^p E\|\xi - f_0(0, x_0)\|^p e^{-at} \\
&\quad + 5^{p-1}M^p b^{1-p}C_0^p \int_0^t e^{-b(t-s)} \int_{-\infty}^0 k(\theta)E\|x(s+\theta)\|^p d\theta\, ds \\
&\quad + 5^{p-1}M^p a^{1-p}C_1^p \int_0^t e^{-a(t-s)} \int_{-\infty}^0 k(\theta)E\|x(s+\theta)\|^p d\theta\, ds \\
&\quad + 5^{p-1}M^p C_2^p \left(\frac{2a(p-1)}{p-2}\right)^{1-(p/2)} \left(\frac{p(p-1)}{2}\right)^{p/2} \int_0^t e^{-a(t-s)} \int_{-\infty}^0 k(\theta)E\|x(s+\theta)\|^p d\theta\, ds,
\end{aligned} \tag{3.28}$$

and it is easily verified that there exists two positive number $M'_* > 0$ and $M''_* > 0$ such that $E\|x(t)\|^2 \le M'_* + M''_*$, for any $t \in (-\infty, 0]$.

By Lemma 3.3, we can derive that

$$\lim_{t \to +\infty} E\|x(t)\|^p = 0. \tag{3.29}$$

To obtain the asymptotical stability in p $(p \geq 2)$-moment, we need to prove that mild solution of system (2.2) is stable in p $(p \geq 2)$-moment. Let $\varepsilon > 0$ be given and choose $\delta > 0$ $(\delta < \varepsilon)$ such that

$$Ce^{-(a\wedge b)t^*} + \Delta\varepsilon < \varepsilon, \tag{3.30}$$

where

$$\begin{gathered} C = 5^{p-1}M^p E\|\varphi\|^p + 5^{p-1}M^p E\|\xi - f_0(0,x_0)\|^p, \\ \Delta = 5^{p-1}M^p\left[b^{-p}C_0^p + a^{-p}C_1^p + C_2^p a^{-p/2}\left(\frac{2(p-1)}{(p-2)}\right)^{1-(p/2)}\left(\frac{p(p-1)}{2}\right)^{p/2}\right]. \end{gathered} \tag{3.31}$$

If $x(t,0,\varphi)$ is a mild solution of system (2.2) with $\sup_{\theta\in(-\infty,0]}E\|\varphi(\theta)\|^p < \delta$, then $x(t)$ is defined in (2.4). Now, we claim that $E\|x(t)\|^p < \varepsilon$ for all $t \geq 0$. Notice that $\sup_{\theta\in(-\infty,0]}E\|\varphi(\theta)\|^p < \varepsilon$. If there exists $t^* > 0$ such that $E\|x(t^*)\|^p = \varepsilon$ and $E\|x(t)\|^p < \varepsilon$, for all $t \in (-\infty, t^*)$, then it follows from (2.4) that

$$\begin{aligned} &E\|x(t^*)\|^p \\ &\leq 5^{p-1}M^p\left(E\|\varphi\|^p + E\|\xi - f_0(0,x_0)\|^p\right)e^{-(a\wedge b)t^*} \\ &\quad + 5^{p-1}M^p b^{1-p}C_0^p\int_0^{t^*} e^{-b(t^*-s)}\int_{-\infty}^0 k(\theta)E\|x(s+\theta)\|^p d\theta\, ds \\ &\quad + 5^{p-1}M^p a^{1-p}C_1^p\int_0^{t^*} e^{-a(t^*-s)}\int_{-\infty}^0 k(\theta)E\|x(s+\theta)\|^p d\theta\, ds \\ &\quad + 5^{p-1}M^p C_2^p\left(\frac{2a(p-1)}{p-2}\right)^{1-(p/2)}\left(\frac{p(p-1)}{2}\right)^{p/2}\int_0^{t^*} e^{-a(t^*-s)}\int_{-\infty}^0 k(\theta)E\|x(s+\theta)\|^p d\theta\, ds \\ &< Ce^{-(a\wedge b)t^*} + \Delta\varepsilon \\ &< \varepsilon, \end{aligned} \tag{3.32}$$

which contradicts the definition of t^*. This shows that the mild solution of system (2.2) is asymptotically stable in p $(p \geq 2)$-moment. The proof of this Theorem is completed. □

4. An Illustrative Example

In this section, we provide an example to illustrate the obtained results above. Let $X = L^2[0,\pi]$ and $Y = R^1$ with the norm $\|\cdot\|$. And let $e_n := \sqrt{2/\pi}\sin(n\xi)$ $(n = 1,2,\ldots)$ denote the completed orthonormal basis in X. Let $w(t) := \sum_{n=1}^{+\infty}\sqrt{\lambda_n}\beta_n(t)e_n$, $(\lambda_n > 0)$, where $\{\beta_n(t)\}$ are one-dimensional standard Brownian motions mutually independent on a usual complete probability space $(\Omega, \Im, \Im_t, P)$. Define $A : X \to X$ by $A = (\partial^2/\partial\xi^2)$ with the domain

$D(A) = \{h \in X : h,\ (\partial/\partial\xi)h$ are absolutely continuous, $(\partial^2/\partial\xi^2)h \in X,\ h(0) = h(\pi) = 0\}$. Then,

$$Ah = \sum_{n=1}^{+\infty} n^2 (h, e_n) e_n, \quad h \in D(A), \tag{4.1}$$

where $e_n,\ n = 1, 2, 3, \ldots$, is also the orthonormal set of eigenvector of A. It is well known that $\|C(t)\| \le \exp(-\pi^2 t)$ and $\|S(t)\| \le \exp(-\pi^2 t),\ t \ge 0$.

Now, we consider the following second-order neutral stochastic partial differential equations with infinite delays:

$$\begin{gathered}
d\left[\frac{\partial}{\partial t} z(t,y) - \frac{a_0}{\pi\sqrt{\pi}} \int_{-\infty}^{0} (-\theta)^{-1/2} e^{\pi^2\theta} z(t+\theta,\xi) d\theta\right] \\
= \left[\frac{\partial^2}{\partial \xi^2} z(t,\xi) + \frac{\alpha_1}{\pi\sqrt{\pi}} \int_{-\infty}^{0} (-\theta)^{-1/2} e^{\pi^2\theta} z(t+\theta,\xi) d\theta\right] dt \\
+ \frac{\alpha_2}{\pi\sqrt{\pi}} \int_{-\infty}^{0} (-\theta)^{-1/2} e^{\pi^2\theta} z(t+\theta,\xi) d\theta\, dw(t), \quad t \ge 0,\ \xi \in [0,\pi], \\
x(t,0) = x(t,\pi) = 0, \quad t \ge 0, \\
z(\theta,\xi) = \varphi(\theta,\xi), \quad \theta \in (-\infty, 0],\ \xi \in [0,\pi], \\
\frac{\partial}{\partial t} z(0,\xi) = \zeta(\xi), \quad \xi \in [0,\pi].
\end{gathered} \tag{4.2}$$

Define

$$\begin{aligned}
f_0(t,z_t) &= \frac{\alpha_0 \pi\sqrt{\pi}}{5} \int_{-\infty}^{0} (-\theta)^{-1/2} e^{\pi^2\theta} z(t+\theta,\xi) d\theta, \quad f_0(t,0) = 0, \\
f_1(t,z_t) &= \frac{\alpha_1 \pi\sqrt{\pi}}{5} \int_{-\infty}^{0} (-\theta)^{-1/2} e^{\pi^2\theta} z(t+\theta,\xi) d\theta, \quad f_1(t,0) = 0, \\
f_2(t,z_t) &= \frac{\alpha_2 \pi}{5} \int_{-\infty}^{0} (-\theta)^{-1/2} e^{\pi^2\theta} z((t+\theta),\xi) d\theta, \quad f_2(t,0) = 0,
\end{aligned} \tag{4.3}$$

for any $z_t \in \mathfrak{B}$.

It is easily verified that

$$\begin{aligned}
\left\| f_0\left(t, z_t^1\right) - f_0\left(t, z_t^2\right) \right\| &\le \frac{\alpha_0 \pi\sqrt{\pi}}{5} \int_{-\infty}^{0} (-\theta)^{-1/2} e^{\pi^2\theta} \left\| z^1(t+\theta,\xi) - z^2(t+\theta,\xi) \right\| d\theta, \quad f_0(t,0) = 0, \\
\left\| f_1\left(t, z_t^1\right) - f_1\left(t, z_t^2\right) \right\| &\le \frac{\alpha_1 \pi\sqrt{\pi}}{5} \int_{-\infty}^{0} (-\theta)^{-1/2} e^{\pi^2\theta} \left\| z^1(t+\theta,\xi) - z^2(t+\theta,\xi) \right\| d\theta, \quad f_1(t,0) = 0, \\
\left\| f_2\left(t, z_t^1\right) - f_2\left(t, z_t^2\right) \right\| &\le \frac{\alpha_2 \pi}{5} \int_{-\infty}^{0} (-\theta)^{-1/2} e^{\pi^2\theta} \left\| z^1(t+\theta,\xi) - z^2(t+\theta,\xi) \right\| d\theta, \quad f_2(t,0) = 0,
\end{aligned} \tag{4.4}$$

for any $z_t^1,\ z_t^2 \in B$.

By virtue of Theorems 3.4 and 3.5, the exponential stability in p $(p \geq 2)$-moment and the asymptotical stability in p $(p \geq 2)$-moment for mild solution to system (4.2) are obtained, provided that the following inequality:

$$\alpha_0^p + \alpha_1^p + \alpha_2 \left(\frac{2(p-1)}{p-2} \right)^{1-(p/2)} \left(\frac{p(p-1)}{2} \right)^{p/2} < 5, \quad p \geq 2, \tag{4.5}$$

holds.

Remark 4.1. Obviously, the result in [18] is ineffective in dealing with this example, and our results are more general than those proposed in [18]. Besides, our results can be easily extended to investigate two cases: (1) the exponential stability and the asymptotic stability for the second-order neutral stochastic partial differential equations with infinite delay and impulses and (2) the exponential stability for the second-order neutral stochastic partial differential equations with time-varying delays; the readers can refer to [12, 22]. Here, we omit them.

References

[1] V. B. Kolmanovskii and A. Myshkis, *Applied Theory of Functional-Differential Equations*, Kluwer Academic, Norwell, Mass, USA, 1992.

[2] R. Khasminskii, *Stochastic Stability of Differential Equations*, Sijthoff & Noordhoff, Amsterdam, The Netherlands, 1980.

[3] C. C. Travis and G. F. Webb, "Cosine families and abstract nonlinear second order differential equations," *Acta Mathematica Academiae Scientiarum Hungaricae*, vol. 32, no. 1-2, pp. 75–96, 1978.

[4] H. Chen, "The existence and uniqueness for the solution of neutral stochastic functional differential equations with infinite delay," *Journal of Mathematical Research and Exposition*, vol. 30, no. 4, pp. 589–598, 2010.

[5] J. Luo and T. Taniguchi, "The existence and uniqueness for non-Lipschitz stochastic neutral delay evolution equations driven by Poisson jumps," *Stochastics and Dynamics*, vol. 9, no. 1, pp. 135–152, 2009.

[6] N. I. Mahmudov, "Existence and uniqueness results for neutral SDEs in Hilbert spaces," *Stochastic Analysis and Applications*, vol. 24, no. 1, pp. 79–95, 2006.

[7] T. Caraballo, J. Real, and T. Taniguchi, "The exponential stability of neutral stochastic delay partial differential equations," *Discrete and Continuous Dynamical Systems. Series A*, vol. 18, no. 2-3, pp. 295–313, 2007.

[8] J. P. Dauer and N. I. Mahmudov, "Integral inequalities and mild solutions of semilinear neutral evolution equations," *Journal of Mathematical Analysis and Applications*, vol. 300, no. 1, pp. 189–202, 2004.

[9] L. Hu and Y. Ren, "Existence results for impulsive neutral stochastic functional integro-differential equations with infinite delays," *Acta Applicandae Mathematicae*, vol. 111, no. 3, pp. 303–317, 2010.

[10] P. Balasubramaniam and P. Muthukumar, "Approximate controllability of second-order stochastic distributed implicit functional differential systems with infinite delay," *Journal of Optimization Theory and Applications*, vol. 143, no. 2, pp. 225–244, 2009.

[11] J. Luo and T. Taniguchi, "Fixed points and stability of stochastic neutral partial differential equations with infinite delays," *Stochastic Analysis and Applications*, vol. 27, no. 6, pp. 1163–1173, 2009.

[12] H. Chen, "Integral inequality and exponential stability for neutral stochastic partial differential equations," *Journal of Inequalities and Applications*, vol. 2009, Article ID 297478, 15 pages, 2009.

[13] K. Balachandran, D. Park, and S. Anthoni, “Existence of solutions of abstract-nonlinear second-order neutral functional integrodifferential equations,” *Computers & Mathematics with Applications*, vol. 46, no. 8-9, pp. 1313–1324, 2003.

[14] K. Balachandran and S. Anthoni, “Controllability of second-order semilinear neutral functional differential systems in Banach spaces,” *Computers & Mathematics with Applications*, vol. 41, no. 10-11, pp. 1223–1235, 2001.

[15] J. Park, Y. Kwun, and H. Lee, “Controllability of second-order neutral functional differential inclusions in Banach spaces,” *Journal of Mathematical Analysis and Applications*, vol. 285, no. 1, pp. 37–49, 2003.

[16] N. I. Mahmudov and M. A. McKibben, “Approximate controllability of second-order neutral stochastic evolution equations,” *Dynamics of Continuous, Discrete & Impulsive Systems. Series B*, vol. 13, no. 5, pp. 619–634, 2006.

[17] M. McKibben, “Second-order neutral stochastic evolution equations with heredity,” *Journal of Applied Mathematics and Stochastic Analysis*, no. 2, pp. 177–192, 2004.

[18] R. Sakthivel, Y. Ren, and H. Kim, “Asymptotic stability of second-order neutral stochastic differential equations,” *Journal of Mathematical Physics*, vol. 51, no. 5, Article ID 052701, 9 pages, 2010.

[19] J. Wu, *Theory And Applications Of Partial Functional-Differential Equations*, Springer, New York, NY, USA, 1996.

[20] G. Da Prato and J. Zabczyk, *Stochastic Equations in Infinite Dimensions*, Cambridge University Press, Cambridge, UK, 1992.

[21] H. O. Fattorini, *Second Order Linear Differential Equations in Banach Spaces*, North Holland Mathematics Studies Series No. 108 Elsevier Science, North-Holland, Amsterdam, The Netherlands, 1985.

[22] H. Chen, “Impulsive-integral inequality and exponential stability for stochastic partial differential equations with delays,” *Statistics & Probability Letters*, vol. 80, no. 1, pp. 50–56, 2010.