Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2011, Article ID 783136, 14 pages
doi:10.1155/2011/783136

Research Article

Stability Analysis of Three-Species Almost Periodic Competition Models with Grazing Rates and Diffusions

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Received 10 May 2011; Accepted 2 June 2011

Academic Editor: Zhengqiu Zhang

Almost periodic solution of a three-species competition system with grazing rates and diffusions is investigated. By using the method of upper and lower solutions and Schauder fixed point theorem as well as Lyapunov stability theory, we give sufficient conditions to ensure the existence and globally asymptotically stable for the strictly positive space homogenous almost periodic solution, which extend and include corresponding results obtained by Q. C. Lin (1999), F. D. Chen and X. X. Chen (2003), Y. Q. Liu, S. L, Xie, and Z. D. Xie (1996).

1. Introduction

In this paper, we study the following three-species competition system with grazing rates and diffusions:

$$\begin{aligned}
\frac{\partial v_1(x,t)}{\partial t} &= k_1(t)\Delta v_1(x,t) + v_1(x,t)[a_1(t) - b_1(t)v_1(x,t) - c_1(t)v_2(x,t) - d_1(t)v_3(x,t)] + f_1(t),\\
\frac{\partial v_2(x,t)}{\partial t} &= k_2(t)\Delta v_2(x,t) + v_2(x,t)[a_2(t) - b_2(t)v_1(x,t) - c_2(t)v_2(x,t) - d_2(t)v_3(x,t)] + f_2(t),\\
\frac{\partial v_3(x,t)}{\partial t} &= k_3(t)\Delta v_3(x,t) + v_3(x,t)[a_3(t) - b_3(t)v_1(x,t) - c_3(t)v_2(x,t) - d_3(t)v_3(x,t)] + f_3(t),
\end{aligned} \tag{1.1}$$

where $(x,t) \in \Omega \times R^{+}$, $\Omega \subseteq R^n$ is the bounded open subset of R^n with smooth boundary $\partial\Omega$, which represent the habitat domain for three species. System (1.1) is supplement with boundary conditions and initial conditions:

$$\frac{\partial v_i(x,t)}{\partial n} = 0, \quad i = 1,2,3, \ (x,t) \in \partial\Omega \times R^{+}, \tag{1.2}$$

$$v_i(x,0) = v_{i0}(x) \geq 0, \quad v_{i0}(x) \not\equiv 0, \ i = 1,2,3, \ x \in \overline{\Omega}, \tag{1.3}$$

where $\partial/\partial n$ denotes the outward normal derivation on $\partial\Omega$, and $v_i(x,t)$ represent the density of ith species at point $x = (x_1,\ldots,x_n)$ and the time of t. Here, $k_i(t)$, $a_i(t)$, $b_i(t)$, $c_i(t), d_i(t)$, and $f_i(t)$ $(i = 1,2,3)$ denote the diffusivity rates, competition rates, and grazing rates, respectively. They are almost periodic functions in real number field R. Δ is a Laplace operator on Ω.

System (1.1)–(1.3) describes the interaction among three species and is an important model in biomathcmatics, which has been intensively investigated, and much attention is carried to the problem [1–8]. When there is no diffusion, Jiang [1] and Lin [2] studied the existence, uniqueness, and stability on periodic solution and almost periodic solution for two-species competition system under the condition that the coefficients are the periodic function and almost periodic function, respectively; F. D. Chen and X. X. Chen [3] extended the results in [2] to n-species case. When there are no diffusion and grazing rates, Zhang and Wang [4, 5] investigated the existence of a positive periodic solution for a two-species nonautonomous competition Lotka-Volterra patch system with time delay and the existence of multiple positive periodic solutions for a generalized delayed population model with exploited term by using the continuation theorem of coincidence degree theory; Hu and Zhang [6] established criteria for the existence of at least four positive periodic solutions for a discrete time-delayed predator-prey system with nonmonotonic functional response and harvesting by employing the continuation theorem of coincidence degree theory. When there are no grazing rates, Pao and Wang [7] proved the stability for invariable coefficient case by utilizing the method of upper and lower solutions. Liu et al. [8] showed the stability on the periodic solution for n-species competition system with grazing rates and diffusions. Nevertheless, generally speaking, the system does not always change strictly according to periodic laws, sometimes it changes according to almost periodic laws, and it is important to survey almost periodic solution for the multispecies competition system with grazing rates and diffusions. To sum up, we pay more attention to almost periodic solution of a three-species competition system (1.1)–(1.3) with grazing rates and diffusions; in this paper, by using the method of upper and lower solutions and Schauder fixed point theorem as well as Lyapunov stability theory, we obtain sufficient conditions which ensure the existence and globally asymptotically stable for the strictly positive space homogenous almost periodic solution, which extend and include corresponding results obtained in [2, 3, 8]. Many other results on the periodic solution and almost periodic solution can be found in [9–16].

2. Preliminary

Firstly, we give out some definitions and lemmas.

Definition 2.1. Suppose that $f(t)$ is a continuous function in R. Then $f(t)$ is said to be almost periodic in $t \in R$ if for every $\varepsilon > 0$ corresponds $T(\varepsilon) > 0$ such that for any interval I whose length is equal to $T(\varepsilon)$ there is at least one $\tau \in I$ such that

$$\left| f(t+\tau) - f(t) \right| \leq \varepsilon, \quad \forall t \in R. \tag{2.1}$$

Definition 2.2. If a smooth function $V(t) = (v_1(t), v_2(t), v_3(t))$ satisfies (1.1) in R^+, and every component of $V(t)$ is the almost periodic function, we called that $V(t)$ is a spatial homogeneity almost periodic solution for (1.1), which is denoted by $V(t, T(\varepsilon))$.

Definition 2.3. For any nonnegative smooth initial data

$$V(x,0) = (v_1(x,0), v_2(x,0), v_3(x,0)) = (v_{10}(x), v_{20}(x), v_{30}(x)) \geq 0, \quad V(x,0) \not\equiv 0, \; x \in \Omega, \tag{2.2}$$

if there exists a unique positive solution $V(x,t) = (v_1(x,t), v_2(x,t), v_3(x,t))$ for the system (1.1) with boundary conditions (1.2), and $\lim_{t\to\infty}(V_i(x,t) - V_i(t, T(\varepsilon))) = 0$, $i = 1,2,3$, uniformly for $x \in \overline{\Omega}$, we called that spatial homogeneity almost periodic solution $V(t, T(\varepsilon))$ is globally asymptotically stable.

Definition 2.4. Suppose that $\overline{V}(x,t) \equiv (\overline{v}_1(x,t), \overline{v}_2(x,t), \overline{v}_3(x,t))$, $\underline{V}(x,t) \equiv (\underline{v}_1(x,t), \underline{v}_2(x,t), \underline{v}_3(x,t))$; if $\overline{V}(x,t) \geq \underline{V}(x,t)$ and

$$\begin{aligned}
\frac{\partial \overline{v}_1(x,t)}{\partial t} &\geq k_1(t)\Delta \overline{v}_1(x,t) + \overline{v}_1(x,t)\left[a_1(t) - b_1(t)\overline{v}_1(x,t) - c_1(t)\underline{v}_2(x,t) - d_1(t)\underline{v}_3(x,t)\right] \\
&\quad + f_1(t), \quad (x,t) \in \Omega \times R^+, \\
\frac{\partial \overline{v}_2(x,t)}{\partial t} &\geq k_2(t)\Delta \overline{v}_2(x,t) + \overline{v}_2(x,t)\left[a_2(t) - b_2(t)\underline{v}_1(x,t) - c_2(t)\overline{v}_2(x,t) - d_2(t)\underline{v}_3(x,t)\right] \\
&\quad + f_2(t), \quad (x,t) \in \Omega \times R^+, \\
\frac{\partial \overline{v}_3(x,t)}{\partial t} &\geq k_3(t)\Delta \overline{v}_3(x,t) + \overline{v}_3(x,t)\left[a_3(t) - b_3(t)\underline{v}_1(x,t) - c_3(t)\underline{v}_2(x,t) - d_3(t)\overline{v}_3(x,t)\right] \\
&\quad + f_3(t), \quad (x,t) \in \Omega \times R^+, \\
&\frac{\partial \overline{v}_i(x,t)}{\partial n} \geq 0, \quad i = 1,2,3, \; (x,t) \in \partial\Omega \times R^+, \\
&\overline{v}_i(x,0) \geq v_{i0}(x), \quad i = 1,2,3, \; x \in \overline{\Omega}, \\
\frac{\partial \underline{v}_1(x,t)}{\partial t} &\leq k_1(t)\Delta \underline{v}_1(x,t) + \underline{v}_1(x,t)\left[a_1(t) - b_1(t)\underline{v}_1(x,t) - c_1(t)\overline{v}_2(x,t) - d_1(t)\overline{v}_3(x,t)\right] \\
&\quad + f_1(t), \quad (x,t) \in \Omega \times R^+, \\
\frac{\partial \underline{v}_2(x,t)}{\partial t} &\leq k_2(t)\Delta \underline{v}_2(x,t) + \underline{v}_2(x,t)\left[a_2(t) - b_2(t)\overline{v}_1(x,t) - c_2(t)\underline{v}_2(x,t) - d_2(t)\overline{v}_3(x,t)\right] \\
&\quad + f_2(t), \quad (x,t) \in \Omega \times R^+, \\
\frac{\partial \underline{v}_3(x,t)}{\partial t} &\leq k_3(t)\Delta \underline{v}_3(x,t) + \underline{v}_3(x,t)\left[a_3(t) - b_3(t)\overline{v}_1(x,t) - c_3(t)\overline{v}_2(x,t) - d_3(t)\underline{v}_3(x,t)\right] \\
&\quad + f_3(t), \quad (x,t) \in \Omega \times R^+, \\
&\frac{\partial \underline{v}_i(x,t)}{\partial n} \leq 0, \quad i = 1,2,3, \; (x,t) \in \partial\Omega \times R^+, \\
&\underline{v}_i(x,0) \leq v_{i0}(x), \quad i = 1,2,3, \; x \in \overline{\Omega},
\end{aligned} \tag{2.3}$$

we called $\overline{V}(x,t)$ and $\underline{V}(x,t)$ a pair of ordered upper and lower solutions for systems (1.1)–(1.3).

Lemma 2.5 (see [12, 17]). *Suppose that $\overline{V}(x,t)$ and $\underline{V}(x,t)$ are a pair of ordered upper and lower solution for systems* (1.1)–(1.3)*, then there exists a unique solution $V(x,t)$ for systems* (1.1)–(1.3)*. Moreover, one has*

$$\overline{V}(x,t) \geq V(x,t) \geq \underline{V}(x,t). \tag{2.4}$$

For the almost periodic function $F(t)$ in R, one denotes $\widetilde{F} = \sup\{F(t),\ t \in R\}$, $\underset{\sim}{F} = \inf\{F(t),\ t \in R\}$, and $M[F] = \lim_{(t-s)\to\infty}\{\int_s^t F(\tau)d\tau/(t-s)\}$. When $F(t)$ is T-periodic function, one denotes $M[F] = \int_0^T F(s)ds/T$.

3. Main Results and Proofs

Now we are in a position to state our main results and give our proofs.

Theorem 3.1. *If $\underset{\sim}{a}_i, \underset{\sim}{b}_i, \underset{\sim}{c}_i, \underset{\sim}{d}_i, \underset{\sim}{f}_i$ are positive numbers, and*

$$\frac{\left(\tilde{b}_i + \tilde{c}_i + \tilde{d}_i\right)}{\underset{\sim}{a}_i} \leq L = \min\left\{\sqrt{\frac{\underset{\sim}{b}_1}{\tilde{f}_1}}, \sqrt{\frac{\underset{\sim}{c}_2}{\tilde{f}_2}}, \sqrt{\frac{\underset{\sim}{d}_3}{\tilde{f}_3}}, \frac{\left(\underset{\sim}{d}_1 + \underset{\sim}{c}_1\right)}{\tilde{a}_1}, \frac{\left(\underset{\sim}{b}_2 + \underset{\sim}{d}_2\right)}{\tilde{a}_2}, \frac{\left(\underset{\sim}{b}_3 + \underset{\sim}{c}_3\right)}{\tilde{a}_3}\right\} \tag{3.1}$$

are satisfied for $i = 1,2,3$, then there exists a strictly positive spatial homogeneity almost periodic solution $V(t) = (\hat{v}_1(t), \hat{v}_2(t), \hat{v}_3(t))$ for (1.1).

Proof. By the conditions in Theorem 3.1, we have

$$0 < \frac{\tilde{c}_1 + \tilde{d}_1}{L\underset{\sim}{a}_1 - \tilde{b}_1} \leq 1, \qquad 0 < \frac{\tilde{b}_2 + \tilde{d}_2}{L\underset{\sim}{a}_2 - \tilde{c}_2} \leq 1, \qquad 0 < \frac{\tilde{b}_3 + \tilde{c}_3}{L\underset{\sim}{a}_3 - \tilde{d}_3} \leq 1. \tag{3.2}$$

Let

$$m = L \cdot \max\left\{\frac{\tilde{c}_1 + \tilde{d}_1}{L\underset{\sim}{a}_1 - \tilde{b}_1}, \frac{\tilde{b}_2 + \tilde{d}_2}{L\underset{\sim}{a}_2 - \tilde{c}_2}, \frac{\tilde{b}_3 + \tilde{c}_3}{L\underset{\sim}{a}_3 - \tilde{d}_3}\right\}. \tag{3.3}$$

Then we have $0 < m \leq L$, and

$$\left(\tilde{c}_1 + \tilde{d}_1\right)\frac{L}{m} \leq L\underset{\sim}{a}_1 - \tilde{b}_1, \qquad \left(\tilde{b}_2 + \tilde{d}_2\right)\frac{L}{m} \leq L\underset{\sim}{a}_2 - \tilde{c}_2, \qquad \left(\tilde{b}_3 + \tilde{c}_3\right)\frac{L}{m} \leq L\underset{\sim}{a}_3 - \tilde{d}_3. \tag{3.4}$$

Therefore

$$\begin{gathered}\tilde{b}_1 + \left(\tilde{c}_1 + \tilde{d}_1\right)\frac{L}{m} - \underset{\sim}{f}_1 m^2 \leq L\underset{\sim}{a}_1, \qquad \tilde{c}_2 + \left(\tilde{b}_2 + \tilde{d}_2\right)\frac{L}{m} - \underset{\sim}{f}_2 m^2 \leq L\underset{\sim}{a}_2, \\ \tilde{d}_3 + \left(\tilde{b}_3 + \tilde{c}_3\right)\frac{L}{m} - \underset{\sim}{f}_3 m^2 \leq L\underset{\sim}{a}_3.\end{gathered} \tag{3.5}$$

Furthermore, by the given conditions in Theorem 3.1, one has

$$\begin{gathered}
\underset{\sim}{b}_1-\widetilde{f}_1L^2\geq 0,\qquad \underset{\sim}{c}_2-\widetilde{f}_2L^2\geq 0,\qquad \underset{\sim}{d}_3-\widetilde{f}_3L^2\geq 0,\qquad \left(\underset{\sim}{c}_1+\underset{\sim}{d}_1\right)\frac{m}{L}\geq m\widetilde{a}_1,\\
\left(\underset{\sim}{b}_2+\underset{\sim}{d}_2\right)\frac{m}{L}\geq m\widetilde{a}_2,\qquad \left(\underset{\sim}{b}_3+\underset{\sim}{c}_3\right)\frac{m}{L}\geq m\widetilde{a}_3.
\end{gathered}\tag{3.6}$$

Thus

$$\begin{gathered}
\underset{\sim}{b}_1+\left(\underset{\sim}{c}_1+\underset{\sim}{d}_1\right)\frac{m}{L}-\widetilde{f}_1L^2\geq m\widetilde{a}_1,\qquad \underset{\sim}{c}_2+\left(\underset{\sim}{b}_2+\underset{\sim}{d}_2\right)\frac{m}{L}-\widetilde{f}_2L^2\geq m\widetilde{a}_2,\\
\underset{\sim}{d}_3+\left(\underset{\sim}{b}_3+\underset{\sim}{c}_3\right)\frac{m}{L}-\widetilde{f}_3L^2\geq m\widetilde{a}_3.
\end{gathered}\tag{3.7}$$

Combining (3.5) and (3.7), we have

$$\begin{aligned}
&\widetilde{b}_1+\left(\widetilde{c}_1+\widetilde{d}_1\right)\frac{L}{m}-\underset{\sim}{f}_1m^2\leq L\underset{\sim}{a}_1, &&\underset{\sim}{b}_1+\left(\underset{\sim}{c}_1+\underset{\sim}{d}_1\right)\frac{m}{L}-\widetilde{f}_1L^2\geq m\widetilde{a}_1,\\
&\widetilde{c}_2+\left(\widetilde{b}_2+\widetilde{d}_2\right)\frac{L}{m}-\underset{\sim}{f}_2m^2\leq L\underset{\sim}{a}_2, &&\underset{\sim}{c}_2+\left(\underset{\sim}{b}_2+\underset{\sim}{d}_2\right)\frac{m}{L}-\widetilde{f}_2L^2\geq m\widetilde{a}_2,\\
&\widetilde{d}_3+\left(\widetilde{b}_3+\widetilde{c}_3\right)\frac{L}{m}-\underset{\sim}{f}_3m^2\leq L\underset{\sim}{a}_3, &&\underset{\sim}{d}_3+\left(\underset{\sim}{b}_3+\underset{\sim}{c}_3\right)\frac{m}{L}-\widetilde{f}_3L^2\geq m\widetilde{a}_3.
\end{aligned}\tag{3.8}$$

Let

$$H_L^m=\left\{\left(\phi(t),\varphi(t),\gamma(t)\right):\phi,\varphi,\gamma\text{ are almost periodic in }t\in R,0<m\leq\phi,\varphi,\gamma\leq L\right\}.\tag{3.9}$$

We consider the following system corresponding to (1.1):

$$\begin{aligned}
\dot{v}_1&=v_1\left(a_1(t)-b_1(t)v_1-c_1(t)v_2-d_1(t)v_3\right)+f_1(t),\quad t\in R^+,\\
\dot{v}_2&=v_2\left(a_2(t)-b_2(t)v_1-c_2(t)v_2-d_2(t)v_3\right)+f_2(t),\quad t\in R^+,\\
\dot{v}_3&=v_3\left(a_3(t)-b_3(t)v_1-c_3(t)v_2-d_3(t)v_3\right)+f_3(t),\quad t\in R^+.
\end{aligned}\tag{3.10}$$

Let $z_i=1/v_i, i=1,2,3$; then (3.10) becomes

$$\begin{aligned}
\dot{z}_1&=b_1(t)-a_1(t)z_1+c_1(t)\frac{z_1}{z_2}+d_1(t)\frac{z_1}{z_3}-f_2(t)z_1^2,\\
\dot{z}_2&=c_2(t)-a_2(t)z_2+b_2(t)\frac{z_2}{z_1}+d_2(t)\frac{z_2}{z_3}-f_2(t)z_2^2,\\
\dot{z}_3&=d_3(t)-a_3(t)z_3+b_3(t)\frac{z_3}{z_1}+c_3(t)\frac{z_3}{z_2}-f_3(t)z_3^2.
\end{aligned}\tag{3.11}$$

For any $(\phi(t),\varphi(t),\gamma(t))\in H_L^m$, by $M[b_1]>0$, $M[c_2]>0$, $M[d_3]>0$, we observe [18] that

$$\begin{aligned}
\dot{z}_1 &= b_1(t)-a_1(t)z_1+c_1(t)\frac{\phi(t)}{\varphi(t)}+d_1(t)\frac{\phi(t)}{\gamma(t)}-f_1(t)\phi^2(t),\\
\dot{z}_2 &= c_2(t)-a_2(t)z_2+b_2(t)\frac{\varphi(t)}{\phi(t)}+d_2(t)\frac{\varphi(t)}{\gamma(t)}-f_2(t)\varphi^2(t),\\
\dot{z}_3 &= d_3(t)-a_3(t)z_3+b_3(t)\frac{\gamma(t)}{\phi(t)}+c_3(t)\frac{\gamma(t)}{\varphi(t)}-f_3(t)\gamma^2(t),
\end{aligned}\tag{3.12}$$

have almost periodic solution:

$$\begin{aligned}
\hat{z}_1(t) &= \int_{-\infty}^{t} e^{-\int_s^t a_1(r)dr}\left[b_1(s)+c_1(s)\frac{\phi(s)}{\varphi(s)}+d_1(s)\frac{\phi(s)}{\gamma(s)}-f_1(s)\phi^2(s)\right]ds,\\
\hat{z}_2(t) &= \int_{-\infty}^{t} e^{-\int_s^t a_2(r)dr}\left[c_2(s)+b_2(s)\frac{\varphi(s)}{\phi(s)}+d_2(s)\frac{\varphi(s)}{\gamma(s)}-f_2(s)\varphi^2(s)\right]ds,\\
\hat{z}_3(t) &= \int_{-\infty}^{t} e^{-\int_s^t a_2(r)dr}\left[d_3(s)+b_3(s)\frac{\gamma(s)}{\phi(s)}+c_3(s)\frac{\gamma(s)}{\varphi(s)}-f_3(s)\gamma^2(s)\right]ds.
\end{aligned}\tag{3.13}$$

By the system (3.13), we define a mapping A:

$$A\big(\phi(t),\varphi(t),\gamma(t)\big)=(\hat{z}_1(t),\hat{z}_2(t),\hat{z}_3(t)),\quad \forall\big(\phi(t),\varphi(t),\gamma(t)\big)\in H_L^m.\tag{3.14}$$

Combining (3.8) and (3.13), we have

$$\begin{aligned}
\hat{z}_1(t) &\ge \int_{-\infty}^{t} e^{-\tilde{a}_1(t-s)}\left[\underset{\sim 1}{b}+\left(\underset{\sim 1}{c}+\underset{\sim 1}{d}\right)\frac{m}{L}-\tilde{f}_1L^2\right]ds=\frac{1}{\tilde{a}_1}\left[\underset{\sim 1}{b}+\left(\underset{\sim 1}{c}+\underset{\sim 1}{d}\right)\frac{m}{L}-\tilde{f}_1L^2\right]\ge m,\\
\hat{z}_1(t) &\le \int_{-\infty}^{t} e^{-\underset{\sim 1}{a}(t-s)}\left[\tilde{b}_1+\left(\tilde{c}_1+\tilde{d}_1\right)\frac{L}{m}-\underset{\sim 1}{f}m^2\right]ds=\frac{1}{\underset{\sim 1}{a}}\left[\tilde{b}_1+\left(\tilde{c}_1+\tilde{d}_1\right)\frac{L}{m}-\underset{\sim 1}{f}m^2\right]\le L,\\
\hat{z}_2(t) &\ge \int_{-\infty}^{t} e^{-\tilde{a}_2(t-s)}\left[\underset{\sim 2}{c}+\left(\underset{\sim 2}{b}+\underset{\sim 2}{d}\right)\frac{m}{L}-\tilde{f}_2L^2\right]ds=\frac{1}{\tilde{a}_2}\left[\underset{\sim 2}{c}+\left(\underset{\sim 2}{b}+\underset{\sim 2}{d}\right)\frac{m}{L}-\tilde{f}_2L^2\right]\ge m,\\
\hat{z}_2(t) &\le \int_{-\infty}^{t} e^{-\underset{\sim 2}{a}(t-s)}\left[\tilde{c}_2+\left(\tilde{b}_2+\tilde{d}_2\right)\frac{L}{m}-\underset{\sim 2}{f}m^2\right]ds=\frac{1}{\underset{\sim 2}{a}}\left[\tilde{c}_2+\left(\tilde{b}_2+\tilde{d}_2\right)\frac{L}{m}-\underset{\sim 2}{f}m^2\right]\le L,\\
\hat{z}_3(t) &\ge \int_{-\infty}^{t} e^{-\tilde{a}_3(t-s)}\left[\underset{\sim 3}{d}+\left(\underset{\sim 3}{b}+\underset{\sim 3}{c}\right)\frac{m}{L}-\tilde{f}_3L^2\right]ds=\frac{1}{\tilde{a}_3}\left[\underset{\sim 3}{d}+\left(\underset{\sim 3}{b}+\underset{\sim 3}{c}\right)\frac{m}{L}-\tilde{f}_3L^2\right]\ge m,\\
\hat{z}_3(t) &\le \int_{-\infty}^{t} e^{-\underset{\sim 3}{a}(t-s)}\left[\tilde{d}_3+\left(\tilde{b}_3+\tilde{c}_3\right)\frac{L}{m}-\underset{\sim 3}{f}m^2\right]ds=\frac{1}{\underset{\sim 3}{a}}\left[\tilde{d}_3+\left(\tilde{b}_3+\tilde{c}_3\right)\frac{L}{m}-\underset{\sim 3}{f}m^2\right]\le L.
\end{aligned}\tag{3.15}$$

Therefore, $(\widehat{z}_1(t), \widehat{z}_2(t), \widehat{z}_3(t)) \in H_L^m$, that is, $AH_L^m \subset H_L^m$. If A is uniformly boundness and equicontinuous, by Ascoli-Arzela theorem, A is compact mapping.

It is obvious to obtain uniformly boundedness. In fact, for any $(\phi(t), \varphi(t), \gamma(t)) \in H_L^m$, by (3.15) we have $(\widehat{z}_1(t), \widehat{z}_2(t), \widehat{z}_3(t)) = A(\phi(t), \varphi(t), \gamma(t)) \in H_L^m$; that is, it satisfies

$$0 < (m, m, m) \leq A\big(\phi(t), \varphi(t), \gamma(t)\big) = (\widehat{z}_1(t), \widehat{z}_2(t), \widehat{z}_3(t)) \leq (L, L, L). \tag{3.16}$$

Next we prove equicontinuous. For any $(\phi(t), \varphi(t), \gamma(t)) \in H_L^m$, we denote $(\widehat{z}_1(t), \widehat{z}_2(t), \widehat{z}_3(t)) = A(\phi(t), \varphi(t), \gamma(t))$, $t_1 < t_2$, and then

$$\begin{aligned} |\widehat{z}_1(t_1) - \widehat{z}_1(t_2)| = \Bigg| &\int_{-\infty}^{t_1} e^{-\int_s^{t_1} a_1(r)dr} \left[b_1(s) + c_1(s)\frac{\phi(s)}{\varphi(s)} + d_1(s)\frac{\phi(s)}{\gamma(s)} - f_1(s)\phi^2(s) \right] \\ &- \int_{-\infty}^{t_2} e^{-\int_s^{t_2} a_1(r)dr} \left[b_1(s) + c_1(s)\frac{\phi(s)}{\varphi(s)} + d_1(s)\frac{\phi(s)}{\gamma(s)} - f_1(s)\phi^2(s) \right] \Bigg|. \end{aligned} \tag{3.17}$$

Let $h_1(t) = b_1(t) + c_1(t)(\phi(t)/\varphi(t)) + d_1(t)(\phi(t)/\gamma(t)) - f_1(t)\phi^2(t)$; we obtain

$$\begin{aligned} |\widehat{z}_1(t_2) - \widehat{z}_1(t_1)| &= \left| \int_{-\infty}^{t_2} e^{-\int_s^{t_2} a_1(r)dr} h_1(s)ds - \int_{-\infty}^{t_1} e^{-\int_s^{t_1} a_1(r)dr} h_1(s)ds \right| \\ &\leq \left| \int_{t_1}^{t_2} e^{-\int_s^{t_2} a_1(r)dr} h_1(s)ds \right| + \left| \int_{-\infty}^{t_1} e^{-\int_s^{t_1} a_1(r)dr} \left(e^{-\int_{t_1}^{t_2} a_1(r)dr} - 1 \right) h_1(s)ds \right|. \end{aligned} \tag{3.18}$$

Recalling $(\phi(t), \varphi(t), \gamma(t)) \in H_L^m$, we deduce that there exists a positive number M such that $|h_1(s)| \leq M$; then (3.18) becomes

$$|\widehat{z}_1(t_2) - \widehat{z}_1(t_1)| \leq M e^{-\int_{\xi_1}^{t_2} a_1(r)dr} |t_2 - t_1| + \frac{1}{\underset{\sim 1}{a}} M \left| 1 - e^{-\int_{t_1}^{t_2} a_1(r)dr} \right|, \tag{3.19}$$

where $\xi_1 \in (t_1, t_2)$.

Similarly, we have

$$|\widehat{z}_2(t_2) - \widehat{z}_2(t_1)| \leq N e^{-\int_{\xi_2}^{t_2} a_2(r)dr} |t_1 - t_2| + \frac{1}{\underset{\sim 2}{a}} N \left| 1 - e^{-\int_{t_1}^{t_2} a_2(r)dr} \right|, \tag{3.20}$$

where $\xi_2 \in (t_1, t_2)$, and N is a positive number.

By a completely analogous argument, we obtain

$$|\widehat{z}_3(t_2) - \widehat{z}_3(t_1)| \leq P e^{-\int_{\xi_3}^{t_2} a_3(r)dr} |t_1 - t_2| + \frac{1}{\underset{\sim 3}{a}} P \left| 1 - e^{-\int_{t_1}^{t_2} a_3(r)dr} \right|, \tag{3.21}$$

where $\xi_3 \in (t_1, t_2)$, and P is a positive number.

By (3.19)–(3.21), for any $(\phi(t),\varphi(t),\gamma(t)) \in H_L^m$, we derive

$$\lim_{\xi\to 0}\sup_{|t_1-t_2|\leq\xi}\left|A(\phi,\varphi,\gamma)(t_1)-A(\phi,\varphi,\gamma)(t_2)\right| = 0, \quad \text{uniformly for } x\in\overline{\Omega}. \tag{3.22}$$

Thus, A is a compact mapping which maps H_L^m into itself; by Schauder fixed point theorem, there exists a fixed point $(\phi(t),\varphi(t),\gamma(t)) \in H_L^m$ for A; namely, (3.11) has a solution; therefore there exists a strictly positive almost periodic solution $(v_1^*(t),v_2^*(t),v_3^*(t)) = (1/\phi(t),1/\varphi(t),1/\gamma(t))$, $t\in R^+$ for system (3.10). It is obvious that $(v_1^*(t),v_2^*(t),v_3^*(t))$, $t\in R^+$ is also the spatial homogeneity almost periodic solution for (1.1). □

Theorem 3.2. *Under the conditions of Theorem 3.1, suppose that system* (1.1) *satisfies the following conditions:*

$$\sup_{t\geq 0}(b_3(t)+b_2(t)-b_1(t)) = -\varepsilon_1 < 0, \qquad \sup_{t\geq 0}(c_3(t)+c_1(t)-c_2(t)) = -\varepsilon_2 < 0,$$
$$\sup_{t\geq 0}(d_1(t)+d_2(t)-d_3(t)) = -\varepsilon_3 < 0. \tag{3.23}$$

Then there exists a strictly positive spatial homogeneity almost periodic solution $(v_1^*(t),v_2^*(t),v_3^*(t))$ *for* (1.1)*, and the corresponding solution for systems* (1.1)–(1.3) *is globally asymptotically stable; that is, the solution* $(v_1(x,t),v_2(x,t),v_3(x,t))$, $(x,t)\in\overline{\Omega}\times R^+$ *satisfies*

$$\lim_{t\to\infty}\left(v_i(x,t)-v_i^*(t)\right) = 0, \quad i=1,2,3, \text{ uniformly for } x\in\overline{\Omega}. \tag{3.24}$$

Proof. We have obtained the existence by Theorem 3.1; next we pay more attention to the stability. Concerning (3.24), we have two cases on initial data $v_{i0}(x), i=1,2,3$.

(1) $v_{i0}(x) > 0, x\in\overline{\Omega}$.

(2) There exists a point $x_0\in\overline{\Omega}$, such that $v_{10}(x_0)=0, v_{20}(x_0)=0$ or $v_{30}(x_0)=0$.

For the case (1), let $l_i = \min_{x\in\overline{\Omega}} v_{i0}(x)$, $r_i = \max_{x\in\overline{\Omega}} v_{i0}(x)$, $i=1,2,3$; then $0 < l_i \leq v_{i0}(x) \leq r_i$. Suppose that $(\overline{v}_1(t),\overline{v}_2(t),\overline{v}_3(t))$ and $(\underline{v}_1(t),\underline{v}_2(t),\underline{v}_3(t))$ are the solution for (3.10) corresponding to initial datum $(\overline{v}_1(0),\overline{v}_2(0),\overline{v}_3(0)) = (r_1,r_2,r_3)$ and $(\underline{v}_1(0),\underline{v}_2(0),\underline{v}_3(0)) = (l_1,l_2,l_3)$, respectively; then there are a pair of ordered upper and lower solutions $(\overline{v}_1(t),\overline{v}_2(t),\overline{v}_3(t))$ and $(\underline{v}_1(t),\underline{v}_2(t),\underline{v}_3(t))$ for (1.1)–(1.3); by Lemma 2.5, there exists a unique solution $(v_1(x,t),v_2(x,t),v_3(x,t)), (x,t)\in\overline{\Omega}\times R^+$ for system (1.1)–(1.3), which satisfies

$$\left(\underline{v}_1(t),\underline{v}_2(t),\underline{v}_3(t)\right) \leq (v_1(x,t),v_2(x,t),v_3(x,t)) \leq (\overline{v}_1(t),\overline{v}_2(t),\overline{v}_3(t)). \tag{3.25}$$

If we have

$$\lim_{t\to\infty}\left[\overline{v}_i(t)-v_i^*(t)\right] = \lim_{t\to\infty}\left[\underline{v}_i(t)-v_i^*(t)\right] = 0, \quad i=1,2,3, \tag{3.26}$$

then (3.24) holds. Therefore, if we want to obtain (3.26), we only need to prove that the solution $(v_1(t), v_2(t), v_3(t))$ for (3.10) with arbitrary positive initial data $(v_1(0), v_2(0), v_3(0)) = (v_{10}, v_{20}, v_{30})$ satisfies

$$\lim_{t\to\infty}\left(v_i(t) - v_i^*(t)\right) = 0, \quad i = 1,2,3. \tag{3.27}$$

Because of the initial datum $(v_{10}, v_{20}, v_{30}) > 0$ and grazing rates $(\underset{\sim}{f}_1, \underset{\sim}{f}_2, \underset{\sim}{f}_3) > 0$, by the practical meaning in biology, we know that $(v_1(t), v_2(t), v_3(t)) > 0$. Now let

$$P_i(t) = \ln v_i(t), \quad Q_i(t) = \ln v_i^*(t), \quad i = 1,2,3. \tag{3.28}$$

Then one has

$$\begin{aligned}
\frac{d}{dt}(P_1(t) - Q_1(t)) &= -b_1(t)\left(e^{P_1(t)} - e^{Q_1(t)}\right) - c_1(t)\left(e^{P_2(t)} - e^{Q_2(t)}\right) - d_1(t)\left(e^{P_3(t)} - e^{Q_3(t)}\right) \\
&\quad + \left(\frac{1}{v_1(t)} - \frac{1}{v_1^*(t)}\right) f_1(t), \\
\frac{d}{dt}(P_2(t) - Q_2(t)) &= -b_2(t)\left(e^{P_1(t)} - e^{Q_1(t)}\right) - c_2(t)\left(e^{P_2(t)} - e^{Q_2(t)}\right) - d_2(t)\left(e^{P_3(t)} - e^{Q_3(t)}\right) \\
&\quad + \left(\frac{1}{v_2(t)} - \frac{1}{v_2^*(t)}\right) f_2(t), \\
\frac{d}{dt}(P_3(t) - Q_3(t)) &= -b_3(t)\left(e^{P_1(t)} - e^{Q_1(t)}\right) - c_3(t)\left(e^{P_2(t)} - e^{Q_2(t)}\right) - d_3(t)\left(e^{P_3(t)} - e^{Q_3(t)}\right) \\
&\quad + \left(\frac{1}{v_3(t)} - \frac{1}{v_3^*(t)}\right) f_3(t).
\end{aligned} \tag{3.29}$$

Namely,

$$\begin{aligned}
\frac{d}{dt}(P_1(t) - Q_1(t)) &= -\left(b_1(t) + \frac{f_1(t)}{v_1(t)v_1^*(t)}\right)\left(e^{P_1(t)} - e^{Q_1(t)}\right) - c_1(t)\left(e^{P_2(t)} - e^{Q_2(t)}\right) \\
&\quad - d_1(t)\left(e^{P_3(t)} - e^{Q_3(t)}\right), \\
\frac{d}{dt}(P_2(t) - Q_2(t)) &= -b_2(t)\left(e^{P_1(t)} - e^{Q_1(t)}\right) - \left(c_2(t) + \frac{f_2(t)}{v_2(t)v_2^*(t)}\right)\left(e^{P_2(t)} - e^{Q_2(t)}\right) \\
&\quad - d_2(t)\left(e^{P_3(t)} - e^{Q_3(t)}\right), \\
\frac{d}{dt}(P_3(t) - Q_3(t)) &= -b_3(t)\left(e^{P_1(t)} - e^{Q_1(t)}\right) - c_3(t)\left(e^{P_2(t)} - e^{Q_2(t)}\right) \\
&\quad - \left(d_3(t) + \frac{f_3(t)}{v_3(t)v_3^*(t)}\right)\left(e^{P_3(t)} - e^{Q_3(t)}\right).
\end{aligned} \tag{3.30}$$

Consider the following Lyapunov function:

$$U(t) = \sum_{i=1}^{3} |P_i(t) - Q_i(t)|, \quad t \geq 0. \tag{3.31}$$

Let D^+U represent the right derivation on function $U(t)$; we have

$$\begin{aligned}
&D^+U(t) \\
&\quad = \sum_{i=1}^{3} D^+|P_i(t) - Q_i(t)| = \sum_{i=1}^{3} \operatorname{sgn}(P_i(t) - Q_i(t)) \frac{d}{dt}(P_i(t) - Q_i(t)) \\
&\quad = \operatorname{sgn}(P_1(t) - Q_1(t)) \\
&\qquad \times \left[-\left(b_1(t) + \frac{f_1(t)}{v_1(t)v_1^*(t)} \right) \left(e^{P_1(t)} - e^{Q_1(t)}\right) - c_1(t)\left(e^{P_2(t)} - e^{Q_2(t)}\right) - d_1(t)\left(e^{P_3(t)} - e^{Q_3(t)}\right) \right] \\
&\qquad + \operatorname{sgn}(P_2(t) - Q_2(t)) \\
&\qquad \times \left[-b_2(t)\left(e^{P_1(t)} - e^{Q_1(t)}\right) - \left(c_2(t) + \frac{f_2(t)}{v_2(t)v_2^*(t)} \right) \left(e^{P_2(t)} - e^{Q_2(t)}\right) - d_2(t)\left(e^{P_3(t)} - e^{Q_3(t)}\right) \right] \\
&\qquad + \operatorname{sgn}(P_3(t) - Q_3(t)) \\
&\qquad \times \left[-b_3(t)\left(e^{P_1(t)} - e^{Q_1(t)}\right) - c_3(t)\left(e^{P_2(t)} - e^{Q_2(t)}\right) - \left(d_3(t) + \frac{f_3(t)}{v_3(t)v_3^*(t)} \right) \left(e^{P_3(t)} - e^{Q_3(t)}\right) \right] \\
&\quad \leq (b_3(t) + b_2(t) - b_1(t)) \left| e^{P_1(t)} - e^{Q_1(t)} \right| + (c_1(t) + c_3(t) - c_2(t)) \left| e^{P_2(t)} - e^{Q_2(t)} \right| \\
&\qquad + \; (d_1(t) + d_2(t) - d_3(t)) \left| e^{P_3(t)} - e^{Q_3(t)} \right| \\
&\quad \leq -\varepsilon_1 \left| v_1(t) - v_1^*(t) \right| - \varepsilon_2 \left| v_2(t) - v_2^*(t) \right| - \varepsilon_3 \left| v_3(t) - v_3^*(t) \right|.
\end{aligned} \tag{3.32}$$

Integrated by the time, we have

$$U(t) + \sum_{i=1}^{3} \varepsilon_i \int_0^t \left| v_i(s) - v_i^*(s) \right| ds \leq U(0). \tag{3.33}$$

By the nonnegative of $U(t)$ and the boundedness of $U(0)$, we obtain that the $U(t)$ is bounded, and

$$\int_0^t \left| v_i(t) - v_i^*(t) \right| ds, \quad i = 1, 2, 3, \tag{3.34}$$

convergences, by (3.32) we get $D^+U(t) < 0$, then the limit

$$\lim_{t \to \infty} U(t) = l \tag{3.35}$$

exists, and $U(t) \geq l$. If $l > 0$, then at least one of the following three inequalities

$$|P_1(t) - Q_1(t)| > \frac{l}{4}, \qquad |P_2(t) - Q_2(t)| > \frac{l}{4}, \qquad |P_3(t) - Q_3(t)| > \frac{l}{4} \tag{3.36}$$

holds. Without loss of generality, we assume that $|P_1(t) - Q_1(t)| > l/4$. Thus there is no point of intersection between $P_1(t)$ and $Q_1(t)$. Suppose that $P_1(t) > Q_1(t)$; then we have $P_1(t) - Q_1(t) > l/4$. Thus

$$\begin{aligned}\int_0^t \left|v_1(t) - v_1^*(t)\right| ds &= \int_0^t \left|e^{P_1(s)} - e^{Q_1(s)}\right| ds = \int_0^t e^{Q_1(s)} \left|e^{P_1(s)-Q_1(s)} - 1\right| ds \\ &\geq m \int_0^t \left(e^{P_1(s)-Q_1(s)} - 1\right) ds > m \int_0^t \left(e^{l/4} - 1\right) ds = m\left(e^{l/4} - 1\right) t \longrightarrow +\infty,\end{aligned} \tag{3.37}$$

which contradicts with the convergence of $\int_0^t |v_i(s) - v_i^*(s)| ds$. Therefore $l = 0$; consequently

$$\lim_{t\to\infty} \left|v_i(t) - v_i^*(t)\right| = 0, \quad i = 1, 2, 3. \tag{3.38}$$

Then we obtain (3.27).

For the case (2), firstly, choose three sufficient large positive numbers $M_1, M_2,$ and M_3, such that

$$\begin{aligned} f_1(t) &\leq -M_1(a_1(t) - b_1(t)M_1), \quad t > 0, \\ f_2(t) &\leq -M_2(a_2(t) - c_2(t)M_2), \quad t > 0, \\ f_3(t) &\leq -M_3(a_3(t) - d_3(t)M_3), \quad t > 0, \end{aligned} \tag{3.39}$$

and $M_i \geq \max_{x\in\overline{\Omega}} v_{i0}(x),\ i = 1, 2, 3$. Let $\underset{\sim}{v}_i = 0,\ \tilde{v}_i = M_i, i = 1, 2, 3$; then we have

$$\begin{aligned}
&\frac{\partial \tilde{v}_1}{\partial t} - k_1(t)\Delta\tilde{v}_1 - \tilde{v}_1\left[a_1(t) - b_1(t)\tilde{v}_1 - c_1(t)\underset{\sim}{v}_2 - d_1(t)\underset{\sim}{v}_3\right] - f_1(t) \geq 0, \\
&\frac{\partial \underset{\sim}{v}_1}{\partial t} - k_1(t)\Delta\underset{\sim}{v}_1 - \underset{\sim}{v}_1\left[a_1(t) - b_1(t)\underset{\sim}{v}_1 - c_1(t)\tilde{v}_2 - d_1(t)\tilde{v}_3\right] - f_1(t) \leq 0, \\
&\frac{\partial \tilde{v}_2}{\partial t} - k_2(t)\Delta\tilde{v}_2 - \tilde{v}_2\left[a_2(t) - b_2(t)\underset{\sim}{v}_1 - c_2(t)\tilde{v}_2 - d_2(t)\underset{\sim}{v}_3\right] - f_2(t) \geq 0, \\
&\frac{\partial \underset{\sim}{v}_2}{\partial t} - k_2(t)\Delta\underset{\sim}{v}_2 - \underset{\sim}{v}_2\left[a_2(t) - b_2(t)\tilde{v}_1 - c_2(t)\underset{\sim}{v}_2 - d_2(t)\tilde{v}_3\right] - f_2(t) \leq 0, \\
&\frac{\partial \tilde{v}_3}{\partial t} - k_3(t)\Delta\tilde{v}_3 - \tilde{v}_3\left[a_3(t) - b_3(t)\underset{\sim}{v}_1 - c_3(t)\underset{\sim}{v}_2 - d_3(t)\tilde{v}_3\right] - f_3(t) \geq 0, \\
&\frac{\partial \underset{\sim}{v}_3}{\partial t} - k_3(t)\Delta\underset{\sim}{v}_3 - \underset{\sim}{v}_3\left[a_3(t) - b_3(t)\tilde{v}_1 - c_3(t)\tilde{v}_2 - d_3(t)\underset{\sim}{v}_3\right] - f_3(t) \leq 0.
\end{aligned} \tag{3.40}$$

Namely, $\underset{\sim}{v}_i = 0$, and $\tilde{v}_i = M_i$, $i = 1,2,3$ are a pair of ordered upper and lower solutions for systems (1.1)–(1.3). By Lemma 2.5, there exists a unique solution $(v_1(x,t), v_2(x,t), v_3(x,t))$ for systems (1.1)–(1.3), which satisfy

$$0 \le v_i(x,t) \le M_i, \quad i = 1,2,3; \ (x,t) \in \overline{\Omega} \times [0,\infty). \tag{3.41}$$

Secondly, we choose positive numbers $\delta_1, \delta_2,$ and δ_3 such that

$$\begin{aligned} &\delta_1 + a_1(t) - b_1(t)v_1(x,t) - c_1(t)v_2(x,t) - d_1(t)v_3(x,t) > 0, \quad (x,t) \in \overline{\Omega} \times [0,\infty), \\ &\delta_2 + a_2(t) - b_2(t)v_1(x,t) - c_2(t)v_2(x,t) - d_2(t)v_3(x,t) > 0, \quad (x,t) \in \overline{\Omega} \times [0,\infty), \\ &\delta_3 + a_3(t) - b_3(t)v_1(x,t) - c_3(t)v_2(x,t) - d_3(t)v_3(x,t) > 0, \quad (x,t) \in \overline{\Omega} \times [0,\infty). \end{aligned} \tag{3.42}$$

Accordingly, we have

$$\begin{aligned} &\frac{\partial v_1}{\partial t} - k_1(t)\Delta v_1 + \delta_1 v_1 = v_1[\delta_1 + a_1(t) - b_1(t)v_1 - c_1(t)v_2 - d_1(t)v_3] + f_1(t) \ge 0, \\ &\frac{\partial v_2}{\partial t} - k_2(t)\Delta v_2 + \delta_2 v_2 = v_2[\delta_2 + a_2 - b_2(t)v_1 - c_2(t)v_2 - d_2(t)v_3] + f_2(t) \ge 0, \\ &\frac{\partial v_3}{\partial t} - k_3(t)\Delta v_3 + \delta_3 v_3 = v_3[\delta_3 + a_3 - b_3(t)v_1 - c_3(t)v_2 - d_3(t)v_3] + f_3(t) \ge 0. \end{aligned} \tag{3.43}$$

Next, we prove $v_i(x,t) > 0$ in $\overline{\Omega} \times (0,\infty)$ for $i = 1,2,3$. Firstly, we show $v_i(x,t) > 0$ in $\Omega \times (0,\infty)$. If there exists one point $(x_0,t_0) \in \Omega \times (0,\infty)$ such that $v_i(x_0,t_0) = 0$, by extremum principle, we have $v_i(x,t) \equiv 0$ in $\overline{\Omega} \times [0,t_0)$. However $v_i(x,0) = v_{i0}(x) \ge 0$, and not being constant zero, we obtain a contradiction. Therefore we have $v_i(x,t) > 0$ in $\Omega \times (0,\infty)$. Then we show $v_i(x,t) > 0$ in $\partial\Omega \times (0,\infty)$. If there exists a point $(x_0,t_0) \in \partial\Omega \times (0,\infty)$ such that $v_i(x_0,t_0) = 0$, by the extremum principle, we have $\partial v_i(x,t)/\partial n < 0$, where $(x,t) \in \partial\Omega \times (0,\infty)$, which is contrary with boundary conditions (1.2). Thus we have $v_i(x,t) > 0$ in $\overline{\Omega} \times (0,\infty)$.

For a fixed number $\lambda > 0$, by (3.41), we have

$$0 < v_i(x,\lambda) \le M_i, \quad i = 1,2,3, \ x \in \overline{\Omega}. \tag{3.44}$$

Because $v_i(x,t+\lambda)$ satisfy system (1.1) in $\overline{\Omega} \times (0,\infty)$ and the conditions (1.2) in $\partial\Omega \times (0,\infty)$, thereby $(v_1(x,t+\lambda), v_2(x,t+\lambda), v_3(x,t+\lambda))$ is regarded as a solution for system (1.1) under initial data $(\widehat{v}_{10}(x), \widehat{v}_{20}(x), \widehat{v}_{30}(x)) = (v_1(x,\lambda), v_2(x,\lambda), v_3(x,\lambda))$, nevertheless, we have $\widehat{v}_{i0} > 0$ in $\overline{\Omega}$, $i = 1,2,3$; combining the conclusions in case (1), we have

$$\lim_{t\to\infty} \left(v_i(x,t+\lambda) - v_i^*(t)\right) = 0, \quad i = 1,2,3, \text{ uniformly for } x \in \overline{\Omega}. \tag{3.45}$$

By the arbitrariness of λ, we obtain

$$\lim_{t\to\infty} \left(v_i(x,t) - v_i^*(t)\right) = 0, \quad i = 1,2,3, \text{ uniformly for } x \in \overline{\Omega}. \tag{3.46}$$

If $k_i(t), a_i(t), b_i(t), c_i(t), d_i(t)$, and $f_i(t)$ $(i = 1,2,3)$ of (1.1) are periodic functions in real number field R, respectively, then we have the following results. □

Corollary 3.3. *If $\underset{\sim i}{a}, \underset{\sim i}{b}, \underset{\sim i}{c}, \underset{\sim i}{d}, \underset{\sim i}{f}$ are positive numbers, and*

$$\frac{\left(\tilde{b}_i + \tilde{c}_i + \tilde{d}_i\right)}{\underset{\sim i}{a}} \leq L = \min\left\{\sqrt{\frac{\underset{\sim 1}{b}}{\tilde{f}_1}}, \sqrt{\frac{\underset{\sim 2}{c}}{\tilde{f}_2}}, \sqrt{\frac{\underset{\sim 3}{d}}{\tilde{f}_3}}, \frac{\left(\underset{\sim 1}{d} + \underset{\sim 1}{c}\right)}{\tilde{a}_1}, \frac{\left(\underset{\sim 2}{b} + \underset{\sim 2}{d}\right)}{\tilde{a}_2}, \frac{\left(\underset{\sim 3}{b} + \underset{\sim 3}{c}\right)}{\tilde{a}_3}\right\} \tag{3.47}$$

are satisfied for $i = 1,2,3$, then there exists a strictly positive spatial homogeneity periodic solution $V(t) = (\hat{v}_1(t), \hat{v}_2(t), \hat{v}_3(t))$ for (1.1).

Corollary 3.4. *Under the conditions of Corollary 3.3, suppose that system (1.1) satisfies the following conditions:*

$$\sup_{t\geq 0}(b_3(t) + b_2(t) - b_1(t)) = -\varepsilon_1 < 0, \qquad \sup_{t\geq 0}(c_3(t) + c_1(t) - c_2(t)) = -\varepsilon_2 < 0,$$
$$\sup_{t\geq 0}(d_1(t) + d_2(t) - d_3(t)) = -\varepsilon_3 < 0. \tag{3.48}$$

Then there exists a strictly positive spatial homogeneity periodic solution $(v_1^*(t), v_2^*(t), v_3^*(t))$ for (1.1), and the corresponding solution for systems (1.1)–(1.3) is globally asymptotically stable; that is, the solution $(v_1(x,t), v_2(x,t), v_3(x,t)), (x,t) \in \overline{\Omega} \times R^+$ satisfies

$$\lim_{t\to\infty}\left(v_i(x,t) - v_i^*(t)\right) = 0, \quad i = 1,2,3, \text{ uniformly for } x \in \overline{\Omega}. \tag{3.49}$$

4. Conclusion

This paper presents the use of upper and lower solutions method for systems of nonlinear reaction-diffusion equations. This method is a powerful tool for solving nonlinear differential equations in mathematical physics, chemistry, and engineering, and so forth. The technique constructing a pair of upper and lower solutions and Lyapunov function provides a new efficient method to handle the nonlinear structure.

We have dealt with the problem of almost periodic solution for a three-species competition system with grazing rates and diffusions. The general sufficient conditions have been obtained to ensure the existence and stability of the strictly positive space homogenous almost periodic solution for the nonlinear reaction-diffusion equations. These criteria generalize and improve some known results. In particular, the sufficient conditions that we obtained are very simple, which provide flexibility for the application and analysis of nonlinear three-species competition system.

Acknowledgments

The authors are grateful to the reviewers for their comments which helped in improving the paper. This work is supported by the Science and Technology Project of Chongqing

Municipal Education Committee (Grant no. kJ110501) of China, the NSFC (Grant nos. 51005264, 40801214) of China, and Natural Science Foundation Project of CQ CSTC (Grant no. 2010BB9401) of China.

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