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Research Article

Complete Convergence for Moving Average Process of Martingale Differences

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Under some simple conditions, by using some techniques such as truncated method for random variables (see e.g., Gut (2005)) and properties of martingale differences, we studied the moving process based on martingale differences and obtained complete convergence and complete moment convergence for this moving process. Our results extend some related ones.

1. Introduction

Let $\{Y_i, -\infty < i < \infty\}$ be a doubly infinite sequence of random variables. Assume that $\{a_i, -\infty < i < \infty\}$ is an absolutely summable sequence of real numbers and

$$X_n = \sum_{i=-\infty}^{\infty} a_i Y_{i+n}, \quad n \geq 1 \tag{1.1}$$

is the *moving average process* based on the sequence $\{Y_i, -\infty < i < \infty\}$. As usual, $S_n = \sum_{k=1}^{n} X_k$, $n \geq 1$, denotes the sequence of partial sums.

For the moving average process $\{X_n, n \geq 1\}$, where $\{Y_i, -\infty < i < \infty\}$ is a sequence of independent identically distributed (i.i.d.) random variables, Ibragimov [1] established the central limit theorem, Burton and Dehling [2] obtained a large deviation principle, and Li et al. [3] gave the complete convergence result for $\{X_n, n \geq 1\}$. Zhang [4] and Li and Zhang [5] extended the complete convergence of moving average process for i.i.d. sequence to φ-mixing sequence and NA sequence, respectively. Theorems A and B are due to Zhang [4] and Kim et al. [6], respectively.

Theorem A. *Suppose that* $\{Y_i, -\infty < i < \infty\}$ *is a sequence of identically distributed* φ*-mixing random variables with* $\sum_{m=1}^{\infty} \varphi^{1/2}(m) < \infty$ *and* $\{X_n, n \ge 1\}$ *is as in* (1.1)*. Let* $h(x) > 0$ $(x > 0)$ *be a slowly varying function and* $1 \le p < 2$, $r \ge 1$*. If* $Y_1 = 0$ *and* $E[|Y_1|^{rp} h(|Y_1|^p)] < \infty$*, then*

$$\sum_{n=1}^{\infty} n^{r-2} h(n) P\left(|S_n| \ge \varepsilon n^{1/p}\right) < \infty, \quad \forall \varepsilon > 0. \tag{1.2}$$

Theorem B. *Suppose that* $\{Y_i, -\infty < i < \infty\}$ *is a sequence of identically distributed* φ*-mixing random variables with* $EY_1 = 0$, $EY_1^2 < \infty$ *and* $\sum_{m=1}^{\infty} \varphi^{1/2}(m) < \infty$ *and* $\{X_n, n \ge 1\}$ *is as in* (1.1)*. Let* $h(x) > 0$ $(x > 0)$ *be a slowly varying function and* $1 \le p < 2$, $r > 1$*. If* $E[|Y_1|^{rp} h(|Y_1|^p)] < \infty$*, then*

$$\sum_{n=1}^{\infty} n^{r-2-1/p} h(n) E\left(|S_n| - \varepsilon n^{1/p}\right)^{+} < \infty, \quad \forall \varepsilon > 0, \tag{1.3}$$

where $x^+ = \max\{x, 0\}$.

Chen et al. [7] and Zhou [8] also studied the limit behavior of moving average process under φ-mixing assumption. Yang et al. [9] investigated the moving average process for AANA sequence. For more works on complete convergence, one can refer to [3–6, 10–13] and the references therein.

Recall that the sequence $\{X_n, n \ge 1\}$ is stochastically dominated by a nonnegative random variable X if

$$\sup_{n \ge 1} P(|X_n| > t) \le C P(X > t) \quad \text{for some positive constant } C,\ \forall t \ge 0. \tag{1.4}$$

Recently, Chen and Li [14] investigated the limit behavior of moving process under martingale difference sequences. They obtained the following theorems.

Theorem C. *Let* $r \ge 1$, $1 \le p < 2$ *and* $rp < 2$*. Assume that* $\{X_n, n \ge 1\}$ *is a moving average process defined in* (1.1)*, where* $\{Y_i, \mathcal{F}_i, -\infty < i < \infty\}$ *is a martingale difference related to an increasing sequence of* σ*-fields* $\mathcal{F}_i$ *and stochastically dominated by a nonnegative random variable* Y*. If* $E[Y^{rp} + Y \log(1+Y)] < \infty$*, then for every* $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1 \le k \le n} |S_k| > \varepsilon n^{1/p}\right) < \infty. \tag{1.5}$$

Theorem D. *Let* $r \ge 1$, $1 \le p < 2$, $rp < 2$ *and* $0 < q < 2$*. Assume that* $\{X_n, n \ge 1\}$ *is a moving average process defined in* (1.1)*, where* $\{Y_i, \mathcal{F}_i, -\infty < i < \infty\}$ *is a martingale difference related to an increasing sequence of* σ*-fields* $\mathcal{F}_i$ *and stochastically dominated by a nonnegative random variable* Y*. If*

$$\begin{cases} E\left[Y^{rp} + Y \log(1+Y)\right] < \infty, & \text{if } q < rp, \\ E\left[Y^{rp} \log(1+Y) + Y \log^2(1+Y)\right] < \infty, & \text{if } q = rp, \\ EY^q < \infty, & \text{if } q > rp, \end{cases} \tag{1.6}$$

then for every $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} n^{r-2-q/p} E\left(\max_{1\le k\le n} |S_k| - \varepsilon n^{1/p}\right)_+^q < \infty, \tag{1.7}$$

where $x_+ = x$ *when* $x > 0$ *and* $x_+ = 0$ *when* $x \le 0$ *and* $x_+^q = (x_+)^q$.

Inspired by Chen and Li [14], Chen et al. [7], Sung [13] and other papers above, we go on to investigate the limit behavior of moving process under martingale difference sequence and obtain some similar results of Theorems C and D, but we only need some simple conditions. Our results extend some results of Chen and Li [14] (see Remark 3.3 in Section 3). Two lemmas and two theorems are given in Sections 2 and 3, respectively. The proofs of theorems are presented in Section 4.

For various results of martingales, one can refer to Chow [15], Hall and Heyde [16], Yu [17], Ghosal and Chandra [18], and so forth. As an application of moving average process based on martingale differences, we can refer to [19–22] and the references therein. Throughout the paper, $I(A)$ is the indicator function of set A, $x^+ = \max\{x, 0\}$ and C, C_1, $C_2, \ldots$ denote some positive constants not depending on n, which may be different in various places.

2. Two Lemmas

The following lemmas are our basic techniques to prove our results.

Lemma 2.1 (cf. Hall and Heyde [16, Theorem 2.11]). *If* $\{X_i, \mathcal{F}_i, 1 \le i \le n\}$ *is a martingale difference and* $p > 0$, *then there exists a constant* C *depending only on* p *such that*

$$E\left(\max_{1\le k\le n}\left|\sum_{i=1}^{k} X_i\right|^p\right) \le C\left\{E\left(\sum_{i=1}^{n} E\left(X_i^2 \mid \mathcal{F}_{i-1}\right)\right)^{p/2} + E\left(\max_{1\le i\le n}|X_i|^p\right)\right\}. \tag{2.1}$$

Lemma 2.2 (cf. Wu [23, Lemma 4.1.6]). *Let* $\{X_n, n \ge 1\}$ *be a sequence of random variables, which is stochastically dominated by a nonnegative random variable* X. *Then for any* $a > 0$ *and* $b > 0$, *the following two statements hold:*

$$\begin{aligned} E\left[|X_n|^a I(|X_n| \le b)\right] &\le C_1\{E[X^a I(X \le b)] + b^a P(X > b)\}, \\ E\left[|X_n|^a I(|X_n| > b)\right] &\le C_2 E[X^a I(X > b)], \end{aligned} \tag{2.2}$$

where C_1 *and* C_2 *are positive constants.*

3. Main Results

Theorem 3.1. *Let* $r > 1$ *and* $1 \le p < 2$. *Assume that* $\{X_n, n \ge 1\}$ *is a moving average processes defined in* (1.1), *where* $\{Y_i, \mathcal{F}_i, -\infty < i < \infty\}$ *is a martingale difference related to an increasing sequence of* σ*-fields* $\mathcal{F}_i$ *and stochastically dominated by a nonnegative random variable* Y. *Let* K *be a* *constant. Suppose that* $EY^{rp} < \infty$ *for* $rp > 1$ *and* $\sup_i E(|Y_i|^{rp} \mid \mathcal{F}_{i-1}) \le K$ *almost surely (a.s.), if* $rp \ge 2$. *Then for every* $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\le k\le n} |S_k| > \varepsilon n^{1/p}\right) < \infty, \tag{3.1}$$

$$\sum_{n=1}^{\infty} n^{r-2} P\left(\sup_{k\ge n} \left|\frac{S_k}{k^{1/p}}\right| > \varepsilon\right) < \infty. \tag{3.2}$$

Theorem 3.2. *Let the conditions of Theorem 3.1 hold. Then for every* $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} n^{r-2-1/p} E\left(\max_{1\le k\le n} |S_k| - \varepsilon n^{1/p}\right)^{+} < \infty, \tag{3.3}$$

$$\sum_{n=1}^{\infty} n^{r-2} E\left(\sup_{k\ge n} \left|\frac{S_k}{k^{1/p}}\right| - \varepsilon\right)^{+} < \infty. \tag{3.4}$$

Remark 3.3. Let $\mathcal{F}_0 \subset \mathcal{F}_1 \subset \ldots$ be an increasing family of σ-algebras and $\{(X_n, \mathcal{F}_n), n \ge 1\}$ be a sequence of martingale differences. Assume that for some $p \ge 2$,

$$E\left(|X_n|^p \mid \mathcal{F}_{n-1}\right) \le K, \quad \text{a.s.}, \tag{3.5}$$

where K is a constant not depending on n, and other conditions are satisfied, Yu [17] investigated the complete convergence of weighted sums of martingale differences. On the other hand, under the condition

$$\sup_{n,k} E\left(X_{n,k}^2 \mid \mathcal{F}_{n,k-1}\right) \le K, \quad \text{a.s.}, \tag{3.6}$$

and other conditions, Ghosal and Chandra [18] obtained the complete convergence of martingale arrays. Thus, if $rp \ge 2$, our assumption $\sup_i E(|Y_i|^{rp} \mid \mathcal{F}_{i-1}) \le K$, a.s., is reasonable. Chen and Li [14] obtained Theorems C and D for the case $1 \le rp < 2$. We go on to investigate this moving average process for the case $rp > 1$, especially for the case $rp \ge 2$ and get the results of (3.1)–(3.4). If $E[Y^{rp} + Y\log(1+Y)] < \infty$ for $r > 1$, $1 \le p < 2$ and $rp < 2$, result (3.1) follows from Theorem C (see Theorem 1.1 of Chen and Li), but we can obtain results (3.1) and (3.2) under weaker condition $EY^{rp} < \infty$. On the other hand, comparing with the conditions of Theorem D, our conditions of Theorem 3.2 are relatively simple.

4. The Proofs of Main Results

Proof of Theorem 3.1. First, we show that the moving average process (1.1) converges a.s. under the conditions of Theorem 3.1. Since $rp > 1$, it has $EY < \infty$, following from $EY^{rp} < \infty$. On the other hand, applying Lemma 2.2 with $a = 1$ and $b = 1$, one has

$$E|Y_i| \le 1 + C_2 E[YI(Y > 1)] \le 1 + C_2 EY < \infty, \quad -\infty < i < \infty. \tag{4.1}$$

Consequently, we have by $\sum_{i=-\infty}^{\infty}|a_i|<\infty$ that

$$\sum_{i=-\infty}^{\infty} E|a_i Y_{i+n}| \le C_3 \sum_{i=-\infty}^{\infty}|a_i| < \infty, \tag{4.2}$$

which implies $\sum_{i=-\infty}^{\infty} a_i Y_{i+n}$ converges a.s.

Note that

$$S_n = \sum_{k=1}^{n} X_k = \sum_{k=1}^{n}\sum_{i=-\infty}^{\infty} a_i Y_{i+k} = \sum_{i=-\infty}^{\infty} a_i \sum_{k=i+1}^{i+n} Y_k. \tag{4.3}$$

Let

$$Y_{nj} = Y_j I\left(\left|Y_j\right| \le n^{1/p}\right) - E\left[Y_j I\left(\left|Y_j\right| \le n^{1/p}\right) \mid \mathcal{F}_{j-1}\right], \quad -\infty < j < \infty. \tag{4.4}$$

Since $Y_j = Y_j I(|Y_j| > n^{1/p}) + Y_{nj} + E[Y_j I(|Y_j| \le n^{1/p}) \mid \mathcal{F}_{j-1}]$, we can see that

$$\begin{aligned}
&\sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\le k\le n}|S_k| > \varepsilon n^{1/p}\right)\\
&\quad\le \sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\le k\le n}\left|\sum_{i=-\infty}^{\infty} a_i \sum_{j=i+1}^{i+k} Y_j I\left(\left|Y_j\right| > n^{1/p}\right)\right| > \frac{\varepsilon n^{1/p}}{2}\right)\\
&\qquad+ \sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\le k\le n}\left|\sum_{i=-\infty}^{\infty} a_i \sum_{j=i+1}^{i+k} E\left[Y_j I\left(\left|Y_j\right| \le n^{1/p}\right) \mid \mathcal{F}_{j-1}\right]\right| > \frac{\varepsilon n^{1/p}}{4}\right)\\
&\qquad+ \sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\le k\le n}\left|\sum_{i=-\infty}^{\infty} a_i \sum_{j=i+1}^{i+k} Y_{nj}\right| > \frac{\varepsilon n^{1/p}}{4}\right)\\
&\quad=: H + I + J.
\end{aligned} \tag{4.5}$$

For H, by Markov's inequality, Lemma 2.2, $\sum_{i=-\infty}^{\infty}|a_i|<\infty$ and $EY^{rp}<\infty$, one has

$$\begin{aligned}
H &\le \frac{2}{\varepsilon}\sum_{n=1}^{\infty} n^{r-2} n^{-1/p} E\left(\max_{1\le k\le n}\left|\sum_{i=-\infty}^{\infty} a_i \sum_{j=i+1}^{i+k} Y_j I\left(\left|Y_j\right| > n^{1/p}\right)\right|\right)\\
&\le C_1 \sum_{n=1}^{\infty} n^{r-2} n^{-1/p} \sum_{i=-\infty}^{\infty}|a_i| E\left(\max_{1\le k\le n}\sum_{j=i+1}^{i+k}\left|Y_j\right| I\left(\left|Y_j\right| > n^{1/p}\right)\right)
\end{aligned}$$

$$
\begin{aligned}
&\leq C_2\sum_{n=1}^{\infty}n^{r-1-1/p}E\left[YI\left(Y>n^{1/p}\right)\right]\\
&= C_2\sum_{n=1}^{\infty}n^{r-1-1/p}\sum_{m=n}^{\infty}E[YI(m<Y^p\leq m+1)]\\
&= C_2\sum_{m=1}^{\infty}E[YI(m<Y^p\leq m+1)]\sum_{n=1}^{m}n^{r-1-1/p}\\
&\leq C_3\sum_{m=1}^{\infty}m^{r-1/p}E[YI(m<Y^p\leq m+1)]\leq C_4E|Y|^{rp}<\infty.
\end{aligned}
\tag{4.6}
$$

Meanwhile, by the martingale property, Lemma 2.2 and the proof of (4.6), it follows that

$$
\begin{aligned}
I &\leq \frac{4}{\varepsilon}\sum_{n=1}^{\infty}n^{r-2}n^{-1/p}E\left(\max_{1\leq k\leq n}\left|\sum_{i=-\infty}^{\infty}a_i\sum_{j=i+1}^{i+k}E\left[Y_jI\left(\left|Y_j\right|\leq n^{1/p}\right)\mid \mathcal{F}_{j-1}\right]\right|\right)\\
&= \frac{4}{\varepsilon}\sum_{n=1}^{\infty}n^{r-2}n^{-1/p}E\left(\max_{1\leq k\leq n}\left|\sum_{i=-\infty}^{\infty}a_i\sum_{j=i+1}^{i+k}E\left[Y_jI\left(\left|Y_j\right|> n^{1/p}\right)\mid \mathcal{F}_{j-1}\right]\right|\right)\\
&\leq C_1\sum_{n=1}^{\infty}n^{r-2}n^{-1/p}\sum_{i=-\infty}^{\infty}|a_i|\sum_{j=i+1}^{i+n}E\left[\left|Y_j\right|I\left(\left|Y_j\right|>n^{1/p}\right)\right]\\
&\leq C_2\sum_{n=1}^{\infty}n^{r-1-1/p}E\left[YI\left(Y>n^{1/p}\right)\right]\leq C_3E|Y|^{rp}<\infty.
\end{aligned}
\tag{4.7}
$$

Obviously, one can find that $\{Y_{nj}, \mathcal{F}_{j-1}, -\infty<j<\infty\}$ is a martingale difference. So, by Markov's inequality, Hölder's inequality, and Lemma 2.1, we get that for any $q\geq 2$,

$$
\begin{aligned}
J &\leq \left(\frac{4}{\varepsilon}\right)^q\sum_{n=1}^{\infty}n^{r-2}n^{-q/p}E\left\{\max_{1\leq k\leq n}\left|\sum_{i=-\infty}^{\infty}a_i\sum_{j=i+1}^{i+k}Y_{nj}\right|^q\right\}\\
&\leq \left(\frac{4}{\varepsilon}\right)^q\sum_{n=1}^{\infty}n^{r-2}n^{-q/p}E\left\{\sum_{i=-\infty}^{\infty}\left(|a_i|^{1-1/q}\right)\left(|a_i|^{1/q}\max_{1\leq k\leq n}\left|\sum_{j=i+1}^{i+k}Y_{nj}\right|\right)\right\}^q\\
&\leq \left(\frac{4}{\varepsilon}\right)^q\sum_{n=1}^{\infty}n^{r-2}n^{-q/p}\left(\sum_{i=-\infty}^{\infty}|a_i|\right)^{q-1}\sum_{i=-\infty}^{\infty}|a_i|E\left\{\max_{1\leq k\leq n}\left|\sum_{j=i+1}^{i+k}Y_{nj}\right|^q\right\}
\end{aligned}
$$

$$
\begin{aligned}
&\le C_1 \sum_{n=1}^{\infty} n^{r-2} n^{-q/p} \sum_{i=-\infty}^{\infty} |a_i| E\left(\sum_{j=i+1}^{i+n} E\left(Y_{nj}^2 \mid \mathcal{F}_{j-1}\right) \right)^{q/2} \\
&\quad + C_1 \sum_{n=1}^{\infty} n^{r-2} n^{-q/p} \sum_{i=-\infty}^{\infty} |a_i| \sum_{j=i+1}^{i+n} E\left|Y_{nj}\right|^q \\
&=: C_1 J_1 + C_1 J_2.
\end{aligned}
\tag{4.8}
$$

If $rp \ge 2$, then we take q large enough such that $q > \max\{(r-1)/(1/p - 1/2), rp\}$. From $\sup_j E(|Y_j|^{rp} \mid \mathcal{F}_{j-1}) \le K$, a.s. and Jensen's inequality for conditional expectation, we have $\sup_j E(Y_j^2 \mid \mathcal{F}_{j-1}) \le K^{2/(rp)}$, a.s. On the other hand,

$$
\begin{aligned}
E\left(Y_{nj}^2 \mid \mathcal{F}_{j-1}\right) &= E\left[Y_j^2 I\left(\left|Y_j\right| \le n^{1/p}\right) \mid \mathcal{F}_{j-1}\right] - \left[E\left(\left|Y_j\right| I\left(\left|Y_j\right| \le n^{1/p}\right) \mid \mathcal{F}_{j-1}\right)\right]^2 \\
&\le E\left[Y_j^2 I\left(\left|Y_j\right| \le n^{1/p}\right) \mid \mathcal{F}_{j-1}\right], \quad \text{a.s.}, \ -\infty < j < \infty.
\end{aligned}
\tag{4.9}
$$

Consequently, we obtain by $\sum_{i=-\infty}^{\infty} |a_i| < \infty$ that

$$
\begin{aligned}
J_1 &\le C_1 \sum_{n=1}^{\infty} n^{r-2} n^{-q/p} \sum_{i=-\infty}^{\infty} |a_i| E\left(\sum_{j=i+1}^{i+n} E\left[Y_j^2 I\left(\left|Y_j\right| \le n^{1/p}\right) \mid \mathcal{F}_{j-1}\right] \right)^{q/2} \\
&\le C_2 \sum_{n=1}^{\infty} n^{r-2-q/p+q/2} < \infty,
\end{aligned}
\tag{4.10}
$$

following from the fact that $q > (r-1)/(1/p - 1/2)$. Meanwhile, by C_r inequality, Lemma 2.2 and $\sum_{i=-\infty}^{\infty} |a_i| < \infty$,

$$
\begin{aligned}
J_2 &\le C_4 \sum_{n=1}^{\infty} n^{r-2} n^{-q/p} \sum_{i=-\infty}^{\infty} |a_i| \sum_{j=i+1}^{i+n} E\left[\left|Y_j\right|^q I\left(\left|Y_j\right| \le n^{1/p}\right)\right] \\
&\le C_5 \sum_{n=1}^{\infty} n^{r-1-q/p} E\left[Y^q I\left(Y \le n^{1/p}\right)\right] + C_6 \sum_{n=1}^{\infty} n^{r-1} P\left(Y > n^{1/p}\right) \\
&\le C_5 \sum_{n=1}^{\infty} n^{r-1-q/p} E\left[Y^q I\left(Y \le n^{1/p}\right)\right] + C_6 \sum_{n=1}^{\infty} n^{r-1-1/p} E\left[Y I\left(Y > n^{1/p}\right)\right] \\
&=: C_5 J_{21} + C_6 J_{22}.
\end{aligned}
\tag{4.11}
$$

Since $q > rp$ and $EY^{rp} < \infty$, one has

$$\begin{aligned} J_{21} &= \sum_{n=1}^{\infty} n^{r-1-q/p} \sum_{i=1}^{n} E\left[Y^q I\left((i-1)^{1/p} < Y \le i^{1/p}\right)\right] \\ &= \sum_{i=1}^{\infty} E\left[Y^q I\left((i-1)^{1/p} < Y \le i^{1/p}\right)\right] \sum_{n=i}^{\infty} n^{r-1-q/p} \\ &\le C_1 \sum_{i=1}^{\infty} E\left[Y^{rp} Y^{q-rp} I\left((i-1)^{1/p} < Y \le i^{1/p}\right)\right] i^{r-q/p} \le C_1 EY^{rp} < \infty. \end{aligned} \tag{4.12}$$

By the proof of (4.6),

$$J_{22} = \sum_{n=1}^{\infty} n^{r-1-1/p} E\left[Y I\left(Y > n^{1/p}\right)\right] \le CEY^{rp} < \infty. \tag{4.13}$$

If $rp < 2$, then we take $q = 2$. Similar to the proofs of (4.8), and (4.11), it has

$$\begin{aligned} J &\le C_1 \sum_{n=1}^{\infty} n^{r-2} n^{-2/p} \sum_{i=-\infty}^{\infty} |a_i| \sum_{j=i+1}^{i+n} EY_{nj}^2 \\ &\le C_2 \sum_{n=1}^{\infty} n^{r-2} n^{-2/p} \sum_{i=-\infty}^{\infty} |a_i| \sum_{j=i+1}^{i+n} E\left[Y_j^2 I\left(\left|Y_j\right| \le n^{1/p}\right)\right] \le C_3 EY^{rp} < \infty, \end{aligned} \tag{4.14}$$

following from $q > rp$, (4.12), and (4.13). Therefore, (3.1) follows from (4.5)–(4.13) and the inequality above.

Inspired by the proof of Theorem 12.1 of Gut [24], it can be checked that

$$\begin{aligned} \sum_{n=1}^{\infty} n^{r-2} P\left(\sup_{k\ge n}\left|\frac{S_k}{k^{1/p}}\right| > 2^{2/p}\varepsilon\right) &= \sum_{m=1}^{\infty} \sum_{n=2^{m-1}}^{2^m-1} n^{r-2} P\left(\sup_{k\ge n}\left|\frac{S_k}{k^{1/p}}\right| > 2^{2/p}\varepsilon\right) \\ &\le 2^{2-r} \sum_{m=1}^{\infty} P\left(\sup_{k\ge 2^{m-1}}\left|\frac{S_k}{k^{1/p}}\right| > 2^{2/p}\varepsilon\right) \sum_{n=2^{m-1}}^{2^m-1} 2^{m(r-2)} \\ &\le 2^{2-r} \sum_{m=1}^{\infty} 2^{m(r-1)} P\left(\sup_{k\ge 2^{m-1}}\left|\frac{S_k}{k^{1/p}}\right| > 2^{2/p}\varepsilon\right) \\ &= 2^{2-r} \sum_{m=1}^{\infty} 2^{m(r-1)} P\left(\sup_{l\ge m} \max_{2^{l-1}\le k<2^l}\left|\frac{S_k}{k^{1/p}}\right| > 2^{2/p}\varepsilon\right) \end{aligned}$$

$$
\begin{aligned}
&\leq 2^{2-r}\sum_{m=1}^{\infty}2^{m(r-1)}\sum_{l=m}^{\infty}P\left(\max_{1\leq k\leq 2^l}|S_k|>\varepsilon 2^{(l+1)/p}\right)\\
&=2^{2-r}\sum_{l=1}^{\infty}P\left(\max_{1\leq k\leq 2^l}|S_k|>\varepsilon 2^{(l+1)/p}\right)\sum_{m=1}^{l}2^{m(r-1)}\\
&\leq C_1\sum_{l=1}^{\infty}2^{l(r-1)}P\left(\max_{1\leq k\leq 2^l}|S_k|>\varepsilon 2^{(l+1)/p}\right)\\
&:=D.
\end{aligned}
\tag{4.15}
$$

If $r<2$, then

$$
\begin{aligned}
D&=2^{2-r}C_1\sum_{l=1}^{\infty}\sum_{n=2^l}^{2^{l+1}-1}2^{(l+1)(r-2)}P\left(\max_{1\leq k\leq 2^l}|S_k|>\varepsilon 2^{(l+1)/p}\right)\\
&\leq 2^{2-r}C_1\sum_{l=1}^{\infty}\sum_{n=2^l}^{2^{l+1}-1}n^{r-2}P\left(\max_{1\leq k\leq n}|S_k|>\varepsilon n^{1/p}\right)\\
&\leq 2^{2-r}C_1\sum_{n=1}^{\infty}n^{r-2}P\left(\max_{1\leq k\leq n}|S_k|>\varepsilon n^{1/p}\right).
\end{aligned}
\tag{4.16}
$$

Otherwise,

$$
\begin{aligned}
D&=C_1\sum_{l=1}^{\infty}\sum_{n=2^l}^{2^{l+1}-1}2^{l(r-2)}P\left(\max_{1\leq k\leq 2^l}|S_k|>\varepsilon 2^{(l+1)/p}\right)\\
&\leq C_1\sum_{l=1}^{\infty}\sum_{n=2^l}^{2^{l+1}-1}n^{r-2}P\left(\max_{1\leq k\leq n}|S_k|>\varepsilon n^{1/p}\right)\\
&\leq C_1\sum_{n=1}^{\infty}n^{r-2}P\left(\max_{1\leq k\leq n}|S_k|>\varepsilon n^{1/p}\right).
\end{aligned}
\tag{4.17}
$$

Combining (3.1) with these inequalities above, we obtain (3.2) immediately. □

Proof of Theorem 3.2. For all $\varepsilon>0$, it has

$$
\begin{aligned}
\sum_{n=1}^{\infty}n^{r-2-1/p}E\left(\max_{1\leq k\leq n}|S_k|-\varepsilon n^{1/p}\right)^{+}&=\sum_{n=1}^{\infty}n^{r-2-1/p}\int_0^{\infty}P\left(\max_{1\leq k\leq n}|S_k|-\varepsilon n^{1/p}>t\right)dt\\
&=\sum_{n=1}^{\infty}n^{r-2-1/p}\int_0^{n^{1/p}}P\left(\max_{1\leq k\leq n}|S_k|-\varepsilon n^{1/p}>t\right)dt\\
&\quad+\sum_{n=1}^{\infty}n^{r-2-1/p}\int_{n^{1/p}}^{\infty}P\left(\max_{1\leq k\leq n}|S_k|-\varepsilon n^{1/p}>t\right)dt
\end{aligned}
$$

$$\leq \sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\leq k\leq n} |S_k| > \varepsilon n^{1/p}\right) + \sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} P\left(\max_{1\leq k\leq n} |S_k| > t\right) dt. \tag{4.18}$$

By Theorem 3.1, in order to proof (3.3), we only have to show that

$$\sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} P\left(\max_{1\leq k\leq n} |S_k| > t\right) dt \ < \infty. \tag{4.19}$$

For $t > 0$, denote

$$Y_{tj} = Y_j I(|Y_j| \leq t) - E[Y_j I(|Y_j| \leq t) \mid \mathcal{F}_{j-1}], \quad -\infty < j < \infty. \tag{4.20}$$

Since $Y_j = Y_j I(|Y_j| > t) + Y_{tj} + E[Y_j I(|Y_j| \leq t) \mid \mathcal{F}_{j-1}]$, it is easy to see that

$$\begin{aligned}
&\sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} P\left(\max_{1\leq k\leq n} |S_k| > t\right) dt \\
&\quad\leq \sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} P\left(\max_{1\leq k\leq n} \left|\sum_{i=-\infty}^{\infty} a_i \sum_{j=i+1}^{i+k} Y_j I(|Y_j| > t)\right| > \frac{t}{2}\right) dt \\
&\qquad+ \sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} P\left(\max_{1\leq k\leq n} \left|\sum_{i=-\infty}^{\infty} a_i \sum_{j=i+1}^{i+k} Y_{tj}\right| > \frac{t}{4}\right) dt \\
&\qquad+ \sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} P\left(\max_{1\leq k\leq n} \left|\sum_{i=-\infty}^{\infty} a_i \sum_{j=i+1}^{i+k} E[Y_j I(|Y_j| \leq t) \mid \mathcal{F}_{j-1}]\right| > \frac{t}{4}\right) dt \\
&\quad=: I_1 + I_2 + I_3.
\end{aligned} \tag{4.21}$$

By Markov's inequality, Lemma 2.2 and $EY^{rp} < \infty$,

$$\begin{aligned}
I_1 &\leq 2\sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} t^{-1} E\left(\max_{1\leq k\leq n} \left|\sum_{i=-\infty}^{\infty} a_i \sum_{j=i+1}^{i+k} Y_j I(|Y_j| > t)\right|\right) dt \\
&\leq 2\sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} t^{-1} \sum_{i=-\infty}^{\infty} |a_i| E\left(\max_{1\leq k\leq n} \sum_{j=i+1}^{i+k} |Y_j| I(|Y_j| > t)\right) dt \\
&\leq C_1 \sum_{n=1}^{\infty} n^{r-1-1/p} \int_{n^{1/p}}^{\infty} t^{-1} E[Y I(Y > t)] dt
\end{aligned}$$

$$
\begin{aligned}
&= C_1 \sum_{n=1}^{\infty} n^{r-1-1/p} \sum_{m=n}^{\infty} \int_{m^{1/p}}^{(m+1)^{1/p}} t^{-1} E\left[Y I\left(Y > m^{1/p}\right)\right] dt \\
&\leq C_2 \sum_{n=1}^{\infty} n^{r-1-1/p} \sum_{m=n}^{\infty} m^{1/p-1-1/p} E\left[Y I\left(Y > m^{1/p}\right)\right] \\
&= C_2 \sum_{m=1}^{\infty} m^{-1} E\left[Y I\left(Y > m^{1/p}\right)\right] \sum_{n=1}^{m} n^{r-1-1/p} \\
&\leq C_3 \sum_{m=1}^{\infty} m^{r-1-1/p} E\left[Y I\left(Y > m^{1/p}\right)\right] \leq C_4 E Y^{rp} < \infty.
\end{aligned}
\tag{4.22}
$$

Since $\{Y_{tj}, \mathcal{F}_{j-1}, -\infty < j < \infty\}$ is a martingale difference, we have by Markov's inequality, Hölder's inequality, and Lemma 2.1 that for any $q \geq 2$,

$$
\begin{aligned}
I_2 &\leq 4^q \sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} t^{-q} E\left(\max_{1\leq k\leq n} \left| \sum_{i=-\infty}^{\infty} a_i \sum_{j=i+1}^{i+k} Y_{tj} \right|^q \right) dt \\
&\leq C \sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} t^{-q} E\left\{ \sum_{i=-\infty}^{\infty} \left(|a_i|^{1-1/q} \right) \left(|a_i|^{1/q} \max_{1\leq k\leq n} \left| \sum_{j=i+1}^{i+k} Y_{tj} \right| \right) \right\}^q dt \\
&\leq C \sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} t^{-q} \left(\sum_{i=-\infty}^{\infty} |a_i| \right)^{q-1} \sum_{i=-\infty}^{\infty} |a_i| E\left\{ \max_{1\leq k\leq n} \left| \sum_{j=i+1}^{i+k} Y_{tj} \right|^q \right\} dt \\
&\leq C_1 \sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} t^{-q} \sum_{i=-\infty}^{\infty} |a_i| E\left(\sum_{j=i+1}^{i+n} E\left(Y_{tj}^2 \mid \mathcal{F}_{j-1} \right) \right)^{q/2} dt \\
&\quad + C_1 \sum_{n=1}^{\infty} n^{r-2-1/p} \int_{n^{1/p}}^{\infty} t^{-q} \sum_{i=-\infty}^{\infty} |a_i| \sum_{j=i+1}^{i+n} E\left|Y_{tj}\right|^q dt \\
&=: C_1 I_{21} + C_1 I_{22}.
\end{aligned}
\tag{4.23}
$$

If $rp \geq 2$, then we take large enough q such that $q > \max\{(r-1)/(1/p - 1/2), rp\}$. By $\sup_j E(|Y_j|^{rp} \mid \mathcal{F}_{j-1}) \leq K$, a.s. and Jensen's inequality for conditional expectation, it has $\sup_j E(Y_j^2 \mid \mathcal{F}_{j-1}) \leq K^{2/(rp)}$, a.s.. Meanwhile,

$$
\begin{aligned}
E\left(Y_{nj}^2 \mid \mathcal{F}_{j-1}\right) &= E\left[Y_j^2 I\left(|Y_j| \leq n^{1/p}\right) \mid \mathcal{F}_{j-1}\right] - \left[E\left(|Y_j| I\left(|Y_j| \leq n^{1/p}\right) \mid \mathcal{F}_{j-1}\right)\right]^2 \\
&\leq E\left[Y_j^2 I\left(|Y_j| \leq n^{1/p}\right) \mid \mathcal{F}_{j-1}\right], \quad \text{a.s.,} \ -\infty < j < \infty.
\end{aligned}
\tag{4.24}
$$

Thus, by $\sum_{i=-\infty}^{\infty}|a_i|<\infty$, one has that

$$\begin{aligned}
I_{21} &= C_1\sum_{n=1}^{\infty}n^{r-2-1/p}\int_{n^{1/p}}^{\infty}t^{-q}\sum_{i=-\infty}^{\infty}|a_i|E\left(\sum_{j=i+1}^{i+n}Z_{nj}\right)^{q/2}dt \\
&\le C_1\sum_{n=1}^{\infty}n^{r-2-1/p}\int_{n^{1/p}}^{\infty}t^{-q}\sum_{i=-\infty}^{\infty}|a_i|E\left(\sum_{j=i+1}^{i+n}2E\left[Y_j^2 I\left(\left|Y_j\right|\le n^{1/p}\right)\mid \mathcal{F}_{j-1}\right]\right)^{q/2}dt \\
&\le C_2\sum_{n=1}^{\infty}n^{r-2-1/p+q/2}\int_{n^{1/p}}^{\infty}t^{-q}dt\le C_3\sum_{n=1}^{\infty}n^{r-2-1/p+q/2}\cdot n^{(-q+1)/p} \\
&= C_3\sum_{n=1}^{\infty}n^{r-2+q/2-q/p}<\infty,
\end{aligned}\tag{4.25}$$

following from the fact that $q>(r-1)/(1/p-1/2)$. We also have by C_r inequality and Lemma 2.2 that

$$\begin{aligned}
I_{22} &\le C_2\sum_{n=1}^{\infty}n^{r-2-1/p}\int_{n^{1/p}}^{\infty}t^{-q}\sum_{i=-\infty}^{\infty}|a_i|\sum_{j=i+1}^{i+n}E\left[\left|Y_j\right|^q I\left(\left|Y_j\right|\le t\right)\right]dt \\
&\le C_3\sum_{n=1}^{\infty}n^{r-1-1/p}\sum_{m=n}^{\infty}\int_{m^{1/p}}^{(m+1)^{1/p}}t^{-q}E[Y^q I(Y\le t)]dt \\
&\quad + C_4\sum_{n=1}^{\infty}n^{r-1-1/p}\int_{n^{1/p}}^{\infty}P(Y>t)dt \\
&=: C_2 I_{22}^{*}+C_3 I_{22}^{**}.
\end{aligned}\tag{4.26}$$

Since $q>rp$ and $EY^{rp}<\infty$, it follows that

$$\begin{aligned}
I_{22}^{*} &\le C_1\sum_{n=1}^{\infty}n^{r-1-1/p}\sum_{m=n}^{\infty}m^{1/p-1-q/p}E\left[Y^q I\left(Y\le (m+1)^{1/p}\right)\right] \\
&= C_1\sum_{m=1}^{\infty}m^{1/p-1-q/p}E\left[Y^q I\left(Y\le (m+1)^{1/p}\right)\right]\sum_{n=1}^{m}n^{r-1-1/p} \\
&\le C_2\sum_{m=1}^{\infty}m^{r-1-q/p}E\left[Y^q I\left(Y\le (m+1)^{1/p}\right)\right] \\
&= C_2\sum_{m=1}^{\infty}m^{r-1-q/p}\sum_{i=1}^{m+1}E\left[Y^q I\left((i-1)^{1/p}<Y\le i^{1/p}\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&= C_2\sum_{m=1}^{\infty} m^{r-1-q/p} E\left[Y^{rp}Y^{q-rp} I\left(m^{1/p} < Y \le (m+1)^{1/p}\right)\right] \\
&\quad + C_2\sum_{m=1}^{\infty} m^{r-1-q/p}\sum_{i=1}^{m} E\left[Y^{q} I\left((i-1)^{1/p} < Y \le i^{1/p}\right)\right] \\
&\le 2^{(q-rp)/p} C_2\sum_{m=1}^{\infty} m^{-1} E\left[Y^{rp} I\left(m^{1/p} < Y \le (m+1)^{1/p}\right)\right] \\
&\quad + C_2\sum_{i=1}^{\infty} E\left[Y^{q} I\left((i-1)^{1/p} < Y \le i^{1/p}\right)\right]\sum_{m=i}^{\infty} m^{r-1-q/p} \\
&\le 2^{(q-rp)/p} C_2\sum_{m=1}^{\infty} m^{-1} E\left[Y^{rp} I\left(m^{1/p} < Y \le (m+1)^{1/p}\right)\right] \\
&\quad + C_2\sum_{i=1}^{\infty} E\left[Y^{rp}Y^{q-rp} I\left((i-1)^{1/p} < Y \le i^{1/p}\right)\right] i^{r-q/p} \\
&\le C_3 EY^{rp} < \infty.
\end{aligned}
\tag{4.27}
$$

From the proof of (4.22),

$$
I_{22}^{**} \le \sum_{n=1}^{\infty} n^{r-1-1/p}\int_{n^{1/p}}^{\infty} t^{-1} EYI(Y > t)\,dt \le CEY^{rp} < \infty. \tag{4.28}
$$

If $rp < 2$, then we take $q = 2$. Similar to the proofs of (4.23) and (4.26), we get that

$$
I_2 \le C_1\sum_{n=1}^{\infty} n^{r-2-1/p}\int_{n^{1/p}}^{\infty} t^{-2}\sum_{i=-\infty}^{\infty} |a_i| \sum_{j=i+1}^{i+n} EY_{tj}^2\,dt \le C_2 EY^{rp} < \infty, \tag{4.29}
$$

following from $q > rp$, (4.27) and (4.28). Consequently, by (4.18)–(4.28), Theorem 3.1 and inequality above, (3.3) holds true.

Now, we turn to prove (3.4). Similar to the proof of (3.2), we have that

$$
\begin{aligned}
&\sum_{n=1}^{\infty} n^{r-2} E\left(\sup_{k\ge n}\left|\frac{S_k}{k^{1/p}}\right| - \varepsilon 2^{2/p}\right)^{+} \\
&\quad = \sum_{n=1}^{\infty} n^{r-2}\int_0^{\infty} P\left(\sup_{k\ge n}\left|\frac{S_k}{k^{1/p}}\right| > \varepsilon 2^{2/p} + t\right) dt \\
&\quad = \sum_{m=1}^{\infty}\sum_{n=2^{m-1}}^{2^m-1} n^{r-2}\int_0^{\infty} P\left(\sup_{k\ge n}\left|\frac{S_k}{k^{1/p}}\right| > \varepsilon 2^{2/p} + t\right) dt
\end{aligned}
$$

$$\begin{aligned}
&\leq 2^{2-r}\sum_{m=1}^{\infty}\int_0^{\infty}P\left(\sup_{k\geq 2^{m-1}}\left|\frac{S_k}{k^{1/p}}\right|>\varepsilon 2^{2/p}+t\right)dt\sum_{n=2^{m-1}}^{2^m-1}2^{m(r-2)}\\
&\leq 2^{2-r}\sum_{m=1}^{\infty}2^{m(r-1)}\int_0^{\infty}P\left(\sup_{k\geq 2^{m-1}}\left|\frac{S_k}{k^{1/p}}\right|>\varepsilon 2^{2/p}+t\right)dt\\
&= 2^{2-r}\sum_{m=1}^{\infty}2^{m(r-1)}\int_0^{\infty}P\left(\sup_{l\geq m}\max_{2^{l-1}\leq k<2^l}\left|\frac{S_k}{k^{1/p}}\right|>\varepsilon 2^{2/p}+t\right)dt\\
&\leq 2^{2-r}\sum_{m=1}^{\infty}2^{m(r-1)}\sum_{l=m}^{\infty}\int_0^{\infty}P\left(\max_{1\leq k\leq 2^l}|S_k|>\left(\varepsilon 2^{2/p}+t\right)2^{(l-1)/p}\right)dt\\
&= 2^{2-r}\sum_{l=1}^{\infty}\int_0^{\infty}P\left(\max_{1\leq k\leq 2^l}|S_k|>\left(\varepsilon 2^{2/p}+t\right)2^{(l-1)/p}\right)dt\sum_{m=1}^{l}2^{m(r-1)}\\
&\leq 2^{2-r}\sum_{l=1}^{\infty}2^{l(r-1)}\int_0^{\infty}P\left(\max_{1\leq k\leq 2^l}|S_k|>\left(\varepsilon 2^{2/p}+t\right)2^{(l-1)/p}\right)dt\quad\left(\text{let } s=2^{(l-1)/p}t\right)\\
&\leq C_1\sum_{l=1}^{\infty}2^{l(r-1-1/p)}\int_0^{\infty}P\left(\max_{1\leq k\leq 2^l}|S_k|>\varepsilon 2^{(l+1)/p}+s\right)ds := F.
\end{aligned}\tag{4.30}$$

If $r<2+1/p$, then

$$\begin{aligned}
F &= 2^{(2+1/p-r)}C_1\sum_{l=1}^{\infty}\sum_{n=2^l}^{2^{l+1}-1}2^{(l+1)(r-2-1/p)}\int_0^{\infty}P\left(\max_{1\leq k\leq 2^l}|S_k|>\varepsilon 2^{(l+1)/p}+s\right)ds\\
&\leq 2^{(2+1/p-r)}C_1\sum_{l=1}^{\infty}\sum_{n=2^l}^{2^{l+1}-1}n^{r-2-1/p}\int_0^{\infty}P\left(\max_{1\leq k\leq n}|S_k|>\varepsilon n^{1/p}+s\right)ds\\
&\leq 2^{(2+1/p-r)}C_1\sum_{n=1}^{\infty}n^{r-2-1/p}E\left(\max_{1\leq k\leq n}|S_k|-\varepsilon n^{1/p}\right)^{+}<\infty.
\end{aligned}\tag{4.31}$$

Otherwise,

$$\begin{aligned}
F &= C_1\sum_{l=1}^{\infty}\sum_{n=2^l}^{2^{l+1}-1}2^{l(r-2-1/p)}\int_0^{\infty}P\left(\max_{1\leq k\leq 2^l}|S_k|>\varepsilon 2^{(l+1)/p}+s\right)ds\\
&\leq C_1\sum_{l=1}^{\infty}\sum_{n=2^l}^{2^{l+1}-1}n^{r-2-1/p}\int_0^{\infty}P\left(\max_{1\leq k\leq n}|S_k|>\varepsilon n^{1/p}+s\right)ds\\
&\leq C_1\sum_{n=1}^{\infty}n^{r-2-1/p}E\left(\max_{1\leq k\leq n}|S_k|-\varepsilon n^{1/p}\right)^{+}<\infty.
\end{aligned}\tag{4.32}$$

Therefore, (3.4) holds true following from (3.3). □

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