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Research Article

Stability Property for the Predator-Free Equilibrium Point of Predator-Prey Systems with a Class of Functional Response and Prey Refuges

Fengde Chen, Yumin Wu, and Zhaozhi Ma

College of Mathematics and Computer Science, Fuzhou University, Ujian, Uzhou 350002, China

Correspondence should be addressed to Fengde Chen, fdchen@fzu.edu.cn

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We investigate the stability property for the predator-free equilibrium point of predator-prey systems with a class of functional response and prey refuges by using the analytical approach. Under some very weakly assumption, we show that conditions that ensure the locally asymptotically stable of the predator-free equilibrium point are consistent with that of the globally asymptotically stable ones. Our result supplements the corresponding result of Ma et al., 2009.

1. Introduction

Stability analysis of a predator-prey system incorporating a given functional response becomes a popular issue in mathematical ecology during the last decade [1–10]. Ma et al. [10] considered the effects of prey refuges on a predator-prey model with a class of functional response, for example,

$$\begin{aligned} \dot{X} &= rX\left(1-\frac{X}{K}\right) - p\varphi(X-\beta X)Y, \\ \dot{Y} &= (q\varphi(X-\beta X)-d)Y, \end{aligned} \tag{1.1}$$

where $X(t)$ and $Y(t)$ denote the density of prey and predator populations at time t, respectively. The term $\varphi(X)$ represents the functional response of the predator population and satisfies the following assumption:

$$\varphi(0)=0, \qquad \varphi'(X)>0 \quad (X>0). \tag{1.2}$$

By using the change of variables:

$$\phi : (R_0^+)^2 \longrightarrow (R_0^+)^2, \qquad \phi(X,Y) = \left(\frac{x}{1-\beta}, (1-\beta)y\right), \tag{1.3}$$

system (1.1) is equivalent to the following model:

$$\begin{aligned} \dot{x} &= rx\left(1 - \frac{x}{(1-\beta)K}\right) - p\varphi(x)y, \\ \dot{y} &= (q\varphi(x) - d)y. \end{aligned} \tag{1.4}$$

Concerned with the stability property of the predator-free equilibrium, by analysing the Jacobian matrix, the authors obtained the following conclusions.

Conclusion 1. $E_1((1-\beta)K, 0)$ is locally asymptotically stable if and only if $q\varphi((1-\beta)K) - d < 0$.

Based on this conclusion, without any other deduction, they declared (see [10, Theorems 4.1(3) and 4.2(3)]).

Conclusion 2. If $1 - \varphi^{-1}(d/q)/K < \beta < 1$, then predator goes extinct while prey population reaches its maximum environment carrying capacity.

Note that Conclusion 1 is local one while Conclusion 2 reflects the globally property of the system (1.4). Obviously there is a gap between these two conclusions. To show the Conclusion 1 implies Conclusion 2, some more detail analysis is needed. The aim of this paper is try to show that under some very weakly assumption on φ, the local asymptotical stability of the predator-free equilibrium point do implies the global ones. More precisely, we obtain the following result.

Theorem 1.1. *Assume that $\varphi(x) = \varphi_1(x)x$ and there exists a positive constant L such that $\varphi_1(x) \le L$ for all $x > 0$ holds, assume further that $1 - \varphi^{-1}(d/q)/K < \beta < 1$ holds. Then predator species will be extinct while prey population reaches its maximum environment carrying capacity.*

Remark 1.2. Many explicit forms for the predator functional response that have been used are satisfy above assumption. For example,

$$\begin{aligned} &\frac{bx}{a+x} \text{ [Holling type II]}, \\ &\frac{bx^2}{a+x^2} \text{ [Holling type III]}, \\ &a(1-\exp\{-cx\}) \text{ [Ivlev]}, \\ &bx^{\gamma}, \quad 0 < \gamma < 1 \text{ [Rosenzweig]}. \end{aligned} \tag{1.5}$$

2. Proof of the Main Result

Proof. We first show that under the assumption of Conclusion 2, predator species will be driven to extinction.

It follows from $1 - \varphi^{-1}(d/q)/K < \beta < 1$ and the continuity of φ, for enough small positive constant ε, the following inequality holds:

$$\beta > 1 - \frac{\varphi^{-1}((d-\varepsilon)/q)}{K+\varepsilon}, \tag{2.1}$$

that is,

$$\varphi^{-1}\left(\frac{d-\varepsilon}{q}\right) > (1-\beta)(K+\varepsilon). \tag{2.2}$$

Since $\varphi'(X) > 0$, $X > 0$, inequality (2.2) is equal to the following inequality:

$$d - \varepsilon > q\varphi((1-\beta)(K+\varepsilon)). \tag{2.3}$$

From the first equation of system (1.4), we have

$$\dot{x} \leq rx\left(1 - \frac{x}{(1-\beta)K}\right). \tag{2.4}$$

Therefore,

$$\limsup_{t\to+\infty} x(t) \leq (1-\beta)K. \tag{2.5}$$

For ε defined by (2.1), inequality (2.5) shows that there exists an enough large T such that

$$x(t) < (1-\beta)(K+\varepsilon), \qquad \forall t \geq T. \tag{2.6}$$

And so, for $t \geq T$, from the second equation of system (1.4) and (2.3), one has

$$\dot{y} \leq (q\varphi((1-\beta)(K+\varepsilon)) - d)y < -\varepsilon y. \tag{2.7}$$

That is,

$$y(t) \leq y(T)\exp\{-\varepsilon(t-T)\} \longrightarrow 0, \qquad \text{as } t \longrightarrow +\infty. \tag{2.8}$$

For any small positive constant $\varepsilon_1 > 0$ which satisfies $\varepsilon_1 \leq r/2p\mathrm{L}$, there exists a $T_1 > T$ such that

$$y(t) \leq \varepsilon_1, \qquad \forall t > T_1. \tag{2.9}$$

On the other hand, since $\varphi(x) = \varphi_1(x)x$ and $\varphi_1(x) \leq L$ for all $x > 0$. Equation (2.9) together with the first equation of (1.4) leads to

$$\begin{aligned}\dot{x} &= rx\left(1 - \frac{x}{(1-\beta)K}\right) - p\varphi_1(x)xy \\ &\geq rx\left(1 - \frac{x}{(1-\beta)K}\right)x - pL\varepsilon_1 x \\ &\geq rx\left(\frac{1}{2} - \frac{x}{(1-\beta)K}\right),\end{aligned} \tag{2.10}$$

for all $t \geq T_1$. From this different inequality, one could easily obtain that,

$$\liminf_{t\to+\infty} x(t) \geq \frac{(1-\beta)K}{2}. \tag{2.11}$$

Now we introducing a transformation $z = x - (1-\beta)k$, then the first equation of system (1.4) is equivalent to

$$\dot{z} = -\frac{rz}{(1-\beta)k}(z + (1-\beta)k) - p\varphi(z + (1-\beta)k)y. \tag{2.12}$$

From (2.5), (2.11), and (2.12) we know that $z(t)$ is bounded differentiable on $(0,\infty)$, Let $\overline{z} = \limsup_{t\to+\infty} z(t), \underline{z} = \liminf_{t\to+\infty} z(t)$, According to Fluctuation lemma [11], there exists sequences $\tau_n \to \infty$, $\sigma_n \to \infty$ such that $\dot{z}(\tau_n) \to 0$, $\dot{z}(\sigma_n) \to 0$, $z(\xi_n) \to \underline{z}$ and $z(\sigma_n) \to \overline{z}$. Also, it follows from (2.12) that

$$\begin{aligned}\dot{z}(\sigma_n) &= -\frac{rz(\sigma_n)}{(1-\beta)k}(z(\sigma_n) + (1-\beta)k) - p\varphi(z(\sigma_n) + (1-\beta)k)y(\sigma_n). \\ \dot{z}(\xi_n) &= -\frac{rz(\xi_n)}{(1-\beta)k}(z(\xi_n) + (1-\beta)k) - p\varphi(z(\xi_n) + (1-\beta)k)y(\xi_n).\end{aligned} \tag{2.13}$$

Since (2.11) implies that

$$\lim_{n\to+\infty}(z(\xi_n) + (1-\beta)k) \geq \frac{(1-\beta)K}{2}. \tag{2.14}$$

Taking limit in (2.13) and (2.14), it follows from (2.8) and (2.16) that

$$0 = \overline{z} = \underline{z}, \tag{2.15}$$

that is

$$\lim_{t\to+\infty} z(t) = 0, \tag{2.16}$$

which is equivalent to say that

$$\lim_{t \to +\infty} x(t) = (1 - \beta)K. \tag{2.17}$$

This ends the proof of Theorem 1.1. □

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