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Research Article

Life Behavior of a System under Discrete Shock Model

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We study the life behavior of a system which is subjected to shocks of random magnitudes over discrete time periods. We obtain the survival function and mean time to failure of the system assuming that the sizes of the shocks follow a discrete probability distribution under cumulative and mixed shock models.

1. Introduction

There are various engineering systems which are subjected to shocks of random magnitudes at random times. The shock models can be classified in different ways. According to the cumulative shock model, the system breaks down because of a cumulative effect of shocks, while in an extreme shock model the system fails because of one single shock with large magnitude. See, for example, [1–9] for various problems on shock models.

Most of the studies on shock models focus on the evaluation of system failure time in a continuous setup, that is, the shocks arrive according to a renewal process, and the times between successive shocks have a continuous probability distribution. Some results on discrete case are in [3, 7, 10].

Consider a system which is subjected to periodic random shocks. A shock occurs with probability p in each period $n = 1, 2, \ldots$. The period should be understood as hour, day, and so forth. The magnitude of the shock which occurs in period j is a random variable denoted by B_j. Assume that such a system fails if and only if the sum of the magnitudes of cumulative shocks exceed, the level k for $k > 0$. Let I_j be a binary random variable representing the shock occurrences that is, $I_j = 1$ if a shock occurs in period j and $I_j = 0$, otherwise. For $j \geq 1$, define

$$Y_j = \begin{cases} B_j, & I_j = 1 \\ 0, & I_j = 0, \end{cases} \tag{1.1}$$

where the random variables I_j and B_j are independent in each time period. The random variable B_j is strictly positive, and $\{B_j, j \geq 1\}$ is a sequence of independent identically distributed (i.i.d.) random variables with cumulative distribution function (c.d.f.) and probability mass function (p.m.f.) f_B.

Thus, under the cumulative shock model, the failure time of the system can be defined by the following waiting time random variable:

$$W_k = \min\left\{ n : \sum_{j=1}^{n} Y_j > k \right\}, \tag{1.2}$$

for $k > 0$.

In the case of a mixed shock model, a system fails if either a single shock with a large magnitude occurs or the sum of cumulative shocks exceeds the critical level. Thus, in this case the time to failure of the system is defined by the following compound waiting time random variable

$$Z_{k,m} = \min(W_k, T_m), \tag{1.3}$$

where

$$T_m = \min\{n : M_n > m\}, \tag{1.4}$$

where $M_n = \max(Y_1, \ldots, Y_n)$ for $k, m > 0$.

Such models can also be applied to insurance, replacing shock with claim and magnitude of the shock with claim amount. In this case, a period can be seen as a week, month, and so forth, and the random variable W_k represents the waiting time until the cumulative sum of claim amounts exceeds the level k. Similarly, the random variable T_m is the waiting time until the first extreme claim size falls above the level m.

The present paper is organized as follows. In Section 2, we derive recurrence formulae for the survival function and the mean time to failure (MTTF) of the system under the cumulative shock model. We also study two related characteristics $N(W_k)$ and $S(W_k)$ which represent, respectively, the number of shocks and the total shock that the system is subjected up to time when the system fails. Section 3 includes the results for mixed shock model.

2. Cumulative Shock Model

In the following, we derive two popular reliability characteristics: survival function and mean time to failure of the system under the cumulative shock model.

It is clear that

$$P\{W_k > n\} = P\{S(n) \leq k\} = F_Y^{*n}(k), \tag{2.1}$$

where $S(n) = Y_1 + \cdots + Y_n$, and F_Y^{*n} denotes the n-fold convolution of F_Y with itself, $F_Y(x) = P\{Y \le x\}$. By conditioning on the claim occurrence, one obtains

$$F_Y(x) = 1 - p + pF_B(x), \tag{2.2}$$

where $F_B(x) = P\{B \le x\}$.

Theorem 2.1. *For $n \ge 1$,*

$$P\{W_k > n\} = p \sum_{b=1}^{\min(k,b^u)} P\{W_{k-b} > n-1\} f_B(b) + (1-p) P\{W_k > n-1\}, \tag{2.3}$$

and $P\{W_k > 0\} = 1$, where b^u is the endpoint of the support of f_B.

Proof. From (2.1), it follows that

$$P\{W_k > n\} = F_Y * F_Y^{*n-1}(k). \tag{2.4}$$

Thus, the proof is immediate from (2.2). □

Proposition 2.2. *For $k > 0$, the MTTF of the system can be computed from*

$$E(W_k) = \frac{1}{p} + \sum_{b=1}^{\min(k,b^u)} E(W_{k-b}) f_B(b), \tag{2.5}$$

with $E(W_0) = 1/p$.

Proof. Using (2.1),

$$E(W_k) = \sum_{n=0}^{\infty} P\{W_k > n\} = \sum_{n=0}^{\infty} P\{S(n) \le k\} = \sum_{n=0}^{\infty} F_Y^{*n}(k) = 1 + F_Y * E(W_k). \tag{2.6}$$

Thus, the proof follows from (2.2) since

$$E(W_k) = 1 + (1-p)E(W_k) + pF_B * E(W_k). \tag{2.7}$$

□

Example 2.3. Let B have a geometric distribution with pmf $f_B(b) = (1-\alpha)\alpha^{b-1}$, $b = 1, 2, \ldots$. Then under the conditions of Proposition 2.2,

$$E(W_k) = \frac{1}{p} + (1-\alpha) \sum_{b=1}^{k} \alpha^{b-1} E(W_{k-b}), \tag{2.8}$$

with $E(W_0) = 1/p$.

2.1. Related Characteristics

For $k > 0$, we define new random variables as follows:

$$N(W_k) = \sum_{j=1}^{W_k} I_j,$$
$$S(W_k) = \sum_{j=1}^{W_k} Y_j = \sum_{j=1}^{N(W_k)} B_j. \tag{2.9}$$

It is clear that the random variables $N(W_k)$ and $S(W_k)$ represent, respectively, the number of shocks and the total shock that the system is subjected up to time when the system fails. These two characteristics might be useful for improvement purposes and can be effectively used in optimal system design.

Theorem 2.4. *For $m \geq 1$,*

$$P\{N(W_k) = m\} = \sum_{n=m}^{\infty} Q(n, m, k), \tag{2.10}$$

where $Q(n, m, k) = P(n, m, k) - R(n, m, k)$, and $P(n, m, k)$ and $R(n, m, k)$ can be computed recursively from

$$P(n, m, k) = pP(n-1, m-1, k) + (1-p)P(n-1, m, k), \tag{2.11}$$

for $n \geq m$ and $P(n, m, k) = 0$ for $n < m$, and

$$R(n, m, k) = p \sum_{b=1}^{\min(k, b^u)} R(n-1, m-1, k-b) f_B(b) + (1-p)R(n-1, m, k), \tag{2.12}$$

for $n \geq m$ and $R(n, m, k) = 0$ for $n < m$.

Proof. By conditioning on W_k,

$$P\{N(W_k) = m\} = \sum_{n=m}^{\infty} P\{N(n) = m, W_k = n\}. \tag{2.13}$$

The probability $Q(n, m, k) = P\{N(n) = m, W_k = n\}$ can be written as follows:

$$Q(n, m, k) = P\{N(n) = m, W_k > n-1\} - P\{N(n) = m, W_k > n\}. \tag{2.14}$$

Thus, we need to get recurrences for $P(n,m,k) = P\{N(n) = m, W_k > n-1\}$ and $R(n,m,k) = P\{N(n) = m, W_k > n\}$. By conditioning on the values of I_n,

$$
\begin{aligned}
P(n,m,k) &= P\{N(n) = m, W_k > n-1\} \\
&= P\left\{\sum_{j=1}^{n} I_j = m, \sum_{j=1}^{n-1} Y_j \le k\right\} = pP\left\{\sum_{j=1}^{n-1} I_j = m-1, \sum_{j=1}^{n-1} Y_j \le k\right\} \\
&\quad + (1-p)P\left\{\sum_{j=1}^{n-1} I_j = m, \sum_{j=1}^{n-1} Y_j \le k\right\} \\
&= pP(n-1,m-1,k) + (1-p)P(n-1,m,k).
\end{aligned}
\tag{2.15}
$$

On the other hand,

$$
\begin{aligned}
R(n,m,k) &= P\{N(n) = m, W_k > n\} = P\left\{\sum_{j=1}^{n} I_j = m, \sum_{j=1}^{n} Y_j \le k\right\} \\
&= P\left\{\sum_{j=1}^{n} I_j = m, \sum_{j=1}^{n} I_j B_j \le k, I_n = 1\right\} \\
&\quad + P\left\{\sum_{j=1}^{n} I_j = m, \sum_{j=1}^{n} I_j B_j \le k, I_n = 0\right\} \\
&= P\left\{\sum_{j=1}^{n-1} I_j = m-1, \sum_{j=1}^{n-1} I_j B_j \le k - B_n\right\} P\{I_n = 1\} \\
&\quad + P\left\{\sum_{j=1}^{n-1} I_j = m, \sum_{j=1}^{n-1} I_j B_j \le k\right\} P\{I_n = 0\} \\
&= p \sum_{b=1}^{\min(k,b^u)} P\{N(n-1) = m-1, W_{k-b} > n-1\} f_B(b) \\
&\quad + (1-p)P\{N(n-1) = m, W_k > n-1\},
\end{aligned}
\tag{2.16}
$$

for $n \ge m$. Thus, the proof is completed. □

Before proceeding with the distribution of $S(W_k)$, it should be noted that the random variable $S(n) = \sum_{j=1}^{N(n)} B_j = \sum_{j=1}^{n} Y_j$ denotes the total shock up to time n and

$$P\{S(n) = s\} = p \sum_{b=1}^{\min(s,b^u)} P\{S(n-1) = s-b\} f_B(b) + (1-p) P\{S(n-1) = s\}, \tag{2.17}$$

for $0 < s \leq n$ and $P\{S(n) = 0\} = (1-p)^n$.

Theorem 2.5. *For $s > k$,*

$$P\{S(W_k) = s\} = \sum_{n=1}^{\infty} Q^*(n, s, k), \tag{2.18}$$

where

$$Q^*(n, s, k) = p \sum_{b=s-k}^{\min(s,b^u)} P\{S(n-1) = s-b\} f_B(b). \tag{2.19}$$

Proof. By the definition of $S(W_k)$,

$$P\{S(W_k) = s\} = \sum_{n=1}^{\infty} P\{S(n) = s, W_k = n\}. \tag{2.20}$$

For $s > k$,

$$\begin{aligned} Q^*(n, s, k) &= P\{S(n) = s, W_k = n\} \\ &= P\{S(n) = s, S(n-1) \leq k, S(n) > k\} \\ &= P\{S(n) = s, S(n-1) \leq k\} \\ &= P\{S(n-1) + Y_n = s, S(n-1) \leq k\} \\ &= pP\{S(n-1) = s - B_n, S(n-1) \leq k\} \\ &\quad + (1-p) P\{S(n-1) = s, S(n-1) \leq k\}. \end{aligned} \tag{2.21}$$

The proof follows by conditioning on B_n and noting that $P\{S(n-1) = s, S(n-1) \leq k\} = 0$ for $s > k$. □

The following result readily follows from the definitions of $N(W_k)$ and $S(W_k)$ and Wald's equation.

Proposition 2.6. *For $k > 0$,*

$$\begin{aligned} E(N(W_k)) &= pE(W_k), \\ E(S(W_k)) &= pE(W_k)E(B). \end{aligned} \tag{2.22}$$

Table 1: $E(W_k), E(N(W_k))$, and $E(S(W_k))$ for geometric shock size distribution.

	$p = 0.1$	$E(B) = 8$		$p = 0.05$	$E(B) = 8$	
k	$E(W_k)$	$E(N(W_k))$	$E(S(W_k))$	$E(W_k)$	$E(N(W_k))$	$E(S(W_k))$
5	16.25	1.625	13	32.5	1.625	13
10	22.50	2.250	18	45	2.250	18
15	28.75	2.875	23	57.5	2.875	23
20	35.00	3.500	28	70	3.500	28
	$p = 0.1$	$E(B) = 5$		$p = 0.05$	$E(B) = 5$	
k	$E(W_k)$	$E(N(W_k))$	$E(S(W_k))$	$E(W_k)$	$E(N(W_k))$	$E(S(W_k))$
5	20	2	10	40	2	10
10	30	3	15	60	3	15
15	40	4	20	80	4	20
20	50	5	25	100	5	25

In Table 1 we compute $MTTF = E(W_k)$, $E(N(W_k))$, and $E(S(W_k))$ whenever the shock size random variable B has a geometric distribution with mean $E(B) = 1/(1-\alpha)$. From Table 1 we observe that an increase in k leads to an increase in MTTF of the system. If the probability of observing a shock in a period increases, then the MTTF decreases. We also observe that MTTF is proportional to p. Therefore, for the same shock size distribution the expected number of shocks $E(N(W_k))$ and expected total shock $E(S(W_k))$ remain the same for different values of p.

3. Mixed Shock Model

For $k \le m$, the mixed shock model is same as the cumulative shock model. Thus we assume that $k > m$. The following is a recursive equation for the survival probability of the system under mixed shock model.

Theorem 3.1. *For $k > m \ge 1$ and $n \ge 1$,*

$$P\{Z_{k,m} > n\} = p \sum_{b=1}^{\min(m,b^u)} P\{Z_{k-b,m} > n-1\} f_B(b) + (1-p) P\{Z_{k,m} > n-1\}, \tag{3.1}$$

and $P\{Z_{k,m} > 0\} = 1$, where $\sum_{b=x}^{y} = 0$ if $x > y$.

Proof. For $n \ge 1$,

$$P\{Z_{k,m} > n\} = P\{W_k > n, T_m > n\} = P\left\{\sum_{j=1}^{n} Y_j \le k, Y_1 \le m, \ldots, Y_n \le m\right\}. \tag{3.2}$$

By conditioning on the values of I_n,

$$\begin{aligned}
P\{Z_{k,m} > n\} &= P\left\{\sum_{j=1}^{n} I_j B_j \le k, Y_1 \le m, \dots, Y_n \le m, I_n = 1\right\} \\
&\quad + P\left\{\sum_{j=1}^{n} I_j B_j \le k, Y_1 \le m, \dots, Y_n \le m, I_n = 0\right\} \\
&= P\left\{\sum_{j=1}^{n-1} I_j B_j \le k - B_n, Y_1 \le m, \dots, Y_{n-1} \le m, B_n \le m\right\} P\{I_n = 1\} \\
&\quad + P\left\{\sum_{j=1}^{n-1} I_j B_j \le k, Y_1 \le m, \dots, Y_{n-1} \le m\right\} P\{I_n = 0\}.
\end{aligned} \tag{3.3}$$

By conditioning on B_n,

$$\begin{aligned}
P\{Z_{k,m} > n\} &= p \sum_{b=1}^{\min(k,m,b^u)} P\left\{\sum_{j=1}^{n-1} I_j B_j \le k - b, Y_1 \le m, \dots, Y_{n-1} \le m\right\} f_{B_n}(b) \\
&\quad + (1-p) P\left\{\sum_{j=1}^{n-1} I_j B_j \le k, Y_1 \le m, \dots, Y_{n-1} \le m\right\}.
\end{aligned} \tag{3.4}$$

Thus, the proof is completed. □

The following result can be proved similar to Proposition 2.2, and hence its proof is omitted.

Proposition 3.2. *For $k > m \ge 1$, the MTTF of the system under mixed shock model can be computed from*

$$E(Z_{k,m}) = \frac{1}{p} + \sum_{b=1}^{\min(m,b^u)} E(Z_{k-b,m}) f_B(b), \tag{3.5}$$

with $E(Z_{0,m}) = 1/p$, where $\sum_{b=x}^{y} = 0$ if $x > y$.

In Table 2, using Proposition 3.2, we compute the MTTF of the system under mixed shock model when the shock size random variable B has a geometric distribution with mean $E(B) = 1/(1-\alpha)$.

Theorem 3.3. *For $n \ge 1$,*

$$P\{N(Z_{k,m}) = n\} = \sum_{s=n}^{\infty} \left[pU(n-1, s-1, k, m) + (1-p)U(n, s-1, k, m) - U(n, s, k, m)\right], \tag{3.6}$$

Table 2: $E(Z_{k,m})$ for geometric shock size distribution.

k	m	$p=0.1, E(B)=8$ $E(Z_{k,m})$	$p=0.05, E(B)=8$ $E(Z_{k,m})$
5	3	14.4705	28.9410
10	3	14.8958	29.7916
10	5	18.4929	36.9858
20	5	19.4064	38.8128
k	m	$p=0.1\ , E(B)=5$ $E(Z_{k,m})$	$p=0.05, E(B)=5$ $E(Z_{k,m})$
5	3	17.7442	35.4944
10	3	19.2442	38.4885
10	5	25.4125	50.8250
20	5	29.3396	58.6792

where

$$U(n,s,k,m) = p\sum_{b=1}^{\min(m,b^u)} U(n-1,s-1,k-b,m)f_B(b) + (1-p)U(n,s-1,k,m). \tag{3.7}$$

Proof. By conditioning on $Z_{k,m}$,

$$P\{N(Z_{k,m})=n\} = \sum_{s=n}^{\infty} P\{N(s)=n, Z_{k,m}=s\}. \tag{3.8}$$

It is clear that

$$P\{N(s)=n, Z_{k,m}=s\} = P\{N(s)=n, Z_{k,m}>s-1\} - P\{N(s)=n, Z_{k,m}>s\}. \tag{3.9}$$

By the definition of $Z_{k,m}$,

$$\begin{aligned}
&U(n,s,k,m)\\
&\quad = P\{N(s)=n, Z_{k,m}>s\}\\
&\quad = P\left\{\sum_{j=1}^{s} I_j = n, \sum_{j=1}^{s} Y_j \le k, Y_1\le m,\ldots,Y_s\le m\right\}\\
&\quad = pP\left\{\sum_{j=1}^{s-1} I_j = n-1, \sum_{j=1}^{s-1} Y_j \le k-B_s, Y_1\le m,\ldots,Y_{s-1}\le m, B_s\le m\right\}\\
&\qquad + (1-p)P\left\{\sum_{j=1}^{s-1} I_j = n, \sum_{j=1}^{s-1} Y_j \le k, Y_1\le m,\ldots,Y_{s-1}\le m\right\}
\end{aligned}$$

$$
\begin{aligned}
&= p \sum_{b=1}^{\min(m,b^u)} P\left\{\sum_{j=1}^{s-1} I_j = n-1, \sum_{j=1}^{s-1} Y_j \le k-b, Y_1 \le m, \dots, Y_{s-1} \le m\right\} f_B(b) \\
&\quad + (1-p) P\left\{\sum_{j=1}^{s-1} I_j = n, \sum_{j=1}^{s-1} Y_j \le k, Y_1 \le m, \dots, Y_{s-1} \le m\right\} \\
&= p \sum_{b=1}^{\min(m,b^u)} P\{N(s-1) = n-1, Z_{k-b,m} > s-1\} f_B(b) \\
&\quad + (1-p) P\{N(s-1) = n, Z_{k,m} > s-1\}.
\end{aligned} \tag{3.10}
$$

On the other hand,

$$
\begin{aligned}
&P\{N(s) = n, Z_{k,m} > s-1\} \\
&\quad = P\left\{\sum_{j=1}^{s} I_j = n, \sum_{j=1}^{s-1} Y_j \le k, Y_1 \le m, \dots, Y_{s-1} \le m\right\} \\
&\quad = pP\{N(s-1) = n-1, Z_{k,m} > s-1\} \\
&\qquad + (1-p) P\{N(s-1) = n, Z_{k,m} > s-1\} \\
&\quad = pU(n-1, s-1, k, m) + (1-p) U(n, s-1, k, m).
\end{aligned} \tag{3.11}
$$

Thus the proof is completed. □

Before the derivation of the distribution of $S(Z_{k,m})$, we note the following recursion which will be useful in the sequel:

$$
\begin{aligned}
V(n,s,m) &= P\{S(n) = s, Y_1 \le m, \dots, Y_n \le m\} \\
&= p \sum_{b=1}^{\min(m,s,b^u)} V(n-1, s-b, m) f_B(b) + (1-p) V(n-1, s, m).
\end{aligned} \tag{3.12}
$$

Theorem 3.4. *For $s > k$,*

$$
P\{S(Z_{k,m}) = s\} = p \sum_{n=1}^{\infty} \sum_{b=s-k}^{b^u} V(n-1, s-b, m) f_B(b), \tag{3.13}
$$

for $m < s \le k$,

$$
P\{S(Z_{k,m}) = s\} = p \sum_{n=1}^{\infty} \sum_{b=m+1}^{\min(s,b^u)} V(n-1, s-b, m) f_B(b). \tag{3.14}
$$

Proof. By the definition of $S(Z_{k,m})$,

$$P\{S(Z_{k,m})=s\}=P\{S(W_k)=s,W_k\le T_m\}+P\{S(T_m)=s,T_m<W_k\}$$
$$=\begin{cases}P\{S(W_k)=s,W_k\le T_m\}, & \text{if } s>k\\ P\{S(T_m)=s,T_m<W_k\}, & \text{if } m<s\le k.\end{cases} \tag{3.15}$$

For $s>k$,

$$\begin{aligned}P\{S(W_k)=s,W_k\le T_m\}&=\sum_{n=1}^{\infty}P\{S(n)=s,T_m\ge n,W_k=n\}\\&=\sum_{n=1}^{\infty}P\{S(n)=s,Y_1\le m,\ldots,Y_{n-1}\le m,W_k=n\}\\&=\sum_{n=1}^{\infty}P\{S(n)=s,Y_1\le m,\ldots,Y_{n-1}\le m,S(n-1)\le k\}\\&=\sum_{n=1}^{\infty}P\{S(n-1)+Y_n=s,S(n-1)\le k,Y_1\le m,\ldots,Y_{n-1}\le m\}\\&=p\sum_{n=1}^{\infty}\sum_{b=s-k}^{b^u}P\{S(n-1)=s-b,Y_1\le m,\ldots,Y_{n-1}\le m\}f_B(b)\\&=p\sum_{n=1}^{\infty}\sum_{b=s-k}^{b^u}V(n-1,s-b,m)f_B(b).\end{aligned} \tag{3.16}$$

Similarly, for $m<s\le k$,

$$\begin{aligned}&P\{S(T_m)=s,T_m<W_k\}\\&=\sum_{n=1}^{\infty}P\{S(n)=s,W_k>n,T_m=n\}\\&=\sum_{n=1}^{\infty}P\{S(n)=s,S(n)\le k,Y_1\le m,\ldots,Y_{n-1}\le m,Y_n>m\}\\&=\sum_{n=1}^{\infty}P\{S(n-1)+Y_n=s,Y_1\le m,\ldots,Y_{n-1}\le m,Y_n>m\}\\&=p\sum_{n=1}^{\infty}\sum_{b=m+1}^{\min(s,b^u)}P\{S(n-1)=s-b,Y_1\le m,\ldots,Y_{n-1}\le m\}f_B(b)\\&=p\sum_{n=1}^{\infty}\sum_{b=m+1}^{\min(s,b^u)}V(n-1,s-b,m)f_B(b).\end{aligned} \tag{3.17}$$

Thus, the proof is completed. □

4. Summary and Conclusions

In this paper, we studied the life behavior of a system under discrete time cumulative and mixed shock models. The probability of getting a shock in any period is p, and the shock occurrences are assumed to be independent over the periods. The size of the shock occuring in a period follows a discrete probability distribution and the system's lifetime coincides with the waiting time random variable which represents the time until the cumulative sum of shocks exceeds a specified level (cumulative shock model). We derived recurrence formulae for the survival function and the MTTF of the system. We also obtained recurrences for the distributions and expected values of the two related quantities which represent the number of shocks and the total shock that the system is subjected until failure. The results were illustrated for the case when the shock size distribution is geometric. We have also obtained a recurrence for the survival function of the system under a mixed shock model. The assumption of discrete shock size distribution enables us to obtain recursive formulae. However, the consideration of continuous shock size distribution might be of special interest in some applications. Therefore, a possible future work can be on discrete time shock models with a continuous shock size distribution.

In the model that was studied in the paper shock occurrence indicators are assumed to be independent and identical with a constant probability p. As a future work, the case in which the shock occurrence indicators form a Markov chain can also be considered.

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