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Research Article

The Dynamics of the Solutions of Some Difference Equations

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This paper is devoted to investigate the global behavior of the following rational difference equation: $y_{n+1} = \alpha y_{n-t}/(\beta + \gamma \sum_{i=0}^{k} y_{n-(2i+1)}^{p} \prod_{i=0}^{k} y_{n-(2i+1)}^{q})$, $n = 0, 1, 2, \ldots$, where $\alpha, \beta, \gamma, p, q \in (0, \infty)$ and k, $t \in \{0, 1, 2, \ldots\}$ with the initial conditions x_0, $x_{-1}, \ldots,$ x_{-2k}, $x_{-2\max\{k,t\}-1} \in (0,\infty)$. We will find and classify the equilibrium points of the equations under studying and then investigate their local and global stability. Also, we will study the oscillation and the permanence of the considered equations.

1. Introduction

The aim of this paper is to study the dynamics of the solutions of the following recursive sequence:

$$y_{n+1} = \frac{\alpha y_{n-t}}{\beta + \gamma \sum_{i=0}^{k} y_{n-(2i+1)}^{p} \prod_{i=0}^{k} y_{n-(2i+1)}^{q}}, \quad n = 0, 1, 2, \ldots, \tag{1.1}$$

where $\alpha, \beta, \gamma, p, q \in (0, \infty)$ and $K \in \{0, 1, 2, \ldots\}$, where $K = \max\{k, t\}$, with the initial conditions x_0, $x_{-1}, \ldots,$ $x_{-2k}, x_{-2K-1} \in (0,\infty)$. We deal with the classification of the equilibrium points of (1.1) in terms of being stable or unstable, where we investigate the global attractor of the solutions of (1.1) as well as the permanence of the equation. Also, we establish some appropriate conditions, which grantee the oscillation of the solutions of (1.1). For more results in the direction of this study, see, for example, [1–23] and the papers therein.

In the sequel, we present some well-known results and definition that will be useful in our investigation of (1.1). Let I be some interval of real numbers and let $f : I^{k+1} \to I$ be a continuously differentiable function. Then, for every set of initial conditions $x_{-k}, x_{-k+1}, \dots, x_0 \in I$, the difference equation

$$x_{n+1} = f(x_n, x_{n-1}, \dots, x_{n-k}), \quad n = 0, 1, \dots, \tag{1.2}$$

has a unique solution $\{x_n\}_{n=-k}^{\infty}$.

Definition 1.1 (permanence). The difference equation (1.2) is said to be permanent if there exist numbers m and M with $0 < m \le M < \infty$ such that, for any initial conditions $x_{-k}, x_{-k+1}, \dots, x_{-1}, x_0 \in (0, \infty)$, there exists a positive integer N which depends on the initial conditions such that $m \le x_n \le M$ for all $n \ge N$.

Definition 1.2 (periodicity). A sequence $\{x_n\}_{n=-k}^{\infty}$ is said to be periodic with period p if $x_{n+p} = x_n$ for all $n \ge -k$.

Definition 1.3 (semicycles). A positive semicycle of a sequence $\{x_n\}_{n=-k}^{\infty}$ consists of a "string" of terms $\{x_l, x_{l+1}, \dots, x_m\}$ all greater than or equal to the equilibrium point $\overline{x}$, with $l \ge -k$ and $m \le \infty$ such that either $l = -k$ or $l > -k$ and $x_{l-1} < \overline{x}$; either $m = \infty$ or $m < \infty$ and $x_{m+1} < \overline{x}$. A negative semicycle of a sequence $\{x_n\}_{n=-k}^{\infty}$ consists of a "string" of terms $\{x_l, x_{l+1}, \dots, x_m\}$ all less than the equilibrium point $\overline{x}$, with $l \ge -k$ and $m \le \infty$ such that either $l = -k$ or $l > -k$ and $x_{l-1} \ge \overline{x}$; either $m = \infty$ or $m < \infty$ and $x_{m+1} \ge \overline{x}$.

Definition 1.4 (oscillation). A sequence $\{x_n\}_{n=-k}^{\infty}$ is called nonoscillatory about the point $\overline{x}$ if there exists $N \ge -k$ such that either $x_n > \overline{x}$ for all $n \ge N$ or $x_n < \overline{x}$ for all $n \ge N$. Otherwise, $\{x_n\}_{n=-k}^{\infty}$ is called oscillatory about $\overline{x}$.

2. Dynamics of (1.1)

The change of variables $y_n = (\beta/\gamma)^{1/[p+(k+1)q]} x_n$ reduces (1.1) to the following difference equation

$$x_{n+1} = \frac{r x_{n-t}}{1 + \sum_{i=0}^{k} x_{n-(2i+1)}^{p} \prod_{i=0}^{k} x_{n-(2i+1)}^{q}}, \quad n = 0, 1, 2, \dots, \tag{2.1}$$

where $r = \alpha/\beta$.

In this section, we study the local stability character and the global stability of the equilibrium points of the solutions of (2.1). Also, we give some results about the oscillation and the permanence of (2.1).

Recall that the equilibrium points of (2.1) are given by

$$\overline{x} = \frac{r\overline{x}}{1 + (k+1)\overline{x}^{p+(k+1)q}}. \tag{2.2}$$

Then, whenever $r \leq 1$, (2.1) has the only equilibrium point $\overline{x} = 0$, and, while at $r > 1$, (2.1) possesses the unique positive equilibrium point $\overline{x} = ((r-1)/(k+1))^{1/(p+(k+1)q)}$.

The following theorem deals with the local stability of the equilibrium point $\overline{x} = 0$ of (2.1).

Theorem 2.1. *The following statements are true.*

(i) *If $r < 1$, then the equilibrium point $\overline{x} = 0$ of* (2.1) *is locally asymptotically stable.*

(ii) *If $r > 1$, then the equilibrium point $\overline{x} = 0$ of* (2.1) *is a saddle point.*

(iii) *If $r = 1$, then the equilibrium point $\overline{x} = 0$ of* (2.1) *is nonhyperbolic with $\lambda = 0 < 1$ and $\lambda = 1$.*

Proof. The linearized equation of (2.1) about $\overline{x} = 0$ is $u_{n+1} - ru_{n-t} = 0$. Then, the associated eigenvalues are $\lambda = 0$ and $\lambda = r$. Then, the proof is complete. □

Theorem 2.2. *Assume that $r < 1$, then the equilibrium point $\overline{x} = 0$ of* (2.1) *is globally asymptotically stable.*

Proof. Let $\{x_n\}_{n=-2k-1}^{\infty}$ be a solution of (2.1). It was shown in Theorem 2.1 that the equilibrium point $\overline{x} = 0$ of (2.1) is locally asymptotically stable. So it suffices to show that

$$\lim_{n\to\infty} x_n = 0. \tag{2.3}$$

Now, it follows from (2.1) that

$$\begin{aligned} x_{n+1} &= \frac{rx_{n-t}}{1 + x_{n-1}^{p+q}x_{n-3}^{q}\cdots x_{n-(2k+1)}^{q} + x_{n-1}^{q}x_{n-3}^{p+q}\cdots x_{n-(2k+1)}^{q} + \cdots + x_{n-1}^{q}x_{n-3}^{q}\cdots x_{n-(2k+1)}^{p+q}} \\ &\leq rx_{n-2t}. \end{aligned} \tag{2.4}$$

Now, assume that $X = \max\{x_{-2t}, x_{-2t+1}, \ldots, x_{-1}, x_0/r\}$. Then, it follows from (1.1) and after some simple computations are achieved that $x_{2(n-1)t+i} \leq r^n X$, $i = 0, 1, \ldots, 2t$. Therefore $x_n \to 0$ as $n \to \infty$, and this completes the proof. □

Theorem 2.3. *Assume that $r > 1$. Then, every solution of* (2.1) *is either oscillatory or tends to the equilibrium point $\overline{x} = ((r-1)/(k+1))^{1/(p+(k+1)q)}$.*

Proof. Let $\{x_n\}_{n=-2K-1}^{\infty}$ be a solution of (2.1). Without loss of generality, assume that $\{x_n\}_{n=-2K-1}^{\infty}$ is a nonoscillatory solution of (2.1), then it is suffices to show that $\lim_{n\to\infty} x_n = \overline{x}$. Assume that $x_n \geq \overline{x}$ for $n \geq n_0$ (the case where $x_n \leq \overline{x}$ for $n \geq n_0$ is similar and will be omitted). It follows from (2.1) that

$$\begin{aligned} x_{n+1} &= \frac{rx_{n-t}}{1 + x_{n-1}^{p+q}x_{n-3}^{q}\cdots x_{n-(2k+1)}^{q} + x_{n-1}^{q}x_{n-3}^{p+q}\cdots x_{n-(2k+1)}^{q} + \cdots + x_{n-1}^{q}x_{n-3}^{q}\cdots x_{n-(2k+1)}^{p+q}} \\ &\leq x_{n-t}\left(\frac{r}{1 + (k+1)\overline{x}^{p+(k+1)q}}\right) = x_{n-t}. \end{aligned} \tag{2.5}$$

Hence, each subsequence $\{x_{(t+1)n+i}\}$, $j = 0, 1, \ldots, t$, of $\{x_n\}_{n=0}^{\infty}$ is decreasing sequence and therefore it has a limit. Let for some $j = 0, 1, \ldots, t$, $\lim_{n\to\infty} x_{(t+1)n+j} = \mu$, and, for the sake of contradiction, assume that $\mu > \overline{x}$. Then, by taking the limit of both sides of

$$x_{(t+1)n+j+1} = \frac{r x_{(t+1)n+j-t}}{1 + \sum_{s=0}^{k} x_{(t+1)n+j-(2s+1)}^{p} \prod_{s=0}^{k} x_{(t+1)n+j-(2s+1)}^{q}}, \quad n = 0, 1, 2, \ldots, \tag{2.6}$$

we obtain $\mu = r\mu/(1 + (k+1)\mu^{p+(k+1)q})$, which contradicts the hypothesis that $\overline{x} = ((r-1)/(k+1))^{1/(p+(k+1)q)}$ is the only positive solution of (2.2). Therefore, $\lim_{n\to\infty} x_{(t+1)n+j} = \mu$, for all $j = 0, 1, \ldots, t$. This means that all the subsequences $\{x_{(t+1)n+i}\}$, $j = 0, 1, \ldots, t$, of $\{x_n\}_{n=0}^{\infty}$ have the same limit, $\overline{x}$, and therefore $\lim_{n\to\infty} x_n = \overline{x}$, which completes the proof. □

Theorem 2.4. *Assume that $t = 0$, $r > 1$, and let $\{x_n\}_{n=-2k-1}^{\infty}$ be a solution of (2.1) which is strictly oscillatory about the positive equilibrium point $\overline{x} = ((r-1)/(k+1))^{1/(p+(k+1)q)}$ of (2.1). Then, the extreme point in any semicycle occurs in one of the first $2(k+1)$ terms of the semicycle.*

Proof. Assume that $\{x_n\}_{n=-2k-1}^{\infty}$ is a strictly oscillatory solution of (2.1). Let $L \geq M \geq N \geq 2(k+1)$, and let $\{x_N, x_{N+1}, \ldots, x_M\}$ be a positive semicycle followed by the negative semicycle $\{x_M, x_{M+1}, \ldots, x_L\}$. Now, it follows from (2.1) that

$$\begin{aligned}
x_{N+2(k+1)} - x_N &= \frac{r x_{N+2k+1}}{1 + x_{N+2k}^{p+q} x_{N+2k-2}^{q} \cdots x_{N}^{q} + x_{N+2k}^{q} x_{N+2k-2}^{p+q} \cdots x_{N}^{q} + \cdots + x_{N+2k}^{q} \cdots x_{N}^{p+q}} - x_N \\
&\leq x_{N+2k+1}\left(\frac{r}{1 + (k+1)\overline{x}^{p+(k+1)q}}\right) - x_N = x_{N+2k+1} - x_N \\
&= \frac{r x_{N+2(k+t)}}{1 + x_{N+2k-1}^{p+q} x_{N+2k-3}^{q} \cdots x_{N-1}^{q} + \cdots + x_{N+2k-1}^{q} \cdots x_{N-1}^{p+q}} - x_N \\
&\leq x_{N+2k}\left(\frac{r}{1 + (k+1)\overline{x}^{p+(k+1)q}}\right) - x_N \\
&= x_{N+2k} - x_N \leq x_{N+2k-1} - x_N \leq \cdots \leq x_{N+1} - x_N \leq x_N - x_N = 0.
\end{aligned} \tag{2.7}$$

Then, $x_N \geq x_{N+2(k+1)}$ for all $N \geq 2(k+1)$.

Similarly, we see from (2.1) that

$$\begin{aligned}
x_{M+2(k+1)} - x_M &= \frac{r x_{M+2k+1}}{1 + x_{M+2k}^{p+q} x_{M+2k-2}^{q} \cdots x_{M}^{q} + x_{M+2k}^{q} x_{M+2k-2}^{p+q} \cdots x_{M}^{q} + \cdots + x_{M+2k}^{q} \cdots x_{M}^{p+q}} - x_M \\
&\geq x_{M+2k+1}\left(\frac{r}{1 + (k+1)\overline{x}^{p+(k+1)q}}\right) - x_M = x_{M+2k+1} - x_M
\end{aligned}$$

$$
\begin{aligned}
&= \frac{r x_{M+2k}}{1 + x_{M+2k-1}^{p+q} x_{M+2k-3}^{q} \cdots x_{M-1}^{q} + \cdots + x_{M+2k-1}^{q} \cdots x_{M-1}^{p+q}} - x_M \\
&\geq x_{M+2k} \left(\frac{r}{1 + (k+1)\overline{x}^{p+(k+1)q}} \right) - x_M \\
&= x_{M+2k} - x_M \leq x_{M+2k-1} - x_M \geq \cdots \geq x_{M+1} - x_M \geq x_M - x_M = 0.
\end{aligned}
\tag{2.8}
$$

Therefore, $x_{M+2(k+1)} \geq x_M$ for all $M \geq 2(k+1)$. The proof is complete. □

Theorem 2.5. *Assume that $t = 2k, r > 1$, and let $\{x_n\}_{n=-2k-1}^{\infty}$ be a solution of (2.1) which is strictly oscillatory about the positive equilibrium point $\overline{x} = ((r-1)/(k+1))^{1/(p+(k+1)q)}$ of (2.1). Then, the extreme point in any semicycle occurs in one of the first $2k$ terms of the semicycle.*

Proof. Assume that $\{x_n\}_{n=-2k-1}^{\infty}$ is a strictly oscillatory solution of (2.1). Let $L \geq M \geq N \geq 2k+1$, and let $\{x_N, x_{N+1}, \ldots, x_M\}$ be a positive semicycle followed by the negative semicycle $\{x_M, x_{M+1}, \ldots, x_L\}$. Now, it follows from (2.1) that

$$
\begin{aligned}
x_{N+2k+1} - x_N &= \frac{r x_N}{1 + x_{N+2k}^{p+q} x_{N+2k-2}^{q} \cdots x_N^{q} + x_{N+2k}^{q} x_{N+2k-2}^{p+q} \cdots x_N^{q} + \cdots + x_{N+2k}^{q} \cdots x_N^{p+q}} - x_N \\
&\leq x_N \left(\frac{r}{1 + (k+1)\overline{x}^{p+(k+1)q}} \right) - x_N = x_N - x_N = 0.
\end{aligned}
\tag{2.9}
$$

Then, $x_N \geq x_{N+2k}$ for all $N \geq 2k+1$.

Similarly, we see from (2.1) that

$$
\begin{aligned}
x_{M+2k+1} - x_M &= \frac{r x_M}{1 + x_{M+2k}^{p+q} x_{M+2k-2}^{q} \cdots x_M^{q} + x_{M+2k}^{q} x_{M+2k-2}^{p+q} \cdots x_M^{q} + \cdots + x_{M+2k}^{q} \cdots x_M^{p+q}} - x_M \\
&\geq x_M \left(\frac{r}{1 + (k+1)\overline{x}^{p+(k+1)q}} \right) - x_M = x_M - x_M = 0.
\end{aligned}
\tag{2.10}
$$

Therefore, $x_{M+2k} \geq x_M$ for all $M \geq 2k+1$. The proof is complete. □

Theorem 2.6. *Assume that $t = 2k+1$, $r > 1$, and let $\{x_n\}_{n=-2k-1}^{\infty}$ be a solution of (2.1) which is strictly oscillatory about the positive equilibrium point $\overline{x} = ((r-1)/(k+1))^{1/(p+(k+1)q)}$ of (2.1). Then, the extreme point in any semicycle occurs in one of the first $2k+1$ terms of the semicycle.*

Proof. Assume that $\{x_n\}_{n=-2k-1}^{\infty}$ be a strictly oscillatory solution of (2.1). Let $L \geq M \geq N \geq 2k+1$ and let $\{x_N, x_{N+1}, \ldots, x_M\}$ be a positive semicycle followed by the negative semicycle $\{x_M, x_{M+1}, \ldots, x_L\}$. Now it follows from (2.1) that

$$\begin{aligned} x_{N+2(k+1)} - x_N &= \frac{r x_N}{1 + x_{N+2k+1}^{p+q} x_{N+2k-1}^{q} \cdots x_N^{q} + x_{N+2k+1}^{q} x_{N+2k-1}^{p+q} \cdots x_N^{q} + \cdots + x_{N+2k+1}^{q} \cdots x_N^{p+q}} - x_N \\ &\leq x_N \left(\frac{r}{1 + (k+1)\overline{x}^{p+(k+1)q}} \right) - x_N = x_N - x_N = 0. \end{aligned} \tag{2.11}$$

Then, $x_N \geq x_{N+2k}$ for all $N \geq 2k+1$.

Similarly, we see from (2.1) that

$$\begin{aligned} x_{M+2(k+1)} - x_M &= \frac{r x_M}{1 + x_{M+2k+1}^{p+q} x_{M+2k-1}^{q} \cdots x_M^{q} + x_{M+2k+1}^{q} x_{M+2k-1}^{p+q} \cdots x_M^{q} + \cdots + x_{M+2k+1}^{q} \cdots x_M^{p+q}} - x_M \\ &\geq x_M \left(\frac{r}{1 + (k+1)\overline{x}^{p+(k+1)q}} \right) - x_M = x_M - x_M = 0. \end{aligned} \tag{2.12}$$

Therefore $x_{M+2k} \geq x_M$ for all $M \geq 2k+1$. The proof is complete. □

Theorem 2.7. *Equation* (2.1) *is permanent.*

Proof. Let $\{x_n\}_{n=-2k-1}^{\infty}$ be a solution of (2.1). There are two cases to consider.

(i) $\{x_n\}_{n=-2k-1}^{\infty}$ is a nonoscillatory solution of (2.1). Then, it follows from Theorem 2.3 that

$$\lim_{n \to \infty} x_n = \overline{x}, \tag{2.13}$$

that is, there is a sufficiently large positive integer N such that $|x_n - \overline{x}| < \varepsilon$ for all $n \geq N$ and for some $\varepsilon > 0$. So $\overline{x} - \varepsilon < x_n < \overline{x} + \varepsilon$, this means that there are two positive real numbers, say C and D, such that $C \leq x_n \leq D$.

(ii) $\{x_n\}_{n=-2k-1}^{\infty}$ is strictly oscillatory about $\overline{x} = ((r-1)/(k+1))^{1/(p+(k+1)q)}$.

Now, let $\{x_{s+1}, x_{s+2}, \ldots, x_t\}$ be a positive semicycle followed by the negative semicycle $\{x_{t+1}, x_{t+2}, \ldots, x_u\}$. If x_V and x_W are the extreme values in these positive and negative semicycles, respectively, with the smallest possible indices V and W, then by Theorem 2.4

we see that $V - s \leq 2(k+1)$ and $W - u \leq 2(k+1)$. Now, for any positive indices μ and L with $\mu < L$, it follows from (2.1) for $n = \mu, \mu+1, \ldots, L-1$ that

$$\begin{aligned}
x_L &= x_{L-1}\left(\frac{r}{1 + x_{L-2}^{p+q}x_{L-4}^{q}\cdots x_{L-2k-2}^{q} + x_{L-2}^{q}x_{L-4}^{p+q}\cdots x_{L-2k-2}^{q} + \cdots + x_{L-2}^{q}x_{L-4}^{q}\cdots x_{L-2k-2}^{p+q}}\right) \\
&= \frac{r^2 x_{L-2}}{\left(1 + x_{L-3}^{p+q}\cdots x_{L-2k-3}^{q} + \cdots + x_{L-3}^{q}\cdots x_{L-2k-3}^{p+q}\right)\left(1 + x_{L-2}^{p+q}\cdots x_{L-2k-2}^{q} + \cdots + x_{L-2}^{q}\cdots x_{L-2k-2}^{p+q}\right)} \\
&\ \ \vdots \\
&= x_{L-\zeta} r^{\zeta} \prod_{\eta=1}^{\zeta}\left(\frac{1}{1 + \sum_{i=0}^{k} x_{L-(2i+1)-\eta}^{p} \prod_{i=0}^{k} x_{L-(2i+1)-\eta}^{q}}\right) \\
&= x_{\mu} r^{L-\mu} \prod_{\eta=\mu}^{L-1}\left(\frac{1}{1 + \sum_{i=0}^{k} x_{\eta-(2i+1)}^{p} \prod_{i=0}^{k} x_{\eta-(2i+1)}^{q}}\right).
\end{aligned} \tag{2.14}$$

Therefore, for $V = L$ and $s = \mu$, we obtain

$$\begin{aligned}
x_V &= x_s r^{V-s} \prod_{\eta=s}^{V-1}\left(\frac{1}{1 + \sum_{i=0}^{k} x_{\eta-(2i+1)}^{p} \prod_{i=0}^{k} x_{\eta-(2i+1)}^{q}}\right) \\
&\leq \overline{x} r^{2k+1} = H.
\end{aligned} \tag{2.15}$$

Again whenever $W = L$ and $\mu = t$, we see that

$$\begin{aligned}
x_W &= x_t r^{W-t} \prod_{\eta=t}^{W-1}\left(\frac{1}{1 + \sum_{i=0}^{k} x_{\eta-(2i+1)}^{p} \prod_{i=0}^{k} x_{\eta-(2i+1)}^{q}}\right) \\
&\geq \overline{x} r^{W-t} \prod_{\eta=t}^{W-1}\left(\frac{1}{1 + (k+1)H^{p+(k+1)q}}\right) \\
&= \overline{x} r^{W-t}\left(\frac{1}{1 + (k+1)H^{p+(k+1)q}}\right)^{W-t-1} \\
&\geq \overline{x}\left(\frac{1}{1 + (k+1)H^{p+(k+1)q}}\right)^{2k+1} = G.
\end{aligned} \tag{2.16}$$

That is $G \leq x_n \leq H$. It follows from (i) and (ii) that $\min\{C, G\} \leq x_n \leq \max\{D, H\}$. Then, the proof is complete. $\square$

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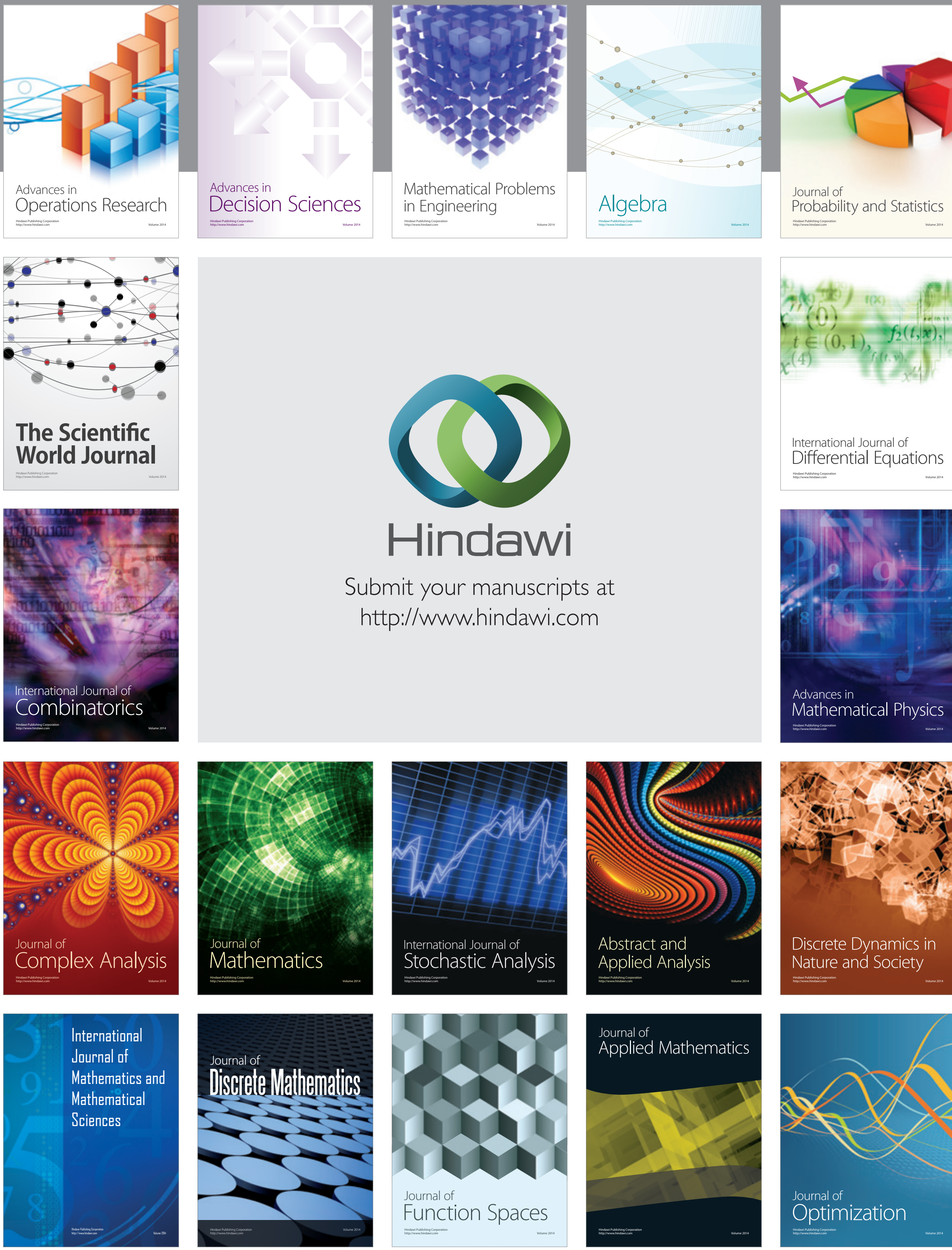
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