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Research Article

Sliding Mode Control for Markovian Switching Singular Systems with Time-Varying Delays and Nonlinear Perturbations

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This paper is devoted to investigating sliding mode control (SMC) for Markovian switching singular systems with time-varying delays and nonlinear perturbations. The sliding mode controller is designed to guarantee that the nonlinear singular system is stochastically admissible and its trajectory can reach the sliding surface in finite time. By using Lyapunov functional method, some criteria on stochastically admissible are established in the form of linear matrix inequalities (LMIs). A numerical example is presented to illustrate the effectiveness and efficiency of the obtained results.

1. Introduction

The sliding mode control (SMC) theory has made rapid progress since it was proposed by Utkin [1]. As an effective robust control strategy, SMC has been successfully applied to a wide variety of practical engineering systems such as robot manipulators, aircrafts, underwater vehicles, spacecrafts, flexible space structures, electrical motors, power systems, and automotive engines [2]. The SMC system has various attractive features such as fast response, good transient performance, and insensitiveness to the uncertainties on the sliding surface [2]. These advantages provide more freedom in designing the controllers for the system models which can be easily modified by introducing virtual disturbances to satisfy some requirements. The SMC strategy has been successfully applied to many kinds of systems, such as uncertain time-delay systems and Markovian jump system [3–14].

Singular systems, also referred to as descriptor systems, generalized state-space systems, differential-algebraic systems, or semistate systems, are more appropriate to describe the behavior of some practical systems, such as economic systems, power systems, and circuits systems, because singular systems mix up dynamic equations and static equations. Basic control theory for singular systems has been widely studied, such as stability and stabilization [14–18], H_∞ control problem [19–22], and optimal control [23] and filtering problem [24–26]. Xu et al. have designed an integral sliding mode controller for singular stochastic hybrid systems [27]. They put up with new sufficient conditions in terms of strict LMIs, which guarantees stochastic stability of the sliding mode dynamics.

In practice, many physical systems may happen to have abrupt variations in their structure, due to random failures or repair of components, sudden environmental disturbances, changing subsystem interconnections, and abrupt variations in the operating points of a nonlinear plant. Therefore, Markovian jump systems have received increasing attention, see [28–33] and the references therein. Wu et al. [28] have probed sliding mode control with bounded H_2 gain performance of Markovian jump singular time-delay systems. Kao et al. [29] have investigated delay-dependent robust exponential stability of Markovian jumping reaction-diffusion Cohen-Grossberg neural networks with mixed delays. Zhang and Boukas have discussed mode-dependent H_∞ filtering for discrete-time Markovian jump linear systems with partly unknown transition probability. To the authors' best knowledge, sliding mode control for a class of Markovian switching singular systems with time-varying delays and nonlinear perturbations has not been properly investigated.

Especially few consider exponential stabilization for this kind of nonlinear singular systems by sliding mode control.

Motivated by the above discussion, we consider exponential stabilization for Markovian switching nonlinear singular systems via sliding mode control. First, we develop two lemmas. Based on these lemmas, delay-dependent sufficient condition on exponential stabilization for singular time-varying delay systems is given in terms of nonstrict LMIs. Some specified matrices are introduced and the non-strict LMIs are translated into strict LMIs which are easy to check by MATLAB LMI toolbox. Second, a sliding surface is derived using an equivalent control approach. A sliding mode controller is developed to drive the systems to the sliding surface in finite time and maintain a sliding motion thereafter. Finally, a numerical example is provided to show the effectiveness of the proposed result.

Notations. $(\Xi, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathscr{P})$ is a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions. $L^P_{\mathscr{F}_0}$ is the family of all $\mathscr{F}_0$-measurable $C([-\tau,0];R^n)$ valued random variables $\xi = \xi(\theta) : -\tau \leq \theta \leq 0$ such that $\sup_{-\tau\leq\theta\leq 0}\mathscr{E}\|\xi(\theta)\|_2^2 < \infty$, where $\mathscr{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathscr{P}$. R^n and $R^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of $n \times m$ real matrices. The superscript T denotes the transpose, and the notation X, Y (resp., $X > Y$) where X and Y are symmetric matrices means that $X - Y$ is positive semi-definite (resp., positive definite). L^2 stands for the space of square integral vector functions. $\|\cdot\|$ will refer to the Euclidean vector norm, and $*$ represents the symmetric form of matrix.

2. System Description and Definitions

Consider the following singular system with time-varying delays, nonlinear perturbations, and Markovian switching:

$$\begin{aligned} E(r(t))\dot{x}(t) &= A(r(t))x(t) + A_d(r(t))x(t-\tau(t)) \\ &\quad + B(r(t))u(t) + f(x(t),t,r(t)), \qquad (1) \\ x(t) &= \phi(t), \quad t \in [-\tau, 0], \end{aligned}$$

where $x(t) \in R^n$ is the state vector; $u(t) \in R^m$ is the control input; $f(x(t),t,r(t)) \in R^n$ represents the system nonlinearity and any model uncertainties in the systems including external disturbances with Markovian switching; $\phi(t) \in L^P_{\mathscr{F}_0}([-\tau,0];R^n)$ is a compatible vector valued continuous function. $A(r(t)), A_d(r(t))$, and $B(r(t))$ are real constant matrices with appropriate dimensions. The matrix $E(r(t)) \in R^{n\times n}$ may be singular, and we assume that $0 < \operatorname{rank}(E(r(t))) = r \leq n$. $\tau(t)$ denotes the time-varying delay and satisfies

$$0 \leq \tau(t) \leq \tau, \dot{\tau}(t) \leq d < 1. \qquad (2)$$

Let $\{r(t), t \geq 0\}$ be a continuous-time discrete-state Markovian process with right continuous trajectories taking value in a finite set $\mathbb{S} = \{1, 2, \ldots, N\}$ with transition probability matrix $\prod = (r_{ij})$, $(i, j \in \mathbb{S})$,

$$P\{r(t+\Delta) = j \mid r(t) = i\} = \begin{cases} r_{ij} \times \Delta + o(\Delta), & i \neq j \\ 1 + r_{ii} \times \Delta + o(\Delta), & i = j, \end{cases} \qquad (3)$$

where $\Delta > 0$ and $\lim_{\Delta\to 0}(o(\Delta))/\Delta = 0$, $r_{ij} > 0$ is the transition rate from i to j if $i \neq j$ and $r_{ii} = -\sum_{j=1, i\neq j}^{N} r_{ij}$.

The nominal Markovian jump singular and time-delay system of system (1) are as follows:

$$E(r(t))\dot{x}(t) = A(r(t))x(t), \qquad (4)$$

$$E(r(t))\dot{x}(t) = A(r(t))x(t) + A_d(r(t))x(t-\tau(t)). \qquad (5)$$

The initial Markovian jump singular system in (1) is assumed to be

$$x(0, \gamma_0) = \phi(0). \qquad (6)$$

Recall that the Markovian process $\{r(t), t \geq 0\}$ takes value in a finite set $\mathbb{S} = \{1, 2, \ldots, N\}$. For simplicity, we write $r(t) = i \in \mathbb{S}$, $E(i) = E_i$, $A(i) = A_i$, $A_d(i) = A_{di}B(r(t)) = B_i$, $f(x(t),t,r(t)) = f_i(x(t),t)$.

Definition 1. (i) The system (4) is said to be regular if $\det(sE_i - A_i) \neq 0$ for every $i \in \mathbb{S}$.

(ii) The system (4) is said to impulse free if $\deg(\det(sE_i - A_i)) = \operatorname{rank}(E_i)$ for every $i \in \mathbb{S}$.

Definition 2. For a given scalar $\tau > 0$, the Markovian jump singular delay system (5) is said to be regular and impulse free for any time delay $\tau(t)$ satisfying $0 \leq \tau(t) \leq \tau$, if the system (4) and the system $E(r(t))\dot{X}(t) = (A(r(t)) + A_d(r(t)))X(t)$ are all regular and impulse free.

The system (5) is said to be stochastically stable if for any $x_0 \in R^n$ and $r_0 \in \mathbb{S}$ there exists a scalar $\widetilde{M}(x_0, r_0) > 0$ such that $\lim_{t\to\infty}\mathscr{E}(\int_0^t x^T(s,x_0,r_0)x(s,x_0,r_0)ds \mid x_0, r_0) \leq \widetilde{M}(x_0, r_0)$, where $x(s, x_0, r_0)$ denotes the solution of system (5) at time t under the initial condition x_0 and r_0.

The system (5) is said to be stochastically admissible if it is regular, impulse free, and stochastically stable.

We will assume the followings to be valid.

Assumption 1. $B(r(t))$ is full-rank:rank $(B(r(t))) = m$.

Assumption 2. The perturbation term $f(x(t),t,r(t))$ is Lipshitz, continuous and satisfies the following matching conditions:

$$f(x(t),t,r(t)) = B(r(t))\overline{f}(x(t),t,r(t)), \qquad (7)$$

with $\overline{f}(x(t),t,r(t)) \in R^m$ bounded by

$$\left\|\overline{f}(x(t),t,r(t))\right\| \leq \epsilon_i \|x(t)\|, \qquad (8)$$

where $\epsilon > 0$ is a constant.

Lemma 3 will support the non-strict LMI to be translated into strict LMI.

Lemma 3 (see [34]). *Let $X \in R^{n\times n}$ be symmetric such that $E_L^T X E_L > 0$ and $T \in R^{(n-r)\times(n-r)}$ nonsingular. Then, $XE + M^T T S^T$ is nonsingular and its inverse is expressed as*

$$\left(XE + M^T T S^T\right)^{-1} = \mathbb{X} E^T + S\mathbb{T}M, \tag{9}$$

where $\mathbb{X}$ is symmetric and $\mathbb{T}$ is a singular matrix with

$$E_R^T \mathbb{X} E_R = \left(E_L^T X E_L\right)^{-1}, \qquad \mathbb{T} = \left(S^T S\right)^{-1} T^{-1} \left(M M^T\right)^{-1}, \tag{10}$$

where M and S are any matrices with full row rank and satisfy $ME = 0$ and $ES = 0$, respectively; E is decomposed as $E = E_L E_R^T$ with $E_L \in R^{n\times r}$ and $E_R \in R^{n\times r}$ are of full column rank.

Lemma 4 (see [35]). *There exists symmetric matrix X such that*

$$\begin{bmatrix} P_1 + X & Q_1 \\ Q_1^T & R_1 \end{bmatrix} < 0, \qquad \begin{bmatrix} P_2 - X & Q_2 \\ Q_2^T & R_2 \end{bmatrix} < 0 \tag{11}$$

if and only if

$$\begin{bmatrix} P_1 + P_2 & Q_1 & Q_2 \\ * & R_1 & 0 \\ * & * & R_2 \end{bmatrix} < 0. \tag{12}$$

Lemma 5. *Let $Q = Q^T$, S, $R = R^T$ be matrices of appropriate dimensions, then $R < 0$, $Q - SR^{-1}S^T < 0$ is equivalent to*

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} < 0. \tag{13}$$

Lemma 6. *Let $X \in R^n$, $Y \in R^n$, and $Q > 0$. Then we have $X^T Y + Y^T X \le Y^T Q Y + X^T Q^{-1} X$.*

We give the following result for the stochastic admissibility of the system (4) without proof, and readers are referred to [36] for detailed proof.

Lemma 7. *The Markovian jump singular system (4) is stochastically admissible if and only if there exists matrices P_i, $i = 1, 2, \ldots, N$ such that*

$$\begin{gathered} E_i^T P_i = P_i^T E_i \ge 0 \\ \sum_{j=1}^{N} r_{ij} E_i^T P_j + P_i^T A_i + A_i^T P_i < 0. \end{gathered} \tag{14}$$

Lemma 8. *For a prescribed scalars $\tau > 0$, d, and any time delay $\tau(t)$ satisfying $0 \le \tau(t) \le \tau$, the Markovian jump singular time-delay system (5) is stochastically admissible, if there exist symmetric positive-definite matrices Q, R and nonsingular matrix P_i for every $i = 1, 2, \ldots, N$, such that*

$$E_i^T P_i = P_i^T E_i \ge 0, \tag{15}$$

$$\mathbf{\Phi_i} = \begin{bmatrix} \mathbf{\Phi_i^{11}} & P_i^T A_{di} & 0 \\ * & -(1-d)Q & 0 \\ * & * & -\dfrac{1}{\tau} R \end{bmatrix} < 0, \tag{16}$$

where

$$\mathbf{\Phi_i^{11}} = P_i^T A_i + A_i^T P_i + Q + \tau R + \sum_{j=1}^{N} r_{ij} E_j^T P_j. \tag{17}$$

Proof. First to prove the system $E(r(t))\dot{x}(t) = [A(r(t)) + A_d(r(t)]x(t)$ is regular and impulse free.

From (16), it is easy to know $\mathbf{\Phi_i^{11}} < 0$ and

$$\begin{bmatrix} \mathbf{\Phi_i^{11}} & P_i^T A_{di} \\ * & -(1-d)Q \end{bmatrix} < 0. \tag{18}$$

By pre- and postmultiplying (18) by $[I, I]$ and $[I, I]^T$, we get

$$P_i^T \left(A_i + A_{di}\right) + \left(A_i + A_{di}\right)^T P_i + dQ + \tau R + \sum_{j=1}^{N} r_{ij} E_j^T P_j < 0, \tag{19}$$

where Q and R are symmetric positive-definite matrices, we have

$$P_i^T \left(A_i + A_{di}\right) + \left(A_i + A_{di}\right)^T P_i + \sum_{j=1}^{N} r_{ij} E_j^T P_j < 0, \tag{20}$$

$$\sum_{j=1}^{N} r_{ij} E_i^T P_j + P_i^T A_i + A_i^T P_i < 0. \tag{21}$$

Based on Lemma 7, (15) and (20) show that the system $E(r(t))\dot{x}(t) = [A(r(t)) + A_d(r(t))]x(t)$ is regular and impulse free, and (15) and (21) ensure that the system $E(r(t))\dot{X}(t) = A(r(t))x(t)$ is regular and impulse free. Hence, according to Definition 2, the system (5) is regular and impulse free for any delay $\tau(t)$ satisfying $0 \le \tau(t) \le \tau$.

Second, to prove the system (5) is stochastically stable. Take a functional candidate for the system as follows:

$$\begin{aligned} V(x(t), r(t)) = {} & X^T(t) E^T(r(t)) P(r(t)) X(t) \\ & + \int_{t-\tau(t)}^{t} X^T(s) Q X(s)\, ds \\ & + \int_{-\tau}^{0} \int_{t+\theta}^{t} X^T(s) R X(s)\, ds\, d\theta. \end{aligned} \tag{22}$$

Then, let $\mathscr{L}$ be the weak infinitesimal generator of the random process $x(t), r(t)$, and for each $i \in \mathbb{S}$, we have

$$\begin{aligned} & \mathscr{L} V(x(t), r(t) = i) \\ & \quad = \lim_{\Delta \to 0} \frac{1}{\Delta} \{\mathscr{E}[V(x(t+\Delta), r(t+\Delta)) \mid x(t), \\ & \qquad r(t) = i] - V(x(t), r(t) = i)\} \end{aligned}$$

$$
\begin{aligned}
&\leq 2x^T(t)P_i^T E_i \dot{x}(t) + x^T(t)\left(\sum_{j=1}^{N} r_{ij}E_j^T P_j\right)x(t) \\
&\quad + x^T(t)Qx(t) \\
&\quad - (1-\dot{\tau}(t))x^T(t-\tau(t))Qx(t-\tau(t)) \\
&\quad + \tau\left(x^T(t)Rx(t)\right) - \int_{t-\tau}^{t} x^T(s)Rx(s)\,ds \\
&\leq 2x^T(t)P_i^T A_i x(t) \\
&\quad + 2x^T(t)P_i^T A_{di}x(t-\tau(t)) \\
&\quad + x^T(t)\left(\sum_{j=1}^{N} r_{ij}E_j^T P_j\right)x(t) + x^T(t)Qx(t) \\
&\quad - (1-d)x^T(t-\tau(t))Qx(t-\tau(t)) \\
&\quad + \tau x^T(t)Rx(t) - \left[\int_{t-\tau}^{t} x^T(s)\,ds\right] \\
&\quad \times \frac{R}{\tau}\left[\int_{t-\tau}^{t} x(s)\,ds\right] \\
&= \xi(t)\boldsymbol{\Phi}_i\xi^T(t),
\end{aligned}
\tag{23}
$$

where $\xi(t) = [x^T(t), x^T(t-\tau(t)), \int_{t-\tau}^{t} x^T(s)ds]$.

With (16), it is easy to know that there exists a scalar $\mu > 0$, such that

$$\mathcal{L}V(x(t), r(t) = i) \leq -\mu x^T(t)x(t). \tag{24}$$

By Dynkin's formula, we get

$$
\begin{aligned}
&\mathcal{E}\{V((x(t), r(t))x(0), r(0)) - V(x(0), r(0))\} \\
&\quad = \mathcal{E}\left\{\int_0^t LV(x(s), r(s))\,ds \mid x(0), r(0)\right\} \\
&\quad \leq -\mu\mathcal{E}\left\{\int_0^t x^T(s)x(s)\,ds \mid x(0), r(0)\right\},
\end{aligned}
\tag{25}
$$

and this means

$$\mathcal{E}\left(\int_0^t x^T(s)x(s)\,ds \mid x(0), r(0)\right) \leq \frac{1}{\mu}V(x(0), r(0)). \tag{26}$$

Therefore by Definition 2, the Markovian jump singular system (5) is stochastically stable. This completes the proof. □

3. Sliding Motion Stability Analysis

3.1. Sliding Surface Design. SMC design involves two basic steps: sliding surface design and controller design. For every $i \in \mathbb{S}$, integral sliding surface with delay and Markovian switching is considered as follows:

$$
\begin{aligned}
S(t) = G_iE_ix(t) &- \int_0^t G_i(A_i + B_iK_i)x(\theta)\,d\theta \\
&- \int_0^t G_iA_{di}x(\theta - \tau(\theta))\,d\theta,
\end{aligned}
\tag{27}
$$

where $K_i \in \mathbf{R}^{m\times n}$ is real matrix to be designed and $G_i \in \mathbf{R}^{m\times n}$ is designed to satisfy that G_iB_i is nonsingular. According to SMC theory, when the system trajectories reach onto the sliding surface, it follows that $S(t) = 0$ and $\dot{S}(t) = 0$.

Therefore, by $\dot{S}(t) = 0$, we get the equivalent control as

$$u_{\mathrm{eqi}}(t) = K_ix(t) - \overline{f}(x(t), t, i). \tag{28}$$

Substituting $u_{\mathrm{eq}}(\mathrm{r}(t) = K(r(t))x(t) - \overline{f}(x(t), t, r(t))$ into (1), we obtain the following sliding mode dynamics:

$$
\begin{aligned}
E(r(t))\dot{x}(t) = {}&[A(r(t)) + B(r(t))K(r(t))]x(t) \\
&+ A_d(r(t))x(t-\tau(t)).
\end{aligned}
\tag{29}
$$

3.2. Sliding Mode Dynamics Analysis. In this section, we pay attention to establishing *LMIs* conditions to check the stochastical admissibility of the system (29). Based on Lemmas 4, it is easy to get the sufficient condition provided in the following theorem.

Theorem 9. *Given scalars τ and d, for any delays $\tau(t)$ satisfying* (2), *the system* (29) *is stochastically admissible if there exist nonsingular matrices P_i and symmetric positive-definite matrix Q and R such that*

$$E_i^TP_i = P_i^TE_i \geq 0, \tag{30}$$

$$
\begin{bmatrix}
\Phi_i^{11} & P_i^TA_{di} & 0 \\
* & -(1-d)Q & 0 \\
* & * & -\frac{1}{\tau}R
\end{bmatrix} < 0,
\tag{31}
$$

where

$$
\begin{aligned}
\Phi_i^{11} = {}&P_i^T(A_i + B_iK_i) + (A_i + B_iK_i)^TP_i \\
&+ Q + \tau R + \sum_{j=1}^{N} r_{ij}E_j^TP_j.
\end{aligned}
\tag{32}
$$

Remark 10. Note that the conditions in Theorem 9 are not strict *LMI* conditions due to matrix equality constraint of (30). According to Lemma 3 and Theorem 9, the strict *LMI* conditions are given as follows.

Theorem 11. *Given scalars τ and d, for any delays $\tau(t)$ satisfying* (2), *the system* (29) *is stochastically admissible if there exist symmetric positive-definite matrices $\mathbb{X}$, Q, Y_i, H, R, and symmetric matrix $\mathbb{T} \in \mathbf{R}^{(n-r)\times(n-r)}$, matrix $\mathbb{L}_i \in R^{m\times n}$,* *and any matrices with full row rank M_i, S_i satisfying $M_iE_i = 0$, $E_iS_i = 0$, respectively, such that*

$$\begin{bmatrix} \Gamma_i^{11} & A_{di} & N_1 \\ * & -(1-d)Q & 0 \\ * & * & N_2 \end{bmatrix} < 0, \quad (33)$$

$$\begin{bmatrix} Q+\tau R - H & 0 & N_3 \\ * & -\frac{1}{\tau}R & 0 \\ * & * & N_4 \end{bmatrix} < 0, \quad (34)$$

$$\begin{bmatrix} -Y_i & 0 & E_i\mathbb{X} + M_i^T\mathbb{T}S_i^T & 0 \\ * & -H & 0 & H \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (35)$$

where

$$\begin{aligned} \Gamma_i^{11} &= A_i\left(\mathbb{X}E_i^T + S_i\mathbb{T}M_i\right) + \left(\mathbb{X}E_i^T + S_i\mathbb{T}M_i\right)^T A_i^T + B_i\mathbb{L}_iE_i^T \\ &\quad + \left(B_i\mathbb{L}_iE_i^T\right)^T + r_{ii}\left(\mathbb{X}E_i^T + S_i\mathbb{T}M_i\right)^T E_i^T + Y_i, \\ N_1 &= \left[B_i\mathbb{L}_i, M_i^T\mathbb{T}S_i^T\right], \qquad N_2 = \operatorname{diag}\left[-\mathbb{X}, -\mathbb{X}\right], \\ N_3 &= \left[E_{1R}, \ldots, E_{i-1,R}, E_{i+1,R}, \ldots, E_{nR}\right], \\ N_4 &= \operatorname{diag}\left[-\frac{1}{r_{i1}}E_{1R}^T\mathbb{X}E_{1R}, \ldots, -\frac{1}{r_{i,i-1}}E_{i-1,R}^T\mathbb{X}E_{i-1,R},\right. \\ &\quad \left. -\frac{1}{r_{i,i+1}}E_{i+1R}^T\mathbb{X}E_{i+1R}, \ldots, -\frac{1}{r_{in}}E_{nR}^T\mathbb{X}E_{nR}\right], \\ \mathbb{L}_i &= K_i\mathbb{X}. \end{aligned} \quad (36)$$

Proof. Let $P_i \triangleq XE_i + M_i^TTS_i^T$ in Theorem 9. We can get

$$\begin{bmatrix} \Phi_i^{11} & \left(XE_i + M_i^TTS_i^T\right)^T A_{di} & 0 \\ * & -(1-d)Q & 0 \\ * & * & -\frac{1}{\tau}R \end{bmatrix} < 0, \quad (37)$$

where

$$\begin{aligned} \Phi_i^{11} &= \left(XE_i + M_i^TTS_i^T\right)^T\left(A_i + B_iK_i\right) \\ &\quad + \left(A_i + B_iK_i\right)^T\left(XE_i + M_i^TTS_i^T\right) \\ &\quad + Q + \tau R + \sum_{j=1}^{N} r_{ij}E_j^TXE_j. \end{aligned} \quad (38)$$

Using Lemma 3, we get

$$\left(XE_i + M_i^TTS_i^T\right)^{-1} = \mathbb{X}E_i^T + S_i\mathbb{T}M_i \triangleq L_i. \quad (39)$$

By pre- and postmultiplying (37) by $\operatorname{diag}[L_i^T, I, I]$ and $\operatorname{diag}[L_i, I, I]$, we get

$$\begin{bmatrix} \Psi_i'^{11} & A_{di} & 0 \\ * & -(1-d)Q & 0 \\ * & * & -\frac{1}{\tau}R \end{bmatrix} < 0, \quad (40)$$

where $\Psi_i'^{11} = (A_i + B_iK_i)L_i + L_i^T(A_i + B_iK_i)^T + L_i^TQL_i + \tau L_i^TRL_i + \sum_{j=1}^{N} r_{ij}L_i^TE_j^TXE_j.$

In light of Lemma 4, there exists symmetric matrix Y_i such that

$$\begin{bmatrix} \left(A_i+B_iK_i\right)L_i + L_i^T\left(A_i+B_iK_i\right)^T + r_{ii}L_i^TE_i^T + Y_i & A_{di} \\ * & -(1-d)Q \end{bmatrix} < 0, \quad (41)$$

$$\begin{bmatrix} L_i^TQL_i + \tau L_i^TRL_i + \sum_{i\neq j=1}^{N} r_{ij}L_i^TE_j^TXE_jL_i - Y_i & 0 \\ * & -\frac{1}{\tau}R \end{bmatrix} < 0. \quad (42)$$

It is easy to know that

$$\begin{aligned} B_iK_iS_i\mathbb{T}M_i + \left(B_iK_iS_i\mathbb{T}M_i\right)^T &= B_iK_i\mathbb{X}\mathbb{X}^{-1}S_i\mathbb{T}M_i \\ &\quad + \left(B_iK_i\mathbb{X}\mathbb{X}^{-1}S_i\mathbb{T}M_i\right)^T \\ &\leq \left(B_iK_i\mathbb{X}\right)\mathbb{X}^{-1}\left(\mathbb{X}K_i^TB_i^T\right) \\ &\quad + \left(M_i^T\mathbb{T}S_i^T\right)\mathbb{X}^{-1}\left(S_i\mathbb{T}M_i\right). \end{aligned} \quad (43)$$

Using Shur's complement, the following inequality can ensure (41) as

$$\begin{bmatrix} A_iL_i + L_i^TA_i^T + B_iK_i\mathbb{X}E_i^T + \left(B_iK_i\mathbb{X}E_i^T\right)^T + r_{ii}L_i^TE_i^T + Y_i & A_{di} & N_1 \\ * & -(1-d)Q & 0 \\ * & * & N_2 \end{bmatrix} > 0, \quad (44)$$

where $N_1 = [B_iK_i\mathbb{X}, M_i^T\mathbb{T}S_i^T]$ and $N_2 = \operatorname{diag}[-\mathbb{X}, -\mathbb{X}]$. Equation (42) is equivalent to

$$\begin{bmatrix} Q + \tau R + \sum_{i\neq j=1}^{N} r_{ij}E_j^TXE_j - P_i^TY_iP_i & 0 \\ * & -\frac{1}{\tau}R \end{bmatrix} < 0. \quad (45)$$

There exists a matrix $H^T = H > 0$ such that

$$\begin{aligned} &\begin{bmatrix} Q + \tau R + \sum_{i\neq j=1}^{N} r_{ij}E_j^TXE_j - P_i^TY_iP_i & 0 \\ * & -\frac{1}{\tau}R \end{bmatrix} \\ &\quad < \begin{bmatrix} Q + \tau R + \sum_{i\neq j=1}^{N} r_{ij}E_j^TXE_j - H & 0 \\ * & -\frac{1}{\tau}R \end{bmatrix} < 0 \end{aligned} \quad (46)$$

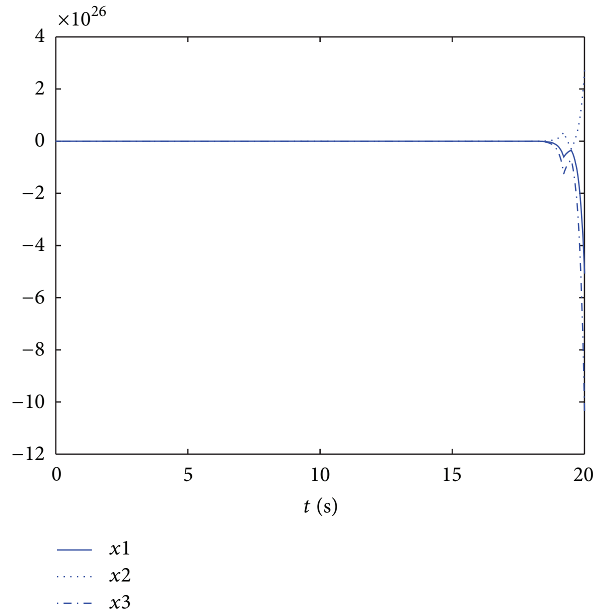

FIGURE 1: State trajectories of the open-loop system.

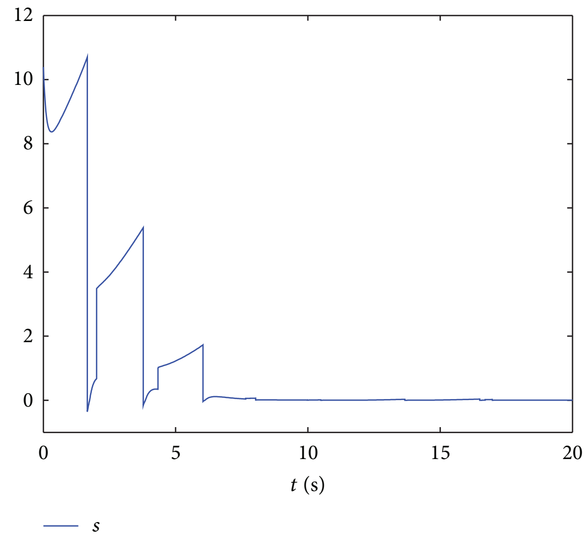

FIGURE 2: Switching surface $s(t)$.

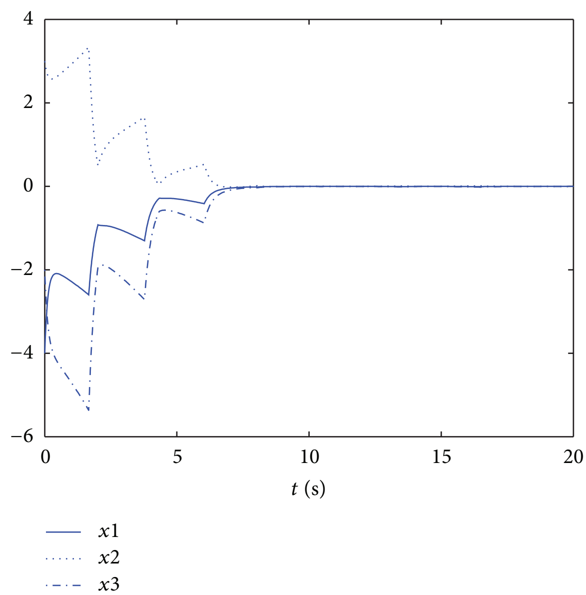

FIGURE 3: State trajectories of the closed-loop system.

if and only if

$$-P_i^T Y_i P_i < -H < 0. \tag{47}$$

The above inequality is equivalent to the following inequality:

$$\begin{bmatrix} -Y_i & L_i^T H \\ * & -H \end{bmatrix} < 0. \tag{48}$$

And the following inequality can guarantee that (48) is true:

$$\begin{bmatrix} -Y_i & 0 & L_i^T & 0 \\ * & -H & 0 & H \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix} < 0. \tag{49}$$

By Shur's complement, the right of (46) is equivalent to

$$\begin{bmatrix} Q+\tau R-H & 0 & N_3 \\ * & -\dfrac{1}{\tau}R & 0 \\ * & * & N_4 \end{bmatrix} < 0, \tag{50}$$

where $N_3 = [E_{1R}, \ldots, E_{i-1,R}, E_{i+1,R}, \ldots, E_{nR}]$ and $N_4 = \operatorname{diag}[-(1/r_{i1})E_{1R}^T \mathbb{X} E_{1R}, \ldots, -(1/r_{i,i-1})E_{i-1,R}^T \mathbb{X} E_{i-1,R}, -(1/r_{i,i+1}) E_{i+1R}^T \mathbb{X} E_{i+1R}, \ldots, -(1/r_{in})E_{nR}^T \mathbb{X} E_{nR}]$. E_i is decomposed as $E_i = E_{iL}E_{iR}^T$ with $E_{iL} \in \mathbf{R}^{n\times r}$ and $E_{iR} \in \mathbf{R}^{n\times r}$ are of full column rank. This completes the proof. □

Remark 12. From the proof of the Theorem 11, it is not difficult to know that Theorem 11 is more easy to compute than Theorem 9.

3.3. Sliding Mode Control Design. After switching surface design, the next important part of sliding mode control is to design a slide mode controller to guarantee the existence of a sliding mode. Now, we design an SMC law, by which the trajectories of singular system (1) can be driven onto the designed sliding surface $S(t) = 0$ in a finite time.

Theorem 13. *With the constant matrix K_i mentioned in Theorem 11 and the integral sliding surface given by* (29)*, the trajectory of the closed-loop system* (1) *can be driven onto the sliding surface in finite time with the control* (51) *as*

$$u_i(t) = K_i x(t) - (\rho_i + \epsilon_i \|x(t)\|) \operatorname{sign}\left(B_i^T G_i^T S(t)\right), \tag{51}$$

where ρ_i is a positive constant.

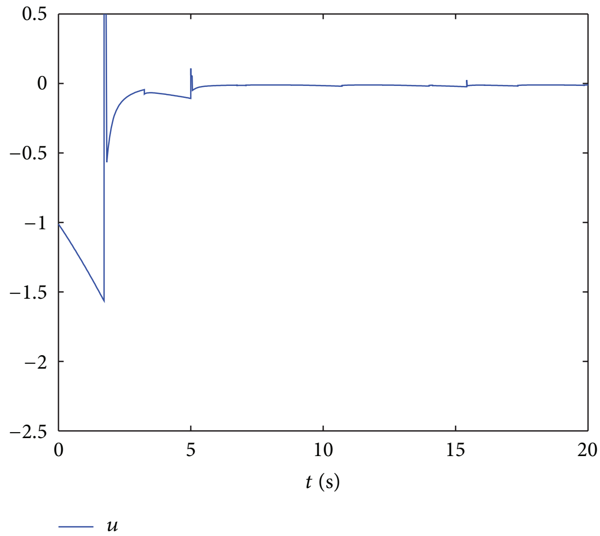

Figure 4: Controller input $u(t)$.

Proof. Choose G_i under the condition of G_iB_i is nonsingular. Consider the following Lyapunov function:

$$V(S(t),t)=\frac{1}{2}S^T(t)S(t). \tag{52}$$

Due to (29), we have

$$dS=G_iB_i\left[-K_ix(t)+u_i(t)+\overline{f}_i\right]dt. \tag{53}$$

Differentiating $V_i(t)$ along the closed-loop trajectories and using (53), we have

$$\begin{aligned}\mathscr{L}V(S(t),(t))&=S_i^T(t)\,dS_i(t)\\&=S_i^T(t)\,G_iB_i\left[-K_ix(t)+u_i(t)+\overline{f}_i\right]\\&=S_i^T(t)\,G_iB_i\left[-(\rho+\epsilon\|x(t)\|)\operatorname{sign}\right.\\&\qquad\left.\times\left(B_i^TG_i^TS(t)\right)+\overline{f}_i\right]\\&\le-(\rho+\epsilon\|x(t)\|)\left\|B_i^TG_i^TS_i(t)\right\|\\&\quad+\left\|\overline{f}_i\right\|\left\|B_i^TG_i^TS_i(t)\right\|\\&\le-\rho\left\|B_i^TG_i^TS_i(t)\right\|\\&\le-\overline{\rho}\left\|S_i(t)\right\|<0,\quad\text{if } S_i(t)\neq0,\end{aligned} \tag{54}$$

where $\overline{\rho}_i>\rho_i\sqrt{\lambda_{\min}(G_iB_iB_i^TG_i^T)}$. Then the state trajectory converges to the surface and is restricted to the surface for all subsequent time. This completes the proof. □

4. Numerical Example

In this section, a numerical example is presented to illustrate the effectiveness of the main results in this paper.

Example 14. Let us consider the system (1) with Markovian process that governs that the mode switching has generator $\prod=(r_{ij})$, $(i,j=1,2)$, $r_{12}=0.25$, $r_{21}=0.2$. The system data are as follows:

$$\begin{gathered}\mathbf{E}_1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\qquad\mathbf{A}_1=\begin{bmatrix}-6.7&1.6&0\\0&-5&0\\0.5&2.2&3.2\end{bmatrix},\\\mathbf{A}_{d1}=\begin{bmatrix}-0.2&0&0\\0&-0.1&0\\0&0&-0.1\end{bmatrix},\qquad\mathbf{B}_1=\begin{bmatrix}1\\1\\0\end{bmatrix},\\\mathbf{M}_1=\begin{bmatrix}0&0&1\end{bmatrix},\qquad\mathbf{S}_1=\begin{bmatrix}0\\0\\1\end{bmatrix},\qquad\mathbf{E}_{1R}=\begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix},\\\mathbf{E}_2=\begin{bmatrix}1&0&0\\0&0&0\\0&0&1\end{bmatrix},\qquad\mathbf{A}_2=\begin{bmatrix}-5.7&-4.2&0\\-3.7&4.8&1.3\\0&2.4&-6\end{bmatrix},\\\mathbf{A}_{d2}=\begin{bmatrix}-0.1&0&0\\0&-0.1&0\\0&0&0\end{bmatrix},\qquad\mathbf{B}_2=\begin{bmatrix}1\\1\\1\end{bmatrix},\\\mathbf{M}_2=\begin{bmatrix}0&1&0\end{bmatrix},\qquad\mathbf{S}_2=\begin{bmatrix}0\\1\\0\end{bmatrix}\qquad\mathbf{E}_{2R}=\begin{bmatrix}1&0\\0&0\\0&1\end{bmatrix}.\end{gathered} \tag{55}$$

In addition, $\tau=0.3,d=0.3,\tau(t)=0.3e^{-t}$ and $f_1(x)=0.65B_1x(t)\sin x(t)$, $f_2(x)=0.65B_2x(t)\sin x(t)$. Solve the LMI (33), (34), and (35) as follows:

$$\begin{aligned}\mathbf{K_1}&=\begin{bmatrix}0.0963&-0.0506&-0.003\end{bmatrix}\\\mathbf{K_2}&=\begin{bmatrix}0.0953&-0.0039&0.1793\end{bmatrix}.\end{aligned} \tag{56}$$

For $\det(G_iB_i)\neq0$, we can choose G_1 and G_2 as

$$G_1=\begin{bmatrix}1.2&1.6&1.1\end{bmatrix};\qquad G_2=\begin{bmatrix}1.3&1.2&1.5\end{bmatrix}. \tag{57}$$

Thus the sliding surface function is

$$s_1(t)=\begin{cases}s_1(t)&=[1.2,1.6,0]\,x(t)\\&\quad-\int_0^t[-7.2204,-3.8017,-3.5116]x(\theta)\,d\theta\\&\quad-\int_0^t[-0.24,-0.016,-0.11]x(\theta-\tau(\theta))\,d\theta,\\&\qquad i=1\\s_2(t)&=[1.3,0,1.5]\,x(t)\\&\quad-\int_0^t[-11.4688,3.8852,-6.7228]\,x(\theta)\,d\theta\\&\quad-\int_0^t[-0.13,-0.12,0]\,x(\theta-\tau(\theta))\,d\theta,\\&\qquad i=2.\end{cases} \tag{58}$$

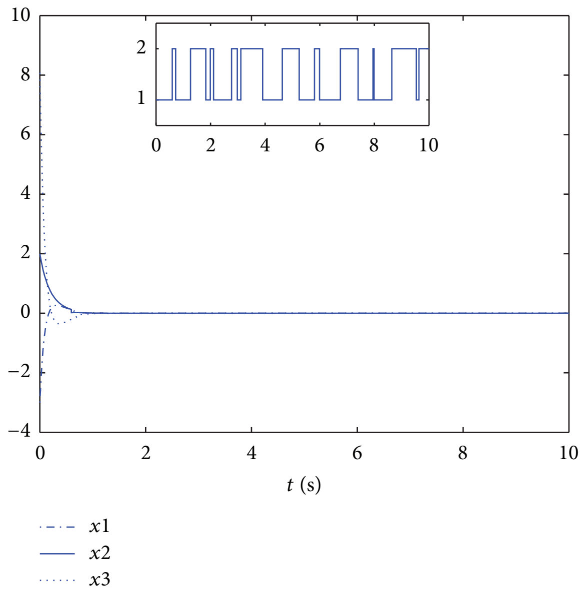

FIGURE 5: State trajectories of sliding mode.

Take $\rho_1 = \rho_2 = 0.64$, then the SMC law designed in (51) can be described as

$$u(t) = \begin{cases} u_1(t) & = [0.0963, -0.0506, -0.003]\, x(t) \\ & \quad -\rho(t)\, \mathrm{sign}\left(2.8 s_1(t)\right), \quad i = 1 \\ u_2(t) & = [0.0953, -0.0037, 0.1793]\, x(t) \\ & \quad -\rho(t)\, \mathrm{sign}\left(4 s_2(t)\right), \quad i = 2, \end{cases} \tag{59}$$

where $\rho(t) = 0.64 + 0.65\|x(t)\|$. The simulation results are given in Figures 1, 2, 3, 4, and 5, which show the validity of the proposed method.

Remark 15. Obviously, our results include Markovian switching and nonlinear perturbation effects, and this model can not be dealt with by the results of [6, 8, 10–14, 16, 18, 20, 22, 26, 27, 34, 37], which show that our results are new.

5. Conclusion

In this paper, the stochastically admissible using sliding mode control for singular system with time-varying delay and nonlinear perturbations is studied by LMI method. The sliding mode control is designed to ensure that the closed-loop system is stochastically admissible. A numerical example demonstrates the effectiveness of the method mentioned above.

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