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Chaotic Dynamics in a Flexible Exchange Rate System: A Study of Noise Effects

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In this paper, we investigate by means of analytical method and numerical simulations the properties of three-dimensional business cycle model, in which foreign exchange rate is flexible and a parameter is fluctuated by noise. The model is a discrete time version of Asada (*Journal of Economics*, 62, 239–269, 1995)'s continuous time open economy model without noise. We show (1) noise may suppress the burst of flexible foreign exchange rate when its behavior begins to burst as a bifurcation parameter (adjustment speed of the goods market) is increased, (2) the windows of cycles can be broken by noise, and (3) noise may reveal the hidden structures.

Keywords: Chaotic dynamics, Flexible exchange rates, Noise effects, Small open economy

1. INTRODUCTION

In general, an economy is not an isolated system but it is subject to the disturbances from other subsystems of the society. This observation is particularly important for theoretical and empirical investigations in international economics or regional sciences, which study the economic interactions between several regions. Asada (1995) presented a dynamic model of small open economy by introducing international trade and international capital movement into the Kaldorian business cycle theory. Asada (1995) investigated both of the fixed exchange rate system and the flexible exchange rate system in a framework of the continuous time model without stochastic disturbance (noise). Asada *et al.* (1998) studied a discrete time version of the fixed exchange rate system with noise effects by means of numerical simulations, and showed that such a system can produce very complex behavior including chaos.

This paper also considers a discrete time version of the Kaldorian business cycle model in a small open economy with noise effects, but in this study

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we concentrate on the system of flexible exchange rates. We investigate by means of analytical method and numerical simulations the properties of three-dimensional system, in which a parameter which represents the adjustment speed of the adaptive expectation of exchange rate is fluctuated by noise. Asada *et al.* (1998) showed in a framework of the fixed exchange rate system that the noise may not only obscure the underlying structures, but also reveal the hidden structures, for example, chaotic attractors near the window. In this paper, we show (1) noise may suppress the burst of flexible exchange rate when its behavior begins to burst as a bifurcation parameter (adjustment speed of the goods market) is increased, (2) the windows of cycle can be broken by noise, and (3) noise may reveal the hidden structures.

2. FORMULATION OF THE MODEL

The basic system of equations is given as follows*:

$$Y_{t+1} - Y_t = \alpha[C_t + I_t + G + J_t - Y_t]; \quad \alpha > 0, \tag{1}$$

$$K_{t+1} - K_t = I_t, \tag{2}$$

$$C_t = c(Y_t - T_t) + C_0; \quad 0 < c < 1, \ C_0 > 0, \tag{3}$$

$$I_t = I(Y_t, K_t, r_t); \quad I_Y \equiv \partial I_t/\partial Y_t > 0, \quad I_K \equiv \partial I_t/\partial K_t < 0, \quad I_t \equiv \partial I_t/\partial r_t < 0, \tag{4}$$

$$T_t = \tau Y_t - T_0; \quad 0 < \tau < 1, \ T_0 > 0, \tag{5}$$

$$M_t/p = L(Y_t, r_t); \quad L_Y \equiv \partial L_t/\partial Y_t > 0, \quad L_t \equiv \partial L_t/\partial r_t < 0, \tag{6}$$

$$J_t = J(Y_t, E_t); \quad J_Y \equiv \partial J_t/\partial Y_t < 0, \quad J_E \equiv \partial J_t/\partial E_t > 0, \tag{7}$$

$$Q_t = \beta\{r_t - r_{\rm f} - (E_t^{\rm e} - E_t)/E_t\}; \quad \beta > 0, \tag{8}$$

$$A_t = J_t + Q_t, \tag{9}$$

$$A_t = 0, \tag{10}$$

$$E_{t+1}^{\rm e} - E_t^{\rm e} = (\gamma + \sigma\varepsilon_t)(E_t - E_t^{\rm e}); \quad \gamma > 0, \ \sigma \gtreqless 0, \tag{11}$$

$$M_t = \bar{M}, \tag{12}$$

where the meanings of the symbols are as follows. Y = net real national income, C = real consumption expenditure, I = net real private investment expenditure, G = real government expenditure (fixed), K = real physical capital stock, T = real income tax, M = nominal money supply, p = price level (fixed), r = nominal domestic rate of interest, $r_{\rm f}$ = nominal foreign rate of interest (fixed), E = value of a unit of foreign currency in terms of domestic currency (exchange rate), $E^{\rm e}$ = expected exchange rate of near future, J = balance of current account (net export) in real terms, Q = balance of capital account in real terms, A = total balance of payments in real terms, α = adjustment speed in the goods market, β = parameter which represents the 'degree of capital mobility', γ = parameter which represents the "speed of adaptation" of the expected exchange rate, ε = normal pseudo random number $N(0, 1)$, σ = standard deviation parameter. The subscript t denotes time period.

Equation (1) represents the quantity adjustment process in the goods market. Equation (2) says that the physical capital stock increases or decreases according as the net investment is positive or negative. Equations (3), (4), and (5) are consumption function, investment function, and income tax function respectively. Equation (6) is the equilibrium condition for the money market. Equation (7) says that the current account is determined by Y_t and E_t, which is a standard type of the current account function. Equation (8) formalizes the idea that the capital account becomes positive or negative according as the difference between the rates of return of domestic and foreign bonds is positive or negative. We can consider β as the index of the degree of the capital mobility. Equation (9) is

* Equations (1)–(7) in this paper are identical to those in a fixed exchange rate system which was presented in Asada *et al.* (1998).

the definition of the total balance of payments. Equation (10) is a characterization of the flexible exchange rate system, i.e., it is assumed that the exchange rate is adjusted instantaneously to keep the equilibrium of the total balance of payments ($A_t = 0$). Equation (11) is a formalization of the 'adaptive expectation hypothesis' concerning the expected exchange rate. It is assumed that the speed of adaptation is fluctuated by noise. Equation (12) says that under flexible exchange rate system the domestic monetary authority can control money supply contrary to the case of fixed exchange rate system, so that we can consider the money supply (M) as an exogenous variable.[†]

We can reduce the above system (1)–(12) to the following system of equations[‡]:

$$\begin{aligned}
&\text{(i)}\ \ Y_{t+1} - Y_t = \alpha[(1-\tau)Y_t + cT_0 + C_0 \\
&\qquad + G + I(Y_t, K_t, r(Y_t, \bar{M})) \\
&\qquad + J(Y_t, E_t) - Y_t]; \quad \alpha > 0, \\
&\text{(ii)}\ \ K_{t+1} - K_t = I(Y_t, K_t, r(Y_t, \bar{M})), \qquad (13) \\
&\text{(iii)}\ \ A_t = J(Y_t, E_t) + \beta\{r(Y_t, \bar{M}) \\
&\qquad - r_{\rm f} - E_t^{\rm e}/E_t + 1\} = 0, \\
&\text{(iv)}\ \ E_{t+1}^{\rm e} - E_t^{\rm e} = (\gamma + \sigma\varepsilon_t)(E_t - E_t^{\rm e}).
\end{aligned}$$

Solving Eq. (13)(iii) with respect to E_t, we obtain

$$\begin{aligned}
&E_t = E(Y_t, E_t^{\rm e}; \beta), \\
&E_Y \equiv \partial E_t/\partial Y_t = (-J_Y - \beta r_Y)/(J_E + \beta E_t^{\rm e}/E_t^2) \\
&\quad = (m - \beta r_Y)/(J_E + \beta E_t^{\rm e}/E_t^2) \qquad (14) \\
&\quad \gtreqless 0 \iff \beta \lesseqgtr m/r_Y, \\
&E_E^{\rm e} = \partial E_t/\partial E_t^{\rm e} = \beta/(J_E E_t + \beta E_t^{\rm e}/E_t) > 0,
\end{aligned}$$

where $m \equiv -J_Y \equiv -\partial J_t/\partial Y_t > 0$, $r_Y \equiv \partial r_t/\partial Y_t > 0$, and $J_E \equiv \partial J_t/\partial E_t > 0$.[¶] Equation (14) implies that E_t is an increasing function of Y_t when the degree of capital mobility (β) is sufficiently small, but it becomes a decreasing function of Y_t when β is sufficiently large.[§]

Substituting Eq. (14) into (13), we obtain the following system of 'fundamental dynamical equations':

$$\begin{aligned}
&\text{(i)}\ \ Y_{t+1} = Y_t + \alpha[(1-\tau)Y_t + cT_0 \\
&\qquad + C_0 + G + I(Y_t, K_t, r(Y_t, \bar{M})) \\
&\qquad + J(Y_t, E(Y_t, E_t^{\rm e}; \beta)) - Y_t] \\
&\qquad \equiv F_1(Y_t, K_t, E_t^{\rm e}; \alpha, \beta), \\
&\text{(ii)}\ \ K_{t+1} = K_t + I(Y_t, K_t, r(Y_t, \bar{M})) \equiv F_2(Y_t, K_t), \\
&\text{(iii)}\ \ E_{t+1}^{\rm e} = E_t^{\rm e} + (\gamma + \sigma\varepsilon_t)\{E(Y_t, E_t^{\rm e}; \beta) - E_t^{\rm e}\} \\
&\qquad \equiv F_3(Y_t, E_t^{\rm e}; \beta, \gamma, \sigma),
\end{aligned} \qquad (\mathrm{S}_1)$$

On the other hand, the continuous time version without noise effect which was formulated in Asada (1995) is read as

$$\begin{aligned}
&\text{(i)}\ \ {\rm d}Y/{\rm d}t = \alpha[(1-\tau)Y + cT_0 + C_0 \\
&\qquad + G + I(Y, K, r(Y, \bar{M})) \\
&\qquad + J(Y, E(Y, E^{\rm e}; \beta)) - Y] \\
&\qquad \equiv f_1(Y, K, E^{\rm e}; \alpha, \beta), \\
&\text{(ii)}\ \ {\rm d}K/{\rm d}t = I(Y, K, r(Y, \bar{M})) \equiv f_2(Y, K), \\
&\text{(iii)}\ \ {\rm d}E^{\rm e}/{\rm d}t = \gamma\{E(Y, E^{\rm e}; \beta) - E^{\rm e}\} \\
&\qquad \equiv f_3(Y, E^{\rm e}; \beta, \gamma).
\end{aligned} \qquad (\mathrm{S}_2)$$

It is easily shown that the equilibrium point $(Y^*, K^*, E^{\rm e*})$ of the system (S_1) is identical to that of the system (S_2), and Asada (1995) showed that there exists an equilibrium point $(Y^*, K^*, E^{\rm e*}) > (0, 0, 0)$ in the system (S_2) under some reasonable conditions. From now on, we *assume* that there exists an economically meaningful equilibrium point in the system (S_1).

[†] Under the fixed exchange rate system, money supply endogenously fluctuates according as the total balance of payments is positive or negative. See Asada (1995) and Asada *et al.* (1998).

[‡] In Eq. (13), $r(Y_t, \bar{M})$ is the 'LM equation' which is the solution of Eq. (6) with respect to r_t.

[¶] Solving Eq. (6) with respect to r_t, we have $r_t = r(Y_t, \bar{M})$; $r_Y \equiv \partial r_t/\partial Y_t = -L_Y/L_r > 0$, where $L_Y \equiv \partial L_t/\partial Y_t > 0$ and $L_r \equiv \partial L_t/\partial r_r < 0$.

[§] The economic implication of this result is very clear. When Y_t increases, the current account (J_t) decreases through the increase of the import, while the capital account (Q_t) increases through the increase of the domestic rate of interest. If β is small, the 'current account effect' dominates the 'capital account effect' so that the total balance of payments (A_t) *decreases*. In this case, the exchange rate (E_t) must *increase* to keep the equilibrium of the balance of payments ($A_t = 0$). On the other hand, if β is large, the 'capital account effect' dominates so that A_t *increases*. In this case, E_t must *decrease* to keep $A_t = 0$.

3. LOCAL STABILITY – INSTABILITY ANALYSIS

Asada (1995) proved the following propositions.

(1) The equilibrium point of the system (S_2) is locally stable if β is sufficiently large.
(2) Suppose that $I_Y + I_r r_Y > 1 - c(1-\tau)$ *at the equilibrium point*. Then, the equilibrium point of the system (S_2) becomes locally unstable when β is sufficiently small and α is sufficiently large.

This proposition implies that in a continuous time version of our model, large capital mobility between countries (or regions) tends to *stabilize* the system. Does this conclusion also apply to the discrete time version? In fact, proposition (1) does not apply to the system (S_1), because in the discrete time version the 'overshooting phenomena' are not negligible so that the system becomes unstable when the degree of capital mobility is too large. Now, let us prove this assertion formally by assuming $\sigma = 0$ (no stochastic disturbance).

We can write the Jacobian matrix of the system (S_1) which is evaluated *at the equilibrium point* as follows:

$$J_1 = \begin{bmatrix} F_{11}(\alpha,\beta) & F_{12}(\alpha) & F_{13}(\alpha,\beta) \\ F_{21} & F_{22} & 0 \\ F_{31}(\beta,\gamma) & 0 & F_{33}(\beta,\gamma) \end{bmatrix}, \qquad (15)$$

where

$$F_{11}(\alpha,\beta) = 1 + \alpha[\underset{(+)}{I_Y} + \underset{(-)}{I_r}\,\underset{(+)}{r_Y} - \{1 - c(1-\tau) + \underset{(+)}{m}\} + \underset{(+)}{J_E}\,\underset{(?)}{E_Y(\beta)}],$$

$$F_{12}(\alpha) = \alpha I_K < 0, \quad F_{13}(\alpha,\beta) = \alpha \underset{(+)}{J_E}\,\underset{(+)}{E_E(\beta)} > 0,$$

$$F_{21} = \underset{(+)}{I_Y} + \underset{(-)}{I_r}\,\underset{(+)}{r_Y}, \quad F_{22} = 1 + \underset{(-)}{I_K},$$

$$F_{31}(\beta,\gamma) = \gamma \underset{(?)}{E_Y(\beta)},$$

$$F_{33}(\beta,\gamma) = 1 + \gamma\{E_E(\beta) - 1\} = 1 - (\underset{(+)}{J_E}\,E\gamma)/(\underset{(+)}{J_E}\,E + \beta).$$

We can write the characteristic equation of this system as

$$\psi(\lambda) = |\lambda I - J_1| = \lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \qquad (16)$$

where

$$\text{(i)}\ a_1 = -\text{trace}\, J_1 = -F_{11}(\alpha,\beta) - F_{22} - F_{33}(\beta,\gamma),$$

$$\text{(ii)}\ a_2 = \begin{vmatrix} F_{22} & 0 \\ 0 & F_{33} \end{vmatrix} + \begin{vmatrix} F_{11} & F_{13} \\ F_{31} & F_{33} \end{vmatrix} + \begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix}$$
$$= F_{22}F_{33}(\beta,\gamma) + F_{11}(\alpha,\beta)F_{33}(\beta,\gamma) - F_{13}(\alpha,\beta)F_{31}(\beta,\gamma) + F_{11}(\alpha,\beta)F_{22} - F_{12}(\alpha,\beta)F_{21},$$

$$\text{(iii)}\ a_3 = -\det J_1 = -F_{11}(\alpha,\beta)F_{22}F_{33}(\beta,\gamma) + F_{13}(\alpha,\beta)F_{22}F_{31}(\beta,\gamma) + F_{12}(\alpha)F_{21}F_{33}(\beta,\gamma). \qquad (17)$$

It follows from the Cohn–Schur conditions for local stability that the system (S_1) is locally stable *if and only if* the following conditions are satisfied‖

$$\text{(i)}\ 1 + a_2 - |a_1 + a_3| > 0,$$
$$\text{(ii)}\ 1 - a_2 + a_1 a_3 - a_3^2 > 0, \qquad (18)$$
$$\text{(iii)}\ a_2 < 3.$$

Therefore, the equilibrium point of the system (S_1) becomes locally unstable if the inequality $a_2 > 3$ is satisfied. The following proposition is a simple corollary of this fact.

PROPOSITION *Suppose that*

$$0 < \underset{(+)}{I_Y} + \underset{(-)}{I_r}\,\underset{(+)}{r_Y} < \underbrace{1 - c(1-\tau)}_{(+)} + \underset{(+)}{m}$$

‖ See, for example, Gandolfo (1996) Chap. 7. In fact, the inequality (18)(iii) is redundant because this inequality can be derived from other two inequalities. Nevertheless, the inequality (18)(iii) as a *necessary* condition for local stability is useful for our purpose.

and

$$I_K < -1.$$

Then, the equilibrium point of the system (S_1) *is locally unstable when either of the following conditions* (A) *or* (B) *is satisfied.*

(A) *The parameters* β *and* α *are sufficiently large.*
(B) *The parameters* β *and* γ *are sufficiently large.*

Proof From Eq. (14), the definitions of $F_{31}(\beta,\gamma)$ and $F_{33}(\beta,\gamma)$, the fact that $E^e = E$ at the equilibrium point we have the following relationships:

$$\lim_{\beta\to\infty} J_E E_Y = \lim_{\beta\to\infty}(m-\beta r_Y)/(1+\beta/J_E E) = -r_Y J_E E < 0, \tag{19}$$

$$\lim_{\beta\to\infty} F_{31}(\beta,\gamma) = \lim_{\beta\to\infty}\gamma(m-\beta r_Y)/(J_E+\beta) = -\gamma r_Y < 0, \tag{20}$$

$$\lim_{\beta\to\infty} F_{33}(\beta,\gamma) = \lim_{\beta\to\infty}[1+\gamma\{\beta/(J_E E+\beta)-1\}] = 1. \tag{21}$$

Therefore, we obtain the following expression

$$A \equiv \lim_{\beta\to\infty} a_2 = \underset{(-)}{F_{22}} + \underset{(-)}{F^*_{11}}(\alpha)\underbrace{(1+F_{22})}_{(-)} - \underset{(+)}{F^*_{13}}(\alpha)\underset{(-)}{F^*_{31}}(\gamma) - F^*_{12}(-)(\alpha)\underset{(+)}{F_{21}}, \tag{22}$$

where $F^*_{ij} = \lim_{\beta\to\infty} F_{ij}$, $\partial F^*_{11}(\alpha)/\partial\alpha < 0$, $\partial F^*_{12}(\alpha)/\partial\alpha < 0$, $\partial F^*_{13}(\alpha)/\partial\alpha > 0$, and $\partial F^*_{31}(\gamma)/\partial\gamma < 0$. It follows from Eq. (22) that we have $A > 3$ for sufficiently large β and α, or alternatively, for sufficiently large β and γ. This proves the proposition.

This proposition shows that under some additional conditions the high degree of capital mobility (β) combined with high adjustment speed in the goods market (α) *or* high speed of adaptation of the expected exchange rate (γ) tends to *destabilize* the system (S_1) because of the overshooting phenomena contrary to the continuous time model.

4. NUMERICAL SIMULATIONS

In this section, we shall present the results of some numerical simulations of the model which was formulated in the previous section. We adopt the following specifications of the functions and parameter values:

$$I(Y_t, K_t, r_t) = f(Y_t) - 0.3K_t - r_t + 100, \tag{23}$$

$$f(Y_t) = (80/3.1415)\text{Arc}\tan\{(2.25\times 3.1415/20)\times(Y_t-112)\} + 35, \tag{24}$$

$$r_t = r(Y_t, M) = 10\sqrt{Y_t} - M, \tag{25}$$

$$J(Y_t, E_t) = -0.3Y_t + 100 - 100/E_t, \tag{26}$$

$$E^e_{t+1} - E^e_t = (\gamma + \sigma\varepsilon_t)(E_t - E^e_t), \tag{27}$$

$$c = 0.8,\quad \tau = 0.2,\quad r_f = 6,\quad cT_0 + C_0 + G = 238,\quad M = 100,\quad \beta = 15,\quad \gamma = 1.2. \tag{28}$$

Equation (24) is a formalization of the Kaldorian S-shaped investment function. Substituting Eqs. (25) and (26) into the equilibrium condition of the balance of payments (Eq. (13)(iii)), we have

$$-0.3Y_t + 100 - 100/E_t + 15(10\sqrt{Y_t} - E^e_t/E_t - 105) = 0. \tag{29}$$

Solving Eq. (29) with respect to E_t, we obtain the following expression for the exchange rate

$$E_t = (100 + 15E^e_t)/(-0.3Y_t + 150\sqrt{Y_t} - 1475). \tag{30}$$

By using these data, we studied the numerical simulation. We selected the parameter α as a bifurcation parameter. When there is no stochastic disturbance ($\sigma = 0$), the equilibrium point is $(Y^*, K^*, E^{e*}) = (Y^*, K^*, E^*) \simeq (112, 450, 1.56)$, which is independent of the value of the parameter α.

In order to investigate the noise effect, the speed of adjustment in adaptive expectation hypothesis of exchange rate (γ) is fluctuated by noise. In general, there are two types of noise, i.e., additive noise and parametric noise. In the case of additive noise, noise is added to a certain deterministic equation, and in the case of parametric noise, a certain parameter of a deterministic equation is fluctuated by noise. Crutchfield *et al.* (1982) studied the effect of noise for logistic equation and showed that the effect of parametric noise is equivalent to that of additive noise. However, it is not always true for high dimensional system. In this study, we treat with the case of parametric noise because it is likely for the foreign exchange rate to be fluctuated by noise. For the details of numerical simulation of the Kaldorian business cycle model with parametric noise in a closed economy, see Dohtani *et al.* (1996). The following statement summarizes the results of our numerical simulation:

(1) The behavior of flexible foreign exchange rate (E) begins to burst as the adjustment speed in the goods market (α) is increased in the system without noise.
(2) Noise may suppress the burst of exchange rate, in other words, noise may reveal the hidden structures.
(3) The windows of periodic solution can be broken by noise and the hidden chaotic structure may appear.

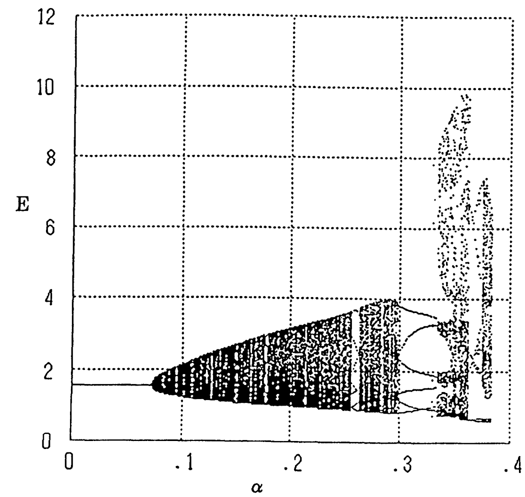

FIGURE 1 The bifurcation diagram of E without noise.

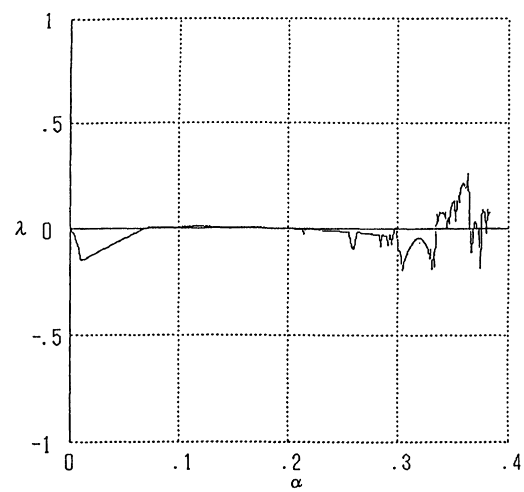

FIGURE 2 The largest Lyapunov exponent without noise.

4.1. The Appearing and the Suppressing of Burst

Figure 1 is the bifurcation diagram of the foreign exchange rate (E) without noise. The bifurcation parameter is the adjustment speed in the goods market (α). Figure 2 shows the largest Lyapunov exponent in this system. These figures indicate that the behavior of the exchange rate begins to burst and chaotic behaviors appear frequently when $\alpha \geq 0.33$. Figure 3 is the chaotic attractor in the E–Y plane without noise when $\alpha = 0.33$.

However, noise can suppress the burst of the exchange rate. Figure 4 is the attractor which is revealed by the small noise ($\sigma = 0.01$) when $\alpha = 0.33$. This attractor is similar to that in the case of $\alpha = 0.32$, which is cyclical and not burst. From the economic point of view, this means that even if the foreign exchange rate fluctuates heavily when there is no stochastic disturbance, this heavy

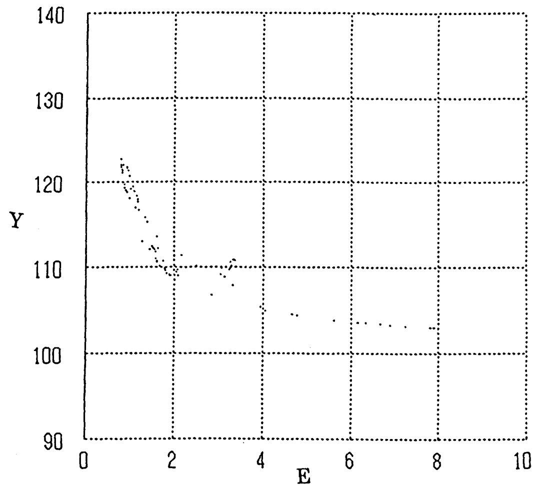

FIGURE 3 The attractor in E–Y plane without noise ($\alpha = 0.33$).

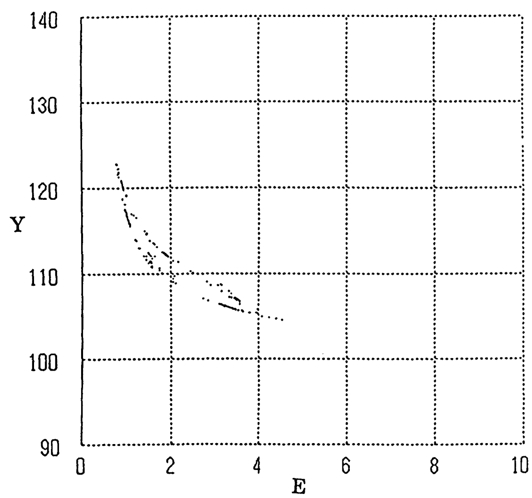

FIGURE 4 The attractor in E–Y plane with noise ($\alpha = 0.33$, $\sigma = 0.01$).

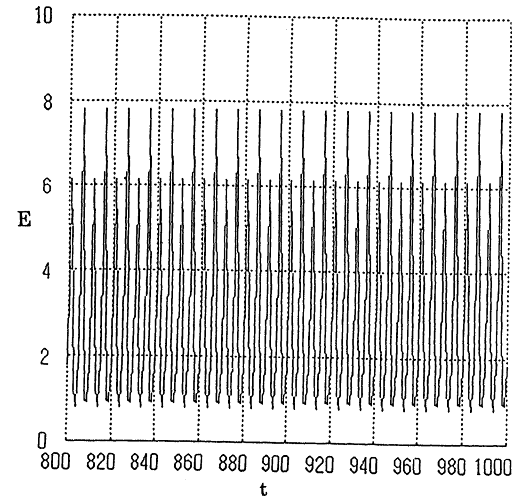

FIGURE 5 A trajectory of E without noise ($\alpha = 0.33$).

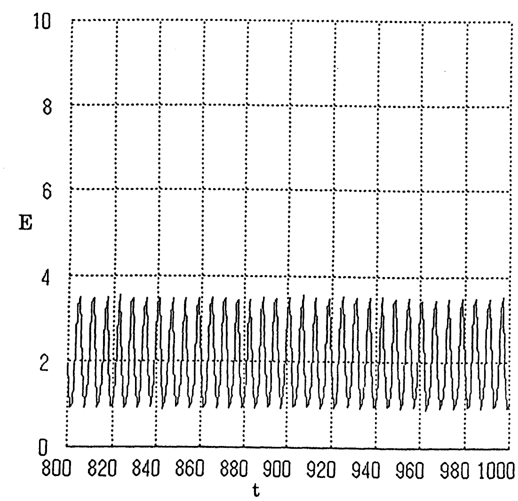

FIGURE 6 A trajectory of E with noise ($\alpha = 0.33$, $\sigma = 0.01$).

fluctuation may be suppressed by a small noise. Figures 5 and 6 compare the trajectory of the exchange rate when $\alpha = 0.33$ without noise to that with small noise ($\sigma = 0.01$).

The above example shows that the noise *may* stabilize the system. But, this does *not* mean that the noise *always* stabilize the system. Figures 7 and 8 are two examples of the experiments with somewhat larger noise ($\sigma = 0.05$). These examples show that the noise can destabilize rather than stabilize the system in some situations. Figure 9 is a revealed attractor in the E–Y plane when $\alpha = 0.33$ and $\sigma = 0.05$, which is chaotic and similar to that for $\alpha = 0.34$ without noise. To sum up, noise can reveal the hidden structures.

4.2. The Broken Windows

By comparing the largest Lyapunov exponent and bifurcation diagram without noise (Figs. 1 and 2)

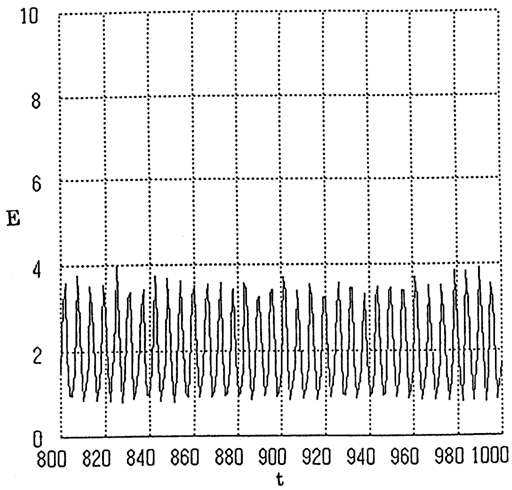

FIGURE 7 A trajectory of E with noise ($\alpha = 0.33$, $\sigma = 0.05$).

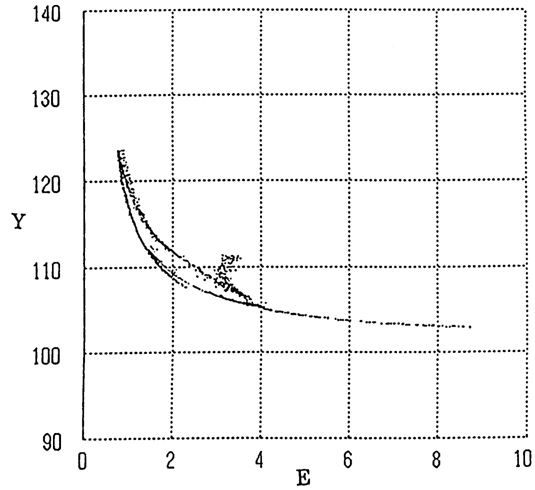

FIGURE 9 The attractor in E–Y plane with noise ($\alpha = 0.33$, $\sigma = 0.05$).

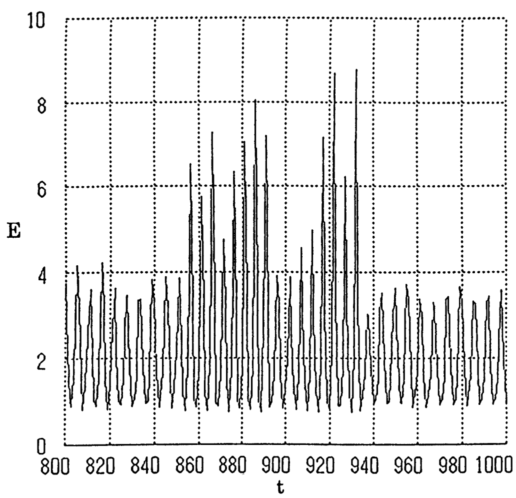

FIGURE 8 A trajectory of E with noise ($\alpha = 0.33$, $\sigma = 0.05$).

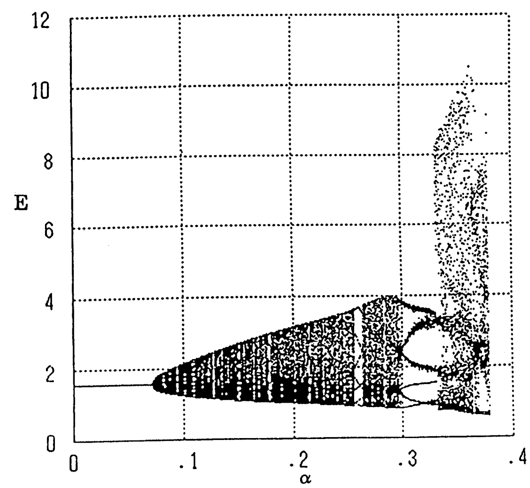

FIGURE 10 The bifurcation diagram of E with noise.

with those with noise (Figs. 10 and 11), we can see that noise may obscure the windows of cycle for the exchange rate (E). This is also true for the behavior of national income (Y). This example shows that noise can reveal the hidden chaotic structure even if the behavior of the system without noise is periodic.

5. CONCLUDING REMARKS

In this paper, we studied the economic implications of the noise effects by using an analytical framework of the discrete time version of Kaldorian business cycle model in a small open economy with flexible exchange rates. This is a good starting point to the study of the economic interactions between countries or regions. The effective range of the model of small open economy is, however, rather restricted, because many variables such as national income or rate of interest of foreign country are supposed to be given outside of the system. Multi-country or multi-regional model may be more appropriate for the study of the

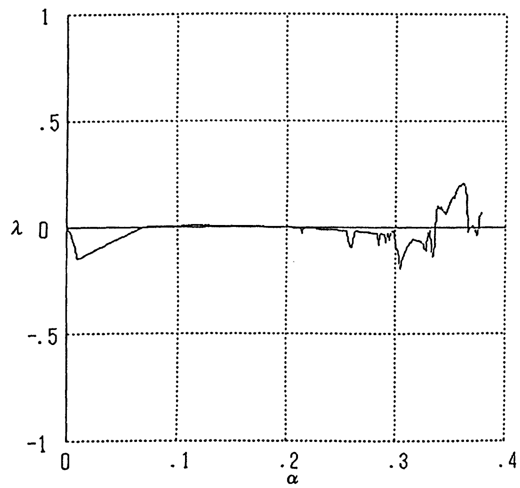

FIGURE 11 The largest Lyapunov exponent with noise.

dynamic interactions between regions. The simplest version of such a model is two country model. The analysis of such a complicated system is beyond the scope of the present paper and it is left for our research in future.

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