COINCIDENCE THEORY FOR SPACES WHICH FIBER OVER A NILMANIFOLD

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Let Y be a finite connected complex and $p : Y \to N$ a fibration over a compact nilmanifold N. For any finite complex X and maps $f,g : X \to Y$, we show that the Nielsen coincidence number $N(f,g)$ vanishes if the Reidemeister coincidence number $R(pf,pg)$ is infinite. If, in addition, Y is a compact manifold and g is the constant map at a point $a \in Y$, then f is deformable to a map $\hat{f} : X \to Y$ such that $\hat{f}^{-1}(a) = \emptyset$.

1. Introduction

The celebrated Lefschetz-Hopf fixed point theorem states that if a selfmap $f : X \to X$ on a compact connected polyhedron X has nonvanishing Lefschetz number $L(f)$, then every map homotopic to f must have a fixed point. On the other hand, if $L(f) = 0$, f need not be homotopic to a fixed point free map. A classical result of Wecken asserts that if X is a triangulable manifold of dimension at least three, then the Nielsen number $N(f)$ is the minimal number of fixed points of maps in the homotopy class of f. Thus, in this case, if $N(f) = 0$, then f is deformable to be fixed point free. For coincidences of two maps $f,g : X \to Y$ between closed oriented triangulable n-manifolds, there is an analogous Lefschetz coincidence number $L(f,g)$, and $L(f,g) \neq 0$ implies $\{x \in X \mid f'(x) = g'(x)\} \neq \emptyset$ for all $f' \sim f$ and $g' \sim g$. Schirmer [14] introduced a Nielsen coincidence number $N(f,g)$ and proved a Wecken-type theorem. While the theory of Nielsen fixed point (coincidence) classes is useful in obtaining multiplicity results in fixed point (coincidence) theory and in other applications, the computation of the Nielsen number remains one of the most difficult and central issues.

One of the major advances in recent development in computing the Nielsen number is a theorem of Anosov who proved that for any selfmap $f : N \to N$ of a compact nilmanifold N, $N(f) = |L(f)|$. By a nilmanifold, we mean a coset space of a nilpotent Lie group by a closed subgroup. Thus, the computation of $N(f)$ reduces to that of the homological trace $L(f)$. Anosov's theorem does not hold in general for selfmaps of solvmanifolds or infranilmanifolds. Meanwhile, the theorem has been generalized to coincidences for

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maps between closed oriented triangulable manifolds of the same dimension. In particular, coincidences of maps from a manifold to a solvmanifold or an infrasolvmanifold have been studied (see, e.g., [8, 10, 15]).

In [9], it was shown that if $f,g : X \to Y$ are maps from a finite complex X to a compact nilmanifold Y, then $R(f,g) = \infty$ implies $N(f,g) = 0$. This result is false in general, for example, when Y is a solvmanifold (see, e.g., [8]). In this work, the main objective is to generalize this result for more general spaces, in particular, for finite connected complexes Y which fiber over a compact nilmanifold N. We should point out that such a space Y necessarily fibers over the unit circle S^1 as every nilmanifold does. The problem of fibering a smooth manifold over S^1 has been settled by Farrell [7] who identified an obstruction which gives the necessary and sufficient condition for fibering over S^1. Since many spaces fiber over S^1 (e.g., the mapping torus T_f of a pseudo-Anosov homeomorphism $f : X \to X$ on a hyperbolic surface X is a hyperbolic 3-manifold which fibers over S^1 (or mapping tori in general) or solvmanifolds), the class of spaces we consider here enlarges the collection of known topological spaces for which calculation of $N(f,g)$ has been studied. In the special case where g is a constant map, we give a sufficient condition which implies that f is deformable to be root free. This work allows us to study situations where the spaces are not necessarily aspherical or manifolds, and the maps need not be fiber-preserving.

For classical Nielsen fixed point theory, the basic references are [4, 12].

2. Main results

Before we present our main results, we first review the appropriate generalization of the classical Nielsen coincidence number using an index-free notion of essentiality due to Brooks (see [1, 3]).

Let $f,g : X \to Y$ be maps between finite complexes and $\mathrm{Coin}(f,g) = \{x \in X \mid f(x) = g(x)\}$. Suppose $x_1,x_2 \in \mathrm{Coin}(f,g)$. Then x_1 and x_2 are *Nielsen equivalent* as coincidences with respect to f and g if there exists a path $\sigma : [0,1] \to X$ such that $\sigma(0) = x_1$, $\sigma(1) = x_2$, and $f \circ \sigma$ is homotopic to $g \circ \sigma$ relative to the endpoints. The equivalence classes of this relation are called the coincidence classes. A coincidence class $\mathcal{F}$ is *essential* if for any $x \in \mathcal{F}$ and for any homotopies $\{f_t\}$, $\{g_t\}$ of $f = f_0$ and $g = g_0$, there exist $x' \in \mathrm{Coin}(f_1,g_1)$ and a path $\gamma : [0,1] \to X$ with $\gamma(0) = x$, $\gamma(1) = x'$ such that $f_t \circ \gamma$ is homotopic to $g_t \circ \gamma$ relative to the endpoints. We say that $x \in \mathcal{F}$ is $\{f_t\},\{g_t\}$-*related* to a coincidence of f_1 and g_1.

The Nielsen coincidence number $N(f,g)$ of f and g is defined to be the number of essential coincidence classes. It is homotopy invariant, finite, and is a lower bound for $\mathrm{Coin}(f',g')$ for $f' \sim f$, $g' \sim g$. By fixing base points in X and in Y, let $f_\sharp$ and $g_\sharp$ be the homomorphisms induced by f and by g, respectively, on the fundamental groups. The Reidemeister coincidence number $R(f,g)$ of f and g is the number of orbits of the action of $\pi_1(X)$ on $\pi_1(Y)$ via $\sigma \bullet \alpha \mapsto g_\sharp(\sigma)\alpha f_\sharp(\sigma)^{-1}$, where $\sigma \in \pi_1(X)$, $\alpha \in \pi_1(Y)$. It is homotopy invariant and is independent of the choice of the base points. Moreover, $N(f,g) \le R(f,g)$. When X and Y are closed oriented n-manifolds, a homological coincidence index $I(f,g;\mathcal{F})$ can be defined for each coincidence class $\mathcal{F}$. It follows that $I(f,g;\mathcal{F}) \neq 0$ implies that $\mathcal{F}$ is essential. In fact, for $n \neq 2$, $I(f,g;\mathcal{F}) \neq 0$ if and only if $\mathcal{F}$

is essential. Thus, the Nielsen number generalizes the classical one [14] defined for oriented n-manifolds. In the special case when g is a constant map, the induced homomorphism $g_\sharp$ is trivial, so $R(f,g) = [\pi_1(Y) : f_\sharp(\pi_1(X))]$, the index of the subgroup $f_\sharp(\pi_1(X))$ in $\pi_1(Y)$.

Let N be a compact nilmanifold and let $\mathscr{C}_N$ denote the family of triples (Y,p,N) where p is a fibration with base N, Y is a finite connected complex, and the typical fiber is path-connected.

Theorem 2.1. *Let $(Y,p,N) \in \mathscr{C}_N$. For any finite complex X and maps $f,g : X \to Y$, if $N(f,g) > 0$, then $R(pf,pg) < \infty$.*

Proof. Since $pf,pg : X \to N$, it suffices to show, by [9, Theorem 3], that $N(f,g) > 0$ implies $N(pf,pg) > 0$. First note that $\mathrm{Coin}(f,g) \subseteq \mathrm{Coin}(pf,pg)$. Moreover, if x_1, x_2 are Nielsen equivalent as coincidences with respect to f and g, then they are Nielsen equivalent as coincidences with respect to pf and pg. Let $\mathscr{F}$ be an essential coincidence class of f and g and let $\mathscr{F}'$ be the unique coincidence class of pf and pg containing $\mathscr{F}$. Suppose $\{H_t'\}$ is a homotopy of pf. Consider the following commutative diagram:

$$\begin{tikzcd} X \times \{0\} \arrow[r, "f"] \arrow[d, "\text{incl.}"'] & X \arrow[d, "p"] \\ X \times [0,1] \arrow[r, "H'"] & N. \end{tikzcd} \tag{2.1}$$

Since p is a fibration, there exists a homotopy H of f covering H', that is, $H' = pH$. Now because N is a manifold, it follows from [1] that the effect of deforming f and g by homotopies $\{f_t\}$, $\{g_t\}$ can be achieved by deforming f and keeping the homotopy $\{g_t\}$ constant. Since $\mathscr{F}$ is essential, every $x \in \mathscr{F}$ is $\{f_t\},\{g_t\}$-related to a coincidence of H_1 and g with $\{g_t\}$ constant as g. Thus, $x \in \mathscr{F} \subseteq \mathscr{F}'$ is $\{pf_t\},\{pg\}$-related to a coincidence of H_1' and pg. It follows that $\mathscr{F}'$ is essential. The proof is complete. □

Remark 2.2. This result clearly generalizes [9, Theorem 3] in that, if Y is already a nilmanifold, then we choose the fibration p to be the identity map. Furthermore, the implication $N(f,g) > 0$ implies $N(pf,pg) > 0$ actually holds for any fibration p without any other assumptions on N. Even when $X = Y$ and g is the identity map, the Nielsen coincidence theory need not be the same as the classical Nielsen fixed point theory in which the identity map remains constant through homotopy. When the target is a manifold, the Nielsen coincidence theory does reduce to that for fixed points (see, e.g., [1]). In order to obtain the next result for fixed points as a consequence of Theorem 2.1, the ability to deform only one of the maps is crucial.

Corollary 2.3. *Let $(Y,p,N) \in \mathscr{C}_N$ and let Y be a topological manifold. For any self-map $f : Y \to Y$, if $R(pf,p) = \infty$, then $N(f) = 0$, where $N(f)$ denotes the classical Nielsen (fixed point) number of f.*

Remark 2.4. If F is the typical fiber of $p: Y \to N$, then the inclusion $F \hookrightarrow Y$ induces an injective homomorphism $\pi_1(F) \to \pi_1(Y)$ since N is aspherical. This result is useful especially when $\pi_1(F)$ is not $f_\sharp$-invariant, that is, f is not homotopic to a fiber-preserving map with respect to the fibration p.

Suppose the map g is the constant map at a point $a \in Y$ and $\bar{a} = p(a) \in N$. We will write $N(f;a) := N(f,g)$ and $R(pf;\bar{a}) := R(pf,pg)$. When Y is a manifold, $N(f;a)$ coincides with the Nielsen root number defined in [2].

Theorem 2.5. *Let $(Y,p,N) \in \mathscr{C}_N$ and let X be a finite complex. Suppose $f: X \to Y$ is a map such that $R(pf;\bar{a}) = \infty$. Then f is homotopic to a map $\hat{f}: X \to Y$ such that $\hat{f}^{-1}(a) = \varnothing$. If, in addition, Y is a closed triangulable n-manifold, then the map $\hat{f}$ can be chosen such that* $\dim \hat{f}(X) \leq n-1$.

Proof. Since $R(pf;\bar{a}) = \infty$ and N is a compact nilmanifold, [9, Theorem 3] asserts that $N(pf;\bar{a}) = 0$. It follows from [9, Theorem 4] that the composite map pf is homotopic to a root-free map $h: X \to N$ such that $h^{-1}(\bar{a}) = \varnothing$. Let $\bar{H}: X \times [0,1] \to N$ be this homotopy with $\bar{H}_0 = pf$ and $\bar{H}_1 = h$. Since p is a fibration, the covering homotopy theorem implies that there exists a homotopy $H: X \times [0,1] \to Y$ such that $H_0 = f$ and $pH = \bar{H}$. Evidently, $H_1^{-1}(a) = \varnothing$. We choose the lift of the homotopy $\bar{H}$ starting from f.

Suppose now that Y is a closed triangulable n-manifold. By the argument above, we have a map $\widehat{\varphi}$, homotopic to f such that $\widehat{\varphi}^{-1}(a) = \varnothing$. Without loss of generality, we may assume that the point a lies in the interior of a maximal n-simplex of Y. Now one can find a compact manifold K of codimension zero in Y with nonempty boundary such that $\widehat{\varphi}(X) \subset \operatorname{int} K$. By collapsing K onto its $(n-1)$-skeleton, $\widehat{\varphi}$ is homotopic to a map $\widehat{f}$ such that $\dim \widehat{f}(X) \leq n-1$ and $a \notin \widehat{f}(X)$. □

Example 2.6. Let Y be the three-dimensional solvmanifold obtained by the relation on $\mathbb{R}^3$ given by

$$(x,y,z) \sim (x+a,(-1)^a y+b,(-1)^a z+c) \tag{2.2}$$

for $a,b,c \in \mathbb{Z}$. The projection $p: Y \to S^1$ on the first factor is a fibration. For any self-map $f: Y \to Y$ of the form

$$[x,y,z] \longmapsto [x,\cdot,\cdot], \tag{2.3}$$

the maps p and pf coincide and thus induce the same epimorphism on fundamental groups. Thus, $R(pf,p)$ is simply the number of conjugacy classes of elements of $\pi_1(S^1) \cong \mathbb{Z}$, and is therefore infinite. By Corollary 2.3, we have $N(f) = 0$.

The map f is in fact fiber-preserving with an induced map, the identity on the base. In general, every self-map of Y is homotopic to a fiber-preserving map with respect to p so that an addition formula can be used to compute $N(f)$ as done in [11]. This example shows the effectiveness of determining $N(f) = 0$ using our result.

Next, we give an example of a coincidence situation where the maps need not be fiber-preserving.

Example 2.7. The three-dimensional solvmanifold Y of Example 2.6 is also a flat manifold whose fundamental group $\pi_1(Y) = \pi \subset \mathbb{R}^3 \rtimes O(3)$ is given by an extension

$$0 \longrightarrow \mathbb{Z}^3 \longrightarrow \pi \longrightarrow \mathbb{Z}_2 \longrightarrow 0, \tag{2.4}$$

where the action of $\mathbb{Z}_2 \cong \langle A \rangle$ on $\mathbb{Z}^3$ is given by

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} p \\ -q \\ -r \end{bmatrix}. \tag{2.5}$$

Here, A is the matrix given by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{2.6}$$

The group π is generated by $\{(e_1,I),(e_2,I),(e_3,I),(\alpha,A)\}$, where e_1, e_2, e_3 are the standard basis for $\mathbb{R}^3$ and

$$\alpha = \begin{bmatrix} \frac{1}{2} \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^3. \tag{2.7}$$

Consider a connected finite complex X such that $\pi_1(X) \cong G \times \langle e \rangle$, where G has a group presentation given by $G = \langle a,b,c,d \mid [a,b][c,d] = 1 \rangle$. For example, X can be chosen to be the 3-manifold $(T^2 \# T^2) \times S^1$, that is, the cartesian product of the connected sum of two 2-tori with the unit circle. The space X may be taken to be nonaspherical so that X need not fiber over S^1. Now let $f : X \to Y$ be a map whose induced homomorphism on π_1 is given by $f_\sharp : \pi_1(X) \to \pi$ via $f_\sharp(a) = (e_3,I)$, $f_\sharp(b) = (e_2,I)^2$, $f_\sharp(c) = (e_3,I)^2$, $f_\sharp(d) = (e_2,I)^{-1}$, and $f_\sharp(e) = (e_2,I)$. It is easy to see that $\langle (e_1,I) \rangle = p_\sharp^{-1}(\pi_1(S^1))$ and $p_\sharp \circ f_\sharp = 0$. Thus, if $a_0 \in Y$ and $\bar{a}_0 = p(a_0)$, then $R(pf;\bar{a}_0) = \infty$. It follows from Theorem 2.5 that $N(f;a_0) = 0$ and hence f is homotopic to a root-free map.

Let N be a compact nilmanifold of dimension k. Then, using a refined upper central series, we obtain a sequence of S^1-principal fibrations p_i, $i = 1,\dots,k-1$,

$$\begin{tikzcd} S^1 \arrow[d] & S^1 \arrow[d] & S^1 \arrow[d] & \cdots & S^1 \arrow[d] & \\ N \arrow[r, "p_{k-1}"'] & N_{k-1} \arrow[r, "p_{k-2}"'] & N_{k-2} \arrow[r] & \cdots \arrow[r] & N_2 \arrow[r, "p_1"'] & N_1 = S^1, \end{tikzcd} \tag{2.8}$$

where N_i is a compact nilmanifold of dimension i. We should point out that not every self-map of N is fiber-preserving with respect to these fibrations p_i.

Let $(Y,p,N) \in \mathscr{C}_N$ and let $p_k : Y \to N$ be a fibration over a compact k-dimensional nilmanifold N with an associated sequence of fibrations as in (2.8). If $f,g : X \to Y$, then we have the following commutative diagram:

$$\begin{tikzcd}
X \arrow[r, equal] \arrow[d, shift right, "f"'] \arrow[d, shift left, "g"] & X \arrow[r, equal] \arrow[d, shift right, "p_k f"'] \arrow[d, shift left, "p_k g"] & X \arrow[r, equal] \arrow[d, shift right, "p_{k-1}p_k f"'] \arrow[d, shift left, "p_{k-1}p_k g"] & \cdots \arrow[r, equal] & X \arrow[d, shift right, "p_1\cdots p_k f"'] \arrow[d, shift left, "p_1\cdots p_k g"] \\
Y \arrow[r, "p_k"'] & N \arrow[r, "p_{k-1}"'] & N_{k-1} \arrow[r] & \cdots \arrow[r, "p_1"'] & N_1.
\end{tikzcd} \tag{2.9}$$

With this setup, together with Theorem 2.1, we have the following theorem.

THEOREM 2.8. *Let $f,g : X \to Y$ and let $p_k : Y \to N$ be as in the previous discussion. Then*

$$\begin{aligned}
N(f,g) > 0 &\Longrightarrow N(p_k f, p_k g) > 0 \\
&\Longrightarrow N(p_{k-1}p_k f, p_{k-1}p_k g) > 0 \\
&\Longrightarrow \cdots \\
&\Longrightarrow N(p_1\cdots p_k f, p_1\cdots p_k g) > 0 \\
&\Longrightarrow R(p_1\cdots p_k f, p_1\cdots p_k g) < \infty.
\end{aligned} \tag{2.10}$$

In particular, for any i, $1 \le i \le k$,

$$R(p_i\cdots p_k f, p_i\cdots p_k g) = \infty \Longrightarrow N(f,g) = 0. \tag{2.11}$$

Remark 2.9. Theorem 2.8 gives an algorithmic procedure of determining the vanishing of $N(f,g)$. To begin, we consider $R(p_1\cdots p_k f, p_1\cdots p_k g)$ whose calculation is done in $\pi_1(N_1) \cong \mathbb{Z}$ since $N_1 = S^1$. In case $R(p_1\cdots p_k f, p_1\cdots p_k g)$ is finite, we then consider $R(p_1\cdots p_{k-1} f, p_1\cdots p_{k-1} g)$ and $\pi_1(N_2)$, and so forth.

The next example illustrates the usefulness of Theorem 2.8.

Example 2.10. Take Y to be the three-dimensional solvmanifold whose fundamental group is the semidirect product $\pi_1(Y) = \mathbb{Z} \rtimes_\theta \mathbb{Z}^2$ where the action $\theta : \mathbb{Z}^2 \to \operatorname{Aut}\mathbb{Z} = \{\pm 1\}$ is given by

$$\theta\big((s^\beta, t^\gamma)\big) = (-1)^\gamma. \tag{2.12}$$

Here, we write $\mathbb{Z} \cong \langle\delta\rangle$ and $\mathbb{Z}^2 \cong \langle s\rangle \times \langle t\rangle$. The projection $\pi_1(Y) \to \mathbb{Z}^2$ via $(\delta^\alpha, (s^\beta, t^\gamma)) \mapsto (s^\beta, t^\gamma)$ gives rise to a fibration $p : Y \to T^2$ of Y over the 2-torus. Let $q : T^2 \to S^1$ be the projection onto the second factor.

Take X to be the same space as in Example 2.7 so that $\pi_1(X) = G \times \langle e\rangle$. Consider the map $f : X \to Y$ whose induced homomorphism on fundamental groups is given by $f_\sharp$ such that $f_\sharp(a) = (1,(1,1)) = f_\sharp(b)$, $f_\sharp(c) = (1,(1,t)) = f_\sharp(d)$, and $f_\sharp(e) = (\delta,(1,1))$.

Let $a \in Y$ be a point. It is straightforward to check that $R(qpf; qp(a)) = 1$ while $R(pf; p(a)) = \infty$ since $q_\sharp p_\sharp f_\sharp(\pi_1(X)) \cong \langle t\rangle \cong \pi_1(S^1)$ but $p_\sharp f_\sharp(\pi_1(X)) \cong 1 \times \langle t\rangle$ has infinite index in $\pi_1(T^2) = \langle s\rangle \times \langle t\rangle$. Thus, by Theorem 2.8, we conclude that $N(f;a) = 0$ and hence f is deformable to be root-free by Theorem 2.5.

3. Concluding remarks

The results in this paper rely on the ability to compute $R(pf, pg)$ or more precisely to determine whether $R(pf, pg)$ is infinite or not. Since the Reidemeister number is computed in the fundamental group of the target space, in this case, in a finitely generated torsion-free nilpotent group, the computation is tractable especially employing powerful computer algebra software such as GAP. Computational aspects concerning infinite polycyclic (and therefore, finitely generated nilpotent) groups have been studied in recent years (see, e.g., [5, 6, 13]). The computation of the Reidemeister number will be the objective of the sequel to this work.

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