GENERIC CONVERGENCE OF ITERATES FOR A CLASS OF NONLINEAR MAPPINGS

SIMEON REICH AND ALEXANDER J. ZASLAVSKI

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Let K be a nonempty, bounded, closed, and convex subset of a Banach space. We show that the iterates of a typical element (in the sense of Baire's categories) of a class of continuous self-mappings of K converge uniformly on K to the unique fixed point of this typical element.

1. Introduction

Let K be a nonempty, bounded, closed, and convex subset of a Banach space $(X, \|\cdot\|)$. We consider the topological subspace $K \subset X$ with the relative topology induced by the norm $\|\cdot\|$. Set

$$\operatorname{diam}(K) = \sup\{\|x-y\| : x, y \in K\}. \tag{1.1}$$

Denote by $\mathcal{A}$ the set of all continuous mappings $A : K \to K$ which have the following property:

(P1) for each $\epsilon > 0$, there exists $x_\epsilon \in K$ such that

$$\|Ax - x_\epsilon\| \le \|x - x_\epsilon\| + \epsilon \quad \forall x \in K. \tag{1.2}$$

For each $A, B \in \mathcal{A}$, set

$$d(A,B) = \sup\{\|Ax - Bx\| : x \in K\}. \tag{1.3}$$

Clearly, the metric space $(\mathcal{A}, d)$ is complete.

In this paper, we use the concept of porosity [1, 2, 3, 4, 5, 6] which we now recall.

Let (Y, ρ) be a complete metric space. We denote by $B(y, r)$ the closed ball of center $y \in Y$ and radius $r > 0$. A subset $E \subset Y$ is called porous in (Y, ρ) if there exist $\alpha \in (0,1)$ and $r_0 > 0$ such that for each $r \in (0, r_0]$ and each $y \in Y$, there exists $z \in Y$ for which

$$B(z, \alpha r) \subset B(y, r) \setminus E. \tag{1.4}$$

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A subset of the space Y is called σ-porous in (Y,ρ) if it is a countable union of porous subsets in (Y,ρ).

Since porous sets are nowhere dense, all σ-porous sets are of the first category. If Y is a finite-dimensional Euclidean space $\mathbb{R}^n$, then σ-porous sets are of Lebesgue measure 0.

To point out the difference between porous and nowhere dense sets, note that if $E \subset Y$ is nowhere dense, $y \in Y$, and $r > 0$, then there are a point $z \in Y$ and a number $s > 0$ such that $B(z,s) \subset B(y,r) \setminus E$. If, however, E is also porous, then for small enough r, we can choose $s = \alpha r$, where $\alpha \in (0,1)$ is a constant which depends only on E.

Our purpose in this paper is to establish the following result.

Theorem 1.1. *There exists a set $\mathcal{F} \subset \mathcal{A}$ such that the complement $\mathcal{A} \setminus \mathcal{F}$ is σ-porous in $(\mathcal{A},d)$ and each $A \in \mathcal{F}$ has the following properties:*

(i) *there exists a unique fixed point $x_A \in K$ such that*

$$A^n x \longrightarrow x_A \quad \text{as } n \longrightarrow \infty, \text{ uniformly } \forall x \in K; \tag{1.5}$$

(ii) $\|Ax - x_A\| \le \|x - x_A\|$ *for all $x \in K$;*

(iii) *for each $\epsilon > 0$, there exist a natural number n and $\delta > 0$ such that for each integer $p \ge n$, each $x \in K$, and each $B \in \mathcal{A}$ satisfying $d(B,A) \le \delta$,*

$$\|B^p x - x_A\| \le \epsilon. \tag{1.6}$$

2. Auxiliary result

In this section, we present and prove an auxiliary result which will be used in the proof of Theorem 1.1 in Section 3.

Proposition 2.1. *Let $A \in \mathcal{A}$ and $\epsilon \in (0,1)$. Then there exist $\bar{x} \in K$ and $B \in \mathcal{A}$ such that*

$$\begin{gathered} d(A,B) \le \epsilon, \\ \|\bar{x} - Bx\| \le \|\bar{x} - x\| \quad \forall x \in K. \end{gathered} \tag{2.1}$$

Proof. Choose a positive number

$$\epsilon_0 < 8^{-1}\epsilon^2 (\operatorname{diam}(K) + 1)^{-1}. \tag{2.2}$$

Since $A \in \mathcal{A}$, there exists $\bar{x} \in K$ such that

$$\|Ax - \bar{x}\| \le \|x - \bar{x}\| + \epsilon_0 \quad \forall x \in K. \tag{2.3}$$

Let $x \in K$. There are three cases:

$$\|Ax - \bar{x}\| < \epsilon; \tag{2.4}$$

$$\|Ax - \bar{x}\| \ge \epsilon, \qquad \|Ax - \bar{x}\| < \|x - \bar{x}\|; \tag{2.5}$$

$$\|Ax - \bar{x}\| \ge \epsilon, \qquad \|Ax - \bar{x}\| \ge \|x - \bar{x}\|. \tag{2.6}$$

First, we consider case (2.4). There exists an open neighborhood V_x of x in K such that

$$\|Ay - \bar{x}\| < \epsilon \quad \forall y \in V_x. \tag{2.7}$$

Define $\psi_x : V_x \to K$ by

$$\psi_x(y) = \bar{x}, \quad y \in V_x. \tag{2.8}$$

Clearly, for all $y \in V_x$,

$$0 = \|\psi_x(y) - \bar{x}\| \le \|y - \bar{x}\|, \qquad \|Ay - \psi_x(y)\| = \|Ay - \bar{x}\| < \epsilon. \tag{2.9}$$

Consider now case (2.5). Since A is continuous, there exists an open neighborhood V_x of x in K such that

$$\|Ay - \bar{x}\| < \|y - \bar{x}\| \quad \forall y \in V_x. \tag{2.10}$$

In this case, we define $\psi_x : V_x \to K$ by

$$\psi_x(y) = Ay, \quad y \in V_x. \tag{2.11}$$

Finally, we consider case (2.6). Inequalities (2.6), (2.2), and (2.3) imply that

$$\|x - \bar{x}\| \ge \|Ax - \bar{x}\| - \epsilon_0 > \frac{7}{8}\epsilon. \tag{2.12}$$

For each $\gamma \in [0,1]$, set

$$z(\gamma) = \gamma Ax + (1-\gamma)\bar{x}. \tag{2.13}$$

By (2.13), (2.6), and (2.12), we have

$$\|z(0) - \bar{x}\| = 0, \qquad \|z(1) - \bar{x}\| = \|Ax - \bar{x}\| \ge \|x - \bar{x}\| > \frac{7}{8}\epsilon. \tag{2.14}$$

By (2.2) and (2.14), there exists $\gamma_0 \in (0,1)$ such that

$$\|z(\gamma_0) - \bar{x}\| = \|x - \bar{x}\| - \epsilon_0. \tag{2.15}$$

It now follows from (2.13), (2.15), and (2.3) that

$$\gamma_0(\|x - \bar{x}\| + \epsilon_0) \ge \gamma_0\|Ax - \bar{x}\| = \|\gamma_0 Ax + (1-\gamma_0)\bar{x} - \bar{x}\| = \|z(\gamma_0) - \bar{x}\| = \|x - \bar{x}\| - \epsilon_0, \tag{2.16}$$

$$\gamma_0 \ge (\|x - \bar{x}\| - \epsilon_0)(\|x - \bar{x}\| + \epsilon_0)^{-1} = 1 - 2\epsilon_0(\|x - \bar{x}\| + \epsilon_0)^{-1} \ge 1 - 2\epsilon_0\|x - \bar{x}\|^{-1}. \tag{2.17}$$

Inequalities (2.17) and (2.12) imply that

$$\gamma_0 \ge 1 - 2\epsilon_0\left(\frac{7}{8}\epsilon\right)^{-1}. \tag{2.18}$$

By (2.13), (1.1), (2.18), and (2.2),

$$\begin{aligned} ||z(\gamma_0) - Ax|| &= ||\gamma_0 Ax + (1-\gamma_0)\bar{x} - Ax|| = (1-\gamma_0)\|Ax - \bar{x}\| \\ &\le (1-\gamma_0)\operatorname{diam}(K) \le 16\epsilon_0(7\epsilon)^{-1}\operatorname{diam}(K) \\ &\le 3\epsilon_0 \operatorname{diam}(K)\epsilon^{-1} \le \frac{3}{8}\epsilon, \end{aligned} \tag{2.19}$$

$$||z(\gamma_0) - Ax|| \le \frac{3}{8}\epsilon. \tag{2.20}$$

Relations (2.15) and (2.20) imply that there exists an open neighborhood V_x of x in K such that for each $y \in V_x$,

$$||z(\gamma_0) - Ay|| < \epsilon, \qquad ||z(\gamma_0) - \bar{x}|| < \|y - \bar{x}\|. \tag{2.21}$$

Define $\psi_x : V_x \to K$ by

$$\psi_x(y) = z(\gamma_0), \quad y \in V_x. \tag{2.22}$$

It is not difficult to see that in all three cases, we have defined an open neighborhood V_x of x in K and a continuous mapping $\psi_x : V_x \to K$ such that for each $y \in V_x$,

$$||Ay - \psi_x(y)|| < \epsilon, \qquad ||\bar{x} - \psi_x(y)|| \le \|y - \bar{x}\|. \tag{2.23}$$

Since the metric space K with the metric induced by the norm is paracompact, there exists a continuous locally finite partition of unity $\{\phi_i\}_{i\in I}$ on K subordinated to $\{V_x\}_{x\in K}$, where each $\phi_i : K \to [0,1]$, $i \in I$, is a continuous function such that for each $y \in K$, there is a neighborhood U of y in K such that

$$U \cap \operatorname{supp}(\phi_i) \neq \varnothing \tag{2.24}$$

only for a finite number of $i \in I$;

$$\sum_{i\in I}\phi_i(x) = 1, \quad x \in K; \tag{2.25}$$

and for each $i \in I$, there is $x_i \in K$ such that

$$\operatorname{supp}(\phi_i) \subset V_{x_i}. \tag{2.26}$$

Here, $\operatorname{supp}(\phi)$ is the closure of the set $\{x \in K : \phi(x) \neq 0\}$. Define

$$Bz = \sum_{i\in I}\phi_i(z)\psi_{x_i}(z), \quad z \in K. \tag{2.27}$$

Clearly, $B : K \to K$ is well defined and continuous.

Let $z \in K$. There are a neighborhood U of z in K and $i_1, \dots, i_n \in I$ such that

$$U \cap \operatorname{supp}(\phi_i) = \varnothing \quad \text{for any } i \in I \setminus \{i_1, \dots, i_n\}. \tag{2.28}$$

We may assume, without loss of generality, that

$$z \in \operatorname{supp}(\phi_{i_p}), \quad p = 1, \dots, n. \tag{2.29}$$

Then

$$\sum_{p=1}^{n} \phi_{i_p}(z) = 1, \qquad Bz = \sum_{p=1}^{n} \phi_{i_p}(z)\psi_{x_{i_p}}(z). \tag{2.30}$$

Relations (2.26), (2.29), and (2.23) imply that for $p = 1, \dots, n$, the following relations also hold: $z \in V_{x_{i_p}}$,

$$||Az - \psi_{x_{i_p}}(z)|| < \epsilon, \qquad ||\bar{x} - \psi_{x_{i_p}}(z)|| \le \|\bar{x} - z\|. \tag{2.31}$$

By (2.31) and (2.30),

$$\begin{gathered} \|Bz - Az\| = \left\| \sum_{p=1}^{n} \phi_{i_p}(z)\psi_{x_{i_p}}(z) - Az \right\| \le \sum_{p=1}^{n} \phi_{i_p}(z) ||\psi_{x_{i_p}}(z) - Az|| < \epsilon, \\ ||\bar{x} - Bz|| = \left\| \bar{x} - \sum_{p=1}^{n} \phi_{i_p}(z)\psi_{x_{i_p}}(z) \right\| \le \sum_{p=1}^{n} \phi_{i_p}(z) ||\bar{x} - \psi_{x_{i_p}}(z)|| \le \|\bar{x} - z\|, \\ \|Bz - Az\| < \epsilon, \qquad \|\bar{x} - Bz\| \le \|\bar{x} - z\|. \end{gathered} \tag{2.32}$$

Proposition 2.1 is proved. □

3. Proof of Theorem 1.1

For each $C \in \mathcal{A}$ and $x \in K$, set $C^0 x = x$. For each natural number n, denote by $\mathcal{F}_n$ the set of all $A \in \mathcal{A}$ which have the following property:

(P2) there exist $\bar{x} \in K$, a natural number q, and a positive number $\delta > 0$ such that

$$\|\bar{x} - Ax\| \le \|\bar{x} - x\| + n^{-1} \quad \forall x \in K, \tag{3.1}$$

and such that, for each $B \in \mathcal{A}$ satisfying $d(B, A) \le \delta$, and each $x \in K$,

$$||B^q x - \bar{x}|| \le n^{-1}. \tag{3.2}$$

Define

$$\mathcal{F} = \cap_{n=1}^{\infty} \mathcal{F}_n. \tag{3.3}$$

Lemma 3.1. *Let $A \in \mathcal{F}$. Then there exists a unique fixed point $x_A \in K$ such that*

(i) *$A^n x \to x_A$ as $n \to \infty$, uniformly on K;*

(ii)

$$||Ax - x_A|| \le ||x - x_A|| \quad \textit{for all } x \in K; \tag{3.4}$$

(iii) *for each* $\epsilon > 0$, *there exist a natural number* q *and* $\delta > 0$ *such that, for each* $B \in \mathcal{A}$ *satisfying* $d(B,A) \le \delta$, *each* $x \in K$, *and each integer* $i \ge q$,

$$||B^i x - x_A|| \le \epsilon. \tag{3.5}$$

Proof. Let n be a natural number. Since $A \in \mathcal{F} \subset \mathcal{F}_n$, it follows from (P2) that there exist $x_n \in K$, an integer $q_n \ge 1$, and a number $\delta_n \ge 0$ such that

$$||x_n - Ax|| \le ||x_n - x|| + n^{-1} \quad \forall x \in K, \tag{3.6}$$

and we have the following property:

(P3) for each $B \in \mathcal{A}$ satisfying $d(B,A) \le \delta_n$, and each $x \in K$,

$$||B^{q_n} x - x_n|| \le \frac{1}{n}. \tag{3.7}$$

Property (P3) implies that for each $x \in K$, $\|A^{q_n} x - x_n\| \le 1/n$. This fact implies, in turn, that for each $x \in K$,

$$||A^i x - x_n|| \le \frac{1}{n} \quad \text{for any integer } i \ge q_n. \tag{3.8}$$

Since n is any natural number, we conclude that for each $x \in K$, $\{A^i x\}_{i=1}^{\infty}$ is a Cauchy sequence and there exists $\lim_{i\to\infty} A^i x$. Inequality (3.8) implies that for each $x \in K$,

$$\left|\left|\lim_{i\to\infty} A^i x - x_n\right|\right| \le \frac{1}{n}. \tag{3.9}$$

Since n is again an arbitrary natural number, we conclude further that $\lim_{i\to\infty} A^i x$ does not depend on x. Hence, there is $x_A \in K$ such that

$$x_A = \lim_{i\to\infty} A^i x \quad \forall x \in K. \tag{3.10}$$

By (3.9) and (3.10),

$$||x_A - x_n|| \le \frac{1}{n}. \tag{3.11}$$

Inequalities (3.11) and (3.6) imply that for each $x \in K$,

$$\begin{aligned} ||Ax - x_A|| &\le ||Ax - x_n|| + ||x_n - x_A|| \le \frac{1}{n} + ||Ax - x_n|| \le \frac{1}{n} + ||x - x_n|| + \frac{1}{n} \\ &\le \frac{2}{n} + ||x - x_A|| + ||x_A - x_n|| \le ||x - x_A|| + \frac{3}{n}, \end{aligned} \tag{3.12}$$

so that

$$||Ax - x_A|| \le ||x - x_A|| + \frac{3}{n}. \tag{3.13}$$

Since n is an arbitrary natural number, we conclude that

$$||Ax - x_A|| \le ||x - x_A|| \quad \text{for each } x \in K. \tag{3.14}$$

Let $\epsilon > 0$. Choose a natural number

$$n > \frac{8}{\epsilon}. \tag{3.15}$$

Property (P3) implies that

$$||B^i x - x_n|| \le \frac{1}{n} \tag{3.16}$$

for each $x \in K$, each integer $i \ge q_n$, and each $B \in \mathscr{A}$ satisfying $d(B,A) \le \delta_n$. Inequalities (3.16), (3.11), and (3.15) imply that for each $B \in \mathscr{A}$ satisfying $d(B,A) \le \delta_n$, each $x \in K$, and each integer $i \ge q_n$,

$$||B^i x - x_A|| \le ||B^i x - x_n|| + ||x_n - x_A|| \le \frac{1}{n} + \frac{1}{n} < \epsilon. \tag{3.17}$$

This completes the proof of Lemma 3.1. □

Completion of the proof of Theorem 1.1. In order to complete the proof of this theorem, it is sufficient, by Lemma 3.1, to show that for each natural number n, the set $\mathscr{A} \setminus \mathscr{F}_n$ is porous in $(\mathscr{A}, d)$.

Let n be a natural number. Choose a positive number

$$\alpha < (16n)^{-1} 2^{-1} \Big((\operatorname{diam}(K) + 1)^2 16 \cdot 8n \Big)^{-1}. \tag{3.18}$$

Let

$$A \in \mathscr{A}, \qquad r \in (0,1]. \tag{3.19}$$

By Proposition 2.1, there exist $A_0 \in \mathscr{A}$ and $\bar{x} \in K$ such that

$$d(A, A_0) \le \frac{r}{8}, \tag{3.20}$$

$$||A_0 x - \bar{x}|| \le \|x - \bar{x}\| \quad \text{for each } x \in K. \tag{3.21}$$

Set

$$\gamma = 8^{-1} r (\operatorname{diam}(K) + 1)^{-1} \tag{3.22}$$

and choose a natural number q for which

$$1 \le q \Big((\operatorname{diam}(K) + 1)^2 16n \cdot 8r^{-1} \Big)^{-1} \le 2. \tag{3.23}$$

Define $\bar{A} : K \to K$ by

$$\bar{A}x = (1 - \gamma) A_0 x + \gamma \bar{x}, \quad x \in K. \tag{3.24}$$

Clearly, the mapping $\bar{A}$ is continuous and, for each $x \in K$,

$$\|\bar{A}x - \bar{x}\| = ||(1-\gamma)A_0x + \gamma\bar{x} - \bar{x}|| = (1-\gamma)||A_0x - \bar{x}|| \le (1-\gamma)\|x - \bar{x}\|. \tag{3.25}$$

Thus, $\bar{A} \in \mathcal{A}$. Relations (1.3), (3.24), (1.1), (3.22), and (3.25) imply that

$$\begin{aligned} d(\bar{A}, A_0) &= \sup\{||\bar{A}x - A_0x|| : x \in K\} = \sup\{\gamma||\bar{x} - A_0x|| : x \in K\} \\ &\le \gamma \operatorname{diam}(K) \le \frac{r}{8}. \end{aligned} \tag{3.26}$$

Together with (3.20), this implies that

$$d(\bar{A}, A) \le d(\bar{A}, A_0) + d(A_0, A) \le \frac{r}{4}. \tag{3.27}$$

Assume now that

$$B \in \mathcal{A}, \qquad d(B, \bar{A}) \le \alpha r. \tag{3.28}$$

Then (3.28), (3.18), and (3.25) imply that for each $x \in K$,

$$||Bx - \bar{x}|| \le ||Bx - \bar{A}x|| + ||\bar{A}x - \bar{x}|| \le \|x - \bar{x}\| + \alpha r \le \|x - \bar{x}\| + \frac{1}{n}. \tag{3.29}$$

In addition, (3.28), (3.27), and (3.18) imply that

$$d(B, A) \le d(B, \bar{A}) + d(\bar{A}, A) \le \alpha r + \frac{r}{4} \le \frac{r}{2}. \tag{3.30}$$

Assume that $x \in K$. We will show that there exists an integer $j \in [0, q]$ such that $\|B^j x - \bar{x}\| \le (8n)^{-1}$. Assume the contrary. Then

$$||B^i x - \bar{x}|| > (8n)^{-1}, \quad i = 0, \dots, q. \tag{3.31}$$

Let an integer $i \in \{0, \dots, q-1\}$. By (3.28) and (3.25),

$$\begin{aligned} ||B^{i+1}x - \bar{x}|| &= ||B(B^i x) - \bar{x}|| \le ||B(B^i x) - \bar{A}(B^i x)|| + ||\bar{A}(B^i x) - \bar{x}|| \\ &\le d(B, \bar{A}) + ||\bar{A}(B^i x) - \bar{x}|| \le \alpha r + (1-\gamma)||B^i x - \bar{x}||, \\ ||B^{i+1}x - \bar{x}|| &\le \alpha r + (1-\gamma)||B^i x - \bar{x}||. \end{aligned} \tag{3.32}$$

When combined with (3.31), (3.18), and (3.22), this inequality implies that

$$\begin{aligned} ||B^i x - \bar{x}|| - ||B^{i+1}x - \bar{x}|| &\ge ||B^i x - \bar{x}|| - \alpha r - (1-\gamma)||B^i x - \bar{x}|| \\ &= \gamma||B^i x - \bar{x}|| - \alpha r > (8n)^{-1}\gamma - \alpha r \ge (16n)^{-1}\gamma, \end{aligned} \tag{3.33}$$

so that

$$||B^i x - \bar{x}|| - ||B^{i+1}x - \bar{x}|| \ge (16n)^{-1}\gamma. \tag{3.34}$$

When combined with (1.1), this inequality implies that

$$\begin{gathered}\operatorname{diam}(K) \geq \|x-\bar{x}\| - ||B^q x-\bar{x}|| \geq \sum_{i=0}^{q-1} (||B^i x-\bar{x}|| - ||B^{i+1}x-\bar{x}||) \geq q(16n)^{-1}\gamma, \\ q \leq \operatorname{diam}(K)\frac{16n}{\gamma},\end{gathered} \tag{3.35}$$

a contradiction (see (3.22) and (3.23)). The contradiction we have reached shows that there exists an integer $j \in \{0,\dots,q-1\}$ such that

$$||B^j x-\bar{x}|| \leq (8n)^{-1}. \tag{3.36}$$

It follows from (3.28) and (3.25) that for each integer $i \in \{0,\dots,q-1\}$,

$$\begin{aligned}||B^{i+1}x-\bar{x}|| &= ||B(B^i x)-\bar{x}|| \leq ||B(B^i x)-\bar{A}(B^i x)|| + ||\bar{A}(B^i x)-\bar{x}|| \\ &\leq d(\bar{A},B) + ||\bar{A}(B^i x)-\bar{x}|| \leq \alpha r + ||B^i x-\bar{x}||, \\ ||B^{i+1}x-\bar{x}|| &\leq ||B^i x-\bar{x}|| + \alpha r.\end{aligned} \tag{3.37}$$

This implies that for each integer s satisfying $j < s \leq q$,

$$||B^s x-\bar{x}|| \leq ||B^j x-\bar{x}|| + \alpha r(s-j) \leq ||B^j x-\bar{x}|| + \alpha r q. \tag{3.38}$$

It follows from (3.36), (3.38), (3.23), and (3.18) that

$$||B^q x-\bar{x}|| \leq \alpha r q + (8n)^{-1} \leq (2n)^{-1}. \tag{3.39}$$

Thus, we have shown that the following property holds: for each B satisfying (3.28) and each $x \in K$,

$$||B^q x-\bar{x}|| \leq (2n)^{-1}, \qquad ||Bx-\bar{x}|| \leq \|x-\bar{x}\| + \frac{1}{n} \tag{3.40}$$

(see (3.29)). Thus

$$\left\{B \in \mathcal{A} : d(B,\bar{A}) \leq \frac{\alpha r}{2}\right\} \subset \mathcal{F}_n \cap \{B \in \mathcal{A} : d(B,A) \leq r\}. \tag{3.41}$$

In other words, we have shown that the set $\mathcal{A} \setminus \mathcal{F}_n$ is porous in $(\mathcal{A},d)$. This completes the proof of Theorem 1.1.

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Simeon Reich: Department of Mathematics, The Technion – Israel Institute of Technology, 32000 Haifa, Israel

E-mail address: sreich@tx.technion.ac.il

Current address: Department of Mathematics, University of California, Santa Barbara, CA 93106-3080, USA

E-mail address: sreich@math.ucsb.edu

Alexander J. Zaslavski: Department of Mathematics, The Technion – Israel Institute of Technology, 32000 Haifa, Israel

E-mail address: ajzasl@tx.technion.ac.il