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Research Article

Fixed Point Theorems for Suzuki Generalized Nonexpansive Multivalued Mappings in Banach Spaces

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In the first part of this paper, we prove the existence of common fixed points for a commuting pair consisting of a single-valued and a multivalued mapping both satisfying the Suzuki condition in a uniformly convex Banach space. In this way, we generalize the result of Dhompongsa et al. (2006). In the second part of this paper, we prove a fixed point theorem for upper semicontinuous mappings satisfying the Suzuki condition in strictly $L(\tau)$ spaces; our result generalizes a recent result of Domínguez-Benavides et al. (2009).

1. Introduction

A mapping T on a subset E of a Banach space X is said to be nonexpansive if

$$\|Tx - Ty\| \leq \|x - y\|, \quad x, y \in E. \tag{1.1}$$

In 2008, Suzuki [1] introduced a condition which is weaker than nonexpansiveness. Suzuki's condition which was named by him the condition (C) reads as follows: a mapping T is said to satisfy the condition (C) on E if

$$\frac{1}{2}\|x - Tx\| \leq \|x - y\| \Longrightarrow \|Tx - Ty\| \leq \|x - y\|, \quad x, y \in E. \tag{1.2}$$

He then proved some fixed point and convergence theorems for such mappings. We shall at times refer to this concept by saying that T is a generalized nonexpansive mapping in the sense of Suzuki. Very recently, the current authors used a modified Suzuki condition for multivalued mappings and proved a fixed point theorem for multivalued mappings satisfying this condition in uniformly convex Banach spaces (see [2]).

In this paper, we first present a common fixed point theorem for commuting pairs consisting of a single-valued and a multivalued mapping both satisfying the Suzuki condition. This result extends a result of Dhompongsa et al. [3].

In the next part, we shall consider a recent result of Domínguez-Benavides et al. [4] on the existence of fixed points in an important class of spaces which are usually called strictly $L(\tau)$ spaces. These spaces contain all Lebesgue function spaces $L_p(\Omega)$ for $p \geq 1$. In this paper, we also generalize results of Domínguez-Benavides et al. [4] to upper semicontinuous mappings satisfying the Suzuki condition.

2. Preliminaries

Given a mapping T on a subset E of a Banach space X, the set of its fixed points will be denoted by

$$\mathrm{Fix}(T) = \{x \in E : Tx = x\}. \tag{2.1}$$

We start by the following definition due to Suzuki.

Definition 2.1 (see [1]). Let T be a mapping on a subset E of a Banach space X. The mapping T is said to satisfy the Suzuki condition (C) if

$$\frac{1}{2}\|x - Tx\| \leq \|x - y\| \Longrightarrow \|Tx - Ty\| \leq \|x - y\|, \quad x, y \in E. \tag{2.2}$$

As the following example shows, the Suzuki condition is weaker than nonexpansiveness. Therefore, it is natural to call these mappings as "generalized nonexpansive mappings". However, we shall at times refer to these mappings as those satisfying the condition (C).

Example 2.2. Let $X = \mathbb{R}$ be equipped with the usual metric $d(x, y) = |x-y|$, and let $E = [0, 7/2]$. We put

$$T(x) = \begin{cases} 0, & x \in [0, 3], \\ 4x - 12, & x \in \left[3, \dfrac{13}{4}\right], \\ -4x + 14, & x \in \left[\dfrac{13}{4}, \dfrac{7}{2}\right]. \end{cases} \tag{2.3}$$

The mapping T is continuous and satisfies the condition (C). However, T is not nonexpansive.

Lemma 2.3 (see [1, Lemma 4]). *Let T be a mapping defined on a closed subset E of a Banach space X. Assume that T satisfies the condition (C). Then* $\mathrm{Fix}(T)$ *is closed. Moreover, if X is strictly convex and E is convex, then* $\mathrm{Fix}(T)$ *is also convex.*

Theorem 2.4 (see [5, Theorem 2.3]). *Let E be a nonempty bounded closed convex subset of a uniformly convex Banach space X. Let $T : E \to E$ be a mapping satisfying the condition (C). Then T has a fixed point.*

Let (X, d) be a metric space. We denote by $CB(X)$ the collection of all nonempty closed bounded subsets of X; we also write $KC(X)$ to denote the collection of all nonempty compact convex subsets of X. Let H be the Hausdorff metric with respect to d, that is,

$$H(A, B) := \max\left\{ \sup_{x\in A} \operatorname{dist}(x, B), \sup_{y\in B} \operatorname{dist}(y, A) \right\}, \tag{2.4}$$

for all $A, B \in CB(X)$ where $\operatorname{dist}(x, B) = \inf_{y\in B} d(x, y)$.

Let $T : X \to 2^X$ be a multivalued mapping. An element $x \in X$ is said to be a fixed point of T provided that $x \in Tx$.

Definition 2.5. A multivalued mapping $T : X \to CB(X)$ is said to be nonexpansive provided that

$$H(Tx, Ty) \le \|x - y\|, \quad x, y \in X. \tag{2.5}$$

Suzuki's condition can be modified to incorporate multivalued mappings. This was done by the current authors in [2]. We call these mappings generalized multivalued nonexpansive mappings in the sense of Suzuki or multivalued mappings satisfying the condition (C). We now state Suzuki's condition for multivalued mappings as follows.

Definition 2.6. A multivalued mapping $T : X \to CB(X)$ is said to satisfy the condition (C) provided that

$$\frac{1}{2} \operatorname{dist}(x, Tx) \le \|x - y\| \Longrightarrow H(Tx, Ty) \le \|x - y\|, \quad x, y \in X. \tag{2.6}$$

Example 2.7. Define a mapping T on $[0, 5]$ by

$$T(x) = \begin{cases} \left[0, \dfrac{x}{5}\right], & x \ne 5, \\ \{1\}, & x = 5. \end{cases} \tag{2.7}$$

It is not difficult to see that T satisfies the Suzuki condition; however, T is not nonexpansive.

The following lemma, proved by Goebel and Kirk [6], plays an important role in the coming discussions.

Lemma 2.8. *Let $\{z_n\}$ and $\{w_n\}$ be two bounded sequences in a Banach space X, and let $0 < \lambda < 1$. If for every natural number n we have $z_{n+1} = \lambda w_n + (1 - \lambda)z_n$ and $\|w_{n+1} - w_n\| \le \|z_{n+1} - z_n\|$, then $\lim_{n\to\infty}\|w_n - z_n\| = 0$.*

Definition 2.9. A multivalued mapping $T : E \to 2^X$ is said to be upper semicontinuous on E if $\{x \in E : Tx \subset V\}$ is open in E whenever $V \subset X$ is open.

We recall that if T is single valued, then T reduces to a continuous function.

3. Fixed Points in Uniformly Convex Banach Spaces

Let E be a nonempty closed convex subset of a Banach space X. Assume that $\{x_n\}$ is a bounded sequence in X. For each $x \in X$, the asymptotic radius of $\{x_n\}$ at x is defined by

$$r(x, \{x_n\}) = \limsup_{n\to\infty} \|x_n - x\|. \tag{3.1}$$

Let

$$\begin{aligned} r = r(E, \{x_n\}) &= \inf\{r(x, \{x_n\}) : x \in E\}, \\ A = A(E, \{x_n\}) &= \{x \in E : r(x, \{x_n\}) = r\}. \end{aligned} \tag{3.2}$$

The number r is known as the *asymptotic radius* of $\{x_n\}$ relative to E. Similarly, the set A is called the *asymptotic center* of $\{x_n\}$ relative to E. In the case that X is a reflexive Banach space and E is a nonempty closed convex bounded subset of X, the set $A(E, \{x_n\})$ is always a nonempty closed convex subset of E. To see this, observe that by the definition of r, for each $\epsilon > 0$, the set

$$C_\epsilon = \left\{x \in E : \limsup_{n\to\infty} \|x_n - x\| \le r + \epsilon\right\} \tag{3.3}$$

is nonempty. It is not difficult to see that each C_ϵ is closed and convex; hence

$$A = \bigcap_{\epsilon>0} C_\epsilon \tag{3.4}$$

is closed and convex. Moreover, it follows from the weak compactness of E that A is nonempty. It is easy to see that if X is uniformly convex and if E is a closed convex subset of X, then A consists of exactly one point.

A bounded sequence $\{x_n\}$ is said to be *regular* with respect to E if for every subsequence $\{x'_n\}$ we have

$$r(E, \{x_n\}) = r(E, \{x'_n\}). \tag{3.5}$$

It is also known that if X is uniformly convex and if E is a nonempty closed convex subset of X, then for any $x \in X$, there exists a unique point $a \in E$ such that $\|x - a\| = \operatorname{dist}(x, E)$.

The following lemma was proved by Goebel and Lim.

Lemma 3.1 (see [7, 8]). *Let $\{x_n\}$ be a bounded sequence in X and let E be a nonempty closed convex subset of X. Then $\{x_n\}$ has a subsequence which is regular relative to E.*

Definition 3.2. Let E be a nonempty closed convex bounded subset of a Banach space X, and let $t : E \to X$ and $T : E \to CB(X)$ be two mappings. Then t and T are said to be commuting mappings if for every $x, y \in E$ such that $x \in Ty$ and $ty \in E$, we have $tx \in Tty$.

Now the time is ripe to state and prove the main result of this section.

Theorem 3.3. *Let E be a nonempty closed convex bounded subset of a uniformly convex Banach space X. Let $t : E \to E$ be a single-valued mapping, and let $T : E \to KC(E)$ be a multivalued mapping. If both t and T satisfy the condition (C) and if t and T are commuting, then they have a common fixed point, that is, there exists a point $z \in E$ such that $z = t(z) \in T(z)$.*

Proof. By Theorem 2.4, the mapping t has a nonempty fixed point set $\operatorname{Fix}(t)$ which is a closed convex subset of X (by Lemma 2.3). We show that for $x \in \operatorname{Fix}(t)$, $Tx \cap \operatorname{Fix}(t) \neq \emptyset$. To see this, let $x \in \operatorname{Fix}(t)$; since t and T are commuting, we have $ty \in Tx$ for each $y \in Tx$. Therefore, Tx is invariant under t for each $x \in \operatorname{Fix}(t)$. Since Tx is a bounded closed convex subset of the uniformly convex Banach space X, we conclude that t has a fixed point in Tx and therefore, $Tx \cap \operatorname{Fix}(t) \neq \emptyset$ for $x \in \operatorname{Fix}(t)$.

Now we find an approximate fixed point sequence for T in Fix(t). Take $x_0 \in \operatorname{Fix}(t)$, since $Tx_0 \cap \operatorname{Fix}(t) \neq \emptyset$, therefore, we can choose $y_0 \in Tx_0 \cap \operatorname{Fix}(t)$. Define

$$x_1 = \frac{1}{2}(x_0 + y_0). \tag{3.6}$$

Since $\operatorname{Fix}(t)$ is a convex set, we have $x_1 \in \operatorname{Fix}(t)$. Let $y_1 \in T(x_1)$ be chosen in such a way that

$$\|y_0 - y_1\| = \operatorname{dist}(y_0, T(x_1)). \tag{3.7}$$

We see that $y_1 \in \operatorname{Fix}(t)$. Indeed, we have

$$\frac{1}{2}\|y_0 - ty_0\| = 0 \leq \|y_0 - y_1\|. \tag{3.8}$$

Since t satisfies the condition (C), we get

$$\|y_0 - ty_1\| = \|ty_0 - ty_1\| \leq \|y_0 - y_1\| \tag{3.9}$$

which contradicts the uniqueness of y_1 as the unique nearest point of y_0 (note that $ty_1 \in Tx_1$). Similarly, put $x_2 = (1/2)(x_1 + y_1)$; again we choose $y_2 \in T(x_2)$ in such a way that

$$\|y_1 - y_2\| = \operatorname{dist}(y_1, T(x_2)). \tag{3.10}$$

By the same argument, we get $y_2 \in \operatorname{Fix}(t)$. In this way, we will find a sequence $\{x_n\}$ in $\operatorname{Fix}(t)$ such that $x_{n+1} = (1/2)(x_n + y_n)$ where $y_n \in T(x_n) \cap \operatorname{Fix}(t)$ and

$$\|y_{n-1} - y_n\| = \operatorname{dist}(y_{n-1}, T(x_n)). \tag{3.11}$$

Therefore, for every natural number $n \geq 1$, we have

$$\frac{1}{2}\left\|x_n - y_n\right\| = \|x_n - x_{n+1}\| \tag{3.12}$$

from which it follows that

$$\frac{1}{2}\operatorname{dist}(x_n, T(x_n)) \leq \frac{1}{2}\left\|x_n - y_n\right\| = \|x_n - x_{n+1}\|, \quad n \geq 1. \tag{3.13}$$

Our assumption now gives

$$H(T(x_n), T(x_{n+1})) \leq \|x_n - x_{n+1}\|, \quad n \geq 1, \tag{3.14}$$

hence

$$\left\|y_n - y_{n+1}\right\| = \operatorname{dist}\left(y_n, T(x_{n+1})\right) \leq H(T(x_n), T(x_{n+1})) \leq \|x_n - x_{n+1}\|, \quad n \geq 1. \tag{3.15}$$

We now apply Lemma 2.8 to conclude that $\lim_{n\to\infty}\|x_n - y_n\| = 0$ where $y_n \in T(x_n)$. Moreover, by passing to a subsequence we may assume that $\{x_n\}$ is regular (see Lemma 3.1). Since X is uniformly convex, $A(\operatorname{Fix}(t), \{x_n\})$ is singleton, say w (note that $w \in \operatorname{Fix}(t)$). Let $r = r(\operatorname{Fix}(t), \{x_n\})$. For each $n \geq 1$, we choose $z_n \in T(w)$ such that

$$\left\|y_n - z_n\right\| = \operatorname{dist}\left(y_n, T(w)\right). \tag{3.16}$$

On the other hand, there is a natural number n_0 such that for every $n \geq n_0$ we have $(1/2)\|x_n - y_n\| \leq \|x_n - w\|$. This implies that

$$\frac{1}{2}\operatorname{dist}(x_n, T(x_n)) \leq \|x_n - w\|, \tag{3.17}$$

and hence from our assumption we obtain

$$H(T(x_n), T(w)) \leq \|x_n - w\|, \quad n \geq n_0. \tag{3.18}$$

Therefore,

$$\left\|y_n - z_n\right\| \leq H(T(x_n), T(w)) \leq \|x_n - w\|, \quad n \geq n_0. \tag{3.19}$$

Moreover, $z_n \in \operatorname{Fix}(t)$ for all natural numbers $n \geq 1$. Indeed, since

$$\left\|y_n - ty_n\right\| = 0 \leq \left\|y_n - z_n\right\|, \quad n \geq 1, \tag{3.20}$$

we have

$$\|y_n - tz_n\| = \|ty_n - tz_n\| \leq \|y_n - z_n\|. \tag{3.21}$$

Since $w \in \mathrm{Fix}(t)$ and $z_n \in T(w)$, by the fact that the mappings t and T are commuting, we obtain $tz_n \in Ttw = Tw$. Now, by the uniqueness of z_n as the nearest point to y_n, we get $tz_n = z_n \in \mathrm{Fix}(t)$.

Since $T(w)$ is compact, the sequence $\{z_n\}$ has a convergent subsequence $\{z_{n_k}\}$ with $\lim_{k\to\infty} z_{n_k} = v \in T(w)$. Because $z_{n_k} \in \mathrm{Fix}(t)$ for all n, and $\mathrm{Fix}(t)$ is closed, we obtain $v \in \mathrm{Fix}(t)$. Note that

$$\|x_{n_k} - v\| \leq \|x_{n_k} - y_{n_k}\| + \|y_{n_k} - z_{n_k}\| + \|z_{n_k} - v\| \tag{3.22}$$

and for $n_k \geq n_0$ we have $\|y_{n_k} - z_{n_k}\| \leq \|x_{n_k} - w\|$. This entails

$$\limsup_{k\to\infty}\|x_{n_k} - v\| \leq \limsup_{k\to\infty}\|x_{n_k} - w\| \leq r. \tag{3.23}$$

Since $\{x_n\}$ is regular, this shows that $w = v \in T(w)$. And hence $w = tw \in Tw$. □

As a consequence, we obtain the theorem already proved by Dhompongsa et al. (see [3, Theorem 4.2]).

Corollary 3.4. *Let E be a nonempty closed convex bounded subset of a uniformly convex Banach space X, $t : E \to E$, and $T : E \to KC(E)$ a single-valued and a multivalued nonexpansive mapping, respectively. Assume that t and T are commuting mappings. Then there exists a point $z \in E$ such that $z = t(z) \in T(z)$.*

Corollary 3.5. *Let E be a nonempty bounded closed convex subset of a uniformly convex Banach space X, and let $T : E \to KC(E)$ be a multivalued mapping satisfying the Suzuki condition (C). Then T has a fixed point.*

Corollary 3.6. *Let E be a nonempty closed convex bounded subset of a uniformly convex Banach space X, and let $T : E \to KC(E)$ be a nonexpansive multivalued mapping. Then T has a fixed point.*

4. Strictly $L(\tau)$ Spaces

Definition 4.1. Let $(X, \|\cdot\|)$ be a Banach space and let τ be a linear topology on X. We say that X is a strictly $L(\tau)$ space if there exists a function $\delta : [0,\infty) \times [0,\infty) \to [0,\infty)$ satisfying the following

(i) δ is continuous;

(ii) $\delta(\cdot, s)$ is strictly increasing;

(iii) $\delta(0, s) = s$, for every $s \in [0,\infty)$;

(iv) $\delta(r, \cdot)$ is strictly increasing;

(v) $\Phi_{(x_n)}(y) = \delta(\Phi_{(x_n)}(0), \|y\|)$, for every $y \in X$ and for every bounded and τ-null sequence (x_n), where $\Phi_{(x_n)}(y) : X \to [0,\infty)$ is defined by

$$\Phi_{(x_n)}(y) := \limsup_{n\to\infty} \|x_n - y\|. \tag{4.1}$$

In this case we also say that X satisfies the strict property $L(\tau)$ with respect to δ.

Example 4.2 (see [9]). Let (Ω, Σ, μ) be a positive σ-finite measure space. For every $1 \le p < +\infty$, consider the Banach space $L_p(\Omega)$ with the usual norm. Let τ be the topology of convergence locally in measure (clm). Then $L_p(\Omega)$ endowed with the clm-topology satisfies the property $L(\tau)$ with $\delta(r,s) = (r^p + s^p)^{1/p}$

Definition 4.3. Let $(X, \|\cdot\|)$ be a Banach space and let τ be a linear topology on X which is weaker than the norm topology. Let E be a closed convex bounded subset of $(X, \|\cdot\|)$; then for $x_0 \in X$, we write $P_E(x_0) = \{x \in E : \|x - x_0\| = \operatorname{dist}(x_0, E)\}$. We say that E has property (P) if for every $x \in \overline{E}^\tau$ the set $P_E(x)$ is a nonempty and norm-compact subset of E.

Theorem 4.4. *Let X be a strictly $L(\tau)$ Banach space and let E be a nonempty closed convex bounded subset of X satisfying the property (P). Suppose, in addition, that $\overline{E}^\tau$ is τ-sequentially compact. If $T : E \to KC(E)$ satisfies the condition (C), and if T is an upper semicontinuous mapping, then T has a fixed point.*

Proof. First, we find an approximate fixed point sequence. Choose $x_0 \in E$ and $y_0 \in T(x_0)$. Define

$$x_1 = \frac{1}{2}(x_0 + y_0). \tag{4.2}$$

Let $y_1 \in T(x_1)$ be chosen in such a way that

$$\|y_0 - y_1\| = \operatorname{dist}(y_0, T(x_1)). \tag{4.3}$$

Similarly, put $x_2 = (1/2)(x_1 + y_1)$; again we choose $y_2 \in T(x_2)$ in such a way that

$$\|y_1 - y_2\| = \operatorname{dist}(y_1, T(x_2)). \tag{4.4}$$

In this way, we will find a sequence $\{x_n\}$ in E such that $x_{n+1} = (1/2)(x_n + y_n)$, where $y_n \in T(x_n)$ and

$$\|y_{n-1} - y_n\| = \operatorname{dist}(y_{n-1}, T(x_n)). \tag{4.5}$$

Therefore, for every natural number $n \ge 1$, we have

$$\frac{1}{2}\|x_n - y_n\| = \|x_n - x_{n+1}\|, \tag{4.6}$$

from which it follows that

$$\frac{1}{2}\operatorname{dist}(x_n, T(x_n)) \le \frac{1}{2}\left\|x_n - y_n\right\| = \|x_n - x_{n+1}\|, \quad n \ge 1. \tag{4.7}$$

Our assumption now gives

$$H(T(x_n), T(x_{n+1})) \le \|x_n - x_{n+1}\|, \quad n \ge 1. \tag{4.8}$$

Hence

$$\left\|y_n - y_{n+1}\right\| = \operatorname{dist}\left(y_n, T(x_{n+1})\right) \le H(T(x_n), T(x_{n+1})) \le \|x_n - x_{n+1}\|, \quad n \ge 1. \tag{4.9}$$

We now apply Lemma 2.8 to conclude that $\lim_{n\to\infty}\|x_n - y_n\| = 0$ where $y_n \in T(x_n)$. Since $\overline{E}^{\tau}$ is τ-sequentially compact, by passing to a subsequence, we may assume that (x_n) is τ-convergent to $x_0 \in \overline{E}^{\tau}$. Now we are going to show that

$$P_E(x_0) \cap T(z) \ne \emptyset, \quad \forall z \in P_E(x_0). \tag{4.10}$$

Taking any $z \in P_E(x_0)$, by the compactness of Tz, we can find $z_n \in Tz$ such that $\|y_n - z_n\| = \operatorname{dist}(y_n, T(z))$. On the other hand, there is a natural number n_0 such that for every $n \ge n_0$ we have $(1/2)\|x_n - y_n\| \le \|x_n - z\|$. This implies that

$$\frac{1}{2}\operatorname{dist}(x_n, T(x_n)) \le \|x_n - z\|, \tag{4.11}$$

and hence from the assumption we obtain

$$H(T(x_n), T(z)) \le \|x_n - z\|, \quad n \ge n_0. \tag{4.12}$$

Therefore,

$$\left\|y_n - z_n\right\| = \operatorname{dist}\left(y_n, T(z)\right) \le H(T(x_n), T(z)) \le \|x_n - z\|, \quad n \ge n_0. \tag{4.13}$$

Since $T(z)$ is compact, the sequence $\{z_n\}$ has a convergent subsequence $\{z_{n_k}\}$ with $\lim_{k\to\infty} z_{n_k} = w_0 \in T(z)$. It follows that

$$\begin{aligned}\Phi_{(x_{n_k})}(w_0) &= \limsup_{k\to\infty}\left\|y_{n_k} - w_0\right\| = \limsup_{k\to\infty}\left\|y_{n_k} - z_{n_k}\right\| \\ &\le \limsup_{k\to\infty}\|x_{n_k} - z\| = \delta\Big(\Phi_{(x_{n_k})}(x_0), \|z - x_0\|\Big).\end{aligned} \tag{4.14}$$

On the other hand, we have that $\Phi_{(x_{n_k})}(w_0) = \delta(\Phi_{(x_{n_k})}(x_0), \|w_0 - x_0\|)$. Since δ is strictly increasing, it follows that $\|w_0 - x_0\| \leq \|z - x_0\|$. Hence $w_0 \in P_E(x_0)$ and so $P_E(x_0) \cap T(z) \neq \emptyset$. Now we define the mapping

$$\widetilde{T} : P_E(x_0) \longrightarrow KC(P_E(x_0)) \tag{4.15}$$

by $\widetilde{T}(z) = P_E(x_0) \cap T(z)$. From [10, Proposition 2.45], we know that the mapping $\widetilde{T}$ is upper semicontinuous. Since $P_E(x_0) \cap T(z)$ is a compact convex set, we can apply the Kakutani-Bohnenblust-Karlin Theorem (see [11]) to obtain a fixed point for $\widetilde{T}$ and hence for T. □

Corollary 4.5. *Let X be a strictly $L(\tau)$ Banach space and let E be a nonempty closed convex bounded subset of X satisfying the property (P). Suppose, in addition, that $\overline{E}^{\tau}$ is τ-sequentially compact. If $T : E \to KC(E)$ is a nonexpansive mapping, then T has a fixed point.*

Corollary 4.6. *Let X be a strictly $L(\tau)$ Banach space and let E be a nonempty closed convex and bounded subset of X satisfying the property (P). Suppose, in addition, that $\overline{E}^{\tau}$ is τ-sequentially compact. If $T : E \to E$ is a continuous mapping satisfying the condition (C), then T has a fixed point.*

Finally we mention that by Example 2.2, this corollary generalizes the recent result of Domínguez-Benavides et al. [4].

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