Hindawi Publishing Corporation
Fixed Point Theory and Applications
Volume 2010, Article ID 718340, 8 pages
doi:10.1155/2010/718340

Research Article

Common Fixed Points for Generalized φ-Pair Mappings on Cone Metric Spaces

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Received 19 October 2009; Revised 12 December 2009; Accepted 10 January 2010

Academic Editor: Massimo Furi

We define the concept of generalized φ-pair mappings and prove some common fixed point theorems for this type of mappings. Our results generalize some recent results.

1. Introduction

Huang and Zhang [1] recently introduced the concept of cone metric spaces and established some fixed point theorems for contractive mappings in these spaces. Afterwards, Rezapour and Hamlbarani [2] studied fixed point theorems of contractive type mappings by omitting the assumption of normality in cone metric spaces. Also, other authors proved the existence of points of coincidence, common fixed point, and coupled fixed point for mappings satisfying different contraction conditions in cone metric spaces (see [1–12]). In [6] Di Bari and Vetro introduced the concept of φ-map and proved a main theorem generalizing some known results. We define the concept of generalized φ-mappings and prove some results about common fixed points for such mappings. Our results generalize some results of Huang and Zhang [1], Di Bari and Vetro [6], and Abbas and Jungck [3]. First, we recall some standard notations and definitions in cone metric spaces.

Let E be a real Banach space and let θ denote the zero element in E. A cone P is a subset of E such that

(i) P is closed, nonempty, and $P \neq \{\theta\}$,

(ii) if a, b are nonnegative real numbers and $x, y \in P$, then $ax + by \in P$,

(iii) $P \cap (-P) = \{\theta\}$.

For a given cone $P \subset E$, the partial ordering $\leq$ with respect to P is defined by $x \leq y$ if and only if $y - x \in P$. The notation $x < y$ will stand for $x \leq y$ but $x \neq y$. Also, we will use $x \ll y$ to indicate that $y - x \in \operatorname{int} P$ where $\operatorname{int} P$ denotes the interior of P. Using these notations, we have the following definition of a cone metric space.

Definition 1.1 (see [1]). Let X be a nonempty set and let E be a real Banach space equipped with the partial ordering $\leq$ with respect to the cone $P \subset E$. Suppose that the mapping $d : X \times X \to E$ satisfies the following conditions:

(d_1) $\theta \leq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$,

(d_2) $d(x, y) = d(y, x)$ for all $x, y \in X$,

(d_3) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a cone metric on X, and (X, d) is called a cone metric space.

The cone P is called normal if there exists a constant $K > 0$ such that for every $x, y \in E$ if $\theta \leq x \leq y$, then $\|x\| \leq K\|y\|$. The least positive number satisfying this inequality is called the normal constant of P. The cone P is called regular if every increasing (decreasing) and bounded above (below) sequence is convergent in E. It is known that every regular cone is normal [1] (see also [2, Lemma 1.1]).

Definition 1.2 (see [1]). Let (X, d) be a cone metric space, let $\{x_n\}$ be a sequence in X, and let $x \in X$.

(i) $\{x_n\}$ is said to be Cauchy sequence if for every $c \in E$ with $\theta \ll c$ there exists $N \in \mathbb{N}$ such that for all $n, m \geq N$, $d(x_n, x_m) \ll c$.

(ii) $\{x_n\}$ is said to be convergent to x, denoted by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$ as $n \to \infty$ if for every $c \in E$ with $\theta \ll c$ there exists $N \in \mathbb{N}$ such that for all $n \geq N$, $d(x_n, x) \ll c$.

(iii) X is said to be complete if every Cauchy sequence in X is convergent in X.

(iv) X is said to be sequentially compact if for every sequence $\{x_n\}$ in X there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $\{x_{n_i}\}$ is convergent in X.

Clearly, every sequentially compact cone metric space is complete (see [1–12]) for more related results about complete cone metric spaces). We also note that the relations $P + \operatorname{int} P \subseteq \operatorname{int} P$ and $\lambda \operatorname{int} P \subseteq \operatorname{int} P (\lambda > 0)$ always hold true.

Definition 1.3 (see [13]). Let T and S be self-mappings of a cone metric space (X, d). One says that S and T are compatible if $\lim_{n\to\infty} d(STx_n, TSx_n) = \theta$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.

The concept of weakly compatible mappings is introduced as follows.

Definition 1.4 (see [13]). The self-mappings T and S of a cone metric space (X, d) are said to be weakly compatible if they commute at their coincidence points, that is, if $Tu = Su$ for some $u \in X$, then $TSu = STu$.

2. Main Results

In this section, we introduce the notation of generalized φ-mapping and a contractive condition called generalized φ-pair. We prove some results on common fixed points of these mappings on cone metric spaces.

Let P be a cone. A nondecreasing mapping $\varphi : P \to P$ is called a φ-mapping [6] if

(φ_1) $\varphi(\theta) = \theta$ and $\theta < \varphi(w) < w$ for $w \in P \setminus \{\theta\}$,

(φ_2) $w - \varphi(w) \in \operatorname{int} P$ for every $w \in \operatorname{int} P$,

(φ_3) $\lim_{n\to\infty}\varphi^n(w) = \theta$ for every $w \in P \setminus \{\theta\}$.

Definition 2.1. Let P be a cone and let $\{w_n\}$ be a sequence in P. One says that $w_n \xrightarrow{\ll} \theta$ if for every $\epsilon \in P$ with $\theta \ll \epsilon$ there exists $N \in \mathbb{N}$ such that $w_n \ll \epsilon$ for all $n \geq N$.

For a nondecreasing mapping $F : P \to P$, we define the following conditions which will be used in the sequal:

(F_1) $F(w) = \theta$ if and only if $w = \theta$,

(F_2) for every $w_n \in P$, $w_n \xrightarrow{\ll} \theta$ if and only if $F(w_n) \xrightarrow{\ll} \theta$,

(F_3) for every $w_1, w_2 \in P$, $F(w_1 + w_2) \leq F(w_1) + F(w_2)$.

Definition 2.2. The self-mappings $f, g : X \to X$ are called generalized φ-pair if there exist a φ-mapping and a mapping F satisfying the conditions (F_1), (F_2), and (F_3) such that

$$F(d(fx, fy)) \leq \varphi(F(d(gx, gy))), \tag{2.1}$$

for every $x, y \in X$.

Now, we are in the position to state the following theorem.

Theorem 2.3. *Let (X, d) be a cone metric space and let $f, g : X \to X$ be a generalized φ-pair. Suppose that f and g are weakly compatible with $fX \subset gX$ such that fX or gX is complete. Then the self-mappings f and g have a unique common fixed point in X.*

Proof. Let $x_0 \in X$ and choose $x_1 \in X$ such that $fx_0 = gx_1$. This can be done, since $fX \subset gX$. Continuing this process, after choosing $x_n \in X$, we choose $x_{n+1} \in X$ such that $gx_{n+1} = fx_n$. Since f and g are generalized φ-pair, by Definition 1.2, there exist a φ-mapping and a mapping F satisfying the conditions (F_1)–(F_3) and the inequality of (2.1). By (2.1), we deduce

$$\begin{aligned} F(d(fx_{n+1}, fx_n)) &\leq \varphi(F(d(gx_{n+1}, gx_n))) = \varphi(F(d(fx_n, fx_{n-1}))) \\ &\leq \varphi^2(F(d(gx_n, gx_{n-1}))) \leq \cdots \leq \varphi^n(F(d(fx_1, fx_0))). \end{aligned} \tag{2.2}$$

Let $\epsilon \in \operatorname{int} P$, then, by ($\varphi_2$), $\epsilon_0 = \epsilon - \varphi(\epsilon) \in \operatorname{int} P$. By ($\varphi_3$),

$$\lim_{n\to\infty} \varphi^n(F(d(fx_1, fx_0))) = \theta. \tag{2.3}$$

Therefore, one can find that $N \in \mathbb{N}$ such that, for all $m \geq N$, $F(d(fx_m, fx_{m+1})) \ll \epsilon - \varphi(\epsilon)$. We show that

$$F(d(fx_m, fx_{n+1})) \ll \epsilon, \tag{2.4}$$

for a fixed $m \geq N$ and $n \geq m$. This holds when $n = m$. Now let (2.4) hold for some $n \geq m$, then we have

$$\begin{aligned} F(d(fx_m, fx_{n+2})) &\leq F(d(fx_m, fx_{m+1})) + F(d(fx_{m+1}, fx_{n+2})) \\ &\ll \epsilon - \varphi(\epsilon) + \varphi(F(d(gx_{m+1}, gx_{n+2}))) \\ &\ll \epsilon - \varphi(\epsilon) + \varphi(F(d(fx_m, fx_{n+1}))) \\ &\ll \epsilon - \varphi(\epsilon) + \varphi(\epsilon) = \epsilon. \end{aligned} \tag{2.5}$$

Therefore, by induction and (F_2) we deduce that $\{fx_n\}$ is a Cauchy sequence. Suppose that fX is a complete subspace of X, then there exists $y \in fX \subset gX$ such that $fx_n \to y$ and also $gx_n \to y$ (This holds also if gX is complete with $y \in gX$.). Let $z \in X$ be such that $gz = y$. We show that $fz = gz$. By (F_2) for $\theta \ll \epsilon$ one can choose a natural number N such that $F(d(y, fx_n)) \ll \epsilon/2$ and $F(d(gx_n, gz)) \ll \epsilon/2$ for all $n \geq N$. Then,

$$\begin{aligned} F(d(y, fz)) &\leq F(d(y, fx_n)) + F(d(fx_n, fz)) \leq F(d(y, fx_n)) + \varphi(F(d(gx_n, gz))) \\ &< F(d(y, fx_n)) + F(d(gx_n, gz)) \ll \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon. \end{aligned} \tag{2.6}$$

Thus, $(\epsilon/m) - F(d(y, fz)) \in P$ for every $m \in \mathbb{N}$. This implies that $-F(d(y, fz)) \in P$, and hence, $F(d(y, fz)) = \theta$. So applying (F_1), we get $d(y, fz) = \theta$ which implies that $y = fz = gz$, that is, y is a point of coincidence of f and g. Now, we use the hypothesis that f and g are weakly compatible to deduce that y is a common fixed point of f and g. From $fz = gz = y$, by compatibility of f and g, it follows that $fy = fgz = gfz = gy$. If $gy \neq y$, then we have

$$F(d(fy, fz)) \leq \varphi(F(d(gy, gz))) < F(d(gy, gz)) = F(d(fy, fz)), \tag{2.7}$$

which implies that $fy = y = gy$. So y is a common fixed point of f and g. The uniqueness of the common fixed point is clear. □

Example 2.4. Let $E = \mathbb{R}$ and let $P = \{x \in \mathbb{R} : x \geq 0\}$ be a normal cone. Let $X = [1, +\infty)$ with usual metric $d(x, y) = |x - y|$. Define $f, g : X \to X$ by $fx = x$ and $gx = 2x - 1$, for all $x \in X$. Also, define $F, \varphi : P \to P$ by $\varphi w = (2/3)w$ and $Fw = (1/2)w$, for all $w \in P$. Then

(1) f and g are weakly compatible,

(2) $fX \subset gX$,

(3) we have $F(d(fx, fy)) \leq \varphi(F(d(gx, gy)))$,

(4) $f1 = g1 = 1$.

Example 2.5. Let $E = \mathbb{R}^2$ and let $P = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0\}$ be a normal cone. Let $X = [1, +\infty)$ with metric $d(x, y) = (|x - y|, (1/2)|x - y|)$. Define $f, g : X \to X$ by $fx = (x + 1)/2$ and $gx = 2x - 1$, for all $x \in X$. Also, define $F, \varphi : P \to P$ by $\varphi(w_1, w_2) = ((1/2)w_1, (1/3)w_2)$ and $F(w_1, w_2) = (w_2, w_1 + w_2)$, for all $w_1, w_2 \in P$. Then

(1) f and g are weakly compatible,

(2) $fX \subset gX$,

(3) we have $F(d(fx, fy)) \leq \varphi(F(d(gx, gy)))$,

(4) $f1 = g1 = 1$.

Example 2.6. Let $E = \mathbb{R}^2$ and let $P = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0\}$ be a normal cone. Let $X = [1, +\infty)$ with metric $d(x, y) = (|x - y|, 2|x - y|)$. Define $f, g : X \to X$ by $fx = (1/2)x + 1$ and $gx = x$, for all $x \in X$. Also, define $F, \varphi : P \to P$ by $\varphi(w_1, w_2) = ((1/2)w_1, (2/3)w_2)$ and $F(w_1, w_2) = (w_1, w_1 + w_2)$, for all $w_1, w_2 \in P$. Then

(1) f and g are weakly compatible,

(2) $fX \subset gX$,

(3) we have $F(d(fx, fy)) \leq \varphi(F(d(gx, gy)))$,

(4) $f2 = g2 = 2$.

If we let the mapping F be the identity mapping in Theorem 2.3, then we obtain the following corollary.

Corollary 2.7. *Let (X, d) be a cone metric space. Suppose that the mappings $f, g : X \to X$ satisfy*

$$d(fx, fy) \leq \varphi(d(gx, gy)), \tag{2.8}$$

for all $x, y \in X$. If $fX \subset gX$, f and g are weakly compatible, and fX or gX is complete, then f and g have a unique common fixed point in X.

Remark 2.8. Corollary 2.7 generalizes Theorem 1 in [6]. Also, if we choose the φ-mapping defined by $\varphi(w) = kw$, where $k \in [0, 1)$ is a constant, then Theorem 2.3 generalizes Theorem 2.1 in [3]. Furthermore, if we let g be the identity map of X, then we obtain Theorem 1 in [1], that is, the extension of the Banach fixed point theorem for cone metric spaces.

If we replace the condition (φ_1) with the following condition:

$(\varphi_1)'$ there exists $k \in [0, 1/2)$ such that $\varphi(w) \leq kw$ for $w \setminus \{\theta\}$ and $\varphi(\theta) = \theta$, then we have the following theorems.

Theorem 2.9. *Let (X, d) be a cone metric space and let $f, g : X \to X$ be self-mappings such that*

$$F(d(fx, fy)) \leq \varphi(F(d(fx, gx) + d(fy, gy))), \tag{2.9}$$

for all $x, y \in X$ where φ is a nondecreasing mapping from P into P satisfying the conditions $(\varphi_1)'$, φ_2, and φ_3, and $F : P \to P$ is a nondecreasing mapping satisfying the conditions (F_1)–(F_3). Suppose that f and g are weakly compatible, $fX \subset gX$, and fX or gX is complete. Then the mappings f and g have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point in X. Choose a point $x_1 \in X$ such that $fx_0 = gx_1$. This can be done since $fX \subset gX$. Continuing this process, after choosing $x_n \in X$ with $fx_n = gx_{n+1}$, by (2.9) and $(\varphi_1)'$, we have

$$\begin{aligned} F(d(fx_{n+1}, fx_n)) &\leq \varphi(F(d(fx_{n+1}, gx_{n+1}) + d(fx_n, gx_n))) \\ &\leq k(F(d(fx_{n+1}, fx_n)) + F(d(fx_n, fx_{n-1}))). \end{aligned} \tag{2.10}$$

Consequently,

$$F(d(fx_{n+1}, fx_n)) \leq hF(d(fx_n, fx_{n-1})), \tag{2.11}$$

where $h = k/(1-k)$. For $n > m$ we have

$$\begin{aligned} F(d(fx_n, fx_m)) &\leq F(d(fx_n, fx_{n-1})) + F(d(fx_{n-1}, fx_{n-2})) \\ &\quad + \cdots + F(d(fx_{m+1}, fx_m)) \\ &\leq \left(h^{n-1} + h^{n-2} + \cdots + h^m\right)F(d(fx_1, fx_0)) \\ &\leq \frac{h^m}{1-h}F(d(fx_1, fx_0)). \end{aligned} \tag{2.12}$$

Then $F(d(fx_n, fx_m)) \xrightarrow{\ll} \theta$ as $n, m \to \infty$, and hence, by (F_2), $\{fx_n\}$ is a Cauchy sequence. Suppose that fX is a complete subspace of X, then there exists $q \in fX \subset gX$ such that $fx_n \to q$ and also $gx_n \to q$ (this holds if gX is complete). Let $p \in X$ be such that $gp = q$. By (F_2), for a fixed $\theta \ll c$ and every $m \in \mathbb{N}$ there exists a natural number N such that $F(d(gx_{n+1}, gx_n)) \ll (c(1-k))/2k$ and $F(d(gx_{n+1}, gp)) \ll (c(1-k))/2$ for all $n \geq N$. Hence,

$$\begin{aligned} F(d(gp, fp)) &\leq F(d(gp, fx_n)) + F(d(fx_n, fp)) \\ &\leq F(d(gp, fx_n)) + \varphi(F(d(fx_n, gx_n) + d(fp, gp))) \\ &\leq F(d(gp, fx_n)) + k(F(d(fx_n, gx_n)) + F(d(gp, fp))), \end{aligned} \tag{2.13}$$

which implies that

$$\begin{aligned} F(d(gp, fp)) &\leq \frac{1}{1-k}F(d(gp, fx_n)) + \frac{k}{1-k}F(d(fx_n, gx_n)) \\ &\ll \frac{c}{2} + \frac{c}{2} = c. \end{aligned} \tag{2.14}$$

Thus, $F(d(gp, fp)) \ll c/m$ for all $m \geq 1$. This implies that $F(d(gp, fp)) = \theta$, and therefore, $gp = fp$. Since f and g are weakly compatible, $fq = fgp = gfp = gq$. If $q \neq gq$, then

$$\begin{aligned} F(d(fq, fp)) &\leq \varphi(F(d(fq, gq) + d(fp, gp))\ ) \\ &\leq k(F(d(fq, gq)) + F(d(fp, gp))), \end{aligned} \tag{2.15}$$

which gives $F(d(fq, fp)) = \theta$, and hence, $gq = fq = q$. So q is a common fixed point for f and g. The uniqueness of common fixed point is clear. □

If in Theorem 2.9 we let F be Id_X and let the φ-mapping be $\varphi(w) = kw$, where $k \in [0, 1/2)$ is a constant, then we obtain the following corollary.

Corollary 2.10. *Let (X, d) be a cone metric space and let $f, g : X \to X$ be self-mappings such that*

$$d(fx, fy) \leq k(d(fx, gx) + d(fy, gy)), \tag{2.16}$$

for all $x, y \in X$, where $k \in [0, 1/2)$ is a constant. Suppose that f and g are weakly compatible, the range of g contains the range of f, and fX or gX is complete. Then the mappings f and g have a unique common fixed point in X.

Remark 2.11. Corollary 2.10 generalizes Theorem 2.3 of [3]. If in Corollary 2.10 we let g be the identity map on X, then we obtain Theorem 3 of [1].

Theorem 2.12. *Let (X, d) be a cone metric space and let $f, g : X \to X$ be self-mappings such that*

$$F(d(fx, fy)) \leq \varphi(F(d(fx, gy) + d(fy, gx))), \tag{2.17}$$

for all $x, y \in X$. Suppose that f and g are weakly compatible, the range of g contains the range of f, and fX or gX is complete. Then the mappings f and g have a unique common fixed point in X.

Proof. Let x_0 be an arbitrary point in X. Choose a point x_1 in X such that $fx_0 = gx_1$. This can be done since $fX \subset gX$. Continuing this process having chosen x_n in X such that $fx_n = gx_{n+1}$, we have

$$\begin{aligned} F(d(fx_{n+1}, fx_n)) &\leq \varphi(F(d(fx_{n+1}, gx_n) + d(fx_n, gx_{n+1}))) \\ &\leq \varphi(F(d(fx_{n+1}, gx_n)) + F(d(fx_n, gx_{n+1}))) \\ &\leq k(F(d(fx_{n+1}, fx_{n-1}))) \\ &\leq k(F(d(fx_{n+1}, fx_n)) + F(d(fx_n, fx_{n-1}))). \end{aligned} \tag{2.18}$$

So,

$$F(d(fx_{n+1}, fx_n)) \leq hF(d(fx_n, fx_{n-1})), \tag{2.19}$$

where $h = k/(1-k) < 1$. Now let $m, n \in \mathbb{N}$ with $n > m$. Then,

$$\begin{aligned} F(d(fx_n, fx_m)) &\leq F(d(fx_n, fx_{n-1})) + F(d(fx_{n-1}, fx_{n-2})) + \cdots + F(d(fx_{m+1}, fx_m)) \\ &\leq \left(h^{n-1} + \cdots + h^m\right) F(d(fx_1, fx_0)) \leq \frac{h^m}{1-h} F(d(fx_1, fx_0)). \end{aligned} \tag{2.20}$$

Following an argument similar to that one given in Theorem 2.9, we obtain a unique common fixed point of f and g. □

If in Theorem 2.12 we let F be the identity map on X and let the φ-map be $\varphi(w) = kw$, where $k \in [0, 1/2)$ is a constant, then we obtain the following corollary.

Corollary 2.13. *Let (X, d) be a cone metric space and let $f, g : X \to X$ be self-mappings such that*

$$d(fx, fy) \leq k(d(fx, gy) + d(fy, gx)), \tag{2.21}$$

for all $x, y \in X$, where $k \in [0, 1/2)$ is a constant. Suppose that f and g are weakly compatible, the range of g contains the range of f, and fX or gX is complete. Then the mappings f and g have a unique common fixed point in X.

Remark 2.14. Corollary 2.13 generalizes Theorem 2.4 of [3] and if in Corollary 2.13 we let g be the identity map on X, then we obtain Theorem 4 of [1].

Acknowledgment

The authors would like to thank the referees for their valuable and useful comments.

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