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RESEARCH NOTES

SOME PROPERTIES OF JORDAN-EHLERS SPACETIME

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ABSTRACT. It is shown that for cylindrically symmetric Jordan-Ehlers spacetime, either the charged scalar meson field associated with meson rest mass M or the charged perfect fluid cannot be the source for generating gravitation.

KEY WORDS AND PHRASES. Gravitation, scalar field, relativistic field equations.
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1. INTRODUCTION.

Jordan et al. [1] and Ehlers [2] studied the cylindrically symmetric spacetime with two degrees of freedom and pointed out that this spacetime is capable of gravitational radiation. Stachel [3] defined a cylindrical gravitational news function to study some relativistic aspects in this spacetime whereas Rao [4] found out some solution representing the propagation of gravitational radiation on a null field background. Singh and Yadav [5] and Letelier [6] studied the problem of equivalency of massless scalar field and irrotational stiff perfect fluid in this spacetime. However, none of the authors tried to study the general nature of this spacetime. Therefore, in this short note we try to reveal the compatibilities of the non-static cylindrically symmetric Jordan-Ehlers spacetime as regards to charged microscopic and charged macroscopic gravitating matter in Sec 2 and 3 respectively. In both the cases the charge density σ does not survive and the electromagnetic field can be completely determined by only two components (viz. ϕ_2 and ϕ_3) of the electromagnetic four potential ϕ_i. In the former case the mass parameter M of the scalar field vanishes and in the latter case the perfect fluid reduces to stiff perfect fluid $(p = w)$.

2. CYLINDRICALLY SYMMETRIC CHARGED MASSIVE SACALR FIELDS.

The cylindrically symmetric metric, in the form given by Jordan and Ehlers [1,2] is

$$ds^2 = e^{2A-2B}(dt^2-dr^2)-r^2e^{-2B}d\theta^2-e^{2B}(cd\theta + dz)^2 \qquad (2.1)$$

where $A = A(r,t)$, $B = B(r,t)$ and $C = C(r,t)$ and the coordinates r, θ, z and t correspond to x^1, x^2, x^3, and x^4 respectively.

The general relativistic field equations for charged massive scalar fields are

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} = -K(T_{ij} + E_{ij}) \tag{2.2}$$

where $K = 8\pi_G$ and the velocity of light is unity. The energy momentum tensors for attractive scalar meson and electromagnetic fields are

$$T_{ij} = \frac{1}{4\pi}\,[V,_i\, V,_j - \frac{1}{2}\, g_{ij}\, (V,_a V'^{a} - M^2V^2)] \tag{2.2a}$$

and

$$E_{ij} = \frac{1}{4\pi}\,(-F_{is}\, F^s_j + \frac{1}{4}\, g_{ij}\, F_{ab}\, F^{ab}) \tag{2.2b}$$

respectively.

The electromagentic field tensor F_{ij} and the scalar field V satisfy the following equations:

$$F_{ij} = \phi_{j,i} - \phi_{i,j} \tag{2.3}$$

$$F^{ij};_j = 4\pi\sigma U^i \tag{2.4}$$

$$g^{ij}V;_{ij} + M^2V = 0 \tag{2.5}$$

Here comma and semicolon denote partial and covariant differentiation resepctively; σ, U^i , and ϕ_i denote the charge density, the four velocity vector and the electromagnetic four potentials respectively.

With the help of the metric (2.1), two of the field equations (2.2), viz., $G_{11} = K(T_{11} + E_{11})$ and $G_{44} = -K(T_{44} + E_{44})$, can be written in the following explicit forms:

$$\begin{aligned} G_{11} &= \frac{1}{4r^2}\,(C_1{}^2 + C_4{}^2)\, e^{4B} + B_1{}^2 + B_4{}^2 - \frac{A_1}{r} \\ &= -\frac{K}{8\pi}\,(V_1{}^2 + V_2{}^2 + g_{11}M^2V^2) - \frac{K}{8\pi}[-g^{22}F_{12}{}^2 - g^{33}\, F_{13}{}^2 - g^{44}F_{14}{}^2 \\ &\quad + g_{11}(g^{22}g^{33} - g^{23}.g^{23})\, F_{23}{}^2 - g^{22}F_{24}{}^2 - g^{33}F_{34}{}^2 \\ &\quad - 2g^{23}(F_{12}F_{13} + F_{24}F_{34})] \end{aligned} \tag{2.6}$$

$$\begin{aligned} G_{44} &= \frac{1}{4r^2}\,(C_1{}^2 + C_2{}^2)\, e^{4B} + B_1{}^2 + B_4{}^2 - \frac{A_1}{r} \\ &= -\frac{K}{8\pi}\,(V_1{}^2 + V_4{}^2 - g_{11}M^2V^2) - \frac{K}{8\pi}\,[-g^{22}F_{12}{}^2 - g^{33}F_{13}{}^2 \\ &\quad - g^{11}F_{14}{}^2 + g^{44}(g^{22}g^{33} - g^{23}.g^{23})\, F_{23}{}^2 - g^{22}F_{24}{}^2 - g^{33}F_{34}{}^2 \\ &\quad - 2g^{23}\,(F_{12}F_{13} + F_{24}F_{34})]. \end{aligned} \tag{2.7}$$

The cylindrical symmetry assumed implies that the scalar potential V shares the same property as A, B and C, as a consequence of which $V_2 = 0 = V_3$. The cylindrically charged massive scalar field will be completely determined by the equations (2.1) to (2.5).

Since $g^{11} = -g^{44}$ for the metric (2.1), the equations (2.6) and (2.7) yield

$$[g_{11}M^2V^2 + g^{11}F_{14}{}^2 + g_{11}(g^{22}g^{33} - g^{23}g^{23})] = 0 \qquad (2.8)$$

Using the metric potentials from (2.1) in the relation (2.8), we obtain

$$M^2V^2 + e^{4B-4A}F_{14}{}^2 + \frac{1}{r^2} F_{23}{}^2 = 0, \qquad (2.9)$$

which implies $M = 0$ and $F_{14} = 0 = F_{23}$. (2.10)

The equations (2.10) and (2.4) for $i = 4$, give

$$\sigma = 0 \text{ (since } U^4 \neq 0). \qquad (2.11)$$

The result obtained here, viz., $\sigma = 0$ and $M = 0$ are invariant statements and hold in all coordinate systems. Thus for the cylindrically symmetric Jordan-Ehlers metric (2.1), there cannot exist charged massive scalar fields.

3. CYLINDRICALLY SYMMETRIC CHARGED PERFECT FLUID DISTRIBUTION.

The relativistic field equations for the region of spacetime containing charged perfect fluid are

$$G_{ij} \equiv R_{ij} - \frac{1}{2}g_{ij}R = -K\,(E_{ij} + M_{ij}) \qquad (3.1)$$

where E_{ij} is given by (2.2b) and

$$M_{ij} = \frac{1}{4\pi}\,[(p+w)\,U_iU_j - pg_{ij}],\quad (g^{ij}\,U_iU_j = 1). \qquad (3.2)$$

Here w, p and U_i are respectively proper mass density, pressure and four velocity vector of the distribution. As before,we can write two of the field equations (3.1), viz., $G_{11} = -K(T_{11} + M_{11})$ and $G_{44} = -K(T_{44} + M_{44})$ in the following explicit forms:

$$\begin{aligned} G_{11} &\equiv \frac{1}{4r^2}\,(C_1{}^2 + C_4{}^2)\,e^{4B} + B_1{}^2 + B_4{}^2 - \frac{A_1}{r} \\ &= -\frac{K}{8\pi}[-g^{22}F_{12}{}^2 - g^{33}F_{13}{}^2 - g^{44}F_{14}{}^2 + g_{11}(g^{22}g^{33} - g^{23}g^{23})F_{23}{}^2 \\ &\quad - g^{22}F_{24}{}^2 - g^{33}F_{34}{}^2 - 2g^{23}(F_{12}F_{13} + F_{24}F_{34})] \\ &\quad - \frac{K}{4\pi}[(p+w)\,U_1{}^2 - pg_{11}]. \end{aligned} \qquad (3.3)$$

$$\begin{aligned} G_{44} &\equiv \frac{1}{4r^2}\,(C_1{}^2 + C_4{}^2)\,e^{4B} + B_1{}^2 + B_4{}^2 - \frac{A_1}{r} \\ &= -\frac{K}{8\pi}\,[-g^{22}F_{12}{}^2 - g^{33}F_{13}{}^2 - g^{11}F_{14}{}^2 + g_{44}(g^{22}g^{33} - g^{23}g^{23})F_{23}{}^2 \\ &\quad - g^{22}F_{24}{}^2 - g^{33}F_{34}^2 - 2g^{23}(F_{12}F_{13} + F_{24}F_{34})] \\ &\quad - \frac{K}{4\pi}\,[(p+w)\,U_4{}^2 - pg_{44}]. \end{aligned} \qquad (3.4)$$

In a co-moving coordinate system for the metric (2.1), we have $U_1 = U_2 = U_3 = 0$ $U_4{}^2 = g_{44}$.

Hence from the field equations (3.3) and (3.4), we obtain

$$[g^{11}F_{14}{}^2 + g_{11}(g^{22}g^{33} - g^{23}g^{23})F_{23}{}^2 + (w-p)g_{11}] = 0. \qquad (3.5)$$

For the metric (2.1), this gives

$$e^{4B-4A}\,F_{14}{}^2 + \frac{1}{r^2}\,F_{23}{}^2 + (w-p) = 0. \qquad (3.6)$$

Since for known physical distribution $w \geq p$, the equation (3.6) is consistent only when

$$F_{14} = 0 = F_{23} \quad \text{and} \quad p = w. \tag{3.7}$$

Again using equation (3.7) in equation (2.4) for $i = 4$, we obtain

$$\sigma = 0 \text{ (since } U^4 \neq 0). \tag{3.8}$$

Thus a charged perfect fluid cannot be a source for generating gravitation in the geometry governed by (2.1).

4. CONCLUSION.

Since the two components of F_{ij}, viz., F_{14} and F_{23} do not survive in both the cases considered before, we can derive rest of the components of F_{ij} using (2.3) as follows:

$$F_{12} = \phi_{2,1}, \; F_{13} = \phi_{3,1}, \; F_{24} = -\phi_{2,4} \; \text{ and } \; F_{34} = -\phi_{3,4} \,.$$

All the surviving components of F_{ij}, therefore, can be determined only by the second and third components of the electromagentic four potential ϕ_i. However, for the spacetime (2.1) the Einstein-Maxwell field equations also indicate the survival of the same components of electromagnetic field tensor as we obtained in interacting cases studied in Sec 2 and Sec. 3.

The conclusions arrived at, viz., $M = 0$ in Sec. 2 and $w = p$ in Sec. 3 for interacting cases can also be established in neutral case (without electromagnetc field).

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