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# CORRIGENDUM

to the paper

## A REMARK ON THE WEIGHTED AVERAGES FOR SUPERADDITIVE PROCESSES (INTERNAT. J. MATH. & MATH. SCI. VOL. 14, NO. 3 (1991) 435-438)

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The proof of Theorem 3.2 in [1] contains an error. It is wrongly stated in [1] thate a $T$-superadditive process is decomposed into the difference of a $T$-additive process $G$ and a positive, purely $T$-superadditive process $H=\{H_n\}=\{\Sigma_{k=0}^{n-1}\ h_k\}$, and $h_k=f_k-T^k\delta$. Actually, the equality $h_k=T^k\delta-f_k$ holds for all $k$, and hence $H$ must be a positive, purely $T$-subadditive process. Consequently, $h_k$'s need not be positive, and the inequality

$$lim_n sup\,|\,S_n(A,H)\,|\ \leq M\ lim_n sup\ \tfrac{1}{n}\ H_n,$$

which is asserted to be true in [1], need not be valid in general. On the other hand, if $h_k$'s are nonnegative for all $k$, then this inequality still holds. Nonetheless, the decomposition results of Section 2 of [1] are still true, with the forementioned corrections, whereas Theorem 3.2 is not true as it stands, but is true if all $h_k$'s are assumed to be nonnegative. Hence, the corrected statement of Theorem 3.2 should read as:

**THEOREM 3.2.** Let $T$ be a positive Dunford-Schwartz operator on $L_1$, or a positive $L_p$-contraction for $1<p<\infty$, and $F$ be a $T$-superadditive process with $h_k\geq 0$, for all $k$, where $\{\Sigma_{k=0}^{n-1}\ h_k\}_n$ is the purely subadditive part of the process. Assume also that

$$T \text{ is Markovian and } sup_{n\geq 1}\,\|\tfrac{1}{n}\ F_n\,\|\ <\infty, \text{ when } p=1, \text{ or}$$

$$lim\ inf_n\,\|\,\tfrac{1}{n}\,\Sigma_{i=1}^{n}\,(F_i-F_{i-1})\,\|_{\,p}<\infty, \text{ when } 1<p<\infty.$$

If $A$ is a bounded sequence such that $(A,T)$ is Birkhoff, then $lim_n\ \frac{1}{n}\ S_n(F,A)$ exists a.e.

It must be noted here that, when $h_k$'s are not necessarily nonnegative, $lim_n\ \frac{1}{n}S_n(H,A)$ may not exist as the following simple example shows.

**EXAMPLE.** Let $h_k=(-1)^k,\ k\geq 0$. Then $\{H_n\}=\{\Sigma_{k=0}^{n-1}h_k\}$ is a subadditive sequence (assuming $H_0=0$). Define the sequence $A=\{a_k\}$ of weights as $a_0=a_1=1$, and for $i\geq 0$,

$$a_k=(-1)^k, \text{ when } 3^i.2\leq k<3^i.4,$$

$$a_k=(-1)^{k+1}, \text{ when } 3^i.4\leq k<3^{i+1}.2.$$

Then $\frac{1}{n}\ S_n(H,A)=0$, when $n=3^i.2$, and is equal to $1/2$ when $n=3^i.4$, for all $i\geq 0$. Therefore we see that

$$lim\ sup\ S_n(H,A)=\tfrac{1}{2} \qquad \text{whereas } lim\ inf\ S_n(H,A)=0.$$

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## REFERENCE

1. COMEZ, D., A remark on the weighted averages for superadditive processes, *Internat. J. Math. & Math. Sci.* 14 (1991), 435-438.