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ABELIAN THEOREMS FOR TRANSFORMABLE BOEHMIANS

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ABSTRACT. A class of generalized functions called transformable Boehmians contains a proper subspace that can be identified with the class of Laplace transformable distributions. In this note, we establish some Abelian theorems for transformable Boehmians.

KEY WORDS AND PHRASES. Abelian theorem, Boehmian, convolution, generalized function, Laplace transform.
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1. INTRODUCTION AND PRELIMINARIES

The Laplace transform has recently been extended to a class of generalized functions called transformable Boehmians [1]. The object of this note is to present some Abelian theorems of the initial type for transformable Boehmians. Such theorems relate the behavior of a transformable Boehmian at zero to the behavior of its transform at infinity.

Our notation is the same as that used in [1]. Let Ω be a subset of the real line R. The space of all continuous complex-valued functions on Ω will be denoted by $C(\Omega)$. Throughout this note it will be assumed that if $\Omega=(a,b)$ then $a<0$ and $b>0$. The space of all functions $f \in C(R)$ such that $f(t)=0$ for $t<0$ will be denoted by $C_+(R)$. The support of a continuous function f, denoted by supp f, is the complement of the largest open set on which f is zero.

The convolution product of two functions $f,g \in C_+(R)$ is given by

$$(f*g)(t) = \int_0^t f(t-u)\, g(u)\, du.$$

A sequence of continuous nonnegative functions $\{\delta_n\}$ will be called a *delta sequence* if

(i) $\int_0^\infty \delta_n(t)\,dt = 1$ for $n=1,2,\ldots$; and (ii) $\operatorname{supp}\delta_n \subseteq [0,\varepsilon_n]$, $\varepsilon_n \to 0$ as $n\to\infty$ $(\varepsilon_n>0)$.

A pair of sequences (f_n,δ_n) is called a *quotient of sequences* if $f_n \in C_+(R)$ for $n=1,2,\ldots$, $\{\delta_n\}$ is a delta sequence, and $f_k*\delta_m = f_m*\delta_k$ for all k and m. Two quotients of sequences (f_n,δ_n) and (g_n,φ_n) are said to be equivalent if $f_k*\varphi_m = g_m*\delta_k$ for all k and m. A straightforward calculation shows that this is an equivalence relation. The equivalence classes are called *Boehmians*. The space of all Boehmians will be denoted by β and a typical element of β will be written as $x = \dfrac{f_n}{\delta_n}$.

By defining addition, multiplication, and scalar multiplication, on β, i.e. $\dfrac{f_n}{\delta_n} + \dfrac{g_n}{\varphi_n} = \dfrac{(f_n*\varphi_n + g_n*\delta_n)}{\delta_n*\varphi_n}$, $\dfrac{f_n}{\delta_n} * \dfrac{g_n}{\varphi_n} = \dfrac{f_n*g_n}{\delta_n*\varphi_n}$, and $\alpha\left(\dfrac{f_n}{\delta_n}\right) = \dfrac{\alpha f_n}{\delta_n}$, where α is a complex number, β becomes a convolution algebra.

Since the Boehmian $\dfrac{\delta_n}{\delta_n}$ corresponds to the Dirac delta distribution, we denote it by δ. Moreover, the n^{th} derivative of δ is given by the formula $D^n\delta = \delta^{(n)} = \dfrac{\delta_k^{(n)}}{\delta_k}$, where $\{\delta_n\}$ is any infinitely differentiable delta sequence. In general, the n^{th} derivative of $x\in\beta$ is given by $D^n x = x*\delta^{(n)}$.

By the translation operator on $C_+(R)$, we mean the operator τ_d, d real, such that $(\tau_d f)(t) = f(t-d)$. The translation operator can be extended to an element $x = \dfrac{f_n}{\delta_n} \in \beta$ by $\tau_d x = \dfrac{\tau_d f_n}{\delta_n}$.

DEFINITION 1.1. Let Ω be an open subset of R. A Boehmian x is said to be equal to a continuous function f on Ω, denoted by $x=f$ on Ω, if there exists a delta sequence $\{\delta_n\}$ such that $x*\delta_n \in C(R)$ for all n and $x*\delta_n \to f$ uniformly on compact subsets of Ω as $n\to\infty$.

Two Boehmians x and y are said to be equal on an open set Ω, denoted by $x=y$ on Ω, if $x-y=0$ on Ω.

The support of $x\in\beta$, written supp x, is the complement of the largest open set on which x is zero. For example, given any delta sequence $\{\delta_n\}$ and $\varepsilon>0$, $\delta_n(t)\to 0$ uniformly for $|t|>\varepsilon$ as $n\to\infty$. Thus, $\operatorname{supp} D^n\delta = \{0\}$ for $n=0,1,2,\ldots$.

For each $x = \dfrac{f_n}{\delta_n} \in \beta$, it is not difficult to show that for each n

$$\operatorname{supp} f_n \subseteq \operatorname{supp} x + \operatorname{supp}\delta_n \tag{1.1}$$

For other results concerning Boehmians see [1], [2], and [3].

2. TRANSFORMABLE BOEHMIANS

A Boehmian x is said to be *transformable* if there exists a delta sequence $\{\delta_n\}$ and a nonnegative number α such that $x*\delta_n \in C_+(R)$ for all n and $x*\delta_n = O(e^{\alpha t})$ as $t\to\infty$ for all n. The space of all transformable Boehmians will be denoted by β_L.

If $f\in C_+(R)$ such that $f(t) = O(e^{\alpha t})$ as $t\to\infty$ for some real number α, then the Laplace transform of f is given by

$$F(s) = \mathcal{L}[f](s) = \int_0^\infty e^{-st} f(t)\,dt \quad \text{for Re } s>\alpha.$$

Throughout this note s will denote a complex variable, while t or σ will denote real variables.

Now, for $x\in\beta_L$ where $x*\delta_n\in C_+(R)$ and $x*\delta_n = O(e^{\alpha t})$ as $t\to\infty$ for all n, the *Laplace Transform* $\mathcal{L}[x](s)=\mathbf{X}(s)$ of x is defined by the equation $\mathcal{L}[x](s)\mathcal{L}[\delta_n](s) = \mathcal{L}[x*\delta_n](s)$ for all n.

It can be shown [1] that the space of transformable distributions $\mathcal{L}_+$ [4] is a proper subspace of β_L.

We state without proof the following theorem.

THEOREM 2.1. For $x,y\in\beta_L$, if $\mathbf{X}(s)$ and $\mathbf{Y}(s)$ are the Laplace transforms of x and y respectively, then:

1. $\mathcal{L}[x+y](s) = \mathbf{X}(s) + \mathbf{Y}(s)$.
2. $\mathcal{L}[\alpha x](s) = \alpha\mathbf{X}(s)$, α a complex number.
3. $\mathcal{L}[D^n x](s) = s^n\mathbf{X}(s)$.
4. $\mathcal{L}[x*y](s) = \mathbf{X}(s)\mathbf{Y}(s)$.
5. $\mathcal{L}[\tau_d x](s) = e^{-ds}\mathbf{X}(s)$, d a real number.
6. If $\mathbf{X}(s) = 0$, then x=0.

The next theorem will be needed in the proof of an Abelian theorem (Theorem 3.2) in the next section. Also, since the Laplace transform of a Boehmian is an analytic function in some half-plane [1], Theorem 2.2 gives a necessary condition for an analytic function to be the Laplace transform of a Boehmian.

THEOREM 2.2. Let $x\in\beta_L$. For each k and $\varepsilon>0$, $\sigma^k e^{-\varepsilon\sigma}\mathbf{X}(\sigma) = o(1)$ as $\sigma\to\infty$.

PROOF. Let $x = \frac{f_n}{\delta_n} \in \beta_L$. We may assume that $f_n\in C^\infty(R)$ for all n. For, if $\{\psi_n\}$ is an infinitely differentiable delta sequence (that is, $\psi_n\in C^\infty(R)$ for all n), then $x = \frac{f_n*\psi_n}{\delta_n*\psi_n}$ and $f_n*\psi_n\in C^\infty(R)$ for all n.

Assume that supp $\delta_n \subseteq [0,a_n]$ for all n, where $a_n\to 0$ as $n\to\infty$. Now, by the Mean Value

Theorem for Integrals, for each n there exists an ξ (which depends on n and σ) such that

$0\le\xi\le a_n$ and $\mathcal{L}[\delta_n](\sigma) = \int_0^\infty e^{-\sigma t}\delta_n(t)\,dt = e^{-\sigma\xi}\int_0^\infty \delta_n(t)\,dt = e^{-\sigma\xi} \ge e^{-a_n\sigma}$

Hence, given an $\varepsilon>0$, we may pick m such that $0<a_m<\varepsilon$. Then,

$$|\mathsf{X}(\sigma)| = \frac{|\mathcal{L}[f_m](\sigma)|}{\mathcal{L}[\delta_m](\sigma)} \le e^{a_m\sigma}|\mathcal{L}[f_m](\sigma)| < e^{\varepsilon\sigma}|\mathcal{L}[f_m](\sigma)|. \tag{2.1}$$

Also, since $f_m\in C^\infty(R)$, for each nonnegative integer k

$$\sigma^k\mathcal{L}[f_m](\sigma) = O(1) \text{ as } \sigma\to\infty \tag{2.2}$$

Hence, (2.1) and (2.2) give

$$e^{-\varepsilon\sigma}\sigma^k\mathcal{L}[x](\sigma) = O(1) \text{ as } \sigma\to\infty. \tag{2.3}$$

The proof is completed by observing that (2.3) is valid for all $\varepsilon>0$ and all nonnegative integers k.

THEOREM 2.3. Suppose that F(s) is an analytic function in some half-plane Re s > α and for some integer k and all $\varepsilon>0$ that $s^k e^{-\varepsilon s}F(s) = O(1)$ as $s\to\infty$, Re s > α. Then, there exists $x\in\beta_L$ such that $\mathcal{L}[x](s)=F(s)$, Re s > α.

PROOF. Suppose that for some integer k and all $\varepsilon>0$

$$s^k e^{-\varepsilon s}F(s) = O(1) \text{ as } s\to\infty, \text{ Re } s > \alpha. \tag{2.4}$$

Let $\{\delta_n\}$ be an infinitely differentiable delta sequence . Define $\varphi_n(t) = \delta_n(t-\frac{1}{n})$ for n=1,2,. . . . Thus, $\{\varphi_n\}$ is an infinitely differentiable delta sequence and for each integer k and all n

$$s^k e^{s/n}\mathcal{L}[\varphi_n](s) = O(1) \text{ as } s\to\infty, \text{ Re } s > \alpha. \tag{2.5}$$

Hence, by (2.4) and (2.5), for each n there exists an M such that $|\mathcal{L}[\varphi_n](s)F(s)| \le \frac{M}{|s|^2}$ for Re s > α. For n=1,2,. . . , define $f_n(t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st}\mathcal{L}[\varphi_n](s)F(s)\,ds$, where $\gamma>\alpha$.

Then, for each n, f_n is a continuous function such that: supp $f_n \subseteq [0,\infty)$, $f_n(t) = O(e^{\gamma t})$ as $t\to\infty$, and $\mathcal{L}[f_n](s) = \mathcal{L}[\varphi_n](s)F(s)$, Re s > α.

Now, $\mathcal{L}[f_n*\varphi_m] = \mathcal{L}[f_n]\mathcal{L}[\varphi_m] = (\mathcal{L}[\varphi_n]F)\mathcal{L}[\varphi_m] = (\mathcal{L}[\varphi_m]F)\mathcal{L}[\varphi_n] = \mathcal{L}[f_m]\mathcal{L}[\varphi_n]$ $= \mathcal{L}[f_m*\varphi_n]$ for all n and m. Thus, $f_n*\varphi_m = f_m*\varphi_n$ for all n and m. Let $x = \frac{f_n}{\varphi_n} \in \beta_L$.

Hence $\mathcal{L}[x](s) = F(s)$, Re s > α.

Although the condition in the previous theorem is sufficient for an analytic function to be the Laplace transform of a Boehmian, as the next example demonstrates it is not necessary .

EXAMPLE 2.4. The transformable Boehmian $x = \sum_{n=0}^{\infty}\frac{\delta^{(n)}}{(2n)!}$ has Laplace transform

$\mathsf{X}(s) = \cosh\sqrt{s}$ [1].

REMARK 2.5. It is not difficult to show that the transform of a Boehmian is an analytic function in some half-plane [1]. Hence, Theorem 2.2 provides a necessary condition for an analytic function to be the Laplace transform of some transformable Boehmian. Thus, the entire function $g(s) = e^s$ is not the transform of a Boehmian. But, for each $\alpha>1$ there is a transformable Boehmian x_α such that $\mathcal{X}_\alpha(s)$ is an entire function and for each $\varepsilon>0$, $\mathcal{X}_\alpha(s)=O\left(\exp|s|^{(\varepsilon\ +(1/\alpha))}\right)$, as $|s|\to\infty$ [1] (where this relation does not hold for any $\varepsilon<0$). An interesting open problem is to characterize the class of transformable Boehmians by their Laplace transforms.

3. INITIAL VALUE THEOREMS

In classical analysis there are many different types of Abelian theorems (see [5] and [6]). Abelian theorems of the final type relate the behavior of a function at infinity to the behavior of its transform at zero, while Abelian theorems of the initial type relate the behavior of a function at zero to the behavior of its transform at infinity. It is both interesting and important to extend such theorems to certain classes of generalized functions (see [4], [7], [8], and [9]). For example, Zemanian [4] has extended two Abelian theorems to transformable distributions. In [1] we presented an Abelian theorem of the final type for transformable Boehmians. In this section we will establish three Abelian theorems of the initial type (Theorems 3.2, 3.5, and 3.8).

A real-valued function $m\in C_+[a,b]$ ($C_+[a,b] = C_+(R)\cap C[a,b]$) is said to be in $\mathcal{M}$ if m does not change sign in [a,b] and $[\mathcal{L}[m](\sigma)]^{-1} = O(\sigma^n e^{o(\sigma)})$ as $\sigma\to\infty$ (for some integer n).

DEFINITION 3.1. $x,y\in\beta_L$. $x\sim y$ as $t\to0^+$ if there exist $f,g\in C_+[a,b]$, $m\in\mathcal{M}$, and an integer n such that $x=D^nf$ and $y=D^ng$ on some neighborhood of [a,b], $\lim\limits_{t\to0^+}\frac{f(t)}{m(t)}=1$, and $\lim\limits_{t\to0^+}\frac{g(t)}{m(t)}=1$.

THEOREM 3.2. Suppose $x,y\in\beta_L$ such that $x\sim y$ as $t\to0^+$. Then $\lim\limits_{\sigma\to\infty}\frac{\mathcal{X}(\sigma)}{\mathcal{Y}(\sigma)}=1$.

PROOF. x can be written in the form

$$x = D^nf + w, \tag{3.1}$$

where $w\in\beta_L$ and $\operatorname{supp} w\subseteq[\alpha,\infty)$ $(\alpha>0)$. Now, by a classical Abelian theorem (see [5]),

$$\frac{\mathcal{L}[D^nf](\sigma)}{\sigma^n\mathcal{L}[m](\sigma)}=\frac{\mathcal{L}[f](\sigma)}{\mathcal{L}[m](\sigma)}\to1 \text{ as } \sigma\to\infty.$$

Since y can also be written in the form of (3.1), to complete the proof it suffices to show that

$$\frac{\mathcal{L}[w](\sigma)}{\sigma^n\mathcal{L}[m](\sigma)}\to0 \text{ as } \sigma\to\infty. \tag{3.2}$$

Now, $w=\frac{f_n}{\delta_n}$ and by (1.1) supp $f_n \subseteq [\alpha,\infty)$ for all n. For each n, let $g_n \in C(R)$ be defined by $\tau_\alpha g_n=f_n$ and let $z=\frac{g_n}{\delta_n} \in \beta_L$. Thus $w=\tau_\alpha z$. Then, (for some $\gamma>0$),

$$\frac{\mathcal{L}[w](\sigma)}{\sigma^n \mathcal{L}[m](\sigma)} = \frac{e^{-\alpha\sigma}\mathcal{L}[z](\sigma)}{\sigma^n \mathcal{L}[m](\sigma)} = O(e^{-\gamma\sigma}\mathcal{L}[z](\sigma)) \quad \text{as } \sigma\to\infty \tag{3.3}$$

By applying Theorem 2.2 to (3.3) we obtain (3.2), and hence the proof is complete.

EXAMPLE 3.3. Let $x\in\beta_L$ such that $x=\delta$ on some neighborhood of [a,b]. Since $D^2t=\delta$, $x\sim\delta$ as $t\to0^+$ and hence $\lim\limits_{\sigma\to\infty} \mathbf{X}(\sigma)=1$.

REMARK 3.4. In Definition 3.1, the condition that the functions f, g, and m be continuous may be relaxed. We need only require that $f,g,m\in L^1(a,b)$. For if $f\in L^1(a,b)$ and $x=D^n f$ in some neighborhood of [a,b], then $x=D^{n+1}(\ell * f)$ in that neighborhood, where ℓ is the Heaviside function ($\ell(t)=1$ for $t>0$ and zero otherwise). Also, if $\frac{f(t)}{m(t)}\to 1$ as $t\to0^+$, then $\frac{(\ell*f)(t)}{(\ell*m)(t)}\to 1$ as $t\to0^+$.

THEOREM 3.5. If $x\in\beta_L$ such that, for some $f\in L^1(a,b)$, $x=D^n f$ on some neighborhood of [a,b] and $\frac{f(t)}{t^\lambda}\to\alpha$ as $t\to0^+$ (α complex and λ real, $\lambda>-1$), then $\lim\limits_{\sigma\to\infty}\frac{\sigma^{\lambda-n+1}\mathbf{X}(\sigma)}{\Gamma(\lambda+1)}=\alpha$ $\left(\text{where } \Gamma(\lambda+1)=\int_0^\infty e^{-t}t^\lambda\,dt\right)$.

PROOF. If $\alpha\neq0$, use Theorem 3.2 with $x=\alpha^{-1}x$ and $y=D^n t^\lambda$. Suppose $\alpha=0$. Now, x can be written in the form $x=D^n f+w$, where $w\in\beta_L$ and supp $w\subseteq[\alpha,\infty)$ $(\alpha>0)$. Thus,

$$\frac{\sigma^{\lambda-n+1}\mathbf{X}(\sigma)}{\Gamma(\lambda+1)} = \frac{\sigma^{\lambda+1}F(\sigma)}{\Gamma(\lambda+1)} + \frac{\sigma^{\lambda-n+1}\mathcal{L}[w](\sigma)}{\Gamma(\lambda+1)}. \tag{3.4}$$

By a classical Abelian theorem (see [5]), the first term on the right hand side of (3.4) tends to zero (as $\sigma\to\infty$). By using a similar argument as in the proof of Theorem 3.2, the second term on the right hand side of (3.4) also tends to zero (as $\sigma\to\infty$). This completes the proof.

EXAMPLE 3.6. Let $x=\alpha t^\lambda+\sum_{n=0}^{\infty}\frac{\tau_\varepsilon\delta^{(n)}}{(2n)!}$ (where $\varepsilon>0$, $\lambda>-1$, and α is complex). Since $x=\alpha t^\lambda$ on $[0,\varepsilon/2)$, Theorem 3.5 yields

$$\lim_{\sigma\to\infty}\frac{\sigma^{\lambda+1}X(\sigma)}{\Gamma(\lambda+1)}=\alpha \qquad \left(\sum_{n=0}^{\infty}\frac{\tau_\varepsilon\delta^{(n)}}{(2n)!}=\delta\text{-}\lim_{n}\sum_{k=0}^{n}\frac{\tau_\varepsilon\delta^{(k)}}{(2k)!}\quad \text{(see [1])}.\right)$$

REMARK 3.7. Example 3.6 maybe used to show that Theorem 3.5 cannot be extended to a half-plane. Indeed, let x = t + \i\su(n=0,∞,\f($\tau_\pi\delta^{(n)}$,(2n)!)) . Then, x = t in [0, 1) and

$$X(s)=\frac{1}{s^2}+\sum_{n=0}^{\infty}\frac{\mathcal{L}[\tau_\pi\delta^{(n)}](s)}{(2n)!}=\frac{1}{s^2}+e^{-\pi s}\sum_{n=0}^{\infty}\frac{s^n}{(2n)!}=\frac{1}{s^2}+e^{-\pi s}\cosh\sqrt{s}\ \text{(see [1])}.$$

Moreover, for Re s = $\gamma_0 > 0$, there exist positive constants M and a such that $|s^2e^{-\pi s}\cosh\sqrt{s}| \geq M|s|^2e^{a\sqrt{|s|}}\to\infty$ as $s\to\infty$. Thus, $s^2X(s) = 1 + s^2e^{-\pi s}\cosh\sqrt{s}\to\infty$ as $s\to\infty$, Re s = $\gamma_0 > 0$. Hence, in order to extend Theorem 3.5 (i.e. not restricting σ to the real axis) we will require some restrictions.

Let $\mathcal{A}$ denote the set of all Boehmians x such that for each $\varepsilon>0$, $e^{-\varepsilon s}X(s) = O(1)$ as $s\to\infty$ for $|\arg s| \leq \psi < \pi/2$.

THEOREM 3.8. Suppose $x\in\beta_L$ such that $x=D^nf + \tau_d y$ (d>0, n an integer) for some $f\in C_+[a,b]$ and $y\in\mathcal{A}$. If \a(,Lim, t→0⁺) \f(f(t),t^λ) = α (α complex and λ real, $\lambda>-1$) then

$$\lim_{s\to\infty}\frac{s^{\lambda-n+1}X(s)}{\Gamma(\lambda+1)}=\alpha,\ |\arg s| \leq \psi < \pi/2.$$

PROOF. $\dfrac{s^{\lambda-n+1}X(s)}{\Gamma(\lambda+1)}=\dfrac{s^{\lambda+1}F(s)}{\Gamma(\lambda+1)}+s^{\lambda-n+1}\mathcal{L}[\tau_d y](s).$

Since $\dfrac{f(t)}{t^\lambda}\to\alpha$ as $t\to0^+$, by a classical Abelian theorem (see [6]), $\dfrac{s^{\lambda+1}F(s)}{\Gamma(\lambda+1)}\to\alpha$ as $s\to\infty$, $|\arg s|\leq\psi<\pi/2$. Thus to complete the proof we need only to show that

$$s^{\lambda-n+1}\mathcal{L}[\tau_d y](s)\to0 \text{ as } s\to\infty, |\arg s|\leq\psi<\pi/2. \tag{3.5}$$

Now, for some positive constants M and a

$$|s^{\lambda-n+1}\mathcal{L}[\tau_d y](s)|\leq M|\mathrm{Re}\, s|^{\lambda-n+1}e^{-a\mathrm{Re}\, s},\ |\arg s|\leq\psi<\pi/2.$$

Since $|\mathrm{Re}\, s|^{\lambda-n+1}e^{-a\mathrm{Re}\, s}\to 0$ as $s\to\infty$ for $|\arg s|\leq\psi<\pi/2$, (3.5) is verified and thus the proof is established.

EXAMPLE 3.9. Let x be as in Example 3.6. Then by Theorem 3.8, $\displaystyle\lim_{s\to\infty}\frac{s^{\lambda+1}X(s)}{\Gamma(\lambda+1)}=\alpha$, $|\arg s| \leq \psi <\pi/2$.

REMARK 3.10. For transformable distributions, Theorem 3.8 may be stated as follows. If $x \in \mathcal{L}_+$ such that $x=D^n f$ in some neighborhood of $[a,b]$ $(f \in L^1(a,b))$ and $\frac{f(t)}{t^\lambda} \to \alpha$ as $t \to 0^+$,

then $\lim\limits_{s \to \infty} \frac{s^{\lambda-n+1} \mathbf{X}(s)}{\Gamma(\lambda+1)} = \alpha$, $|\arg s| \le \psi < \pi/2$.

This follows by observing that x can be written as $x=D^n f + y$, where $y \in \mathcal{L}_+$ and $\operatorname{supp} y \subseteq [\xi,\infty)$ $(\xi>0)$. Thus, $y \in \mathcal{A}$ (see [4]).

Since $\mathcal{L}_+$ is a subspace of β_L, the Abelian theorem of the initial type for $\mathcal{L}_+$ proved in [4] (Theorem 8.6–2) is a special case of Theorem 3.5. It can be shown [1] that the transformable Boehmian x in Remark 3.7 has Laplace transform $\mathbf{X}(s) = \frac{1}{s^2} + e^{-\pi s}\cosh\sqrt{s}$, which shows that (i) Theorem 3.5 extends the theorem of Zemanian (since x is not a transformable distribution); and (ii) Theorem 3.8 cannot be extended to a half- plane (see Remark 3.7).

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