

THE UNITARY AMICABLE PAIRS TO 10^8

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ABSTRACT: We present an exhaustive list of the 185 unitary amicable pairs whose smaller number is less than 10^8 and a new unitary sociable set of four numbers.

KEY WORDS AND PHRASES. Unitary amicable pairs, unitary sociable sets.

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1. INTRODUCTION.

A unitary amicable pair (UAP) is a pair of integers m, n such that

$$\sigma^*(m) = m + n = \sigma^*(n) \quad (1.1)$$

where σ^* is the sum of unitary divisors function. In 1971, Hagis [4] published a table of UAP's, including the results of an exhaustive computer search for all pairs with smaller number less than 10^6 . There were thirty-two UAP's in the table, none of which were simultaneously ordinary amicable pairs; i.e., which were not square-free. The definitions of unitary amicable pairs and ordinary amicable pairs overlap on square-free pairs. Hagis did not list Wall's dissertation [7] which included several hundred UAP's. Since Hagis, several writers (see, for example [3] and [5]) have added to the known UAP's.

2. UNITARY AMICABLE PAIRS.

The present search for UAP's was conducted on a NeXT station using *Mathematica*.. Let

$$s^*(n) = \sigma^*(n) - n. \quad (2.1)$$

The program searched for numbers k such that

$$s^*(s^*(k)) = k. \quad (2.2)$$

As part of a verification of the program, it initially picked out fixed points of the first ten iterates of s^* , thus finding the first four unitary perfect numbers.

The search agreed with Hagis's in finding the nineteen UAP's less than 10^6 and the additional four, pairs number 20, 24, 25, and 111 of this list, less than 10^8 that Hagis listed. The table follows.

Square free UAP's are designated by an asterisk. The square-free UAP's agree with the ordinary AP's found in te Riele [6]. Since Wall's listing [7] was not in numerical order, this list was not compared against his. Total running time is estimated to have exceeded 100 hours.

TABLE OF UNITARY AMICABLE PAIRS TO 10^8

1.	114 = 2.3.19;	126 = 2.3(2).7
2.	1140 = 2(2).3.5.19;	1260 = 2(2).3(2).5.7
3.	18018 = 2.3(2).7.11.13;	22302 = 2.3(3).7.59
4.	32130 = 2.3(3).5.7.17;	40446 = 2.3(3).7.107
5.	44772 = 2(2).3.7.13.41;	49308 = 2(2).3.7.587
6.	56430 = 2.3(3).5.11.19;	64530 = 2.3(3).5.239
7.	67158 = 2.3(2).7.13.41;	73962 = 2.3(2).7.587
8.	*142310 = 2.5.7.19.107;	168730 = 2.5.47.359
9.	180180 = 2(2).3(2).5.7.11.13;	223020 = 2(2).3(3).5.7.59
10.	197340 = 2(2).3.5.11.13.23;	286500 = 2(2).3.5(3).191
11.	241110 = 2.3(3).5.19.47;	242730 = 2.3(3).5.29.31
12.	296010 = 2.3(2).5.11.13.23;	429750 = 2.3(2).5(3).191
13.	308220 = 2(2).3.5.11.467;	365700 = 2(2).3.5(2).23.53
14.	462330 = 2.3(2).5.11.467;	548550 = 2.3(2).5(2).23.53
15.	591030 = 2.3(3).5.11.199;	618570 = 2.3(3).5.29.79
16.	669900 = 2(2).3.5(2).7.11.29;	827700 = 2(2).3.5(2).31.89
17.	671580 = 2(2).3(2).5.7.13.41;	739620 = 2(2).3(2).5.7.587
18.	785148 = 2(2).3.7.13.719;	827652 = 2(2).3.7.59.167
19.	815100 = 2(2).3.5(2).11.13.19;	932100 = 2(2).3.5(2).13.239
20.	1004850 = 2.3(2).5(2).7.11.29;	1241550 = 2.3(2).5(2).31.89
21.	*1077890 = 2.5.11.41.239;	1099390 = 2.5.17.29.223
22.	1080150 = 2.3.5(2).19.379;	1291050 = 2.3(2).5(2).19.151
23.	*1156870 = 2.5.11.13.809;	1292570 = 2.5.19.6803
24.	1177722 = 2.3(2).7.13.719;	1241478 = 2.3(2).7.59.167
25.	1222650 = 2.3(2).5(2).11.13.19;	1398150 = 2.3(2).5(2).13.239
26.	1281540 = 2(2).3.5.13.31.53;	1621500 = 2(2).3.5(3).23.47
27.	1475810 = 2.5.7.29.727;	1669150 = 2.5(2).7.19.251
28.	*1511930 = 2.5.7.21599;	1598470 = 2.5.19.47.179
29.	1571388 = 2(2).3.7.13.1439;	1654212 = 2(2).3.7.47.419
30.	1610700 = 2(2).3.5(2).7.13.59;	1883700 = 2(2).3(2).5(2).7.13.23
31.	*1669910 = 2.5.11.17.19.47;	2062570 = 2.5.239.863
32.	1707720 = 2(3).3.5.7.19.107;	2024760 = 2(3).3.5.47.359
33.	1908420 = 2(2).3.5.17.1871;	2135100 = 2(2).3.5(2).11.647
34.	1922310 = 2.3(2).5.13.31.53;	2432250 = 2.3(2).5(3).23.47
35.	1997520 = 2(4).3.5.7.29.41;	2115120 = 2(4).3.5.7.1259
36.	*2236570 = 2.5.7.89.359;	2429030 = 2.5.23.59.179
37.	2357082 = 2.3(2).7.13.1439;	2481318 = 2.3(2).7.47.419
38.	*2728726 = 2.7.11.13.29.47;	3077354 = 2.7.19.23.503
39.	2862630 = 2.3(3).5.17.1871;	3202650 = 2.3(2).5(2).11.647
40.	3406116 = 2(2).3.7.23.41.43;	3690204 = 2(2).3.7.197.223
41.	3482700 = 2(2).3.5(2).13.19.47;	3506100 = 2(2).3.5(2).13.29.31
42.	3951990 = 2.3(4).5.7.17.41;	4974858 = 2.3(4).7.41.107

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|------|-----------------------------------|----------------------------------|
| 43. | 4198236 = 2(2).3.7.23.41.53; | 4510884 = 2(2).3.7.83.647 |
| 44. | *4246130 = 2.5.7.60659; | 4488910 = 2.5.23.29.673 |
| 45. | 4439890 = 2.5.7(2).13.17.41; | 5085710 = 2.5.7(2).97.107 |
| 46. | *4532710 = 2.5.7.13.17.293; | 6135962 = 2.7.71.6173 |
| 47. | 4918550 = 2.5(2).7.13.23.47; | 5145322 = 2.7.13.17.1663 |
| 48. | 5073570 = 2.3(3).5.19.23.43; | 5570910 = 2.3(3).5.47.43 |
| 49. | 5109174 = 2.3(2).7.23.41.43; | 5535306 = 2.3(2).7.197.223 |
| 50. | *5123090 = 2.5.7.163.449; | 5504110 = 2.5.19.59.491 |
| 51. | 5191290 = 2.3(4).5.13.17.29; | 5967270 = 2.3(4).5.53.139 |
| 52. | 5224050 = 2.3(2).5(2).13.19.47; | 5259150 = 2.3(2).5(2).13.29.31 |
| 53. | 5341620 = 2(2).3.5.127.701; | 5441100 = 2(2).3.5(2).7.2591 |
| 54. | *5385310 = 2.5.7.107.719; | 5812130 = 2.5.17.179.191 |
| 55. | 5431188 = 2(2).3.7.19.41.83; | 5858412 = 2(2).3.7.97.719 |
| 56. | 6297354 = 2.3(2).7.23.41.53; | 6766326 = 2.3(2).7.83.647 |
| 57. | 6381732 = 2(2).3.7.17.41.109; | 6923868 = 2(2).3.7.139.593 |
| 58. | 6433476 = 2(2).3.7.19.29.139; | 7006524 = 2(2).3.7.239.349 |
| 59. | 6940890 = 2.3(4).5.11.19.41; | 7937190 = 2.3(4).5.41.239 |
| 60. | *6993610 = 2.5.13.23.2339; | 7158710 = 2.5.13.53.1039 |
| 61. | 7051590 = 2.3(3).5.7(2).13.41; | 7766010 = 2.3(3).5.7(2).587 |
| 62. | *7288930 = 2.5.11.23.43.67; | 8221598 = 2.7.11.197.271 |
| 63. | 7929012 = 2(2).3.7.13.53.137; | 8763468 = 2(2).3.7.104327 |
| 64. | 8012430 = 2.3(2).5.127.701; | 8161650 = 2.3(2).5(2).7.2591 |
| 65. | 8095620 = 2(2).3.5.13.97.107; | 9685500 = 2(2).3.5(3).11.587 |
| 66. | 8146782 = 2.3(2).7.19.41.83; | 8787618 = 2.3(2).7.97.719 |
| 67. | 8194230 = 2.3(3).5.11.31.89; | 9224010 = 2.3(3).5.127.269 |
| 68. | 8235810 = 2.3(3).5.11.47.59; | 9182430 = 2.3(3).5.71.479 |
| 69. | 8537100 = 2(2).3.5(2).11.13.199; | 8934900 = 2(2).3.5(2).13.29.79 |
| 70. | *8619765 = 3.5.7.11.17.439; | 9627915 = 3.5.11.23.43.59 |
| 71. | *8754130 = 2.5.7.11.11369; | 10893230 = 2.5.757.1439 |
| 72. | *8826070 = 2.5.11.19.41.103; | 10043690 = 2.5.31.179.181 |
| 73. | 9057300 = 2(2).3.5(2).7.19.227; | 9912300 = 2(2).3.5(2).19.37.47 |
| 74. | *9478910 = 2.5.7.19.7127; | 11049730 = 2.5.71.79.197 |
| 75. | 9572598 = 2.3(2).7.17.41.109; | 10385802 = 2.3(2).7.139.593 |
| 76. | 9650214 = 2.3(2).7.19.29.139; | 10509786 = 2.3(2).7.239.349 |
| 77. | 9680310 = 2.3(4).5.17.19.37; | 10511370 = 2.3(4).5.19.683 |
| 78. | 9701076 = 2(2).3.7.11.10499; | 10458924 = 2(2).3.7.89.1399 |
| 79.* | 10254970 = 2.5.11.53.1759; | 10273670 = 2.5.11.59.1583 |
| 80. | 11777220 = 2(2).3(2).5.7.13.719; | 12414780 = 2(2).3(2).5.7.59.167 |
| 81. | 11893518 = 2.3(2).7.13.53.137; | 13145202 = 2.3(2).7.104327 |
| 82. | 12143430 = 2.3(2).5.13.97.107; | 14528250 = 2.3(2).5(3).11.587 |
| 83. | 12805650 = 2.3(2).5(2).11.13.199; | 13402350 = 2.3(2).5(2).13.29.79 |
| 84. | 12934680 = 2(3).3.5.11.41.239; | 13192680 = 2(3).3.5.17.29.223 |
| 85. | 13321490 = 2.5.7.13.14639; | 16192750 = 2.5(3).7.19.487 |
| 86. | 13585950 = 2.3(2).5(2).7.19.227; | 14868450 = 2.3(2).5(2).19.37.47 |
| 87. | 13882440 = 2(3).3.5.11.13.809; | 15510840 = 2(3).3.5.19.6803 |
| 88. | 14257020 = 2(2).3.5.127.1871; | 14496900 = 2(2).3.5(2).11.23.191 |
| 89. | *14426230 = 2.5.7.13.83.191; | 18087818 = 2.7.31.71.587 |
| 90. | 14551614 = 2.3(2).7.11.10499; | 15688386 = 2.3(2).7.89.1399 |

91	14634270 = 2.3(4).5.7.29.89;	17247330 = 2.3(4).5.107.199
92.	16045722 = 2.3(3).7.11.17.227;	17048934 = 2.3(3).7.23.37.53
93.	16326090 = 2.3(3).5.11.23.239;	18510390 = 2.3(3).5.179.383
94.	*17041010 = 2.5.7.31.7853;	19150222 = 2.7.13.43.2447
95.	*17257695 = 3.5.7.13.47.269;	17578785 = 3.5.7.23.29.251
96.	17278548 = 2(2).3.7.29.41.173;	17799852 = 2(2).3.7.29.7307
97.	17468730 = 2.3(3).5.23.29.97;	18093510 = 2.3(3).5.19.3527
98.	17709720 = 2(3).3.5.7.29.727;	20029800 = 2(3).3.5(2).7.19.251
99.	18143160 = 2(3).3.5.7.21599;	19181640 = 2(3).3.5.19.47.179
100.	20038920 = 2(3).3.5.11.17.19.47;	24750840 = 2(3).3.5.239.863
101.	21385530 = 2.3(2).5.127.1871;	21745350 = 2.3(2).5(2).11.23.191
102.	*21448630 = 2.5.7.131.2339;	23030090 = 2.5.19.53.2287
103.	21705684 = 2(2).3.7.11.13(2).139;	23990316 = 2(2).3.7.285599
104.	23570820 = 2(2).3(2).5.7.13.1439;	24813180 = 2(2).3(2).5.7.47.419
105.	24542700 = 2(2).3.5(2).7.13.29.31;	31367700 = 2(2).3(2).5(2).7.13.383
106.	25425876 = 2(2).3.7.19.89.179;	26414124 = 2(2).3.11.17.79.149
107.	25917822 = 2.3(2).7.29.41.173;	26699778 = 2.3(2).7.29.7307
108.	26355084 = 2(2).3.7.29.31.349;	27404916 = 2(2).3.7(2).11.19.223
109.	26791830 = 2.3(3).5.13.17.449;	30361770 = 2.3(3).5.139.809
110.	26838840 = 2(3).3.5.7.89.359;	29148360 = 2(3).3.5.23.59.179
111.	27287260 = 2(2).5.7.11.13.29.47;	30773540 = 2(2).5.7.19.23.503
112.	29408130 = 2.3(3).5.17.43.149;	30467070 = 2.3(3).5.19.5939
113.	29656530 = 2.3(4).5.19.41.47;	29855790 = 2.3(4).5.29.31.41
114.	*30724694 = 2.7.11.13.103.149;	32174506 = 2.7.13.17.10399
115.	31035550 = 2.5(2).7.13.19.359;	31863650 = 2.5(2).7.13.47.149
116.	32558526 = 2.3(2).7.11.13(2).139;	35985474 = 2.3(2).7.285599
117.	32744712 = 2(3).3.7.11.13.29.47;	36928248 = 2(3).3.7.19.23.503
118.	*34256222 = 2.7.11.13.71.241;	35997346 = 2.7.11.23.10163
119.	*35361326 = 2.7.11.13.17.1039;	40117714 = 2.7.13.53.4159
120.	*37784810 = 2.5.7.539783;	39944086 = 2.7.13.41.53.101
121.	38138310 = 2.3(3).5.7.17.1187;	48081978 = 2.3(3).7.131.971
122.	38138814 = 2.3(2).7.19.89.179;	39621186 = 2.3(2).11.17.79.149
123.	39532626 = 2.3(2).7.29.31.349;	41107374 = 2.3(2).7(2).11.19.223
124.	40880532 = 2(2).3.7.11.151.293;	44920428 = 2(2).3.7.607.881
125.	42740880 = 2(4).3.5.7.13.19.103;	52306800 = 2(4).3.5(2).7.13.479
126.	43181292 = 2(2).3.7.11.17.2749;	51858708 = 2(2).3.11.131.2999
127.	48339228 = 2(2).3.7.17.33851;	49154532 = 2(2).3.7.53.61.181
128.	49117590 = 2.3(5).5.17.29.41;	50492970 = 2.3(5).5.11.1889
129.	50953560 = 2(3).3.5.7.60659;	53866920 = 2(3).3.5.23.29.673
130.	51091740 = 2(2).3(2).5.7.23.41.43;	55353060 = 2(2).3(2).5.7.197.223
131.	53278680 = 2(3).3.5.7(2).13.17.41;	61028520 = 2(3).3.5.7(2).97.107
132.	54392520 = 2(3).3.5.7.13.17.293;	73631544 = 2(3).3.7.71.6173
133.	58062480 = 2(4).3.5.7.17.19.107;	68841840 = 2(4).3.5.17.47.359
134.	59022600 = 2(3).3.5(2).7.13.23.47;	61743864 = 2(3).3.7.13.17.1663
135.	59037132 = 2(2).3.7.11.181.353;	64664628 = 2(2).3.7.251.3067
136.	59604468 = 2(2).3.7.11.251.257;	65226252 = 2(2).3.7(3).13.23.53
137.	60477900 = 2(2).3.5(2).7.31.929;	63323700 = 2(2).3.5(2).11.31.619
138.	61320798 = 2.3(2).7.11.151.293;	67380642 = 2.3(2).7.607.881

139.	61477080 = 2(3).3.5.7.163.449;	66049320 = 2(3).3.5.19.59.491
140.	62973540 = 2(2).3(2).5.7.23.41.53;	67663260 = 2(2).3(2).5.7.83.647
141.	63022806 = 2.3(3).7.11.23.659;	64710954 = 2.3(3).7.11.79.197
142.	63813036 = 2(2).3.7.17.44687;	64888404 = 2(2).3.7.41.83.227
143.	64623720 = 2(3).3.5.7.107.719;	69745560 = 2(3).3.5.17.179.191
144.	64771938 = 2.3(2).7.11.17.2749;	77788062 = 2.3(2).11.31.299
145.	65156700 = 2(2).3.5(2).7.19.23.71;	78612900 = 2(2).3.5(2).31.79.107
146.	*66595130 = 2.5.7.31.30689;	74824390 = 2.5.31.59.4091
147.	69405490 = 2.5.7.11.23.3919;	93164750 = 2.5(3).7.139.383
148.	69662970 = 2.3(3).5.13.89.223;	72585990 = 2.3(3).5.41.79.83
149.	70370412 = 2(2).3.7.17.49279;	71555988 = 2(2).3.7.41.79.263
150.	71151210 = 2.3(4).5.13.29.233;	73910070 = 2.3(4).5.13.7019
151.	*71241830 = 2.5.11.19.89.383;	78057370 = 2.5.17.359.1279
152.	71620500 = 2(2).3.5(3).7.19.359;	73531500 = 2(2).3.5(3).7.47.149
153.	72508842 = 2.3(2).7.17.33851;	73731798 = 2.3(2).7.53.61.181
154.	72696690 = 2.3(4).5.11.41.199;	76084110 = 2.3(4).5.29.41.79
155.	73284900 = 2(2).3.5(2).13.19.23.43;	80468700 = 2(2).3.5(2).13.47.439
156.	75139680 = 2(5).3.5.7.11.19.107;	89089440 = 2(5).3.5.11.47.359
157.	75729654 = 2.3(4).7.11.13.467;	79002378 = 2.3(4).7.13.23.233
158.	76487964 = 2(2).3.7.17.29.1847;	83179236 = 2(2).3.7.131.7559
159.	*78447010 = 2.5.17.19.149.163;	80960990 = 2.5.11.491.1499
160.	79509870 = 2.3(3).5.11.19.1409;	91043730 = 2.3(3).5.449.751
161.	81467820 = 2(2).3(2).5.7.19.41.83;	87876180 = 2(2).3(2).5.7.97.719
162.	83093388 = 2(2).3.7.23.41.1049;	86250612 = 2(2).3.7.83.89.139
163.	83846532 = 2(2).3.7.11.103.881;	92271228 = 2(2).3.7.701.1567
164.	83923320 = 2(3).3.5.13.23.2339;	85904520 = 2(3).3.5.13.53.1039
165.	84650130 = 2.3(3).5.19.29.569;	87717870 = 2.3(3).5.19.17099
166.	85305948 = 2(2).3.7.13.191.409;	91026852 = 2(2).3.7.59.18367
167.	86075730 = 2.3(3).5.13.137.179;	89195310 = 2.3(3).5.19.17387
168.	86490978 = 2.3.7(2).37.7951;	94814622 = 2.3(2).7.283.2659
169.	87467160 = 2(3).3.5.11.23.43.67;	98659176 = 2(3).3.7.11.197.271
170.	*87998470 = 2.5.7.29.67.647;	102358010 = 2.5.47.89.2447
171.	88555698 = 2.3(2).7.11.181.353;	96996942 = 2.3(2).7.251.3067
172.	89406702 = 2.3(2).7.11.251.257;	97839378 = 2.3(2).7(3).13.23.53
173.	90062700 = 2(2).3.5(2).7.13.3299;	102129300 = 2(2).3(2).5(2).7.13.29.43
174.	90716850 = 2.3(2).5(2).7.31.929;	94985550 = 2.3(2).5(2).11.31.619;
175.	91250124 = 2(2).3.7.29.47.797;	92609076 = 2(2).3.7.37.83.359
176.	91885920 = 2(5).3.5.7.23.29.41;	99714720 = 2(5).3.5.7.59.503
177.	94283700 = 2(2).3.5(2).7.17.19.139;	115380300 = 2(2).3.5(2).7(2).47.167
178.	94327860 = 2(2).3.5.11.131.1091;	113239500 = 2(2).3.5(3).11.6863
179.	95719554 = 2.3(2).7.17.44687;	97332606 = 2.3(2).7.41.83.227
180.	95725980 = 2(2).3(2).5.7.17.41.109;	103858020 = 2(2).3(2).5.7.139.593
181.	*95791430 = 2.5.7.17.101.797;	115187002 = 2.7.17.113.4283
182.	96502140 = 2(2).3(2).5.7.19.29.139;	105097860 = 2(2).3(2).5.7.239.349
183.	97292058 = 2.3.43.439.859;	102503142 = 2.3(2).7.43.18919
184.	97735050 = 2.3(2).5(2).7.19.23.71;	117919350 = 2.3(2).5(2).31.79.107
185.	98324226 = 2.3(3).7.11.13.17.107;	121145598 = 2.3(3).7.53.6047

Hagis posed five questions of which two bear comment in terms of the UAP's found.

Question 3. Is every odd pair of unitary amicable numbers simultaneously amicable?

Only two pairs listed here, numbers 70 and 95, are odd and both are amicable. The evidence suggests that the answer is "Yes."

Question 4. If $m = 2^a M$ and $n = 2^b N$, where MN is odd, is it always the case that $a = b$?

The condition holds for all pairs listed here. Again, the evidence, meager as it is, suggests "Yes."

3. NEW UNITARY SOCIABLE SETS.

Flammenkamp [2] discovered eleven new sets of sociable numbers. Eight sets have four members; two, eight; and one, nine. We may use

$$2^3/\sigma(2^3) = 2^{10}.3.5.7.41/\sigma^*(2^{10}.3.5.7.41) \quad (3.1)$$

on Flammenkamp's fifth set to produce a new four element set of unitary sociable numbers.

1. 1 58682 84023 96160 = 2(10).3.5.7.11.19.41.1722307 ;
2. 1 82978 97587 55840 = 2(10).3.5.7.17.41.71.343891;
3. 1 95405 61761 43360 = 2(10).3.5.7.31.41.13567133;
4. 1 73443 99131 49440 = 2(10).3.5.7.41.47.8371211.

The remaining sociable sets in [2] and those listed by Cohen [1] are not amenable to this type of conversion since a necessary condition of relative primeness fails.

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