Internat. J. Math. & Math. Sci.
Vol. 1 (1978) 391-392

RESEARCH NOTES

ON THE POST-WIDDER TRANSFORM

RICHARD BELLMAN

Departments of Mathematics,
Electrical Engineering, and Medicine
University of Southern California
Los Angeles, California 90007 U.S.A.

(Received June 18, 1978)

1. INTRODUCTION.

The Laplace transform

$$L(f) = \int_0^\infty f(t)e^{-st}dt \tag{1.1}$$

is one of the most powerful tools of analysis. A complex inversion formula exists which allows us to make use of the theory of a complex variable to obtain asymptotic relations and expansions [1]. It is of theoretical interest to obtain an inversion formula which depends only on real value. Such a formula is furnished by the post-Widder inversion formula [2].

This formula is not useful computationally because the inverse is an unbounded operator. A great care has to be exercised and different methods have to be employed [3].

The usual method for establishing the post-Widder formula depends upon Laplace's asymptotic evaluation of an integral. Here we shall follow a different route which permits many generalizations.

2. THE PARSEVAL-PLANCHEREL FORMULA.

Let F be the Fourier transform of f. Then we have

$$\int_0^\infty f(t)e^{-st}dt = \int_0^\infty \frac{f(x)}{s+ix}\,dx, \tag{2.1}$$

We see then that any linear operator which transforms $\frac{1}{1+ixt}$ into e^{-ixt} furnishes an inversion formula using the standard results for the inversion of the fourier transform [4]. The post-Widder formula is one such transformation. In Widder's book, other transformations are given.

3. GENERALIZATIONS.

We see that there are several possible generalizations. In the first place, we can divide other transformations. In the second place, any orthogonal expansion has a Parseval relation. If we take the continuous analogue, we obtain an analogue of the Parseval-Plancheral forumla. We thus have a method for obtaining inversion formulas for general transforms.

REFERENCES

1. Bellman, R. and K. L. Cooke. Differential-Difference Equations, Academic Press, Inc., New York, 1963.

2. Widder, D. V. The Laplace Transform, Princeton University Press, Princeton, New Jersey, 1941.

3. Bellman, R., R. Kalaba and J. Lockett. Numerical Inversion of the Laplace Transform, American Elsevier Publishing Company, Inc., New York, 1966.

4. Titchmarsh, E. C. The Fourier Integral, Oxford, Clarendon Press, 1959.

KEY WORDS AND PHRASES. *Post-Widder and transform.*

AMS (MOS) SUBJECT CLASSIFICATION (1970) CODES. *44A15, 44A10.*